A.2 Trigonometrische Funktionen

Quadrant	I	II	III	IV
$\sin x$	+	+	–	–
$\cos x$	+	–	–	+
$\tan x$	+	–	+	–

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
α	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	–	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	–	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

Verschiebung des Sinus	Verschiebung des Kosinus
$\sin\left(x+\frac{\pi}{2}\right) = \cos x$	$\cos\left(x+\frac{\pi}{2}\right) = -\sin x$
$\sin\left(x-\frac{\pi}{2}\right) = -\cos x$	$\cos\left(x-\frac{\pi}{2}\right) = \sin x$
$\sin\left(\frac{\pi}{2}-x\right) = \cos x$	$\cos\left(\frac{\pi}{2}-x\right) = \sin x$
$\sin\left(-\frac{\pi}{2}-x\right) = -\cos x$	$\cos\left(-\frac{\pi}{2}-x\right) = -\sin x$
$\sin(x \pm \pi) = -\sin x$	$\cos(x \pm \pi) = -\cos x$
$\sin(\pm\pi - x) = \sin x$	$\cos(\pm\pi - x) = -\cos x$

Additionstheoreme	Doppelwinkelformeln
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\sin(2x) = 2\sin x \cos x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\cos(2x) = \cos^2 x - \sin^2 x$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\tan(2x) = \dfrac{2\tan x}{1-\tan^2 x}$
$\cot(x \pm y) = \dfrac{\cot x \cot y \mp 1}{\cot y \pm \cot x}$	$\cot(2x) = \dfrac{\cot^2 x - 1}{2\cot x}$

A.3 Ableitungen

Funktion $f(x)$	Ableitung $f'(x)$	Funktion $f(x)$	Ableitung $f'(x)$
$\frac{1}{x}$	$-\frac{1}{x^2}$	e^x	e^x
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	$a^x \quad (a>0)$	$(\ln a)\, a^x$
$x^a \quad (a \in \mathbb{R})$	$a\, x^{a-1}$	$\ln x$	$\frac{1}{x}$
		$\log_a x \quad (a>0, a\neq 1)$	$\frac{1}{(\ln a)\, x}$
$\sin x$	$\cos x$	$\sinh x$	$\cosh x$
$\cos x$	$-\sin x$	$\cosh x$	$\sinh x$
$\tan x$	$1+\tan^2 x$	$\tanh x$	$1-\tanh^2 x$
$\cot x$	$-1-\cot^2 x$	$\coth x$	$1-\coth^2 x$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	$\operatorname{arsinh} x$	$\frac{1}{\sqrt{1+x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$	$\operatorname{arcosh} x$	$\frac{1}{\sqrt{x^2-1}}$
$\arctan x$	$\frac{1}{1+x^2}$	$\operatorname{artanh} x$	$\frac{1}{1-x^2}$
$\operatorname{arccot} x$	$-\frac{1}{1+x^2}$	$\operatorname{arcoth} x$	$-\frac{1}{x^2-1}$

A.4 Ableitungsregeln

Regel	Formel
Faktorregel	$(C\, f(x))' = C\, f'(x)$
Summenregel	$(f(x) \pm g(x))' = f'(x) \pm g'(x)$
Produktregel	$(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
Quotientenregel	$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g(x)^2}$
Kettenregel	$(f(u(x)))' = f'(u(x)) \cdot u'(x)$
Umkehrfunktion	$\left(f^{-1}(y)\right)' = \frac{1}{f'(x)}$

A.5 Integrale

Funktion	Stammfunktion (ohne Konstante C)	Funktion	Stammfunktion (ohne Konstante C)
$\int \frac{1}{x}\,\mathrm{d}x$	$= \ln\lvert x\rvert$	$\int \mathrm{e}^x\,\mathrm{d}x$	$= \mathrm{e}^x$
$\int \sqrt{x}\,\mathrm{d}x$	$= \frac{2}{3}x\sqrt{x}$	$\int a^x\,\mathrm{d}x$	$= \frac{1}{\ln a}a^x \quad (a>0)$
$\int x^a\,\mathrm{d}x$	$= \frac{1}{a+1}x^{a+1} \quad (a\neq -1)$	$\int \ln x\,\mathrm{d}x$	$= x(\ln x - 1)$
		$\int \log_a x\,\mathrm{d}x$	$= x(\log_a x - \log_a \mathrm{e}) \quad (a>0, a\neq 1)$
$\int \sin x\,\mathrm{d}x$	$= -\cos x$	$\int \sinh x\,\mathrm{d}x$	$= \cosh x$
$\int \cos x\,\mathrm{d}x$	$= \sin x$	$\int \cosh x\,\mathrm{d}x$	$= \sinh x$
$\int \tan x\,\mathrm{d}x$	$= -\ln\lvert\cos x\rvert$	$\int \tanh x\,\mathrm{d}x$	$= \ln\lvert\cosh x\rvert$
$\int \cot x\,\mathrm{d}x$	$= \ln\lvert\sin x\rvert$	$\int \coth x\,\mathrm{d}x$	$= \ln\lvert\sinh x\rvert$
$\int \arcsin x\,\mathrm{d}x$	$= x\arcsin x + \sqrt{1-x^2}$	$\int \operatorname{arsinh} x\,\mathrm{d}x$	$= x\operatorname{arsinh} x - \sqrt{x^2+1}$
$\int \arccos x\,\mathrm{d}x$	$= x\arccos x - \sqrt{1-x^2}$	$\int \operatorname{arcosh} x\,\mathrm{d}x$	$= x\operatorname{arcosh} x - \sqrt{x^2-1}$
$\int \arctan x\,\mathrm{d}x$	$= x\arctan x - \frac{\ln(1+x^2)}{2}$	$\int \operatorname{artanh} x\,\mathrm{d}x$	$= x\operatorname{artanh} x + \frac{\ln(1-x^2)}{2}$
$\int \operatorname{arccot} x\,\mathrm{d}x$	$= x\operatorname{arccot} x + \frac{\ln(1+x^2)}{2}$	$\int \operatorname{arcoth} x\,\mathrm{d}x$	$= x\operatorname{arcoth} x + \frac{\ln(x^2-1)}{2}$
$\int \frac{1}{x-a}\,\mathrm{d}x$	$= \ln\lvert x-a\rvert$	$\int \frac{1}{x^2+a^2}\,\mathrm{d}x$	$= \frac{1}{a}\arctan\frac{x}{a} \quad (a\neq 0)$
$\int \frac{1}{(x-a)^n}\,\mathrm{d}x$	$= -\frac{1}{(n-1)(x-a)^{n-1}} \quad (n\neq 1)$	$\int \frac{2ax+b}{ax^2+bx+c}\,\mathrm{d}x$	$= \ln\lvert ax^2+bx+c\rvert \quad (a\neq 0)$
$\int x\,\mathrm{e}^{ax}\,\mathrm{d}x$	$= \frac{ax-1}{a^2}\mathrm{e}^{ax}$	$\int x^2\,\mathrm{e}^{ax}\,\mathrm{d}x$	$= \frac{a^2x^2-2ax+2}{a^3}\mathrm{e}^{ax}$
$\int x\sin ax\,\mathrm{d}x$	$= \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax$	$\int x\cos ax\,\mathrm{d}x$	$= \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$
$\int x^2\sin ax\,\mathrm{d}x$	$= \frac{2x}{a^2}\sin ax - \frac{a^2x^2-2}{a^3}\cos ax$	$\int x^2\cos ax\,\mathrm{d}x$	$= \frac{2x}{a^2}\cos ax + \frac{a^2x^2-2}{a^3}\sin ax$
$\int \mathrm{e}^{ax}\sin bx\,\mathrm{d}x$	$= \frac{\mathrm{e}^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$	$\int \mathrm{e}^{ax}\cos bx\,\mathrm{d}x$	$= \frac{\mathrm{e}^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$
$\int \sin^2 x\,\mathrm{d}x$	$= \frac{x}{2} - \frac{1}{2}\sin x\cos x$	$\int \cos^2 x\,\mathrm{d}x$	$= \frac{x}{2} + \frac{1}{2}\sin x\cos x$

A.6 Integralregeln

Regel	Formel
Faktorregel	$\int C\, f(x)\,\mathrm{d}x = C \int f(x)\,\mathrm{d}x$
Summenregel	$\int f(x) \pm g(x)\,\mathrm{d}x = \int f(x)\,\mathrm{d}x \pm \int g(x)\,\mathrm{d}x$
Substitution	$\int f(u(x)) \cdot u'(x)\,\mathrm{d}x = \int f(u)\,\mathrm{d}u$
	$\int f(x) \cdot f'(x)\,\mathrm{d}x = \frac{1}{2} f^2(x)$
	$\int \frac{f'(x)}{f(x)}\,\mathrm{d}x = \ln \lvert f(x) \rvert$
Partielle Integration	$\int f(x) \cdot g'(x)\mathrm{d}x = f(x) \cdot g(x) - \int f'(x) \cdot g(x)\,\mathrm{d}x$
Vertauschen	$\int_a^b f(x)\,\mathrm{d}x = -\int_b^a f(x)\,\mathrm{d}x$
Integrationsbereich	$\int_a^b f(x)\,\mathrm{d}x = \int_a^c f(x)\,\mathrm{d}x + \int_c^b f(x)\,\mathrm{d}x$
Hauptsatz I	$\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_a^t f(x)\,\mathrm{d}x\right)' = f(t)$
Hauptsatz II	$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$

A.7 Potenzreihen

Funktion	Potenzreihe	Konvergenzradius
$\frac{1}{1-x}$	$= \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + \ldots$	1
e^x	$= \sum_{k=0}^{\infty} \frac{1}{k!} x^k = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \ldots$	∞
$\ln(1+x)$	$= \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} x^k = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \pm \ldots$	1
$\sin x$	$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \pm \ldots$	∞
$\cos x$	$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k} = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} \pm \ldots$	∞
$\arctan x$	$= \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} x^{2k+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} \pm \ldots$	1
$\sinh x$	$= \sum_{k=0}^{\infty} \frac{1}{(2k+1)!} x^{2k+1} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ldots$	∞
$\cosh x$	$= \sum_{k=0}^{\infty} \frac{1}{(2k)!} x^{2k} = 1 + \frac{x^2}{2} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots$	∞

A.8 Fourier-Reihen

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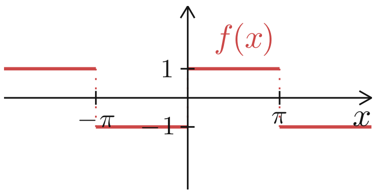

$$f(x)=\begin{cases}-1 & -\pi\le x<0\\ 1 & 0\le x<\pi\end{cases}$$
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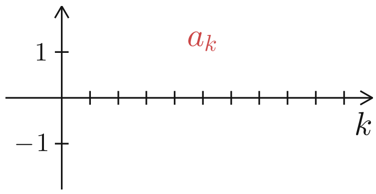

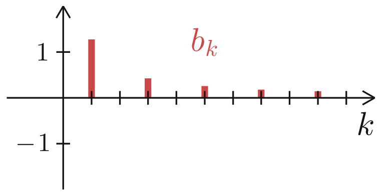

$$f(x)=\frac{4}{\pi}\left(\sin x+\frac{\sin 3x}{3}+\frac{\sin 5x}{5}+\dots\right)$$
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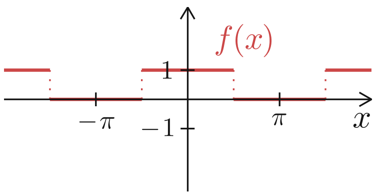

$$f(x)=\begin{cases}0 & -\pi\le x<-\frac{\pi}{2}\\ 1 & -\frac{\pi}{2}\le x<\frac{\pi}{2}\\ 0 & \frac{\pi}{2}\le x<\pi\end{cases}$$
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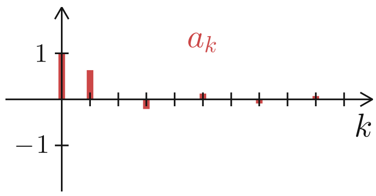

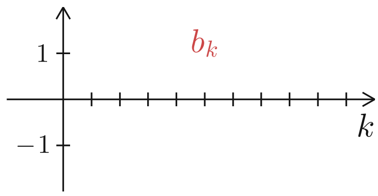

$$f(x)=\frac{1}{2}+\frac{2}{\pi}\left(\cos x-\frac{\cos 3x}{3}+\frac{\cos 5x}{5}+\dots\right)$$
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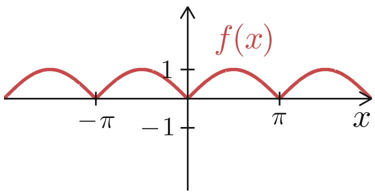

$$f(x)=|\sin x| \qquad -\pi\le x<\pi$$
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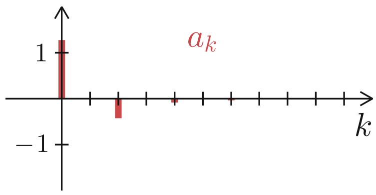

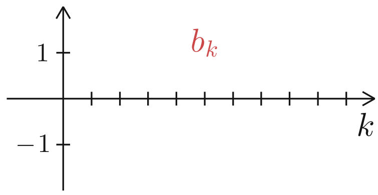

$$f(x)=\frac{2}{\pi}-\frac{4}{\pi}\left(\frac{\cos 2x}{1\cdot 3}+\frac{\cos 4x}{3\cdot 5}+\frac{\cos 6x}{5\cdot 7}+\dots\right)$$
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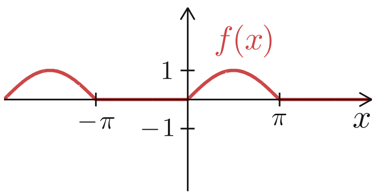

$$f(x)=\begin{cases}0 & -\pi\le x<0\\ \sin x & 0\le x<\pi\end{cases}$$
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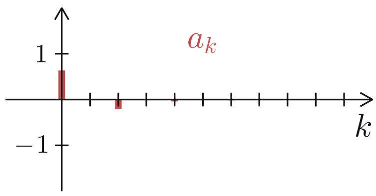

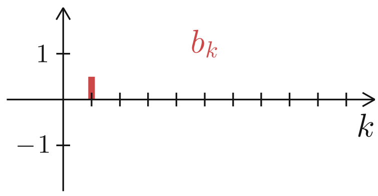

$$f(x)=\frac{1}{\pi}-\frac{2}{\pi}\left(\frac{\cos 2x}{1\cdot 3}+\frac{\cos 4x}{3\cdot 5}+\frac{\cos 6x}{5\cdot 7}+\dots\right)+\frac{\sin x}{2}$$
</td></tr>
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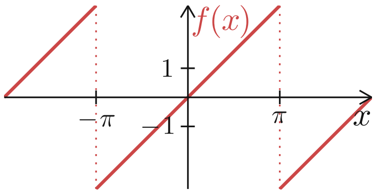
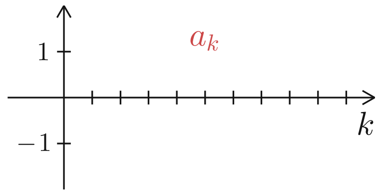
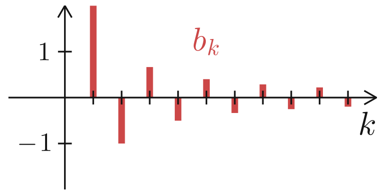
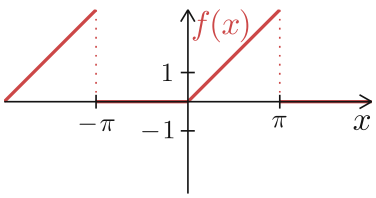
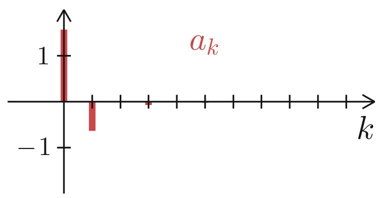
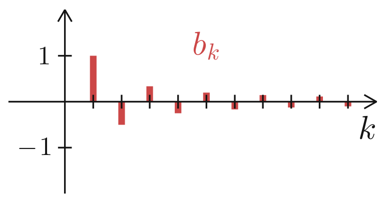
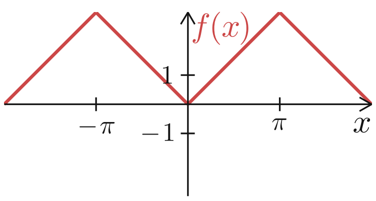
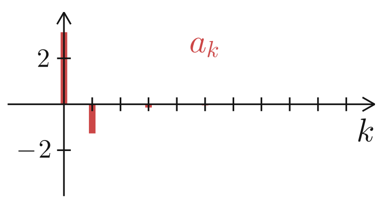
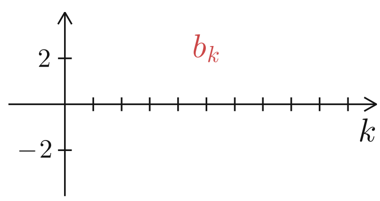
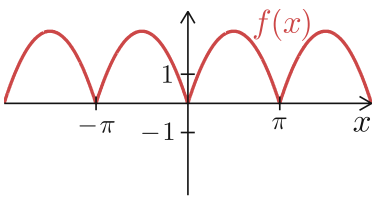
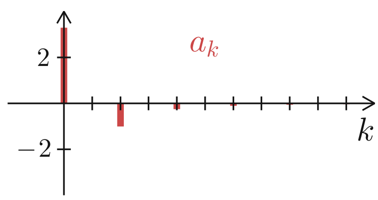
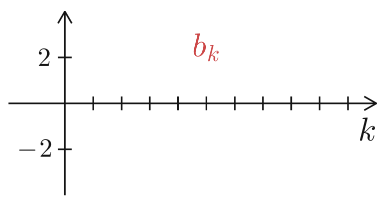
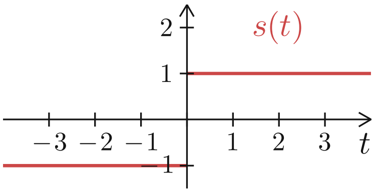
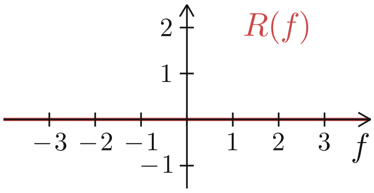
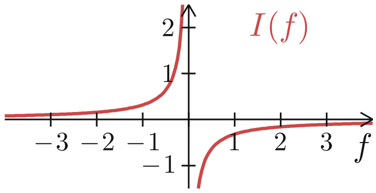
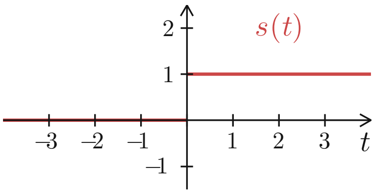
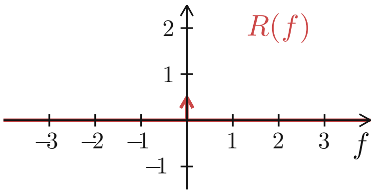
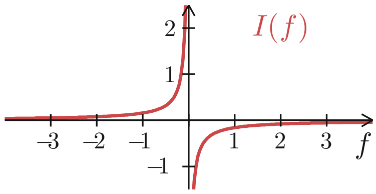
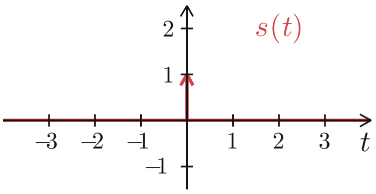
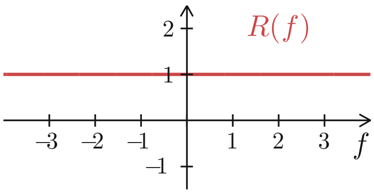
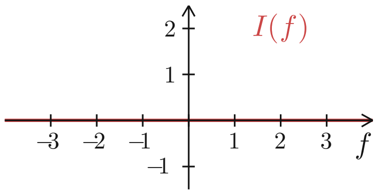
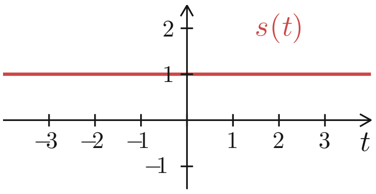
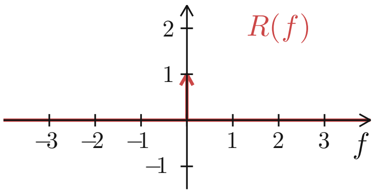
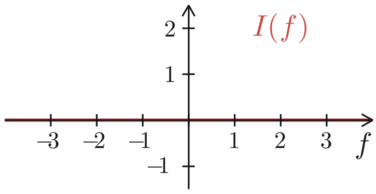
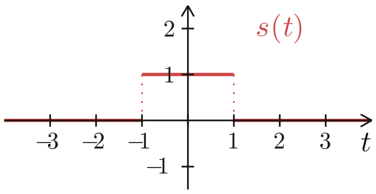
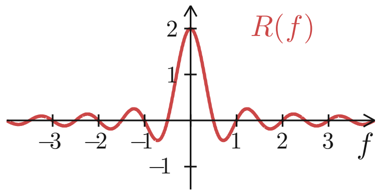
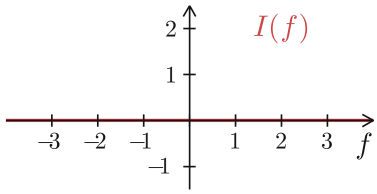
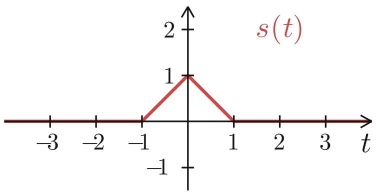
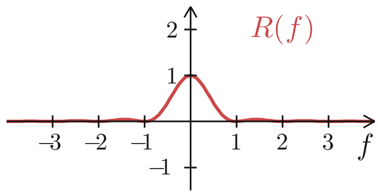
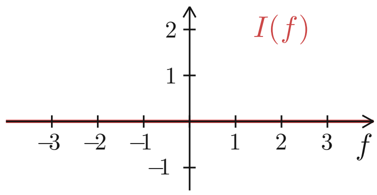
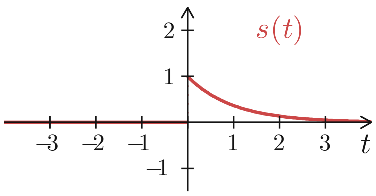
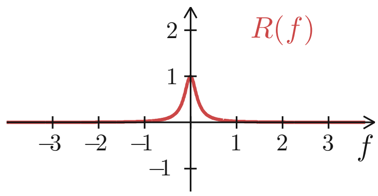
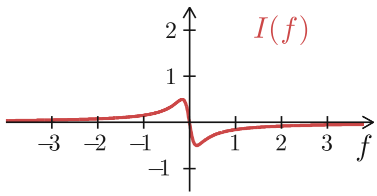
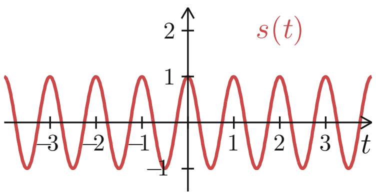
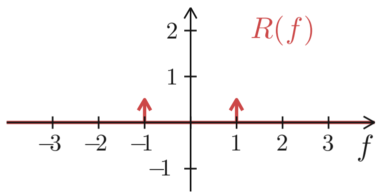
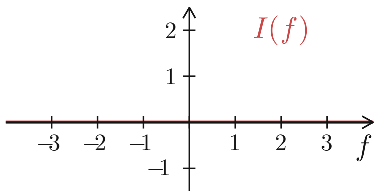
Funktion	Fourier-Reihe
 $f(x) = x \quad -\pi \le x < \pi$	  $f(x) = 2\left(\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} \mp \dots\right)$
 $f(x) = \begin{cases} 0 & -\pi \le x < 0 \\ x & 0 \le x < \pi \end{cases}$	  $f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left(\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \dots\right)$ $+ \sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} \mp \dots$
 $f(x) = \lvert x \rvert \quad -\pi \le x < \pi$	  $f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left(\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \dots\right)$
 $f(x) = \begin{cases} -x(x+\pi) & -\pi \le x < 0 \\ -x(x-\pi) & 0 \le x < \pi \end{cases}$	  $f(x) = \frac{\pi^2}{6} - \left(\cos 2x + \frac{\cos 4x}{2^2} + \frac{\cos 6x}{3^2} + \dots\right)$

A.9 Korrespondenzen der Fourier-Transformation

Zeitfunktion $s(t)$	Fourier-Transformation $S(f) = R(f) + \mathrm{i}\, I(f)$
 $s(t) = \operatorname{sgn}(t)$	  $S(f) = -\mathrm{i}\,\dfrac{1}{\pi f}$
 $s(t) = \sigma(t)$	  $S(f) = \dfrac{1}{2}\,\delta(f) - \mathrm{i}\,\dfrac{1}{2\pi f}$
 $s(t) = \delta(t)$	  $S(f) = 1$
 $s(t) = 1$	  $S(f) = \delta(f)$

Zeitfunktion $s(t)$	Fourier-Transformation $S(f) = R(f) + \mathrm{i}\, I(f)$	
		
$s(t) = \sigma(t+1) - \sigma(t-1)$	$S(f) = 2\,\dfrac{\sin(2\pi f)}{2\pi f}$	
		
$s(t) = (1+t)\big(\sigma(t+1) - \sigma(t)\big) + (1-t)\big(\sigma(t) - \sigma(t-1)\big)$	$S(f) = \left(\dfrac{\sin(\pi f)}{\pi f}\right)^2$	
		
$s(t) = \mathrm{e}^{-t}\sigma(t)$	$S(f) = \dfrac{1}{1+\mathrm{i}\,2\pi f} = \dfrac{1}{1+4\pi^2 f^2} - \mathrm{i}\,\dfrac{2\pi f}{1+4\pi^2 f^2}$	
		
$s(t) = \cos(2\pi t)$	$S(f) = \dfrac{1}{2}\big(\delta(f-1) + \delta(f+1)\big)$	

A.10 Eigenschaften der Fourier-Transformation

Eigenschaft	Zeitfunktion	Bildfunktion
Linearität	$C_1\, s_1(t) + C_2\, s_2(t)$	$C_1\, S_1(f) + C_2\, S_2(f)$
Zeitverschiebung	$s(t - t_0)$	$\mathrm{e}^{-\mathrm{i}\, 2\,\pi\, f\, t_0}\, S(f)$
Frequenzverschiebung	$\mathrm{e}^{\mathrm{i}\, 2\,\pi\, f_0\, t}\, s(t)$	$S(f - f_0)$
Amplitudenmodulation	$s(t)\, \cos(2\,\pi\, f_0\, t)$	$\frac{1}{2}\left(S(f - f_0) + S(f + f_0)\right)$
Ähnlichkeit	$s(a\, t)$	$\frac{1}{\lvert a\rvert}\, S\left(\frac{f}{a}\right)$
Zeitumkehr	$s(-t)$	$S(-f)$
Differenziation in t	$\dot{s}(t)$ $\ddot{s}(t)$ $\vdots$ $\frac{\mathrm{d}^n}{\mathrm{d}t^n} s(t)$	$\mathrm{i}\, 2\,\pi\, f\, S(f)$ $(\mathrm{i}\, 2\,\pi\, f)^2\, S(f)$ $\vdots$ $(\mathrm{i}\, 2\,\pi\, f)^n\, S(f)$
Differenziation in f	$(-\mathrm{i}\, 2\,\pi\, t) s(t)$ $(-\mathrm{i}\, 2\,\pi\, t)^2 s(t)$ $\vdots$ $(-\mathrm{i}\, 2\,\pi\, t)^n s(t)$	$S'(f)$ $S''(f)$ $\vdots$ $S^{(n)}(f)$
Multiplikation in t	$t\, s(t)$ $t^2\, s(t)$ $\vdots$ $t^n\, s(t)$	$S'(f)$ $\frac{S''(f)}{-\mathrm{i}\, 2\,\pi}$ $\vdots$ $\frac{S^{(n)}(f)}{(-\mathrm{i}\, 2\,\pi)^n}$
Integration	$\int_{-\infty}^{t} s(\tau)\, \mathrm{d}\,\tau$	$\frac{1}{\mathrm{i}\, 2\,\pi\, f}\, S(f) + \frac{1}{2}\, S(0)\, \delta(f)$
Faltung in t	$s_1(t) \star s_2(t)$	$S_1(f) \cdot S_2(f)$
Faltung in f	$s_1(t) \cdot s_2(t)$	$S_1(f) \star S_2(f)$

A.11 Korrespondenzen der Laplace-Transformation

Bildfunktion $F(s)$	Zeitfunktion $f(t)$	Bildfunktion $F(s)$	Zeitfunktion $f(t)$
1	$\delta(t)$	$\frac{a}{s^2+a^2}$	$\sin at$
$\frac{1}{s}$	1	$\frac{s}{s^2+a^2}$	$\cos at$
$\frac{1}{s^2}$	t	$\frac{a}{s^2-a^2}$	$\sinh at$
$\frac{n!}{s^{n+1}}$	t^n	$\frac{s}{s^2-a^2}$	$\cosh at$
$\frac{1}{s-a}$	e^{at}	$\frac{a}{(s-b)^2+a^2}$	$\mathrm{e}^{bt}\sin at$
$\frac{1}{(s-a)^2}$	$t\,\mathrm{e}^{at}$	$\frac{s-b}{(s-b)^2+a^2}$	$\mathrm{e}^{bt}\cos at$
$\frac{a}{s(s-a)}$	$\mathrm{e}^{at}-1$	$\frac{a}{(s-b)^2-a^2}$	$\mathrm{e}^{bt}\sinh at$
$\frac{a-b}{(s-a)(s-b)}$	$\mathrm{e}^{at}-\mathrm{e}^{bt}$	$\frac{s-b}{(s-b)^2-a^2}$	$\mathrm{e}^{bt}\cosh at$
$\frac{a}{1+as}$	$\mathrm{e}^{-\frac{t}{a}} \quad (a \neq 0)$	$\frac{2as}{(s^2+a^2)^2}$	$t\sin at$
$\frac{a^2}{(1+as)^2}$	$t\,\mathrm{e}^{-\frac{t}{a}} \quad (a \neq 0)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$	$t\cos at$
$\frac{1}{s(1+as)}$	$1-\mathrm{e}^{-\frac{t}{a}} \quad (a \neq 0)$	$\frac{2as}{(s^2-a^2)^2}$	$t\sinh at$
$\frac{a-b}{(1+as)(1+bs)}$	$\mathrm{e}^{-\frac{t}{a}}-\mathrm{e}^{-\frac{t}{b}} \quad (a,b \neq 0)$	$\frac{s^2+a^2}{(s^2-a^2)^2}$	$t\cosh at$
$\frac{s}{(s-a)^2}$	$(1+at)\mathrm{e}^{at}$	$\frac{2}{(s-a)^3}$	$t^2\,\mathrm{e}^{at}$
$\frac{(a-b)s}{(s-a)(s-b)}$	$a\,\mathrm{e}^{at}-b\,\mathrm{e}^{bt}$	$\frac{2s}{(s-a)^3}$	$(at^2+2t)\mathrm{e}^{at}$
$\frac{a^3 s}{(1+as)^2}$	$(a-t)\mathrm{e}^{-\frac{t}{a}} \quad (a \neq 0)$	$\frac{2s^2}{(s-a)^3}$	$(a^2t^2+4at+2)\mathrm{e}^{at}$
$\frac{ab(a-b)s}{(1+as)(1+bs)}$	$a\,\mathrm{e}^{-\frac{t}{b}}-b\,\mathrm{e}^{-\frac{t}{a}} \; (a,b \neq 0)$	$\frac{a^2}{s^2(s-a)}$	$\mathrm{e}^{at}-at-1$

A.12 Eigenschaften der Laplace-Transformation

Eigenschaft	Zeitfunktion	Bildfunktion
Linearität	$C_1\,f_1(t) + C_2\,f_2(t)$	$C_1\,F_1(s) + C_2\,F_2(s)$
Ähnlichkeit $(a > 0)$	$f(a\,t)$	$\frac{1}{a}\,F\left(\frac{s}{a}\right)$
Zeitverschiebung	$\sigma(t - t_0)\,f(t - t_0)$	$\mathrm{e}^{-t_0\,s}\,F(s)$
Dämpfung	$\mathrm{e}^{-s_0\,t}\,f(t)$	$F(s + s_0)$
Differenziation in t	$f'(t)$ $f''(t)$ $\vdots$ $f^{(n)}(t)$	$s\,F(s) - f(0)$ $s^2\,F(s) - s\,f(0) - f'(0)$ $\vdots$ $s^n\,F(s) - \sum_{k=0}^{n-1} s^{n-k-1}\,f^{(k)}(0)$
Differenziation in s	$-t\,f(t)$ $t^2\,f(t)$ $\vdots$ $(-t)^n\,f(t)$	$F'(s)$ $F''(s)$ $\vdots$ $F^{(n)}(s)$
Multiplikation mit t	$t\,f(t)$ $t^2\,f(t)$ $\vdots$ $t^n\,f(t)$	$-F'(s)$ $F''(s)$ $\vdots$ $(-1)^n\,F^{(n)}(s)$
Integration im Zeitbereich	$\int_0^t f(\tau)\,\mathrm{d}\,\tau$	$\frac{1}{s}\,F(s)$
Integration im Bildbereich	$\frac{1}{t}\,f(t)$	$\int_s^\infty F(u)\,\mathrm{d}\,u$
Faltung im Zeitbereich	$f_1(t) \star f_2(t)$	$F_1(s) \cdot F_2(s)$
Periodische Funktion	$f(t + T) = f(t)$	$\frac{1}{1 - \mathrm{e}^{-T\,s}} \int_0^T f(t)\,\mathrm{e}^{-s\,t}\,\mathrm{d}\,t$

A.13 Korrespondenzen der z-Transformationen

Bildfunktion $F(z)$	Zeitfolge (f_k)	Bildfunktion $F(z)$	Zeitfolge (f_k)
1	δ_k	$\frac{1}{z^n}$	1 für $k=n$, 0 sonst
$\frac{z}{z-1}$	1	$\frac{z}{(z-1)^2}$	k
$\frac{z}{z-a}$	a^k	$\frac{az}{(z-a)^2}$	$k\,a^k$

A.14 Eigenschaften der z-Transformationen

Eigenschaft	Zeitfolge	Bildfunktion
Linearität	$C_1\,(f_k)+C_2\,(g_k)$	$C_1\,F(z)+C_2\,G(z)$
Dämpfung	$(a^{-k}\,f_k)$	$F(a\,z)$
Indexverschiebung	(f_{k-n})	$z^{-n}F(z)$
	(f_{k+1})	$z(F(z)-f_0)$
	(f_{k+2})	$z^2(F(z)-f_0-f_1z^{-1})$
	$\vdots$	$\vdots$
	(f_{k+n})	$z^n\left(F(z)-\sum\limits_{k=0}^{n-1} f_k z^{-k}\right)$
Differenzen	$(\Delta\, f_k)$	$(z-1)F(z)-z\,f_0$
	$\left(\Delta^2\, f_k\right)$	$(z-1)^2F(z)-z((z-1)f_0+\Delta f_0)$
	$\vdots$	$\vdots$
	$(\Delta^n\, f_k)$	$(z-1)^nF(z)-z\sum\limits_{k=0}^{n-1}(z-1)^{n-k-1}\Delta^k f_0$
Multiplikation mit k	$(k\,f_k)$	$-z\,F'(z)$
	$\left(k^2\,f_k\right)$	$z\,F'(z)-z^2\,F''(z)$
	$\vdots$	$\vdots$
Faltung im Zeitbereich	$(f_k)\star(g_k)$	$F(z)\cdot G(z)$