HRDD Risk Assessment Tool

Mathematical Formulation & Implementation Guide

Enhanced Focus-Based Risk Allocation with Budget Optimization
Version 7.0 - December 2024

> 🔧 **Enhanced with Budget Optimization**
>
> This version includes both rank preservation mechanisms and a sophisticated multi-algorithm budget optimization system that maximizes risk reduction per dollar spent while maintaining total budget constraints.

1) Overview

This document provides the complete mathematical formulation for a coverage-based Human Rights Due Diligence (HRDD) risk assessment system with enhanced rank preservation mechanisms and budget optimization. The methodology incorporates country-specific coverage distribution, focus-based targeting, real-time effectiveness tracking, and cost-aware optimization to maximize risk reduction within budget constraints across multinational supply chains.

2) Portfolio Risk Foundation

1 Volume-Weighted Baseline Risk

$$B = (\Sigma_i \; w_i \cdot b_i) \; / \; \Sigma_i \; w_i$$

Where:

- w_i = volume weight for country i
- b_i = baseline risk score for country i (0-100)
- B = portfolio baseline risk

2 Risk Concentration Factor

$$C = (\Sigma_i \; (w_i/W) \cdot b_i^2) \; / \; B^2$$

Properties:

- $C \geq 1$ always (mathematical constraint)
- $C = 1$ when all countries have identical risk
- $C > 1$ indicates risk concentration in specific countries
- Higher C values amplify focus effectiveness

3) Enhanced Focus-Based Coverage Distribution

Coverage is distributed individually to each country based on their risk profile, focus settings, and resource conservation constraints, with new rank preservation mechanisms.

1 Modified Focus Exponent Calculation

> **MODIFICATION 1: Reduced Maximum Exponent**
>
> Maximum focus exponent reduced from 3.5 to 2.0 to prevent extreme biasing that could cause rank reversals.

$$\varphi = \begin{cases} 1 + f \cdot (\varphi_{max} - 1) \cdot (f/0.5) & \text{if } f \leq 0.5 \\ 1 + (\varphi_{max} - 1) \cdot [0.5 + (f-0.5)^{1.8} \cdot 0.5] & \text{if } f > 0.5 \end{cases}$$ Where: $\varphi_{max} = 2.0$ (FURTHER REDUCED from 2.8)

Characteristics:

- Linear growth up to 50% focus threshold
- Accelerated exponential growth beyond 50% (exponent 1.8)
- **Modified:** Maximum exponent of 2.0 for controlled targeting

2 Enhanced Biased Risk Ratio with Dual Compression

> **MODIFICATION 2: Enhanced Compression System**
>
> Added secondary compression layer for extreme cases and reduced maximum ratio from 5.0 to 2.5.

$$r_i' = \max(r_{min}, \min(r_{max}, (b_i/B)^{\varphi})) \text{ If } f > 0.6 \text{ AND } b_i/B < 0.8: r_i' = r_i' \cdot [0.3 + 0.7 \cdot (b_i/B)^2] \text{ NEW: If } f > 0.7 \text{ AND } r_i' > 1.5: r_i' = r_i' \cdot [0.5 + 0.5 \cdot (1.5/r_i')^{0.5}]$$

Further Reduced Parameters:

Parameter	Original	First Revision	Current Value	Impact

r_{min}	0.05	0.05	0.08	Reduced low-risk penalties
r_{max}	5.0	4.0	2.5	Significantly reduced maximum benefits
Secondary Compression Threshold	None	2.0	1.5	Earlier compression activation
Compression Strength	None	0.6 + 0.4×√	0.5 + 0.5×√	Stronger compression factor

3 Modified Country-Specific Coverage Allocation

MODIFICATION 3: Gradual High-Risk Boost

Replaced sharp threshold (risk ≥ 60) with gradual curve starting at risk ≥ 40, and reduced maximum boost from 80% to 30%.

$$c_{ij} = c_j \cdot [(1-f) + f \cdot r_i'] \cdot \eta_i \quad \text{MODIFIED: If } f > 0.3 \text{ AND } b_i \geq 40: \text{riskNormalized} = \min(1, (b_i - 40) / 40) \quad \text{focusNormalized} = \min(1, (f - 0.3) / 0.7) \quad \text{boostFactor} = 1 + (\text{riskNormalized} \times \text{focusNormalized} \times 0.3) \quad c_{ij} = c_{ij} \cdot \text{boostFactor}$$

Gradual Boost Characteristics:

- `Risk 40-80`: Normalized to 0-1 scale
- `Focus 0.3-1.0`: Normalized to 0-1 scale
- `Maximum boost`: 30% (reduced from 80%)
- Prevents sudden jumps in coverage allocation

Resource Conservation

The system maintains total resource usage close to original levels while allowing up to 30% expansion for high-focus scenarios to accommodate targeting benefits. Conservation factors (η_i) now apply the gradual boost system as well.

4) Rank Preservation Mechanisms

> **NEW SECTION: Critical Rank Preservation Features**
>
> These mechanisms ensure that risk rankings remain logically consistent after strategy application.

1 Progressive Effectiveness Caps

$$\text{For each country } i:$$

$$\text{CapEffectiveness}_i = 0.50 + 0.20 \times (1 - b_i/100)$$

Strengthened Progressive Cap System:

Country Risk Level	Original Max Reduction	Current Max Reduction	Minimum Managed Risk
Very High Risk (80+)	65%	50%	40+ points
High Risk (60-79)	69%	55%	27-36 points
Medium Risk (40-59)	73%	60%	16-24 points
Low Risk (20-39)	81%	70%	6-12 points

2 Enhanced Risk Floor Preservation

$$\text{riskFloor}_i = b_i \times 0.25 \text{ (INCREASED from 0.12)}$$

$$m_i^{final} = \max(m_i^{calculated}, \text{riskFloor}_i)$$

Strengthened Floor Protection:

- No country can have managed risk below 25% of baseline (increased from 12%)
- Prevents unrealistic low-risk outcomes
- Maintains stronger proportional relationships between countries

3 NEW: Direct Rank Preservation Constraint

> After calculating all managed risks: Sort countries by baseline risk: $b_1 \geq b_2 \geq \ldots \geq b_n$ For each country i (from highest to lowest baseline risk): If $m_i \geq m_{i+1}$: $m_i = \max(b_i \times$

> 0.25, $m_{i+1} - 0.5$) Recalculate portfolio: $M = (\Sigma_i w_i \cdot m_i^{corrected}) / \Sigma_i w_i$

Mathematical Guarantee:

- Countries are sorted by baseline risk in descending order
- Each country's managed risk must be at least 0.5 points lower than the previous
- Still respects the 25% risk floor constraint
- Provides absolute guarantee against rank inversions
- Portfolio risk is recalculated using corrected values

5) Budget Optimization System (Panel 6)

> 🚀 **Advanced Multi-Algorithm Optimization**
>
> The budget optimization system employs a sophisticated multi-strategy approach combining three algorithms to find the optimal allocation of HRDD tools and response methods that maximizes risk reduction within budget constraints.

1 Cost Model Formulation

```
Total Cost = Σⱼ (External Costsⱼ + Internal Costsⱼ) For each tool j in Panel 3: External Costsⱼ = Annual Programmeⱼ × (cⱼ/100) + Per Supplierⱼ × Sⱼ Internal Costsⱼ = Internal Hoursⱼ × Sⱼ × Hourly Rate Where: cⱼ = Coverage percentage for tool j (0-100) Sⱼ = Number of suppliers using tool j = ⌈Total Suppliers × (cⱼ/100)⌉
```

Cost Components:

Component	Description	Unit	Typical Range
Annual Programme Cost	Fixed annual cost for tool infrastructure	USD/year	$0 - $50,000
Per Supplier Cost	Variable cost per supplier using the tool	USD/supplier/year	$0 - $2,000
Internal Hours	Staff time required per supplier	hours/supplier/year	0 - 500
Hourly Rate	Internal staff cost rate	USD/hour	$20 - $200

2 Optimization Problem Definition

```
Minimize: M (Managed Risk) Subject to: 1. Total Cost ≤ Target Budget × (1 + tolerance) 2. 0 ≤ cⱼ ≤ 100 for all tools j 3. 0 ≤ rₖ ≤ 100 for all response methods k 4. c_o = r_o (Voice linkage constraint) 5. Improvement ≥ 0.1 percentage points Where: tolerance = 0.02 (2% budget flexibility)
```

3 Multi-Strategy Optimization Algorithm

The optimizer employs a sophisticated three-phase approach with multiple restart strategies:

Phase 1: Enhanced Simulated Annealing

> Temperature Schedule: T(i) = T_0 × 0.95^i Acceptance Probability: P(ΔE) = exp(-ΔE/T) if ΔE > 0, else 1 Neighbor Generation: - Step size: [-6, +6] for standard tools - Voice bonus: +3 additional for voice-priority strategy - Budget pre-check before expensive evaluation

Phase 2: Budget-Constrained Genetic Algorithm

> Population Size: 20 individuals Selection: Tournament selection preferring valid solutions Crossover: α-blending with α ∈ [0.3, 0.9] Mutation: Gaussian noise with σ = 6 Elitism: Top 3 valid solutions preserved Budget Enforcement: After each generation, adjust allocations to meet budget

Phase 3: Local Search Refinement

> Search Strategy: Coordinate descent with adaptive step size Step Size: max(3, 10 × (1 - iteration/max_iterations)) For each dimension: - Try ±step_size changes - Accept if improves fitness and maintains budget - Terminate at local optimum

4 Optimization Strategies

The system employs four distinct optimization strategies in rotation:

Strategy	Priority	Budget Adjustment Method	Focus
Balanced	Equal reduction across tools	Proportional scaling	Overall effectiveness
Voice Priority	Preserve worker voice tools	Reduce non-voice first	Worker engagement
Efficiency Focused	Maximize effectiveness/cost ratio	Cut inefficient tools first	Cost-effectiveness
Preserve Voice	Maintain voice coverage	Adjust other tools only	Worker voice protection

5 Budget Adjustment Algorithm

> adjustToBudget(allocation, target_budget): While |current_cost - target_budget| > tolerance: If current_cost > target_budget: // Reduce allocation strategically If strategy == 'preserve_voice': Reduce non-voice tools by 8% Else if strategy == 'efficiency_focused': Reduce least efficient tool by 10% Else: Reduce highest allocation by 5-8% Else: // Increase allocation to use budget If voice < 85%: Increase voice by 6-8% Else: Increase most effective tool by 6%

6 Performance Metrics

```
Fitness Function: F = M + λ × |Cost - Budget|/Budget Where: M = Managed risk after allocation λ = 2 (penalty weight for budget violation) Success Criteria: - Risk reduction improvement ≥ 0.1 percentage points - Budget violation ≤ 2% - At least 12 valid solutions found
```

7 Convergence and Termination

The optimization terminates when:

- Maximum time reached (4.5 seconds)
- Sufficient valid solutions found (≥12) with good fitness
- 5 restart attempts completed
- No improvement possible (re-optimization returns same solution)

Re-optimization Handling

The system caches optimization results using a state hash. If re-optimization is attempted with identical parameters, the system checks if the new solution meets improvement criteria:

- **Criteria A:** Lower budget with same or better risk reduction
- **Criteria B:** At least 0.1 percentage point better risk reduction within budget

If neither criterion is met, the previous solution is retained as optimal.

6) Country-Specific Transparency Calculation

1 Per-Country Category Transparency

$$\text{For each country } i \text{ and category } k:$$

$$T_{ik} = 1 - \prod_{j \in J_k} (1 - (c_{ij}/100) \cdot (e_j^{base} + e_j^{user})/2)$$

Tool Categories:

Category	Tools	Base Effectiveness	Category Weight (α_k)
Worker Voice	Continuous (j=0), Surveys (j=1)	90%, 45%	1.0
Audit	Unannounced (j=2), Announced (j=3)	25%, 15%	0.85
Passive	Evidenced Self-Report (j=4), Unevidenced Self-Report (j=5)	12%, 5%	0.70

2 Combined Country Transparency

$$T_i = \min(1 - \prod_{k=1}^{3} (1 - T_{ik} \cdot \alpha_k), 0.90)$$

3 Portfolio-Wide Transparency

$$T = (\Sigma_i w_i \cdot T_i) / \Sigma_i w_i$$

This represents the volume-weighted average transparency across all countries in the portfolio, accounting for their individual focus-adjusted coverage allocations.

7) Focus Multiplier System

1 Portfolio Focus Multiplier

$$F_o = (1 - f \cdot \gamma) + f \cdot \gamma \cdot C$$

$$\text{Where: } \gamma = 2.75 \text{ (focus concentration weight)}$$

2 Country-Specific Focus Multipliers

> For each country i: If $f > 0.6$ AND $b_i \geq 70$: $F_i = F_o \cdot [1 + (f-0.6) \cdot 0.5]$ Else: $F_i = (1 - f \cdot \gamma) + f \cdot \gamma \cdot r_i'$

Ultra-High-Risk Bonus: Countries with baseline risk ≥70 and focus >60% receive additional multiplier benefits up to 20% extra effectiveness.

8) Response Effectiveness

$$R = (\Sigma_j \, r_j \cdot \rho_j) \, / \, \Sigma_j \, r_j$$

Where:

- r_j = response strategy weight for method j
- ρ_j = response effectiveness rate for method j (0-100%)

9) Final Managed Risk Calculation with Rank Preservation

1 Enhanced Individual Country Managed Risk

> **MODIFICATION 4: Multi-Layer Protection System**
>
> Managed risk calculation now includes progressive caps and floor protection to prevent rank reversals.

```
Step 1: Calculate base reduction factor baseReduction_i = T_i · R · F_i Step 2: Apply progressive effectiveness cap cap_i = 0.50 + 0.20 × (1 - b_i/100) cappedReduction_i = min(baseReduction_i, cap_i) Step 3: Calculate managed risk m_i^calc = b_i · (1 - cappedReduction_i) Step 4: Apply risk floor protection floor_i = b_i × 0.25 m_i = max(m_i^calc, floor_i)
```

2 Portfolio Managed Risk

$$M = (\Sigma_i \, w_i \cdot m_i) \, / \, \Sigma_i \, w_i$$

10) Focus Effectiveness Metrics

The system provides comprehensive metrics to evaluate how effectively the focus strategy is targeting high-risk countries while preserving ranking.

Risk Tier Classification

- High Risk : $b_i \geq 60$
- Medium Risk : $40 \leq b_i < 60$
- Low Risk : $b_i < 40$

Enhanced Effectiveness Measures

```
For each risk tier: ē_tier = (Σ_i∈tier (b_i - m_i)/b_i · 100) / |tier| Differential Benefit = ē_high - ē_low Focus Effectiveness = min(100, (Differential Benefit)/(f · 50) · 100) NEW: Rank Preservation Check = (Countries maintaining relative ranking / Total countries) × 100
```

Enhanced Performance Indicators:

- **Focus Effectiveness ≥70%:** Excellent targeting with preserved ranking
- **Focus Effectiveness 40-70%:** Moderate targeting with good preservation
- **Focus Effectiveness <40%:** Poor targeting or rank violations detected

- **Rank Preservation ≥95%:** Successful rank maintenance

11) Implementation Parameters

Enhanced Implementation Parameters

Parameter	Original	First Revision	Current Value	Description
φ_{min}	1.0	1.0	1.0	Minimum focus exponent (unchanged)
φ_{max}	3.5	2.8	2.0	Maximum focus exponent (further reduced)
r_{min}	0.05	0.05	0.08	Minimum risk ratio (increased)
r_{max}	5.0	4.0	2.5	Maximum risk ratio (further reduced)
High-risk boost threshold	60	40	40	Gradual boost starting point (unchanged)
Maximum boost percentage	80%	30%	30%	Maximum coverage boost (unchanged)
Risk floor	None	12%	25%	Minimum managed risk retention (increased)
Progressive cap range	None	65%-85%	50%-70%	Maximum reduction limits (strengthened)
Direct rank constraint	None	None	0.5 pt separation	Mathematical ranking guarantee (new)
Budget tolerance	N/A	5%	2%	Optimization budget flexibility
Min improvement	N/A	N/A	0.1 pp	Minimum risk reduction improvement

Default Coverage Values

- **Continuous Worker Voice:** 35% (rare, expensive)
- **Worker Surveys:** 15% (moderate deployment)
- **Unannounced Audits:** 25% (targeted approach)
- **Announced Audits:** 60% (common practice)
- **Evidenced Supplier Self-Reporting:** 80% (widespread)
- **Unsupported Supplier Self-Reporting:** 90% (universal baseline)

Enhanced Focus Strategy Guidelines

Focus Level	Strategy Type	Rank Preservation	Recommended Use
f = 0.1 - 0.3	Legacy audit programs	Excellent	Calendar-driven, even spread
f = 0.4 - 0.6	Risk-led programs	Very Good	Periodic risk-based allocation
f = 0.7 - 0.9	Continuous voice + risk triage	Good	Highly targeted with preservation
f = 0.9 - 1.0	Crisis response mode	Monitored	Emergency surge with safeguards

12) Budget Optimization Example

Initial Setup

Portfolio with 500 suppliers, $40/hour internal rate, initial budget of $500,000:

Step 1: Calculate Current Cost

```
Current allocation: [35%, 15%, 25%, 60%, 80%, 90%] (Panel 3 tools)
Current response: [35%, 5%, 25%, 25%, 10%, 10%] (Panel 4 methods) Tool
costs: - Worker Voice: $12,000 × 0.35 + $120 × 175 suppliers = $25,200
- Surveys: $0 × 0.15 + $0 × 75 suppliers + 20hrs × 75 × $40 = $60,000
- Unannounced Audits: $0 × 0.25 + $1000 × 125 = $125,000 ... Total
current budget: $486,000 Current risk reduction: 42.3%
```

Step 2: Run Multi-Strategy Optimization

```
Strategy 1 - Balanced: Phase 1 (Simulated Annealing): 200 iterations
Phase 2 (Genetic Algorithm): 120 generations Phase 3 (Local Search):
100 iterations Result: 48.1% risk reduction at $488,000 Strategy 2 -
Voice Priority: [Same phases] Result: 49.2% risk reduction at $492,000
Strategy 3 - Efficiency Focused: [Same phases] Result: 50.3% risk
reduction at $485,000 ← BEST Strategy 4 - Preserve Voice: [Same
phases] Result: 47.8% risk reduction at $490,000
```

Step 3: Optimized Allocation

```
Optimized tool allocation: [52%, 8%, 45%, 35%, 65%, 72%] Optimized
response: [52%, 12%, 35%, 18%, 8%, 5%] Key changes: - Worker Voice:
+17% (leverage high effectiveness) - Unannounced Audits: +20% (cost-
effective detection) - Announced Audits: -25% (reduce low-
effectiveness spend) - Response reallocation to match detection
increases Total optimized budget: $485,000 (-$1,000) Optimized risk
reduction: 50.3% (+8.0 percentage points)
```

Step 4: Validation

```
✓ Budget constraint met: $485,000 ≤ $500,000 × 1.02 ✓ Minimum
improvement met: 8.0% > 0.1% ✓ All allocations within bounds: 0% ≤ all
≤ 100% ✓ Voice linkage maintained: c_o = r_o = 52% ✓ Valid solutions
found: 127 > 12 Result: OPTIMIZATION SUCCESSFUL Improvement: 8.0
percentage points better risk reduction Budget savings: $1,000
Efficiency gain: 19% more risk reduction per dollar
```

Document Version: *7.0 Enhanced with Budget Optimization* | ***Last Updated:*** *December 2024* |
Implementation: *HRDD Risk Assessment Tool with Rank Preservation and Cost Optimization*

Technical Notes: *All calculations maintain numerical stability through bounded parameters, progressive safeguards, and multi-strategy optimization. The budget optimizer employs sophisticated algorithms to*

maximize risk reduction within financial constraints while preserving logical risk rankings. The system prevents rank reversals while finding cost-effective allocations through intelligent multi-phase optimization.