Intermittency model description and examples

This livescript contains the following information:

- Descriptions of model parameters
- Descriptions of model equations
- Examples of how to run the model and plot results

Additional description and contextualization of the model can be found in:

- Tilsen, S. *Phrasal rhythmicity and the sources of temporal intermittency in speech*
- Tilsen, S. (2019) *Syntax with oscillators and energy levels.* Language Science Press.

Please note that this model is a work-in-progress, and a number of future improvements are planned.

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Model parameters

Below are described the parameters used in `intermittency_model.m`. The defaults are defined in the function `model_default_params.m`, but can be replaced by passing them as fields in an input structure (see how to run the model below). The variable names, default values, and short descriptions are provided for each parameter. The effects of their values can be better understood by examining the model equations. Note the following abbreviations:

- C,c : concept system
- S,s : syntactic system
- F : feedback system
- E : excitation potential levels
- A : annealer
- M : excitation monitoring system

```
addpath('.\helpers');
P = model_default_params;
dp = @(ptype)disp_pars(P,ptype);
```

Simulation parameters:

```
dp('simulation');
```

name	val	desc
"T"	5	"maximal run time"
"stop_on_completion"	0.001	"stop if the sequence is completed"
"dt"	0.001	"time step"

Systems parameters:

```
p = dp('systems');
```

name	val	desc
"Nc"	{[100]}	"number of c-systems"
"Ns"	{[3]}	"number of s-systems"
"Sx0"	{[0.5000 0.3000 0.1000]}	"initial syntactic system activation"
"S_S"	{3×3 double }	"s-to-s coupling matrix"
"C_S_prop"	{[0.3333]}	"proportion of systems coupled to verbal S-system"
"Ne"	{[5]}	"number of initial energy levels (usually Ns + 2 extra for ground and
"f"	{[8]}	"oscillator intrinsic frequency (Hz)"

This is the syntactic system-to-syntactic system coupling matrix for an SVO utterance, where the rows/cols correspond to N+ (S), V, and N- (O) s-systems:

```
disp(p.val{p.name=='S_S'})
```

```
     0     1     0
     1     0    -1
     0    -1     0
```

For further detail, see Tilsen (2019). *Syntax with oscillators and energy levels*.

Noise amplitudes:

```
dp('noise amplitude');
```

name	val	desc
"eta_Cp"	0.1	"c-system phase noise"
"eta_Cx"	0.1	"c-system activation noise"
"eta_Sp"	0.1	"s-system phase noise"
"eta_Sx"	0.5	"s-system activation noise"
"eta_Sx0"	0.1	"s-system activation reset noise"

Activation coupling strength parameters. Currently the default model includes only c-to-s activation coupling.

```
dp('activation coupling strength');
```

name	val	desc
"chi_SC"	0	"s-to-c coupling"
"chi_SS"	0	"s-to-s coupling"

```
"chi_CC"     0     "c-to-c coupling"
"chi_CS"     4     "c-to-s coupling"
```

Phase coupling strength parameters:

```
dp('phase coupling strength');
```

```
  name        val         desc
________     ____     _________________

"psi_SC"       10     "s-to-c coupling"
"psi_SS"       10     "s-to-s coupling"
"psi_CC"        0     "c-to-c coupling"
"psi_CS"     0.05     "c-to-s coupling"
```

Global excitation potential parameters:

```
dp('global excitation potential');
```

```
  name        val        desc
_________     ___     _____________

"gw_ub"       1.1     "upper limit"
"gw_lb"         0     "lower limit"
"gain_gw"     0.1     "gain"
```

Syntactic-system excitation potential parameters:

```
dp('s-system excitation potential');
```

```
   name        val                   desc
__________     ____     __________________________________

"sigma_VE"     0.05     "width of force regions"
"gain_VE"         1     "strength of excitation potential"
```

Annealing parameters:

```
dp('annealer');
```

```
  name        val                            desc
_________     ___     ___________________________________________________

"gain_Ai"     1.2     "gain of annealer in initial organization phase"
"gain_Ar"       6     "gain of annealer after initial organization phase"
```

Excitation monitor parameters:

```
dp('excitation monitor');
```

```
      name            val                       desc
__________________    ____    _______________________________________________

"Me_unocc_cost"        15     "cost of unoccupied level"
```

```
"Me_multiocc_cost"      15    "cost of multiply occupied level"
"Me_decay"              10    "decay rate"
"sigme_VMe"           0.05    "width of excitation monitor potential"
"reset_Me"             0.5    "threshold for Monitor-induced reset"
"reorg_Me"            -0.5    "threshold for Monitor-induced reorganization"
"eps_Sx"              0.01    "supression value for syntactic systems"
"eps_Cx"              0.01    "degeneracy reset factor for concept systems"
```

Feedback parameters:

```
dp('feedback');
```

```
     name                  val                                  desc
____________    ________________________    ________________________________________________

"tau_Fx_dem"    {[0.5000 0.5000 0.5000]}    "feedback threshold for syntactic systems"
"tau_Sx_sel"    {[                   1]}    "threshold for syntactic system feedback growth"
```

Model equations

The model is implemented with a simple numerical method (first-order Euler). At each time step every dynamical variable is updated with an equation of the form:

$$x(t) = x(t-1) + \frac{\Delta x}{\Delta t} \cdot \delta t$$

with time indices $t = 1, 2, ..., (T/\delta t)$

For convenience in presenting the equations below, $\dot{x}$ is substituted for $\frac{\Delta x}{\Delta t}$, hence:

$$x(t) = x(t-1) + \dot{x}(t) \cdot \delta t$$

The specific update equations in the model are the following:

$\theta_i^C(t) = \theta_i^C(t-1) + \dot{\theta}_i^C \cdot \delta t$ phase for concept system i

$\theta_m^S(t) = \theta_m^S(t-1) + \dot{\theta}_m^S \cdot \delta t$ phase for syntactic system m

$x_i^C(t) = x_i^C(t-1) + \dot{x}_i^C \cdot \delta t$ activation for concept system i

$x_m^S(t) = x_m^S(t-1) + \dot{x}_m^S \cdot \delta t$ activation for syntactic system m

$x_m^F(t) = x_m^F(t-1) + \dot{x}_m^F \cdot \delta t$ feedback for syntactic system m

$E_l(t) = E_l(t-1) + \dot{E}_l \cdot \delta t$ value of excitation level l

$A(t) = A(t-1) + \dot{A} \cdot \delta t$ annealer state

$M(t) = M(t-1) + \dot{M} \cdot \delta t$ excitation monitor

For phase variables, 2π-periodicity is imposed by taking the updated value modulo 2π:

$$\theta_i^C(t) = \theta_i^C(t) \pmod{2\pi}$$

$$\theta_m^S(t) = \theta_m^S(t) \pmod{2\pi}$$

Note the following:

- Concept systems are indexed by i and j
- Syntactic systems are indexed by m and n. For convenience, the indices correspond to the order in which syntactic systems are selected.
- Excitation potential levels are indexed by l
- U is a random number from a uniform distribution on the interval (0,1)
- η parameters are the noise terms described above.

Concept system phase:

$$\dot{\theta}_i^C = \eta^{C\theta} \cdot U + \omega + \psi_{SC} \cdot \sum_m \left(-\sin(\phi_{mi}^{SC}) \cdot w_{mi}^{SC}\right) + \psi_{CC} \cdot \sum_j \left(-\sin(\phi_{ij}^{CC}) \cdot w_{ij}^{CC}\right)$$

where:

$\omega = 2\pi f$ is the intrinsic oscillator frequency in radians per second

$\phi_{mi}^{SC} = \theta_i^C - \theta_m^S$ is the phase difference between the concept system i and syntactic system m

w_{mi}^{SC} is the syntactic-conceptual system coupling matrix

$\phi_{ij}^{CC} = \theta_i^C - \theta_j^C$ is the phase difference between the concept system i and concept system j

w_{ij}^{CC} is the conceptual-conceptual system coupling matrix

Syntactic system phase:

$$\dot{\theta}_m^S = \eta^{S\theta} \cdot U + \omega + \psi_{SS} \cdot \sum_n \left(-\sin(\phi_{mn}^{SS}) \cdot w_{mn}^{SS}\right) + \psi_{CS} \cdot \sum_i \left(-\sin(\phi_{im}^{CS}) \cdot w_{im}^{CS}\right)$$

where:

$\phi_{mn}^{SS} = \theta_n^S - \theta_m^S$ is the phase difference between the syntactic system m and syntactic system n

w_{mn}^{SS} is the syntactic-syntactic system coupling matrix

$\phi_{im}^{CS} = \theta_m^S - \theta_i^C$ is the phase difference between the syntactic system m and concept system i

w_{im}^{CS} is the concept-syntactic system coupling matrix

Concept system activation:

$$\dot{x}_i^C = -\left(x_i^C - \left(\mathscr{E}_i + \eta^{Cx} \cdot U\right)\right)$$

where:

$\mathscr{E}_i$ is the driving force from the environment on concept system i

Syntactic system activation:

$$\dot{x}_m^S = \eta^{Sx} \cdot U + V_m^S + W\left(x_m^S\right) + \chi_{CS} \cdot \sum_i w_{im}^{CS} \cdot (F_m^{Sx})$$

where:

$V_m^E = \alpha_{Ve} \cdot \dfrac{A}{\delta t} \cdot \sum_l V(x_m^S, E_l)$ is the force on syntactic system m associated with excitation potential forces

α_{Ve} is the gain of the excitation potential forces

A is the current state of the annealing system

$V(x_m^S, E_l) = -(x_m^S - E_l) \cdot e^{-\frac{\left(x_m^S - E_l\right)^2}{2\sigma_{VE}^2}}$ is the excitation potential force between level l and syntactic system m.

$W\left(x_m^S\right) = \alpha_W \cdot \left(W^{\text{lower}}(x_m^S) + W^{\text{upper}}(x_m^S)\right)$ is the global activation potential

$W^{\text{lower}}(x_m^S) = \dfrac{1}{x_m^S - W_{lower}}$ is the lower bound component of the global activation potential

$W^{\text{upper}}(x_m^S) = \dfrac{-1}{W_{upper} - x_m^S}$ is the upper bound component of the global activation potential

$F_m^{Sx} = \begin{cases} 1, & \text{if } F_m^x < \tau_{dem} \\ 0, & \text{if } F_m^x \geq \tau_{dem} \end{cases}$ This factor enforces the condition that demoted syntactic systems are no longer coupled to concepts.

Syntactic system feedback:

$$\dot{F}^x_m = \begin{cases} 1, & \text{if } x^S_m > \tau_{sel} \\ 0, & \text{if } x^S_m \leq \tau_{sel} \end{cases}$$

This states that feedback for syntactic system m increases when system m is selected.

Excitation levels:

$\dot{E}_l = 0$ Excitation level values are constant in this version of the model, although note that the number of above-ground levels changes upon reorganization (see below).

Excitation monitoring system:

$$\dot{M} = M^{unocc} \cdot \mathscr{E}^{unocc} + M^{multiocc} \cdot \mathscr{E}^{multiocc} + M^{decay} \cdot (-1 - M)$$

where:

$\mathscr{E}^{multiocc} = \sum_{mn, m \neq n} x^S_m x^S_n \cdot V^M(x^S_m, x^S_n)$ is the cost associated with multiply-occupied levels, where:

$$V^M(x^S_m, x^S_m) = e^{\frac{-\left(x^S_m - x^S_n\right)}{2\sigma^2_{V^M}}}$$

$\mathscr{E}^{unocc} = \sum_l E_l \cdot \min_m \sqrt{(E_l - x^S_m)^2}$ is the cost associated with unoccupied levels

Annealing system:

$$\dot{A} = \begin{cases} \begin{cases} A^{init}, & \text{before any selection, or after degeneracy} \\ A^{reorg}, & \text{otherwise} \end{cases}, & A < 1 \\ 0, & A \geq 1 \end{cases}$$

This posits that the cooling rate is A^{init} before any systems have been selected and A^{reorg} after systems have been selected, and that the annealer state stops increases when $A \geq 1$.

Excitation operations:

After the above updates, excitation operations are applied, based on various conditions described below. Note that the following logical values are calculated each time step:

$sel_m = x^S_m > 1$: indicates whether syntactic system m is currently selected

$dem_m = sel_m \wedge F^x_m \geq \tau_{dem}$: flag syntactic system m for demotion if it is selected and if feedback for system m is greater than or equal to the demotion threshold

$sup_m = \neg sel_m \ \wedge \ F^x_m \geq \tau_{dem}$: indices whether syntactic m is currently supressed (i.e., it is not selected but is has feedback state $\geq \tau_{dem}$)

Condition: degeneracy before any selection

$M(t) > M^{reset} \ \wedge \ \forall_m \neg sup_m$:

$A \to 0$ the annealer is reset to 0

$M \to 0$ the excitation monitoring system is reset to 0

$x^S_m \to x^S_0 + \eta^{Sx0} \cdot U$ syntactic systems are reset to their initial values with a random perturbation

$C \to \epsilon_{Cx} \cdot C$ concept system values are diminished

Condition: degeneracy after any system has been selected and supressed

$M(t) > M^{reset} \ \wedge \ \exists_m \ sup_m$:

$A \to 0$ the annealer is reset to 0

$M \to 0$ the excitation monitoring system is reset to 0

$$x^S_m \to \begin{cases} \eta^{Sx0} \cdot U + \dfrac{m}{N_{unsel}}, & \neg sup_m \\ \epsilon_{Sx}, & sup_m \end{cases}$$

The above states that the activations of syntactic systems that have not been supressed are reset to evenly spaced values on the interval $\left[\frac{1}{N_{unsel}}, 1\right]$, where N_{unsel} is the number of systems that have not yet been selected, with a random perturbation $\eta^{Sx0} \cdot U$. The activations of syntactic systems that have been supressed are set to ϵ_{Sx}. Note that this formulation of the operation requires that syntactic systems be indexed in the order in which they are canonically selected.

$C \to \epsilon_{Cx} \cdot C$ concept system values are diminished by the factor ϵ_{Cx}

Condition: initial coherence

$M(t) < M^{reorg} \ \wedge \ \forall m \ \neg sel_m$: when the monitor reaches a coherence threshold and not any syntactic systems have yet been selected:

$A \to 0$ the annealer is reset to 0

$M \to 0$ the excitation monitoring system is reset to 0

$E \to r^C(E)$ the canonical reorganization operation is applied to the excitation potential levels. This operation eliminates the lowest non-ground level from the potential, reflecting the hypothesis that the potential itself emerges from the excitation of conceptual-syntactic systems.

$x_m^{Sx} \to \epsilon_{Sx}, \text{if } \ dem_m$ the activations of demoted syntactic systems are set to ϵ_{Sx}

Condition: within sequence coherence

$\exists m \ \ dem_m$: when a syntactic system reaches the demotion state

$A \to 0$ the annealer is reset to 0

$M \to 0$ the excitation monitoring system is reset to 0

$E \to r^C(E)$ the canonical reorganization operation is applied to the excitation potential levels. This operation eliminates the lowest non-ground level.

$x_m^{Sx} \to \epsilon_{Sx}, \text{if } \ dem_m$ the activations of demoted syntactic systems are set to ϵ_{Sx}

Examples

How to run the model and plot the results

```
sim = intermittency_model;
```

All of the dynamical variables and parameters are in the output structure (here named `sim`):

```
sim
```

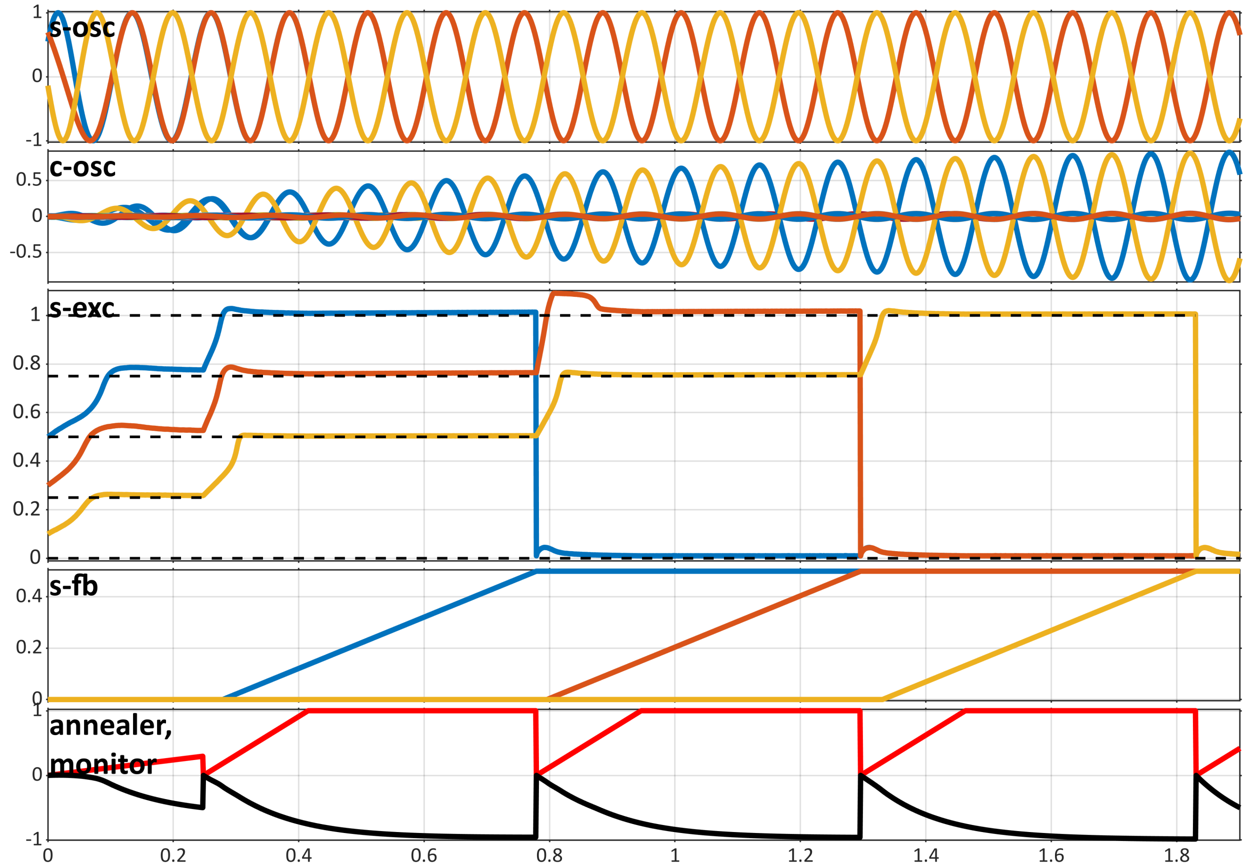
```
sim = struct with fields:
                    An: [5000×1 double]
                   C_C: [100×100 double]
                   C_S: [3×100 logical]
              C_S_prop: 0.3333
                    Cp: [5000×100 double]
                    Cx: [5000×100 double]
                 Cx_eq: [0.0402 0.0261 0.0222 0.0051 0.0040 0.0780 0.0630 1.0588 0.0166 0.0407 0.0527 0.0746 0.0963
                    EV: {6×2 cell}
                    Ex: [5000×5 double]
                    Fx: [5000×3 double]
                    Me: [5000×1 double]
              Me_decay: 10
      Me_multiocc_cost: 15
         Me_unocc_cost: 15
                 N_ixs: [1 2 3 4 6 7 8 9 10 11 12 13 14 15 16 17 18 20 21 22 24 25 26 27 28 30 31 32 33 34 35 36 39
                    Nc: 100
                    Ne: 5
                    Ns: 3
                     P: [42×4 table]
                PHI_CC: [100×100 double]
                PHI_CS: [100×3 double]
                PHI_SC: [3×100 double]
```

```
            PHI_SS: [3×3 double]
               S_S: [3×3 double]
                Sp: [5000×3 double]
                Sx: [5000×3 double]
               Sx0: [0.5000 0.3000 0.1000]
                 T: 5
              VE_S: [-6.4267 -6.4247 -6.4822]
               V_E: @(x,c)-(x-c).*(exp(-((x-c).^2)./(2*sigma_VE^2)))
              V_Me: @(x)exp(-(x.^2)./(2*sigme_VMe^2))
             V_ixs: [5 19 23 29 37 38 47 48 56 65 72 78 79 80 91 96 97 98 100]
            c_coup: [0 0 0]
            chi_CC: 0
            chi_CS: 4
            chi_SC: 0
            chi_SS: 0
               dAn: 6
               dCp: [50.3289 50.3471 50.3613 50.2713 50.3335 50.3344 50.3443 50.2921 50.3250 50.3442 50.3564 50.
               dCx: [-0.0018 -0.0163 -0.0202 -0.0370 -0.0401 0.0345 0.0215 0.1648 -0.0260 -0.0027 0.0100 0.0321
               dEx: 0
               dFx: [0 0 0]
               dMe: -5.0095
               dSp: [50.3339 50.3640 50.2763]
               dSx: [-0.2360 -0.0580 -0.4801]
              dgex: [0 NaN NaN NaN NaN]
              dgsx: [3×3 double]
                dt: 1.0000e-03
         env_num_N: 2
         env_num_V: 1
            eps_Cx: 0.0100
            eps_Sx: 0.0100
            eta_Cp: 0.1000
            eta_Cx: 0.1000
            eta_Sp: 0.1000
            eta_Sx: 0.5000
           eta_Sx0: 0.1000
                 f: 8
           gain_Ai: 1.2000
           gain_Ar: 6
           gain_VE: 1
           gain_gw: 0.1000
                gw: [6.0130 6.0152 5.9545]
             gw_lb: 0
             gw_ub: 1.1000
                 i: 1901
         init_eorg: 0
            ix_dem: [0 0 0]
      ix_diff_sign: [100×100 logical]
          ix_env_C: [0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
            ix_sel: [0 0 0]
            ix_sup: [1 1 1]
                nT: 5000
             omega: 50.2655
            psi_CC: 0
            psi_CS: 0.0500
            psi_SC: 10
            psi_SS: 10
            rN_ixs: [64 8]
            rV_ixs: 80
          reorg_Me: -0.5000
          reset_Me: 0.5000
          sigma_VE: 0.0500
         sigme_VMe: 0.0500
stop_on_completion: 1.0000e-03
                 t: [0 1.0000e-03 0.0020 0.0030 0.0040 0.0050 0.0060 0.0070 0.0080 0.0090 0.0100 0.0110 0.0120 0
```

```
         tau_Fx_dem: [0.5000 0.5000 0.5000]
         tau_Sx_sel: 1
               wC_S: [100×3 double]
               wS_C: [3×100 double]
```

Passing the output structure to the function `plot_sim` will plot some of the variables. Other variables can be plotted by accessing the output structure above.

```
plot_sim(sim)
```



Changing parameters

Recall that the default parameters are specified in:

```
P = model_default_params;
head(P)
```

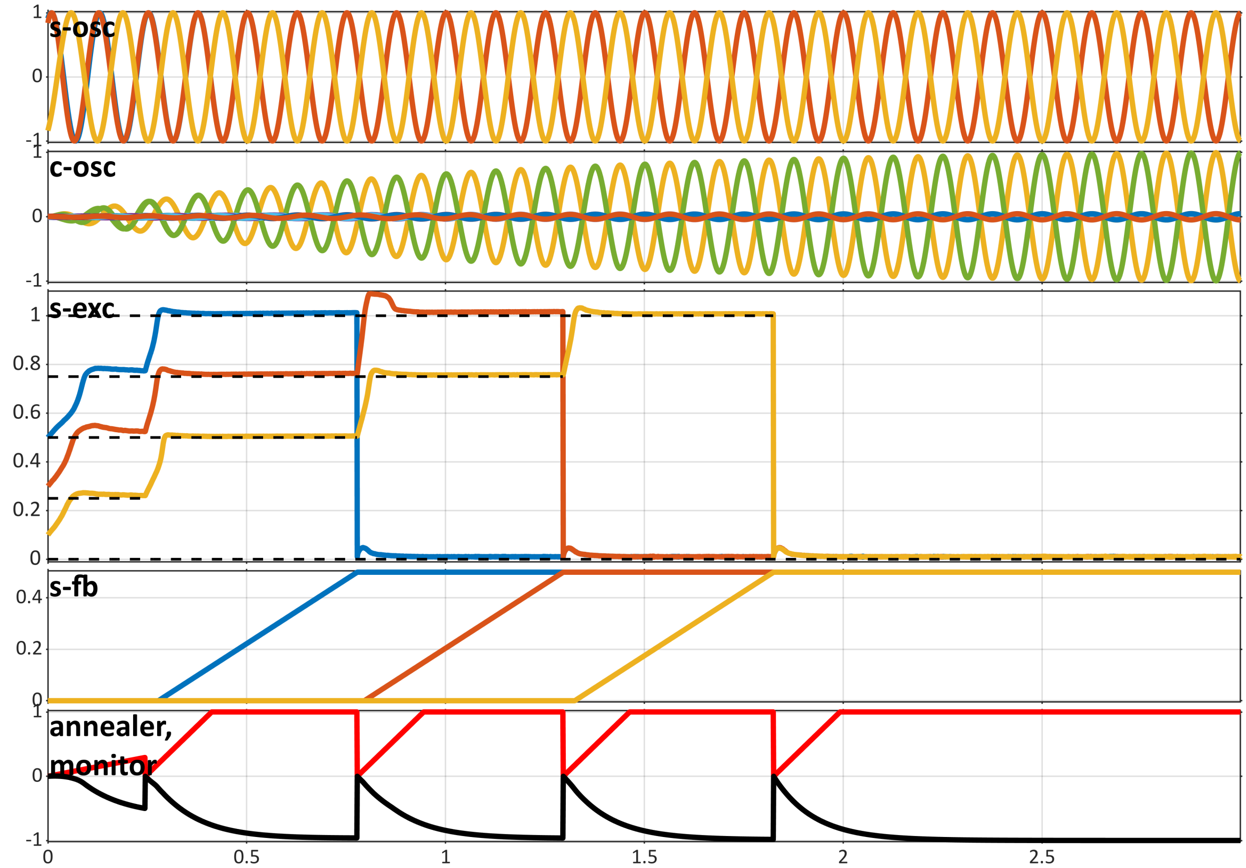
name	val	type	desc
{'T' }	{[5]}	{'simulation'}	{'maximal run time'
{'stop_on_completion'}	{[1.0000e-03]}	{'simulation'}	{'stop if the sequence is completed'
{'dt' }	{[1.0000e-03]}	{'simulation'}	{'time step'
{'Nc' }	{[100]}	{'systems' }	{'number of c-systems'
{'Ns' }	{[3]}	{'systems' }	{'number of s-systems'
{'Sx0' }	{[0.5000 0.3000 0.1000]}	{'systems' }	{'initial syntactic system activation'

```
{'S_S'              }    {3×3 double            }    {'systems'   }    {'s-to-s coupling matrix'
{'C_S_prop'         }    {[              0.3333]}    {'systems'   }    {'proportion of systems coupled to verba
```

All of the parameters can be overwritten by providing them as fields of a single input structure. For example, if we wanted to (i) prevent the simulation from stopping upon completion of the sequence, (ii) decrease the maximal run time to 3 s, and (iii) increase the noise amplitude for syntactic system activation, we would do the following:

```
par.stop_on_completion = false;   %(i)
par.T = 3;                        %(ii)
par.eta_Sx = 1;                   %(iii)

sim = intermittency_model(par);
plot_sim(sim)
```



Helper functions for this livescript:

```
function [P] = disp_pars(P,ptype)
P = P(ismember(P.type,ptype),{'name' 'val' 'desc'});
P.name = string(P.name);
P.desc = string(P.desc);
try
   P.val=cell2mat(P.val);
catch
```

```
end
disp(P);
end
```