Beliefs (evidence)

Beliefs in Markets and Experiments, Over-/Under-Inference

Tilman Fries

From *Normative* to *Positive*

Last time we learned how beliefs *should* update.

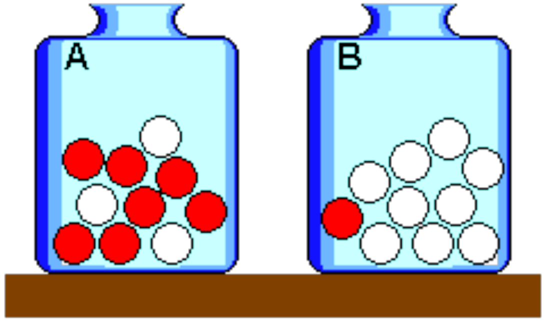
Now, we will learn how human beliefs **actually** update.

- Lab evidence: *bookbags-and-pokerchips* paradigm
- Field evidence: betting & forecasting data
- A unifying model: **over- vs. under-reaction** to information

Direct Tests – Bookbags and Pokerchips

Experimental setup

- Two bags, each with 100 chips
  - **Bag A:** q% red, $1-q$ % blue
  - **Bag B:** $1-q$ % red, q% blue
  - $q > 0.5$ → red is *diagnostic* of A
- Bag A is chosen with probability λ.
- n draws revealed to participant.
- Task: report $p \equiv P(\text{Bag} = A \mid n \text{ draws})$.



Many such experiments since the 1960s…

Journal of Experimental Psychology
1966, Vol. 72, No. 3, 346–354

CONSERVATISM IN A SIMPLE PROBABILITY INFERENCE TASK[1]

LAWRENCE D. PHILLIPS AND WARD EDWARDS

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3 experiments investigated the effects on posterior probability estimates of (a) prior probabilities, amount of data, and diagnostic impact of the data; (b) payoffs; and (c) response modes. In all the experiments Ss usually behaved conservatively, i.e., the difference between their prior

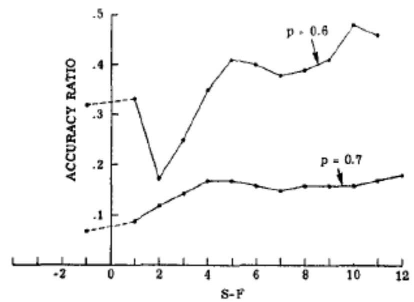


FIG. 1. Effect of bag composition on accuracy ratio.

The posterior odds in favor of the correct hypothesis is given by Ω_1, while Ω_0 represents

sion of his subjective probabilities is less than the amount calculated from Bayes' theorem.

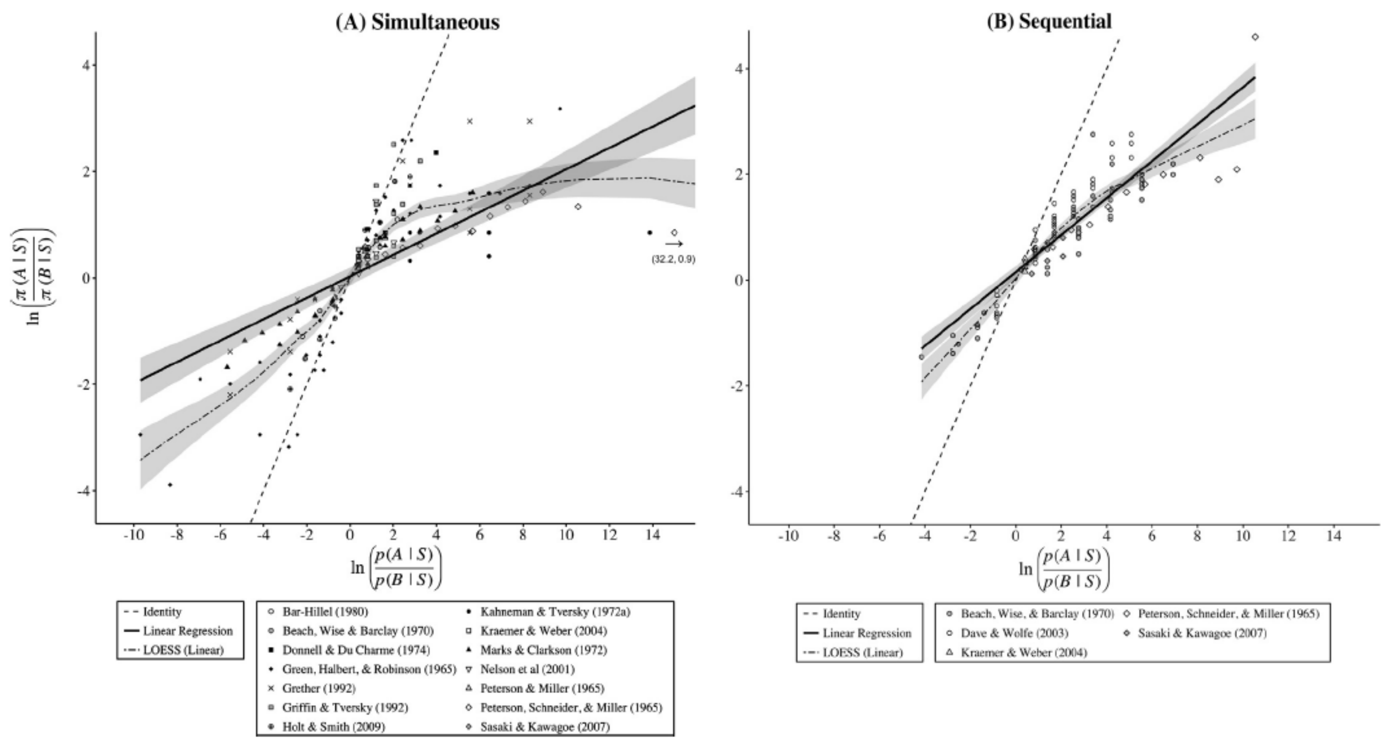
Results

Plots of inferred log likelihood ratios (averaged across *S*s and sequences) as a function of $s - f$ showed no systematic effects from prior probabilities. Accuracy ratios based on the mean inferred log likelihood ratios are shown in Fig. 1. (The accuracy ratio is not defined for $s - f = 0$. Consequently, the plots from $s - f = -1$ to $s - f = 1$ are connected with dotted lines.) For all values of $s - f$ and for both bag

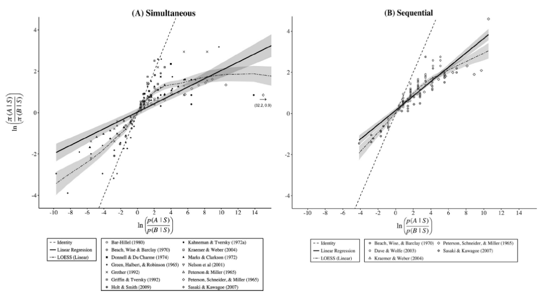
...let's add another

Meta-analytical Results

Benjamin (2019) provides a *meta-analysis* summarizing more than 50 years of research.



Meta-analytical Results



The figure plots participant estimates (y-axis) vs. Bayesian estimates (x-axis).

Participant estimates are compressed towards their prior.

- This is commonly known as **conservatism bias:** People revise their beliefs in the correct direction but **not far enough.**

Limits and Advantages

The **bookbags and pokerchips** paradigm allows for a precise comparison of beliefs to a Bayesian benchmark.

But the beliefs that we are more interested in do not allow for the same, precise comparison:

- Eg., it may be relevant to ask whether *financial markets* over- or underreact to information, or whether people learn rationally from *reading the news.* Finding the normative benchmark (how much people *should* react) is often impossible in these cases.

But some did still try (next).

Testing Beliefs in the Wild

Challenges when moving outside the lab

In the lab, we have full control over the signal-generating process. This allows us to compare subjective estimates of p to the Bayesian estimate.

In the field, we often do not know the Bayesian estimate, so it is difficult to say how wrong beliefs are.

But we can test other properties of probability measures. A nice candidate is:

The Martingale Property

Potential Empirical Setup

Suppose we can identify an empirical setting with a binary state that can be *true* or *false*, $\Omega = \{T, F\}$. Many potential scenarios:

- Whether the EU lifts sanctions on Russia by 2026.
- Whether inflation will be higher than GDP growth this year.
- Whether LMU mensa will accept digital payments by 2035...

Potential Empirical Setup

Suppose we can identify an empirical setting with a binary state that can be *true* or *false*, $\Omega = \{T, F\}$.

In such a setting, we could repeatedly ask participants for $P_t(\omega = T)$ at different t, denoted as p_t.

- Or we could simply use betting market data.

The nice thing when the state is binary is that eliciting p_t] down the whole belief distribution:

$$\mu_t(\omega = T) = p_t,\ \mu_t(\omega = F) = 1 - p_t,\ \mathrm{Var}_{\mu_t} = p_t(1 - p_t).$$

Potential Empirical Setup

We can additionally define *belief movement* and *uncertainty reduction* between $t = 1$ and $t = 2$.

Definition. *Belief movement* M denotes by how much beliefs change between the evaluation periods **1** and **2**: $M = (p_2 - p_1)^2$.

Potential Empirical Setup

We can additionally define *belief movement* and *uncertainty reduction* between $t = 1$ and $t = 2$.

Definition. *Uncertainty reduction* R denotes the reduction in belief variance between periods **1** and **2**:

$$R = p_1(1 - p_1) - p_2(1 - p_2).$$

The insight of Augenblick and Rabin (QJE, 2021)

A&R observe the following result:

> **Proposition.** If μ_t is a probability measure, then $\mathbb{E}[M] = \mathbb{E}[R]$.

This is a direct consequence of the martingale property.

(Proof on board)

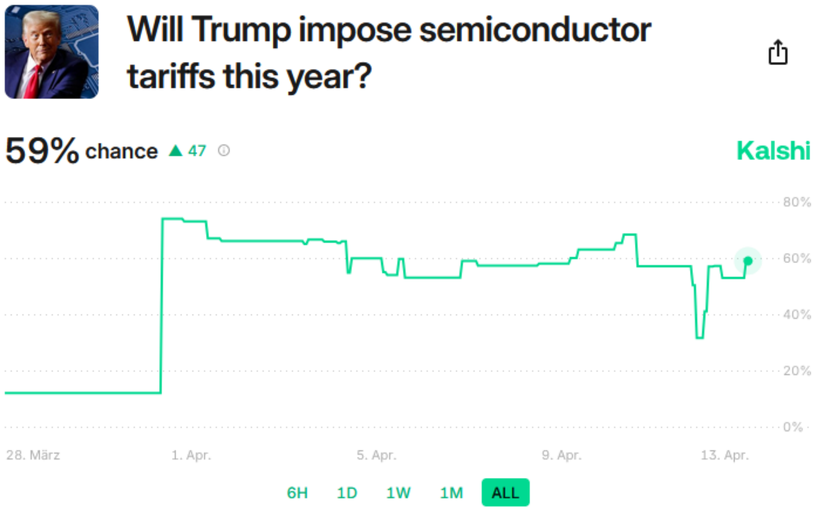
Illustration w/ betting market data

On the left, belief $\mu_t(\text{BAR wins})$ starts at >50% and moves towards 100% over time. As beliefs move, uncertainty is reduced...

Illustration w/ betting market data

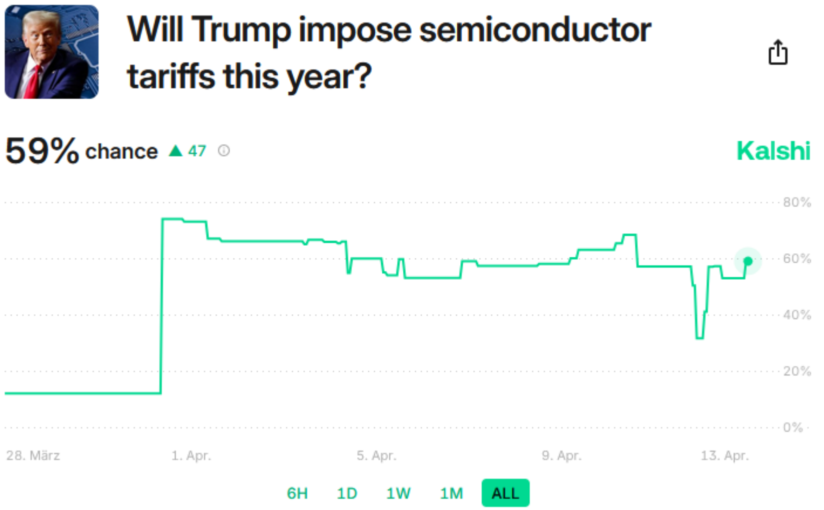
On the right, belief starts at <20% and moves to 60% over time. Here, we observe belief movement and an *increase* in uncertainty.

Illustration w/ betting market data



The proposition tells us that beliefs should move like the beliefs on the left *most of the time.*

Illustration w/ betting market data



While belief uncertainty can increase sometimes, this should be an exception, not the rule.

Testing with real forecasts

A&R test their predictions:

- Using a panel of experts making probabilistic forecasts.
  - E.g., *“Will Greece remain a member of the EU through June 2012?”*
- Using a baseball prediction algorithm.
- Using sports betting market data.

Testing with real forecasts

	Individual (1)	Fangraphs (2)	Betfair (3)
Excess movement X	0.0137***	–0.00022***	0.00036***
Z	(4.22)	(9.40)	(19.55)
Norm. excess M. X_{norm}	1.20	0.931	1.046
Belief observations	593,218	926,083	7,422,530
Event observations	291	11,753	286,257
Individuals	3,408	–	–
Total movement	33,178	2,724	61,382

Excess movement is defined as $M - R$.

Experts (Col. 1) **overshoot**: $\mathbb{E}[M] > \mathbb{E}[R]$ by $\approx 20\%$.

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Algorithm & betting markets (cols 2 + 3) are much closer to the benchmark.

But betting market movement still overshoots by $\approx 4.6\%$.

A puzzle?

Lab experiments on bookbags and pokerchips suggest that individual beliefs move too little in response to new information.

A&R conceive a comparable test to test for belief movements outside the lab. They find that individual forecasters move their predictions by too much.

We will learn about a unifying explanation next.

Over-
vs. underinference

Augenblick, Lazarus, Thaler (QJE, 2025) study why individuals sometimes over- and sometimes underinfer from data.

Idea: Distinguishing **good news** from **bad news** is often easy. It is much harder to say *how much better* one piece of good news is to another piece.

- E.g., beating earnings expectations is good news for a firm's valuation. But difficult to assess by *how much* its value should increase.

In other words, a signal's **qualitative valence** ($+$ or $-$) is easier to assess than the **quantitative size** of the signal.

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$\rightarrow$ Developing this idea in a model we get overinference from weak signals and underinference from strong signals. (Next)

Setup

Binary state of the world, $\omega \in \{H, L\}$; an agent has prior $P(\omega = H) = \lambda$.

The agent receives a signal $s \in \{g, b\}$ with signal strength $P(s = g|\omega = H) = P(s = b|\omega = L) = q$.

However, the agent does not know q for sure. They have some idea that the average good signal has a q distributed according to $\mathcal{N}(\bar{q}_0, \sigma_0)$.

Upon observing s, the agent receives an additional signal *about the size of* q, $q_s|q \sim \mathcal{N}(q, \sigma_q)$.

Setup

This setup combines a binary-binary updating model with a normal-normal model.

- The agent receives a binary signal about a binary state.
- They receive a normal signal about the normally distributed signal strength.

Updating follows a two-step procedure:

1. Observe signal valence and estimate signal strength.
2. Use the estimated signal strength to update beliefs about the state of the world.

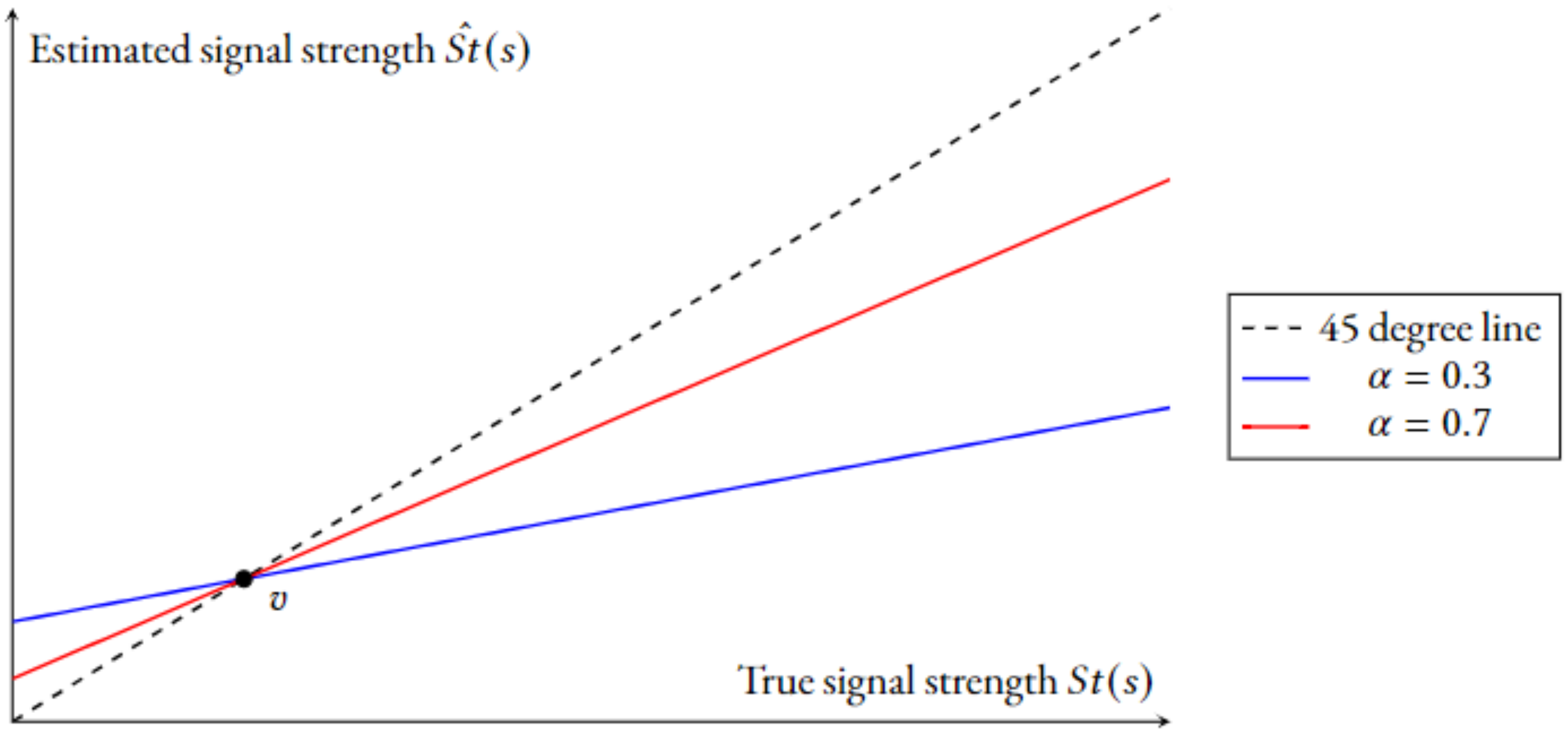
Over- vs. underinference

Proposition. The agent's estimated signal strength is equal to

$$\bar{q}_1 = \alpha q_s + (1-\alpha)\bar{q}_0, \text{ where } \alpha = \frac{\sigma_0}{\sigma_0 + \sigma_q}$$

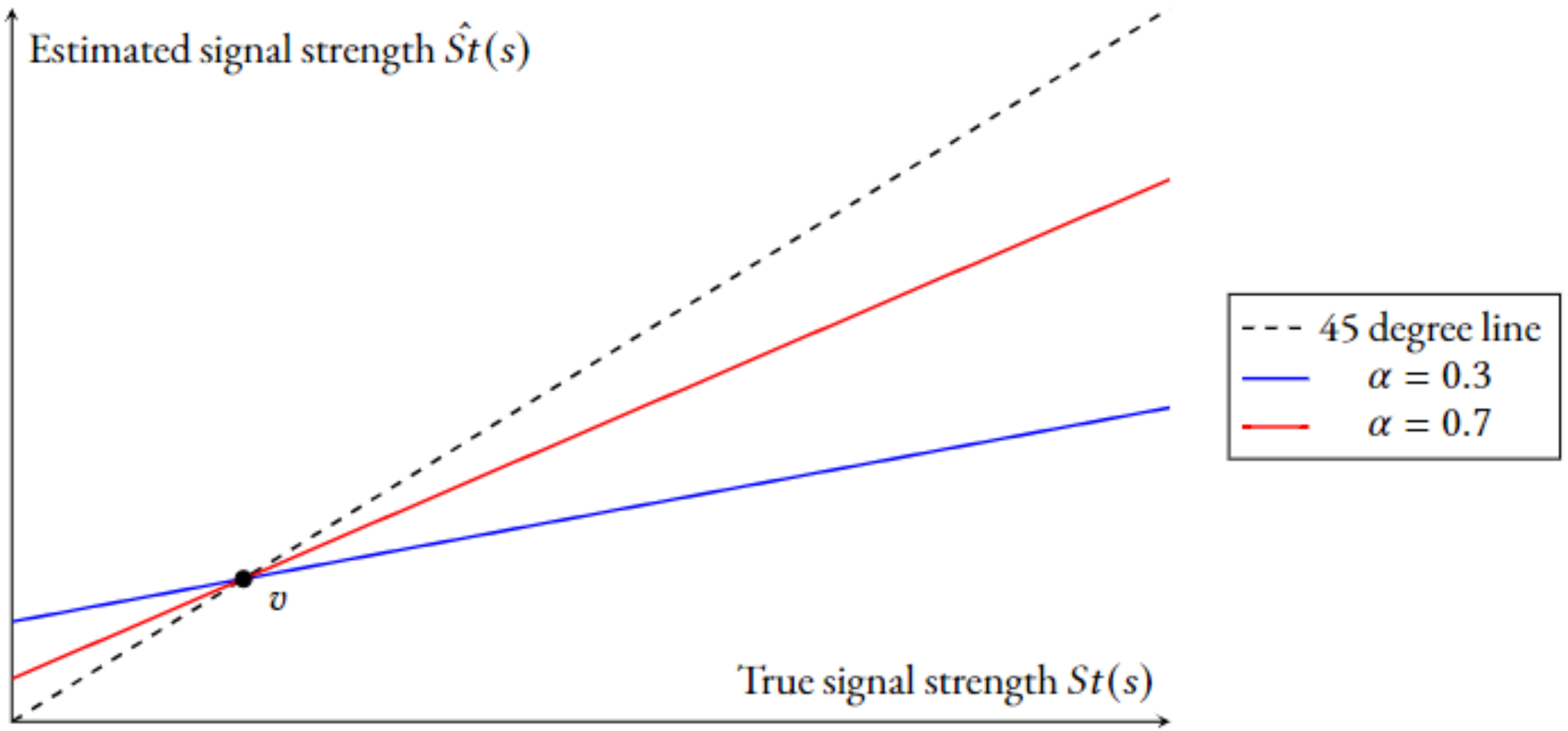
This directly follows from the normal-normal model.

Over- vs. underinference



Implication: Individuals overestimate weak signals and they underestimate strong signals.

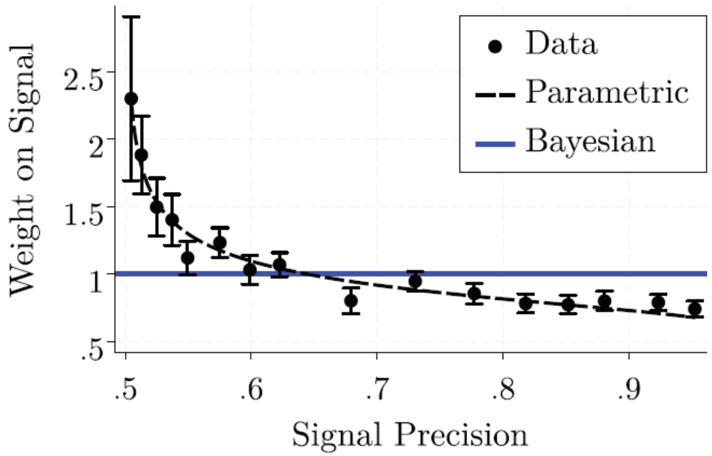
Over- vs. underinference



Key prediction: Individuals overinfer from weak signals and underinfer from strong signals.

Bookbags & pokerchips revisited

ALT conduct a lab belief updating experiment and find overinference if $q < 0.6$ and underinference if $q > 0.6$.



- The y-axis is normalized. y>1 means overinference and y<1 means underinference.

- The metastudy by Benjamin (2019) we saw earlier could not find this as all experiments it considered had $q \geq 0.6$!

Betting-market evidence

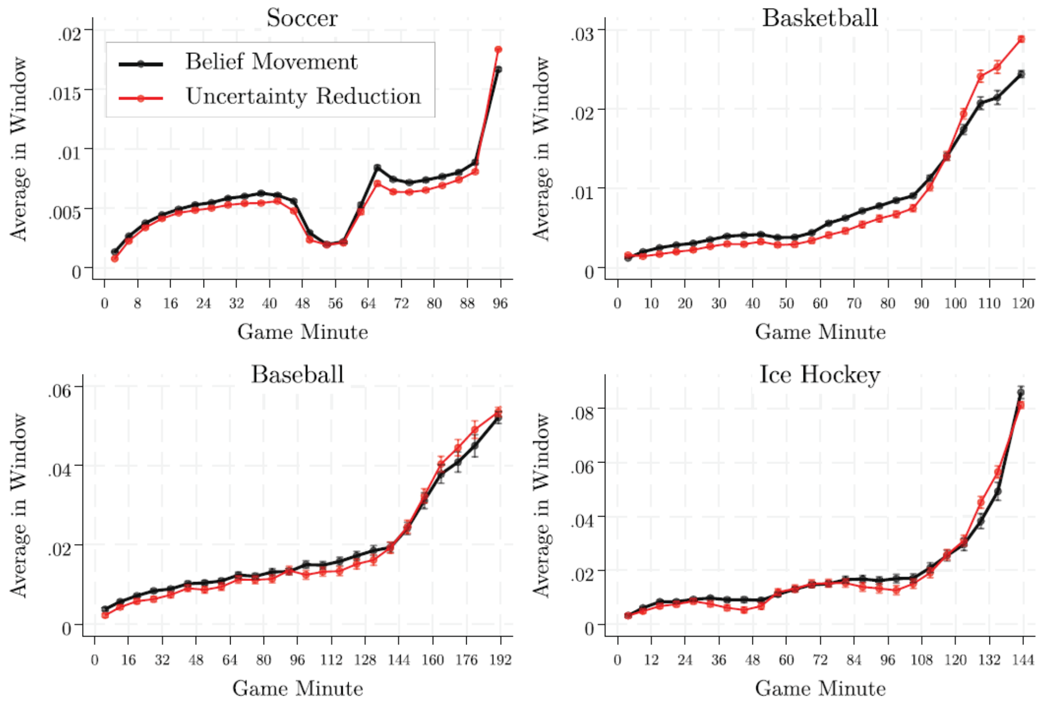
ALT also aim for a test of reactions towards weak and strong signals in sports betting and financial markets.

They propose that, in sports, news that are revealed early in the game are less informative about the game outcome than equivalent news revealed later.

- E.g., in football, scoring the first goal in min. 1 is less informative than scoring the first goal in min. 89.

Based on this, they predict that **belief movement > uncertainty reduction** early in the game and **belief movement < uncertainty reduction** late in the game.

Betting-market evidence



- Markets move **too much** early, **too little** late.
- Similar for long- vs. short-maturity stock options.

Takeaways

1. **Lab tests** (bookbags & pokerchips) → *conservatism* on average.
2. **Field tests** (forecast panels, betting) → *excess belief movement*.
3. Aggregation in markets partly corrects individual biases.
4. A unified view: People find it easier to judge the valence than the size of a signal. ⇒ they **over-react to weak** and **under-react to strong** information.