The Math:

The sieve of Eratosthenes is an easy method to generate primes.
You simply start at 2, and mark the multiples as composite numbers.
This is repeated with the next number, which was not marked as a composite, i.e. the next prime.
With this algorithm one can specify the prime-counting function as:

$$\pi(n) = |\{2, ..., n\} \setminus \bigcup_{i=2}^{n-1} \bigcup_{k=2}^{n+1-i} i \cdot k|$$

If you shift the indices, the set of composites, which is substracted from the set $\{2, ..., n\}$ could also be written as:

$$\bigcup_{i=2}^{n-1} \bigcup_{k=2}^{n+1-i} i \cdot k = \bigcup_{i=3}^{n} \bigcup_{k=2}^{n+2-i} (i-1) \cdot k = \bigcup_{i=3}^{n} \bigcup_{k=2}^{i-1} (n+2-i) \cdot k =: \varphi(n)$$

Now consider the next function:

$$\bigcup_{i=3}^{n} \bigcup_{k=2}^{i-1} (i+1-k) \cdot k =: \psi(n)$$

The claim is: $\varphi(n) = \psi(n) \ \forall n \geq 2$, i.e. the two functions generate the same sets.

Proof by Mathematical Induction:

Base Case: $n = 3$ ($\psi(2)$ is an empty set)

$$\varphi(3) = \bigcup_{i=3}^{3} \bigcup_{k=2}^{i-1} (n+2-i) \cdot k = \{4\} = \bigcup_{i=3}^{3} \bigcup_{k=2}^{i-1} (i+1-k) \cdot k = \psi(3)$$

Assumption:

$$\varphi(n-1) = \bigcup_{i=3}^{n-1} \bigcup_{k=2}^{i-1} (n+1-i) \cdot k = \bigcup_{i=3}^{n-1} \bigcup_{k=2}^{i-1} (i+1-k) \cdot k = \psi(n-1)$$

Induction Step: $n-1 \rightarrow n$

The following tables show the development of the sets of composites for $\varphi(n)$ and $\psi(n)$, starting at $n = 3$:

It shows that

$\varphi(n)$

k \ i	3	4	5	6	7
2	4				
2	6	4			
3		6			
2	8	6	4		
3		9	6		
4			8		
2	10	8	6	4	
3		12	9	6	
4			12	8	
5				10	
2	12	10	8	6	4
3		15	12	9	6
4			16	12	8
5				15	10
6					12

$\psi(n)$

k \ i	3	4	5	6	7
2	4				
2	4	6			
3		6			
2	4	6	8		
3		6	9		
4			8		
2	4	6	8	10	
3		6	9	12	
4			8	12	
5				10	
2	4	6	8	10	12
3		6	9	12	15
4			8	12	16
5				10	15
6					12

$$\varphi(n) = \bigcup_{i=3}^{n} \bigcup_{k=2}^{i-1} (n+2-i)\cdot k = \varphi(n-1) \cup \bigcup_{i=3}^{n} (n+2-i)\cdot(i-1) := \varphi(n-1) \cup \varphi'$$

and

$$\psi(n) = \bigcup_{i=3}^{n} \bigcup_{k=2}^{i-1} (i+1-k)\cdot k = \psi(n-1) \cup \bigcup_{k=2}^{n-1} (n+1-k)\cdot k := \psi(n-1) \cup \psi'$$

Now we show that the added sets are equal:

$$\varphi' = \bigcup_{i=3}^{n} (n+2-i)\cdot(i-1) = \bigcup_{i=2}^{n-1} (n+2-(i+1))\cdot((i+1)-1) = \bigcup_{i=2}^{n-1} (n+1-i)\cdot i = \bigcup_{k=2}^{n-1} (n+1-k)\cdot k = \psi'$$

$\Rightarrow \varphi(n) = \psi(n)\ \forall n \geq 2$

$\square$

We see that all composite numbers can be described by the polynomial $(i+1-k)\cdot k$.
These polynomials are also visible within the application (activate View > Polys to draw the polynomials).

Applications:

The prime-counting function:

$\pi(n) = |\{2, ..., n\} \setminus \psi(n)|$

The following function calculates the number of Goldbach partitions for the even number $2 \cdot n$:

$$G(n) = |\{2, ..., n\} \setminus \bigcup_{b=0}^{1} \bigcup_{i=3}^{n} \bigcup_{k=2}^{i-1} n + (-1)^b \cdot (n - (i + 1 - k) \cdot k)|$$

The function $\psi(n)$ could be further generalized to use a constanct factor $c \in \mathbb{N}$:

$$\psi_c(n) = \bigcup_{j=2-c}^{1} \bigcup_{i=3}^{n} \bigcup_{k=2}^{i-1} c \cdot (i + \tfrac{j}{c} - k) \cdot k$$

The prime-counting function could now be expressed by

$\pi_c(n) = |\{2, ..., n\} \setminus \psi_c(n)|$

And the Goldbach function as

$$G_c(n) = |\{2, ..., n\} \setminus \bigcup_{b=0}^{1} \bigcup_{j=2-c}^{1} \bigcup_{i=3}^{n} \bigcup_{k=2}^{i-1} n + (-1)^b \cdot (n - c \cdot (i + \tfrac{j}{c} - k) \cdot k)|$$

It holds that:

$\pi(n) = \pi_c(n) \ \forall_{n,c \in \mathbb{N}}$
$G(n) = G_c(n) \ \forall_{n,c \in \mathbb{N}}$