Random variables and probability distributions: Exercises

Exercises 1

Sample from the distribution with PDF

$$p(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}\sin x & \text{otherwise} \\ 0 & x > \pi \end{cases}$$

- Plot the PDF and CDF
- Sample from the distribution using invere transform sampling
- Compare the histogram of the samples to the PDF

Exercise 2

Show the expressions for the mean and variance:

- For some constant a: $\mathrm{E}[x+a] = \mu + a$
- For some constant c: $\mathrm{E}[cx] = c\mu$
- $\mathrm{Var}[x] = \mathrm{E}[(x - \mathrm{E}[x])^2] = \mathrm{E}[x^2] - \mathrm{E}[x]^2$
- For some constant a: $\mathrm{Var}[x+a] = \sigma^2$
- For some constant c: $\mathrm{Var}[cx] = c^2\sigma^2$

Exercises 3

- Confirm by simulation that the probability of k out of n iid $X_i \sim \mathcal{U}(0,1)$ being in a small interval Δx follows a Poisson distribution.
- Derive the Poisson distribution from the binomial distribution.

Exercises 4

- Make plots of the PDFs of the distributions we covered so far, varying their parameters
- Compute the distribution of the sum of two independent Gaussian RVs.
- What is the distribution of the sum of two independent RVs $X \sim f$ and $Y \sim g$?
- Derive the PDF of the $\chi^2_{\nu=1}$ distribution
- Bonus:
  - What is the product of two independent RVs U and V?
  - Compute the distribution of the ratio of two Gaussian RVs.