# Oligopoly in Networked Markets

Tilman Jacobs

June 20, 2024

**Abstract**

Oligopolists frequently sell their product and compete in more than just one market. Often this leads to collusive behavior and lower intensity of competition. This is an example of a networked market. I present different approaches from the literature to reconcile such a setup with two of the three canonical models of Oligopoly: Cournot and Stackelberg. I then use the example of four major Chinese cities to illustrate how network effects might influence the provision of electricity. In a next step, I consider whether a social planner can improve welfare by acting as a Stackelberg leader.

# Introduction

Traditionally, we think of a firm as competing in a single market. There it faces a unique set of competitors and unique demand. In reality large firms often sell their product in more then just one market. These markets are most often separated along regional lines and differences in local demand, market structure and other characteristics can lead to wide variations in prices and traded quantities. Examples of industries that feature firms competing across many different markets include utilities such as water, natural gas and electricity as well as hospitality, banking and air travel. An electricity producer may produce at many different generators and sell its electricity on a number of regional power markets. Cement is very difficult to transport over long distances on land. Thus, a producer of cement will usually operate plants in different parts of the world that serve their own regional markets. The hospitality industry too features a dominant group of multinational firms that compete regionally. In industries like banking a group of small banks that only offer their services regionally compete with larger firms with global operations. Most clearly differentiated are the departure-destination pairs along the lines of which airlines compete. One airline has to decide which connections to operate on and what prices or quantities to set.

Naturally, we would like to simplify this situation by only considering the aggregate of all the regional markets a firm competes in. In doing so we could simply model firms competion by considering a Cournot, Stackelberg or Bertrand game on the unified market. However, the presence of capacity constraints or non-linear cost functions mean that firms profits are not separable across regional markets. To overcome this we might consider a network of firms and markets.

In Section 1, I review the literature on networked markets and introduce the setup I will work with for the rest of the essay. In Section 2, I present a number of works on Cournot competition on a networked market with a particular focus on the model by Bimpikis et al. (2019). I further illustrate the model using the example of regional electricity markets. In section 3, I use the work of Neto et al. (2016) to introduce a welfare-oriented market maker that acts as a Stackelberg leader with profit maximizing firms following.

# 1 Networked Markets

## 1.1 Literature

Networked markets first emerged as a concept when economists and computer scientists used graph theory to describe what happens when competing firms make contact across multiple markets. The economics literature on multi-market contact predates the first mention of networked markets by several years. A lot of the early research into multi-market contact was motivated by the banking industry. Interstate banking was long highly restricted in the USA. From the mid-1980s these restrictions were gradually rolled back. Heggestad and Rhoades (1978) show empirical evidence for anti-competitive effects of multi-market contact between large banks. Pilloff (1999) provides additional evidence for reductions in competition amongst firms that share multiple markets. Scott (1982) studies how in a more general setting multi-market contact facilitates collusion which counteracts the positive effects of diversification on firms' productivity. In a seminal paper Evans and Kessides (1994) find that multi-market contact between airlines leads to higher fares on routes served by those airlines. They point out that this supports the widely held industry belief that firms will not try to compete by setting lower prices when they can expect their competitors to retaliate in other shared markets. Such collusive behavior is also referred to as "mutual forebearance" or the "golden rule". More recently Schmitt (2018) studied multi-market contact in the hospital industry. Bernheim and Whinston (1990) describe the conditions under which multi-market competitors collude to lower competition.

These strong implications for the competitiveness of markets explain the need for formal models of networked competition or linked oligopolies. Due to the relevance to the emerging internet markets as well as the methodological synergies, computer scientists have contributed a lot to the networked markets literature. Chawla and Roughgarden (2008) model how internet service providers compete by setting prices for bandwidth as a Bertrand game on a network. Bimpikis et al. (2019) describe networked Cournot competition and notably provide an implicit characterization of equilibria for games with quadratic costs. Neto et al. (2016) present a model of networked Stackelberg competition. With the establishment of a theoretical foundation of networked markets there have come several papers that experimentally test this theory. (Cassar et al., 2010; Comola and Fafchamps, 2018)

### 1.2 A general mathematical foundation

To make the concept of oligopolists competing in multiple interdependent markets more workable we introduce a formal model of networked markets. Consider a bipartite graph $G(M \cup F, E)$ with vertices in $M \cup F$ and edges in $E$. We refer to $m \in M$ as markets and to $f \in F$ as firms. A firm $f_i$ has a presence in market $m_k$ exactly if $(f_i, m_k) \in E$. Further we denote $M_k = \{f_i \in F : (f_i, m_k) \in E\}$ the set of all firms competing in a market $k$ and $F_i = \{m_k \in M : (f_i, m_k) \in E\}$ the set of all markets a given firm $i$ is active in.

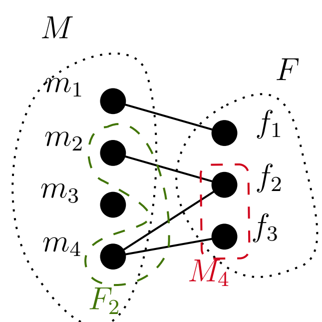


Figure 1: A Networked Market

## 2 Regional Electricity Markets as a Cournot Game

### 2.1 Bimpikis et al. (2019)

Bimpikis et al. (2019) present a general model of quantity competition in networked markets. They denote $q_{ik}$ the quantity a firm $f_i$ supplies to a market $m_k$. The demand function

$\mathcal{P}_k\left(\sum_{j\in M_k} q_{jk}\right)$ gives the price at market $m_k$. Crucially, all $\mathcal{P}_k$ are twice differentiable, strictly decreasing and concave. Further, firms face costs of production given by $\mathcal{C}_i(\sum_{j\in F_i} q_{ij})$. The cost function too is twice differentiable but strictly increasing and convex. Therefore, profits are

$$\pi_i(q) = \sum_{m_k\in F_i} q_{ik}\cdot \mathcal{P}_k\left(\sum_{j\in M_k} q_{jk}\right) - \mathcal{C}_i\left(\sum_{j\in F_i} q_{ij}\right).$$

Due to the convexity of the costs the profit function is not separable across markets. Otherwise we could simply consider each market as a separate Cournot game. From here firms set individual market quantities to maximize their profits given the competitors actions. The induced game is denoted $\mathcal{CG}\left(\{\mathcal{P}_k\}_{1\le k\le m}, \{\mathcal{C}_i\}_{1\le i\le n}, G\right)$.

The authors prove a number of useful results about the equilibria of such a game. First, $\mathcal{CG}\left(\{\mathcal{P}_k\}_{1\le k\le m}, \{\mathcal{C}_i\}_{1\le i\le n}, G\right)$ has a unique Nash equilibrium in pure strategies.

**Lemma 1** *Game* $\mathcal{CG}\left(\{\mathcal{P}_k\}_{1\le k\le m}, \{\mathcal{C}_i\}_{1\le i\le n}, G\right)$ *has a unique Nash equilibrium when* $\{\mathcal{P}_k\}_{1\le k\le m}$ *are twice differentiable, concave, and strictly decreasing,* $\{\mathcal{C}_i\}_{1\le i\le n}$ *are twice differentiable, convex, and increasing.*

If we restrict the parameters of the game to only allow linear demands and quadratic costs we can also characterize this equilibrium implicitly. Let $\mathcal{P}_k = \alpha_k - \beta_k\cdot\sum_{j\in M_k} q_{jk}$ and $\mathcal{C}_i = c_i\cdot(\sum_{k\in F_i} q_{ik})^2$. We write $\mathcal{CG}\left(\alpha,\beta,c,G\right)$, where $\alpha = (\alpha_1,...,\alpha_m)^T, \beta = (\beta,...,\beta)^T$ and $c = (c_1,...,c_n)^T$. Additionally, we define a $|E|\times 1$ vector $\bar{\alpha}$ with entries

$$\bar{\alpha}_{ik} = \begin{cases} \alpha, \text{ if } (i,k\in E) \\ 0, \text{ otherwise.} \end{cases}$$

Lemma 2 now gives us the unique equilibrium of the game $\mathcal{CG}\left(\alpha,\beta,c,G\right)$ as the solution to the *linear complementarity problem* $LCP(w,Q)$ of finding a vector $z\ge 0$ with $Qz+w\ge 0$ and $z^t(Qz+w)=0$.

**Lemma 2** *The unique equilibrium $q$ of $\mathcal{CG}(\alpha, \beta, c, G)$ is given by the unique solution of $LCP(-\bar{\alpha}, D)$, where $D$ is the following $|E| \times |E|$ matrix*

$$D_{ik,jl} = \begin{cases} 2(\beta_k + c_i) & , \textit{if } i = j, k = l \\ 2c_i & , \textit{if } i = j, k = l \\ \beta_k(\beta_k + c_i) & , \textit{if } i = j, k = l \\ 0 & , \textit{otherwise.} \end{cases}$$

The authors prove number of further results which are omitted for brevity. They discuss mergers and market entry implications which notably differ from those in a single market. Particularly, the paper shows that the entire network structure must be considered to determine whether a merger has positive effects on welfare.

## 2.2 Example: Three Firms compete on the electricity market

Next I will illustrate the model using a highly simplified approximation of a networked electricity market.

### 2.2.1 The Game

Consider a game of three producers of electricity competing to sell their kilowatt-hours on the markets of four major Chinese cities: Beijing, Shanghai, Chengdu and Guangzhou. Firm one is only interested in serving the very biggest cities. Thus, they only have a presence in Beijing and Shanghai. Firm two, a national provider sells in all four cities. Lastly, Firm three is a regional, southern producer and does not sell in the northern Beijing. Each city has its own demand for electricity, which is linear and decreasing.

$$P_i = \alpha_i - \beta_i Q_i$$

The costs of the producers in terms of the produced quantity are quadratic as below.

$$C_i = c_i Q_i^2$$

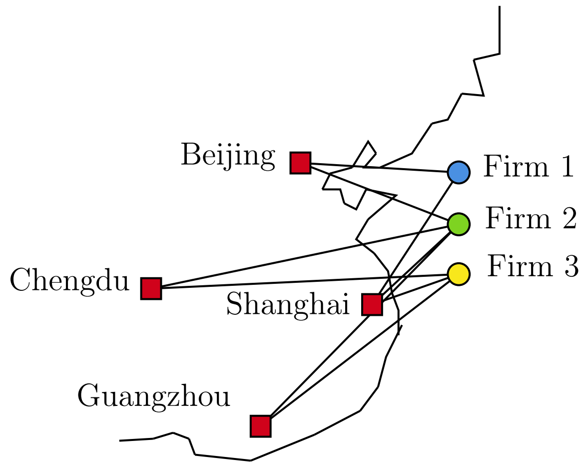


Figure 2: The graph of the electricity game

Using the formalism of Bimpikis et al. (2019), the game is

$$\mathcal{CG}(\alpha, \beta, c, G).$$

Where $\alpha = (\alpha_1, ..., \alpha_4)^t, \beta = (\beta_1, ..., \beta_4)^t, c = (c_1, c_2, c_3)^t$. The set of firms is

$$F = \{f_1, f_2, f_3\}$$

and the set of markets will from now on be simply written as

$$M = \{\text{Beijing, Shanghai, Chengdu, Guangzhou}\} = \{m_1, m_2, m_3, m_4\}.$$

And we have

$$F_1 = \{m_1, m_2\}, F_2 = \{m_1, m_2, m_3, m_4\}, F_3 = \{m_2, m_3, m_4\}$$

as well as

$$M_1 = \{f_1, f_2\}, M_2 = \{f_1, f_2, f_3\}, M_3 = \{f_2, f_3\}, M_4 = \{f_2, f_3\}.$$

### 2.2.2 Wefare Maximization

We assume for example that $\alpha = (20, 17, 13, 12)^t, \beta = \left(1, \frac{1}{2}, 2, \frac{3}{4}\right)^t, c = (1, 1, 1)^t$. Before we consider the Cournot game on this networked electricity market we will first consider the welfare maximizing assignment. If a social planner could dictate the assignment

$$q^w = (q^w_{11}, q^w_{12}, q^w_{21}, q^w_{22}, q^w_{23}, q^w_{24}, q^w_{32}, q^w_{33}, q^w_{34}),$$

they would do so by optimizing the welfare function

$$w = \sum_{f_i \in F} \pi_i + CS.$$

Where $CS$ is the aggregate consumer welfare of all four cities. Since we have no general result for this assignment we determine computationally [1]. We find that the maximal achieveable welfare is $w_{max} = 293.16\bar{6}$ with consumer welfare $CS = 139.4921$ and aggregate profits of $\Pi = 153.6746$. The associated assignment is

$$q^{w_{max}} = (5.0000, 4.8571, 5.0000, 4.8571, 2.1667, 3.4286, 4.8571, 2.1667, 3.4286).$$

### 2.2.3 Cournot Equilibrium

By Lemma 2 (Bimpikis et al., 2019) the unique equilibrium of the game $\mathcal{CG}(\alpha, \beta, c, G)$ is given by the vector

$$q^* = (q^*_{11}, q^*_{12}, q^*_{21}, q^*_{22}, q^*_{23}, q^*_{24}, q^*_{32}, q^*_{33}, q^*_{34})$$

[1] A MATLAB script with all the computations is attached.

that solves the associated linear complemetarity problem. We can solve the problem and obtain

$$q^* = (2.9586, 2.6347, 2.8964, 2.5105, 0.1805, 0.0369, 3.8473, 0.5147, 0.9281).$$

Therefore the traded quantities of electricity in each of the four ctites are

$$Q_{\text{Beijing}} = \sum_{f_i \in M_1} q^*_{i,1} = 2.9586 + 2.8964 = 5.855$$

$$Q_{\text{Shanghai}} = \sum_{f_i \in M_2} q^*_{i,2} = 2.6347 + 2.5105 + 3.8473 = 8.9925$$

$$Q_{\text{Chengdu}} = \sum_{f_i \in M_3} q^*_{i,3} = 0.1805 + 0.5147 = 0.6952$$

$$Q_{\text{Guangzhou}} = \sum_{f_i \in M_4} q^*_{i,4} = 0.0369 + 0.9281 = 0.9650.$$

This gives the following equilibrium prices

$$P_1 = 14.145, P_2 = 12.50375, P_3 = 11.6096, P_4 = 11.27625.$$

The vector of the firms equilibrium quantities is

$$q^*_f = (q^*_{f_1}, q^*_{f_2}, q^*_{f_3}) = (5.5933, 5.6243, 5.2901).$$

Lastly, their aggregate profits are

$$\pi^* = (59.0977, 60.1463, 48.6188),$$

consumer surplus is

$$CS = 38.1888$$

and aggregate welfare

$$w = 206.0515.$$

Notably, the Cournot outcome strongly underperformed in terms of welfare. If we compare the quantities produced in every market with the socially optimal ones we see that provision of electricity was socially too low in all four cities. Especially in the cities with lower demand only a fraction of the socially optimal quantities was provided as firms seemingly move their resources to more profitable markets. Prices and profits are much higher while consumer surplus drops drastically in the Cournot assignment. While this example is not sophisticated enough to make any conclusions about network incentives to collude it illustrates that even in very simple scenarios multi-market contact can induce rather sub-optimal outcomes for consumers in a Cournot game.

# 3 Market Makers as Stackelberg Leaders in Electricity Markets

As Bimpikis et al. (2019) point out in their paper and as our example illustrates, a networked market can be highly competitive, in the sense that many firms compete, and still not be very close to welfare maximizing. This result matches the empirical findings of negative effects on competition due to multi-market contact. It also offers us an interesting opportunity to consider a unique variation of Stackelberg competition.

## 3.1 Neto et al. (2016)

As opposed to our previous setup access to an electricity market must not be free. A market maker could charge access fees to potential sellers. This is the assumption under which Neto et al. (2016) present their model. Further, the model considers where firms produce their electricity and how much of it they can make accessible where, at what time and at what cost. This model is quite a lot more detailed than the previous one because it is fine tuned to the particularities of electricity markets. It is not specialized enough to preclude it from being used in any other contexts but doing so will require a lot more work than the Bimpikis et al. (2019) model and a good amount of familiarity with the new context to make the proper adjustments.

Similarly to the previous model the proposed Stackelberg model comes down to a complementarity problem. However, due to the presence of the leader it is no longer a linear optimization problem. The structure of the graph remains mostly the same but edges no have capacity constraint. That is an individual generator is limited in how much electricity it can deliver to a given city at a time. Further, due to the dynamic nature of the Stackelberg game the authors present both a continuous as well as discrete time application. They present several numerical examples from a naive three node game to a 15 node approximation of the european electric grid. They find that when congestion, that is binding capacity constraints, are not a factor the Stackelberg outcomes are very similar to the Cournot outcomes in terms of social welfare and produced quantities. Interestingly, while overall welfare remains steady the leaders action did have a tendency to shift surplus from producers to the consumers. When congestion is a factor though the their Stackelberg leaders can significantly improve welfare by setting the appropriate prices. Congested networks are not a very strong assumption though as unlimited capacity does not exist in reality. Thus, the networked Stackelberg game provides a promising approach for regulators trying to counteract the anti-competitive effects of multi-market contact.

# 4 Conclusion

Multimarket contact is well established in the empirical literature to negatively affect competition. This essay discusses how the traditional models of Cournot and Stackelberg competition hold up to the context of networked markets with only a reasonable amount of modification. Those estensions allow us to study oligopoly dynamics in more complex and realistic markets than we we were able to before. Crucially, many results that are generally true in a single market do not carry over when considering a networked market. This is very interesting from an anti-trust standpoint as regulators must question wether their assumptions truly reflect the markets they govern. If not a networked markets approach might be helpful in making more informed policy decisions. The Cournot model presented in Section 2 shows that anti-competitive effects exist in networked markets in general and when firms merge specifically. However, it does not provide a direct explanation for active or passive collusion. For possible policy implications this is a very promising direction for future research. Also relevant to policy makers and future researchers should be the evidence for the effectiveness of congestion pricing measures. More general theoretical results in this direction as well as empirical work are needed.

The electricity market example demonstrates how the models work using a very simple approximation of a real world market. Unfortunately, including a section on Bertrand competition in this essay proved itself beyond the scope of this essay in time, space and the available literature. To the best of my knowledge no model as sound as the Bimpikis et al. (2019) model exists for Bertrand games currently. However, computer scientists have studied the existence of Bertrand equilibria in similar contexts. Chawla and Roughgarden (2008) propose a model of Bertrand competition where internet service providers sell bandwidth to internet users. The Model is unfortunately very specific to the internet context and not satisfying as a general extension of Bertrand competition to networked markets. Also, the authors do not provide an implicit characterization of equilibria like Bimpikis et al. (2019). Due to the more complicated structure of the profit function it is unlikely that an extension can be found by following a similar argument to Bimpikis et al. (2019) who rely heavily on the concavity of the game in their proofs. Thus, extending Bertrand competition to networks remains a promising but, it seems, challenging direction for future research.

# References


B. Douglas Bernheim and Michael D. Whinston. Multimarket contact and collusive behavior. *The RAND Journal of Economics*, pages 1–26, 1990.

Kostas Bimpikis, Shayan Ehsani, and Rahmi İlkılıç. Cournot Competition in Networked Markets. *Management Science*, 65(6):2467–2481, June 2019. ISSN 0025-1909, 1526-5501. doi: 10.1287/mnsc.2018.3061.

Alessandra Cassar, Daniel Friedman, and Patricia Higino Schneider. A laboratory investigation of networked markets. *The Economic Journal*, 120(547):919–943, 2010.

Shuchi Chawla and Tim Roughgarden. Bertrand Competition in Networks. In Burkhard Monien and Ulf-Peter Schroeder, editors, *Algorithmic Game Theory*, volume 4997, pages 70–82, Berlin, Heidelberg, 2008. Springer Berlin Heidelberg. ISBN 978-3-540-79308-3 978-3-540-79309-0. doi: 10.1007/978-3-540-79309-0_8.

Margherita Comola and Marcel Fafchamps. An experimental study on decentralized networked markets. *Journal of Economic Behavior & Organization*, 145:567–591, 2018.

William N. Evans and Ioannis N. Kessides. Living by the “golden rule”: Multimarket contact in the US airline industry. *The Quarterly Journal of Economics*, 109(2):341–366, 1994.

Arnold A. Heggestad and Stephen A. Rhoades. Multi-market interdependence and local market competition in banking. *The Review of Economics and Statistics*, pages 523–532, 1978.

Pedro A. Neto, Terry L. Friesz, and Ke Han. Electric Power Network Oligopoly as a Dynamic Stackelberg Game. *Networks and Spatial Economics*, 16(4):1211–1241, December 2016. ISSN 1566-113X, 1572-9427. doi: 10.1007/s11067-016-9337-7.

Steven J. Pilloff. Multimarket contact in banking. *Review of Industrial Organization*, 14(2): 163–182, 1999. ISSN 0889938X. doi: 10.1023/A:1007779814751.

Matt Schmitt. Multimarket contact in the hospital industry. *American Economic Journal: Economic Policy*, 10(3):361–387, 2018.

John T. Scott. Multimarket contact and economic performance. *The Review of Economics and Statistics*, pages 368–375, 1982.