OLD DOMINION UNIVERSITY

Constrained Optimization: Constraints

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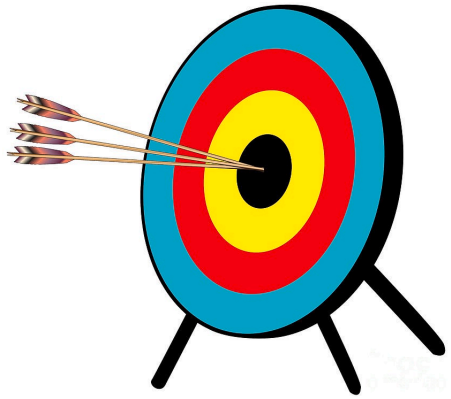
2 Core Ideas in Economics

Agents: Optimization

- Agents have **objects** they value
- Agents face **constraints**
- Make **tradeoffs** to maximize objectives within constraints



Markets: Equilibrium

- Agents **compete** with others over **scarce** resources
- Agents **adjust** behaviors based on prices
- **Stable outcomes** when adjustments stop

Rational Choice Theory

Consumer Behavior

- How do people decide:
  - what to buy
  - what activities to dedicate time to
- Answers to these questions are building blocks for **demand curves**

Consumer Behavior

- Consumer purchasing decisions is only our paradigmatic *example*
  - Really about how **individuals** make choices in almost *any* context!

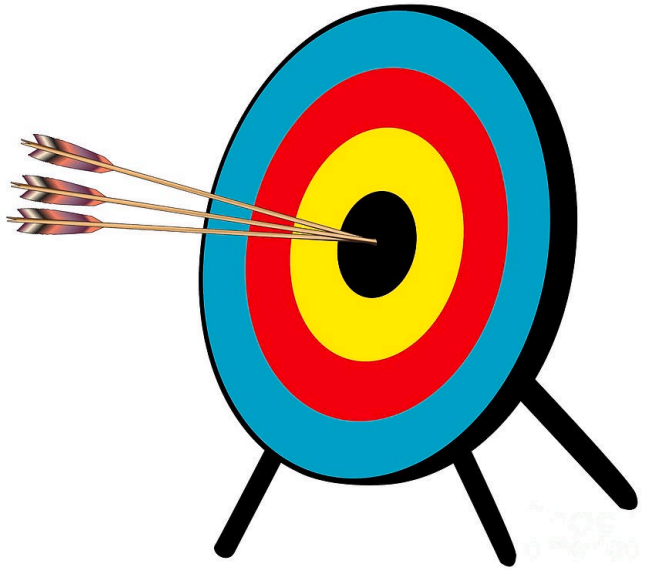
Consumer Behavior

- Consumer purchasing decisions is only our paradigmatic *example*
  - Really about how **individuals** make choices in almost *any* context!
  - Model is **widely applicable**, but a **caricature of reality**

Constrained Optimization

Constrained Optimization

- We model most situations as a **constrained optimization problem**:



- People **optimize**: make tradeoffs to achieve their **objective** *as best as they can*

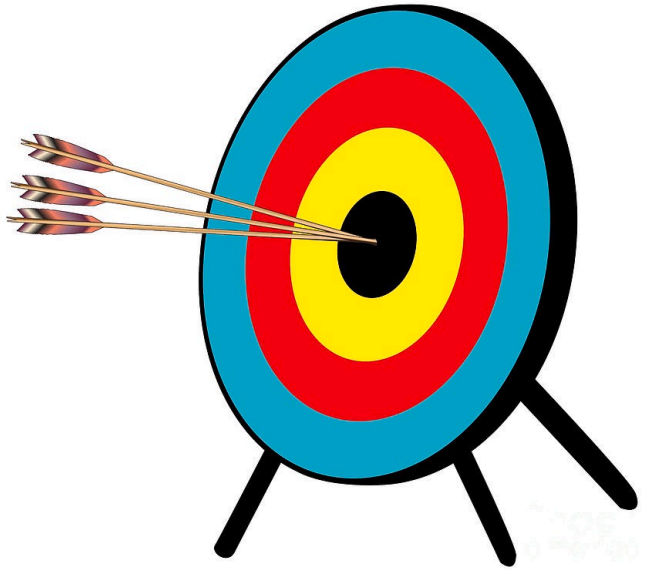
- Subject to **constraints**: limited resources (income, time, attention, etc)

Constrained Optimization

- *Endless applications*:

- Consumer, firms, parent, child, politician, judge, bureaucrat, voter, dictator, pirate, drug cartel, etc.

- **Key economic skill: recognizing how to apply the model to a situation**



Constrained Optimization

- All constrained optimization models have three moving parts:

1) Choose: < some alternative >

2) In order to maximize: < some objective >

3) Subject to: < some constraints >

Constrained Optimization

- **Example:** A student picking courses hoping to achieve the highest GPA while getting an Econ major.

1) Choose:

2) In order to maximize:

3) Subject to:

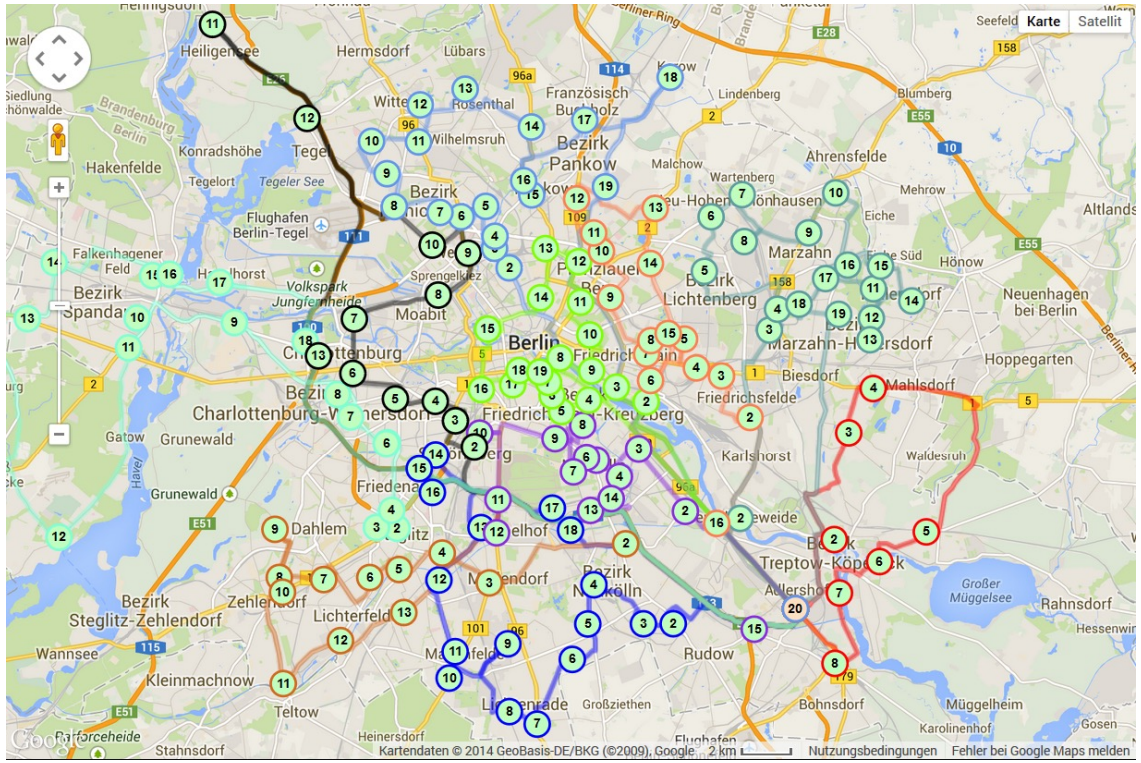
Constrained Optimization

- **Example:** How should Amazon plan its delivery route?

1) Choose:

2) In order to maximize:

3) Subject to:



Constrained Optimization

- **Example:** How do elected officials make decisions in politics?

1) Choose:

2) In order to maximize:

3) Subject to:

Constrained Optimization

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The Consumer's Problem

- The **consumer's constrained optimization problem** is:

- **Choose: < a consumption bundle >**
- **In order to maximize: < utility >**
- **Subject to: < income and market prices >**

Consumer Behavior: Basic Framework

Consumption Bundles

- Imagine your out for an evening for food *(x)* and beverages *(y)*

- Your choices:
- amounts of *(x, y)* to buy as a **bundle**

Consumption Bundles

- How to represent a bundle

a = (x,y)

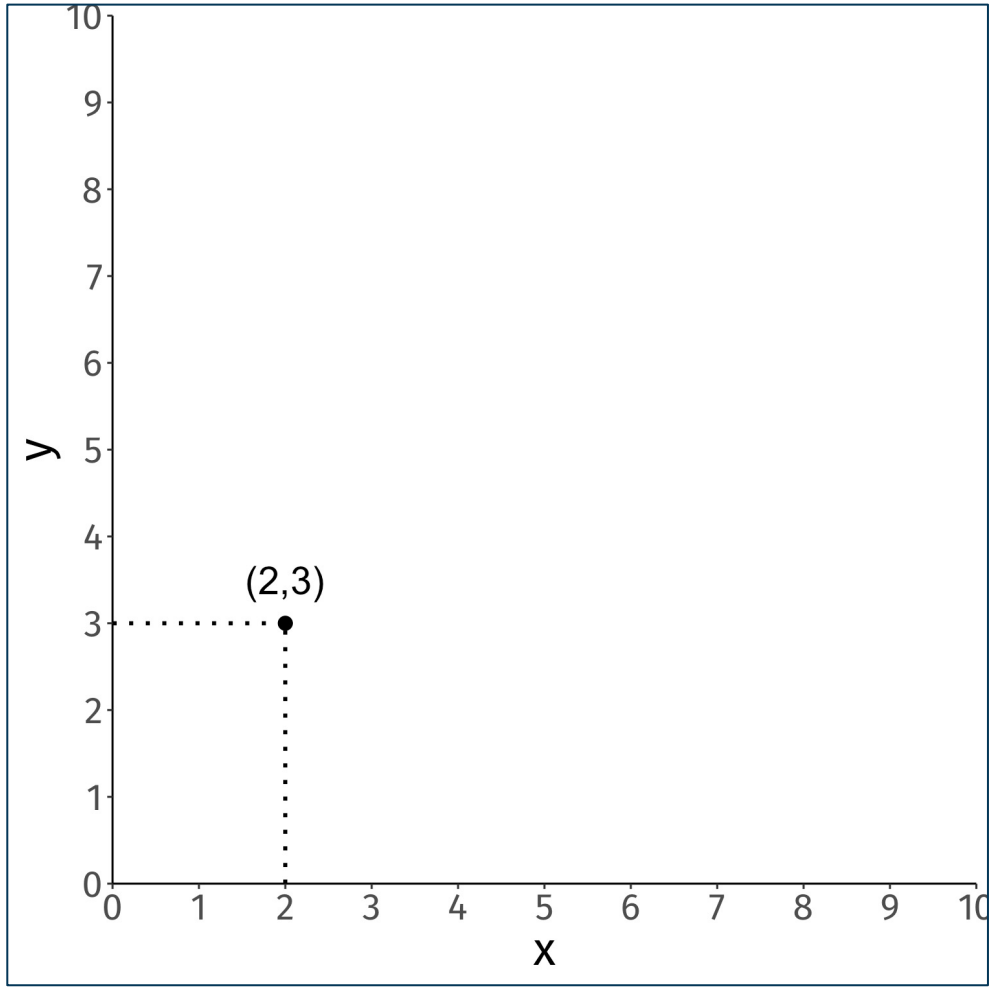
a = (3, 6)

b = (8, 2)

c = (5, 5)

Consumption Bundles

- We can represent bundles graphically
- We'll stick with 2 goods *(x,y)* in 2-dimensions



Affordability

- If you had $100 to spend, what bundles of goods (x,y) would you buy?
- Only those bundles that are **affordable**
- Denote prices of each good as (p_x, p_y)
- Let m be the amount of income a consumer has

The Budget Set

- The set of *all* affordable bundles that a consumer can choose is called the **budget set** or **choice set**

$$p_x*X+ p_y*Y \leq m$$

The **budget *constraint*** is the set of all bundles that spend *all income* m:

$$p_x*X+ p_y*Y = m$$

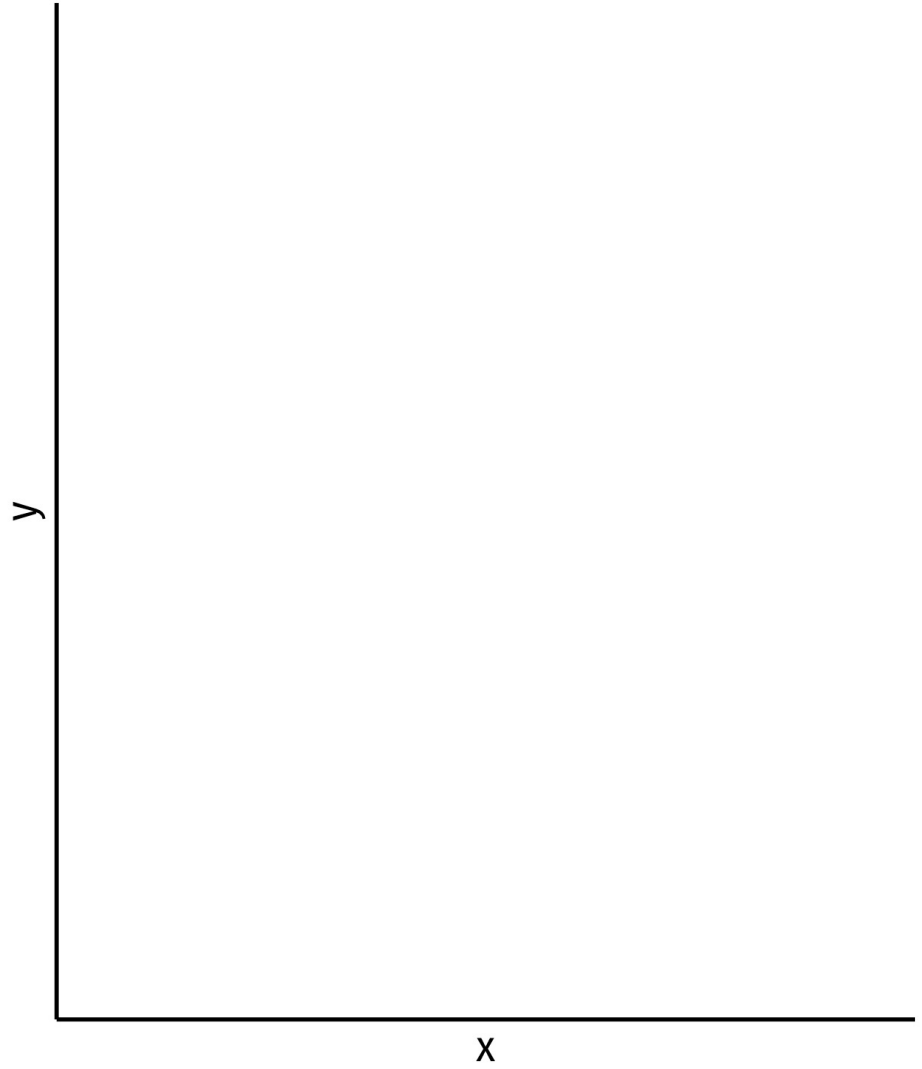
The Budget Set & the Budget Constraint

- For 2 goods *(x,y)*

$$m = p_x x + p_y y$$

Solve for y to graph

$$y = \frac{m}{p_y} - \frac{p_x}{p_y} x$$

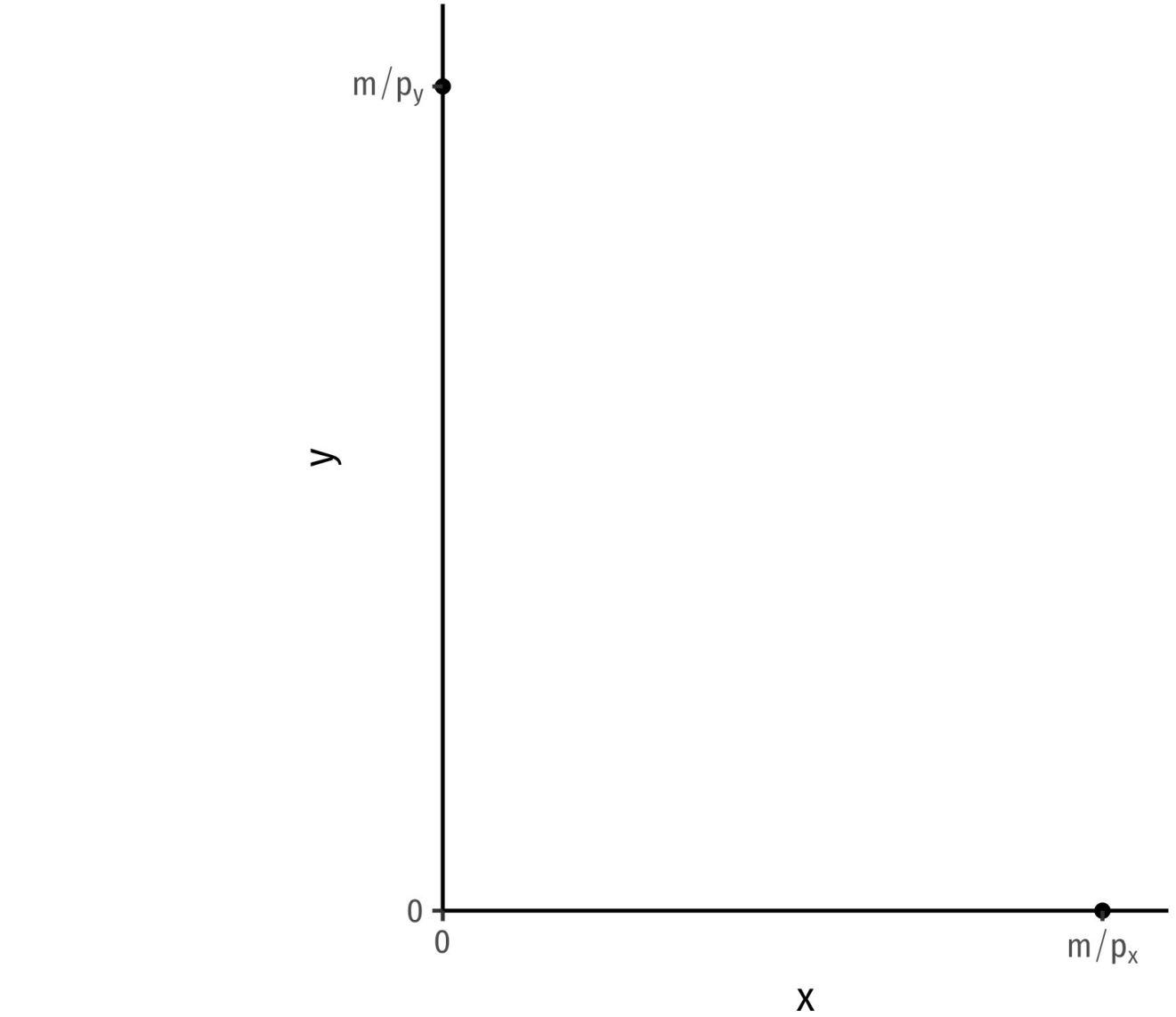


The Budget Set & the Budget Constraint

$$y = \frac{m}{p_y} - \frac{p_x}{p_y}x$$

y – intercept $\frac{m}{p_y}$

x – intercept $\frac{m}{p_x}$



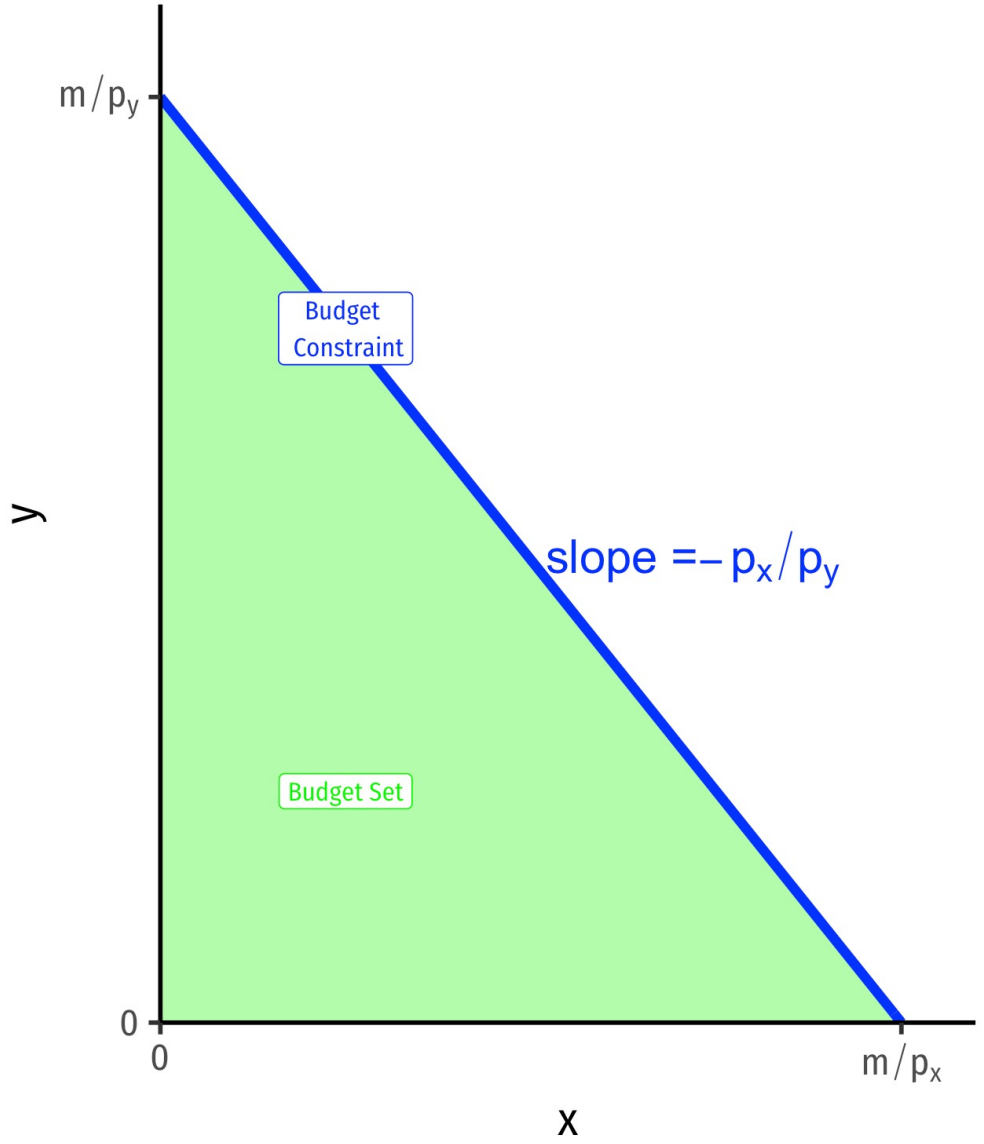
The Budget Set & the Budget Constraint

$$y = \frac{m}{p_y} - \frac{p_x}{p_y}x$$

y – intercept = $\frac{m}{p_y}$

x – intercept = $\frac{m}{p_x}$

Slope = $\frac{p_x}{p_y}$



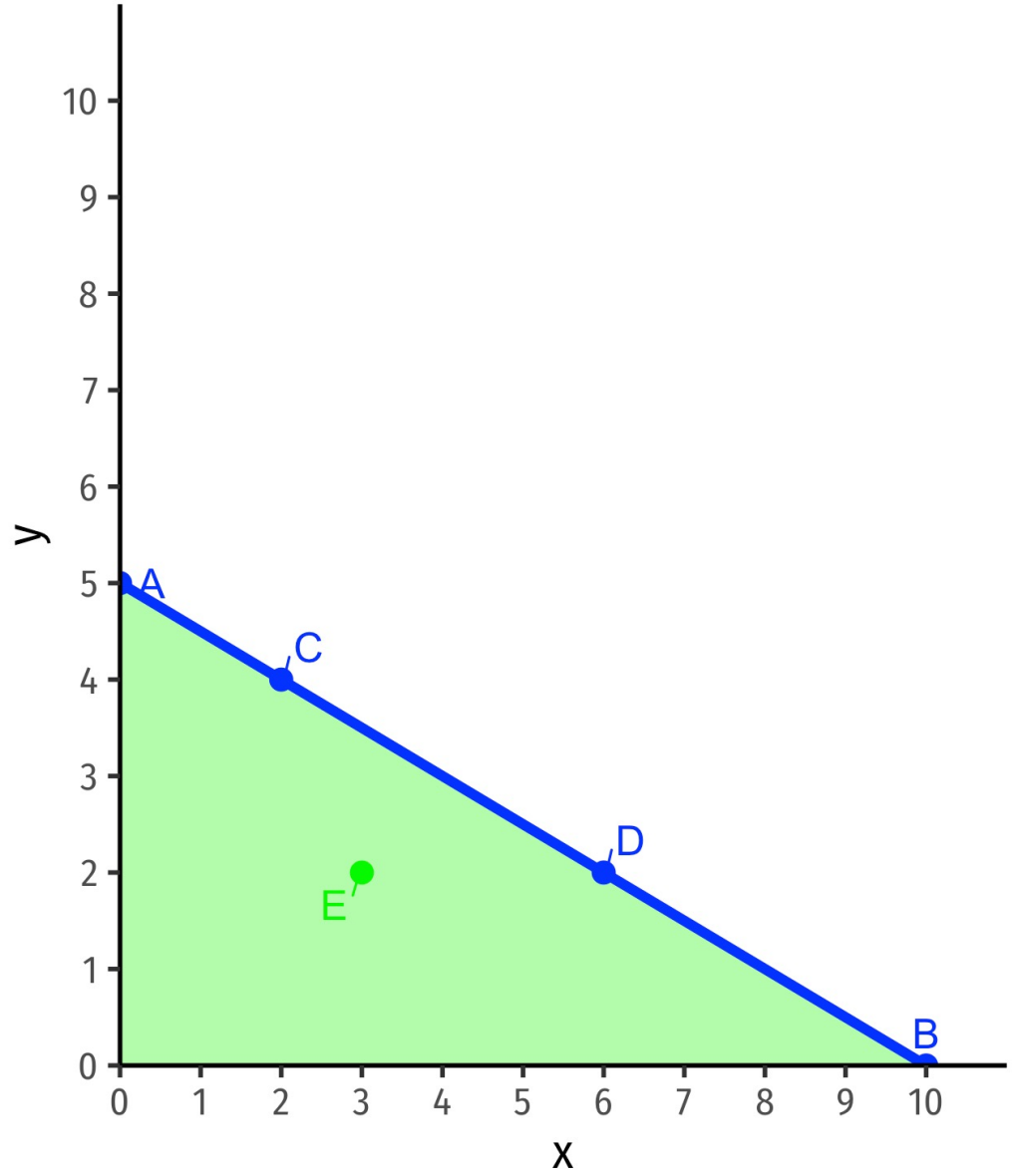
The Budget Set & the Budget Constraint

- **Example**: Suppose you have an income of $50 to spend on food (F) and beverages *(B)*. The price of food is $5 and the price of beverages is $10.
- Let F be on the horizontal axis and B be on the vertical axis.

1) Write an equation for the budget constraint (in graphable form).

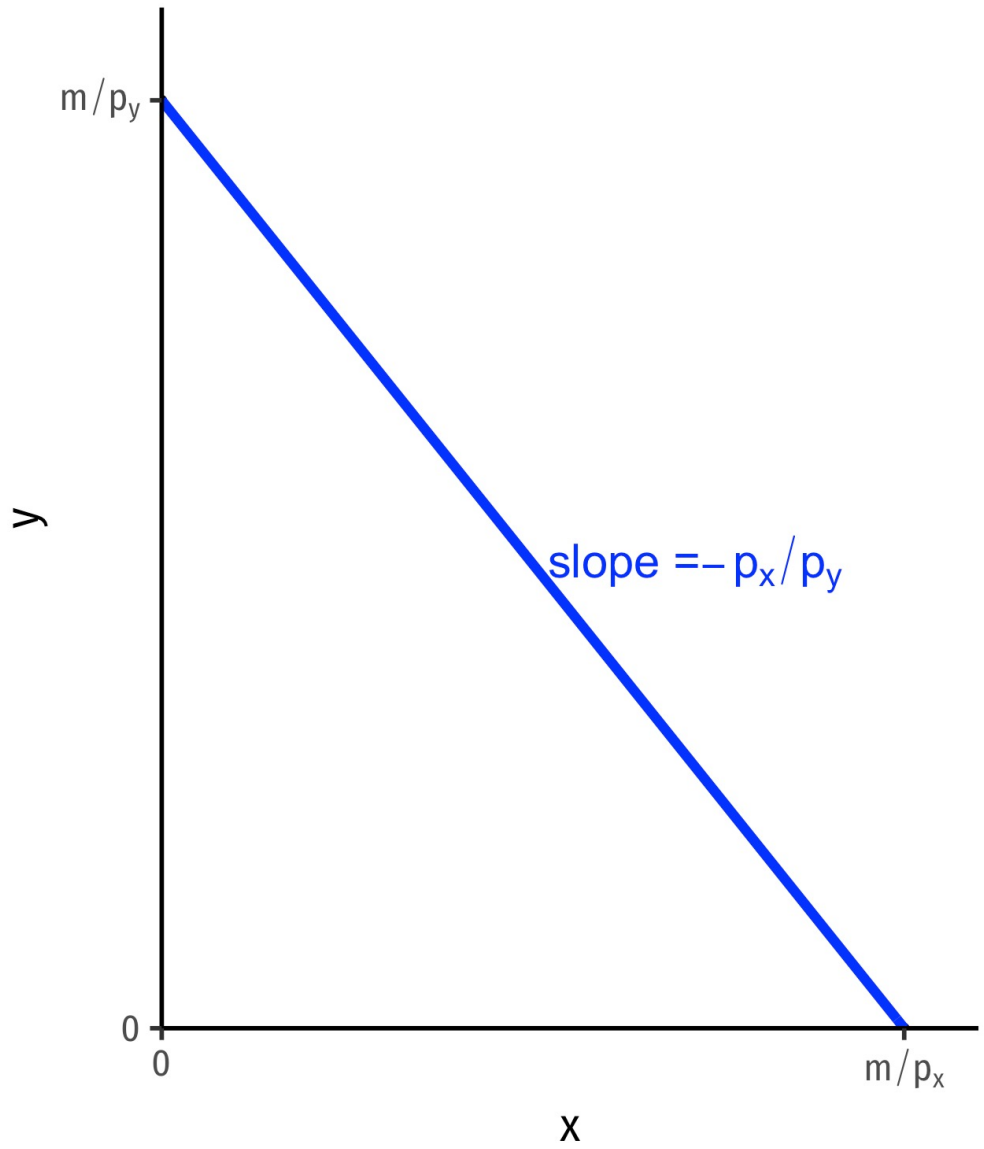
2) Graph the budget constraint.

Interpreting the Budget Constraint

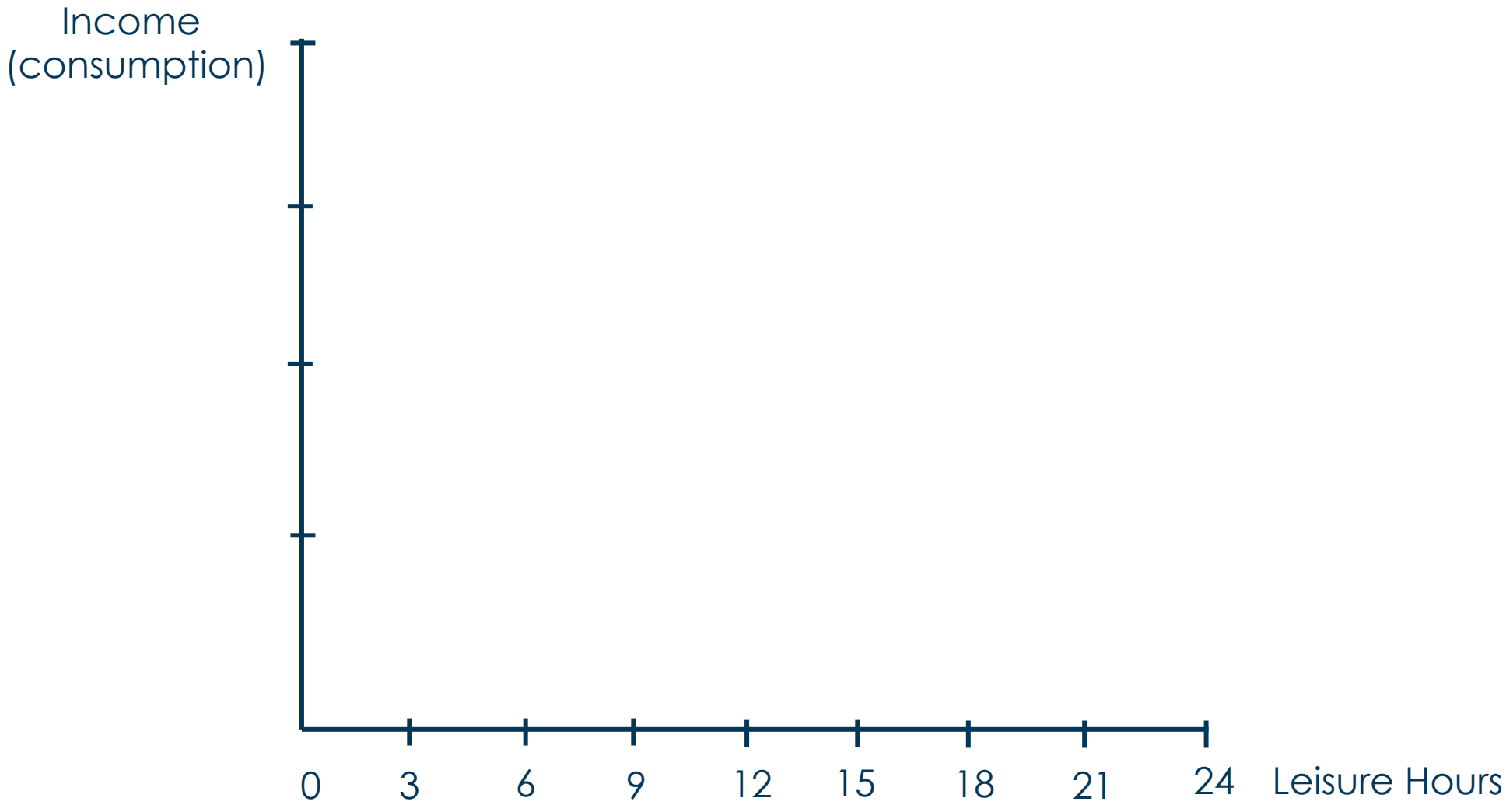


Interpreting the Slope

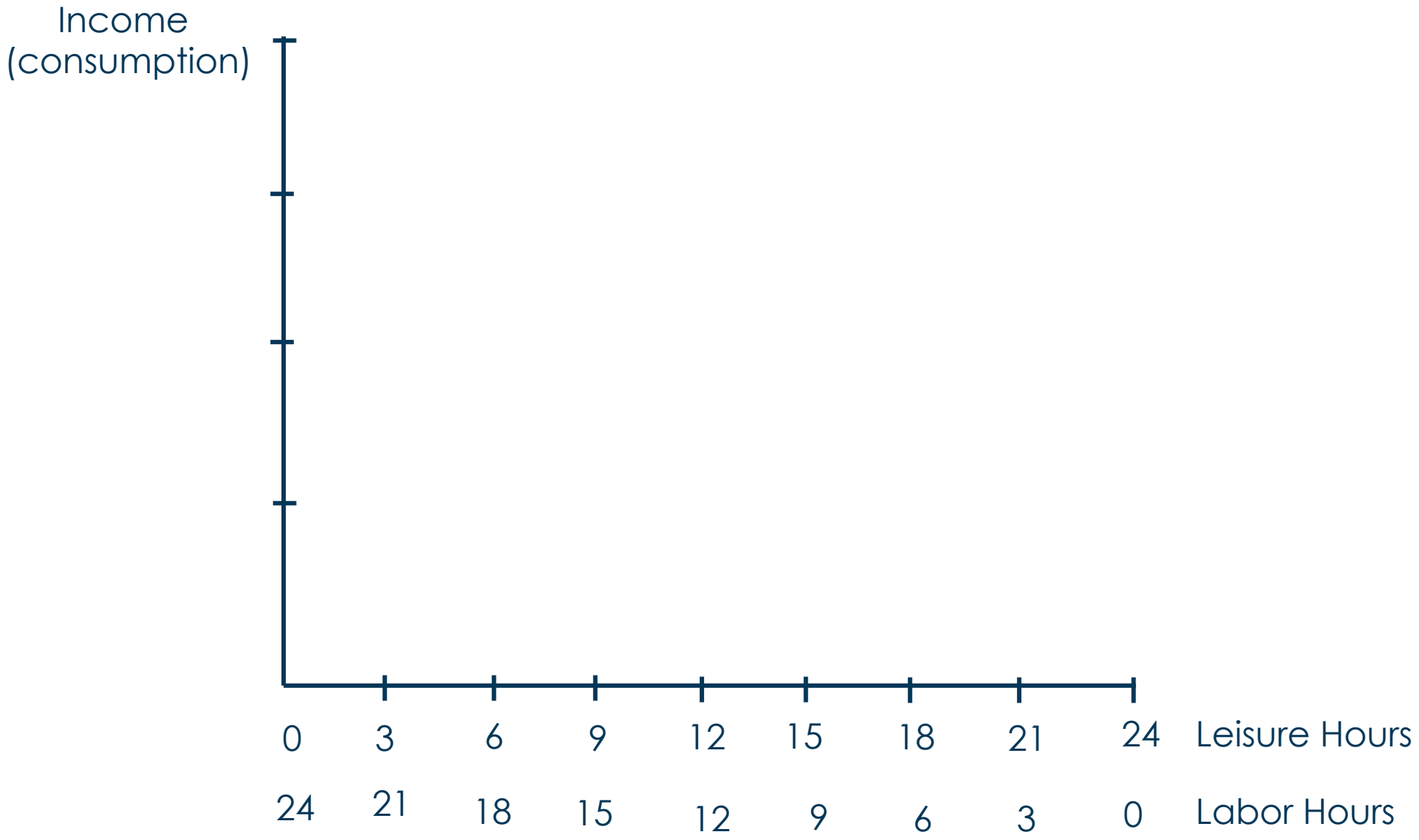
- **Slope**: market-rate of **tradeoff** between x and y
- **Relative price** of x or its **opportunity cost**:
  - Consuming 1 more unit of x requires giving up $\frac{p_x}{p_y}$ units of y
- Foreshadowing:
- Is **your** valuation of the tradeoff between x and y the same as the market rate?



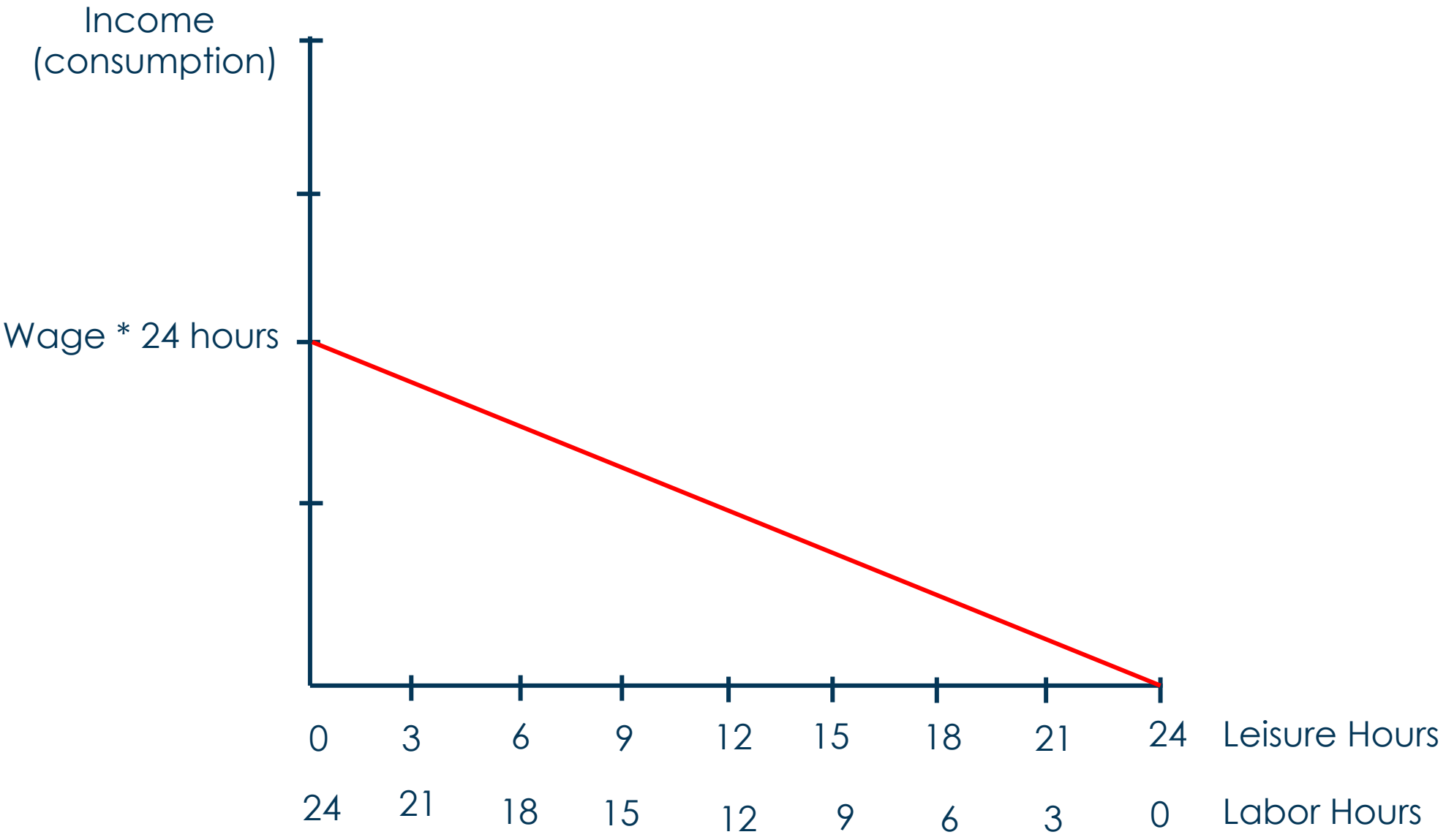
Example: Labor and Leisure



Example: Labor and Leisure



Example: Labor and Leisure

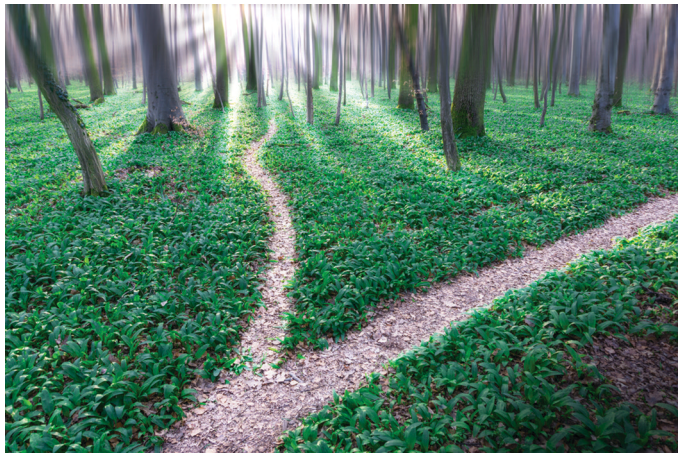


Change in parameters

Change in parameters

$$m = p_x x + p_y y$$

$$y = \frac{m}{p_y} - \frac{p_x}{p_y} x$$



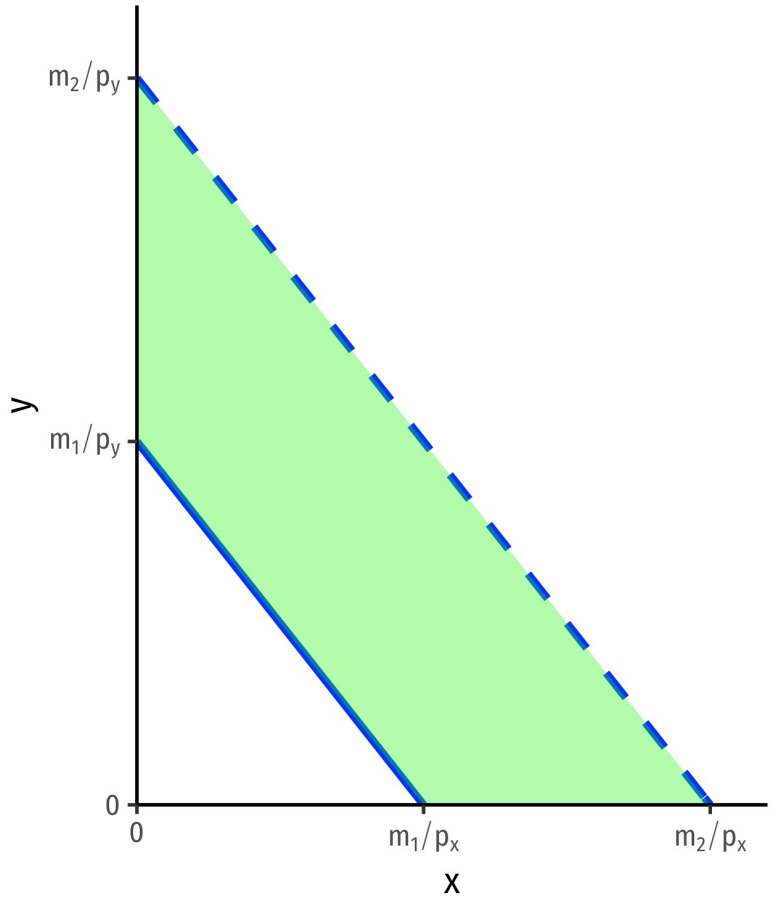
- Budget constraint is a function of specific **parameters**
  - m: income
  - p_x, p_y: market prices
- Economics: **how changes in constraints affect people's choices**

Changes in Income, *m*

- Changes in **income**: a *parallel* shift in budget constraint

- **Example**: An increase in income
- Same slope (relative prices don't change!)

- **Gain of affordable bundles**



Changes in Income, *m*

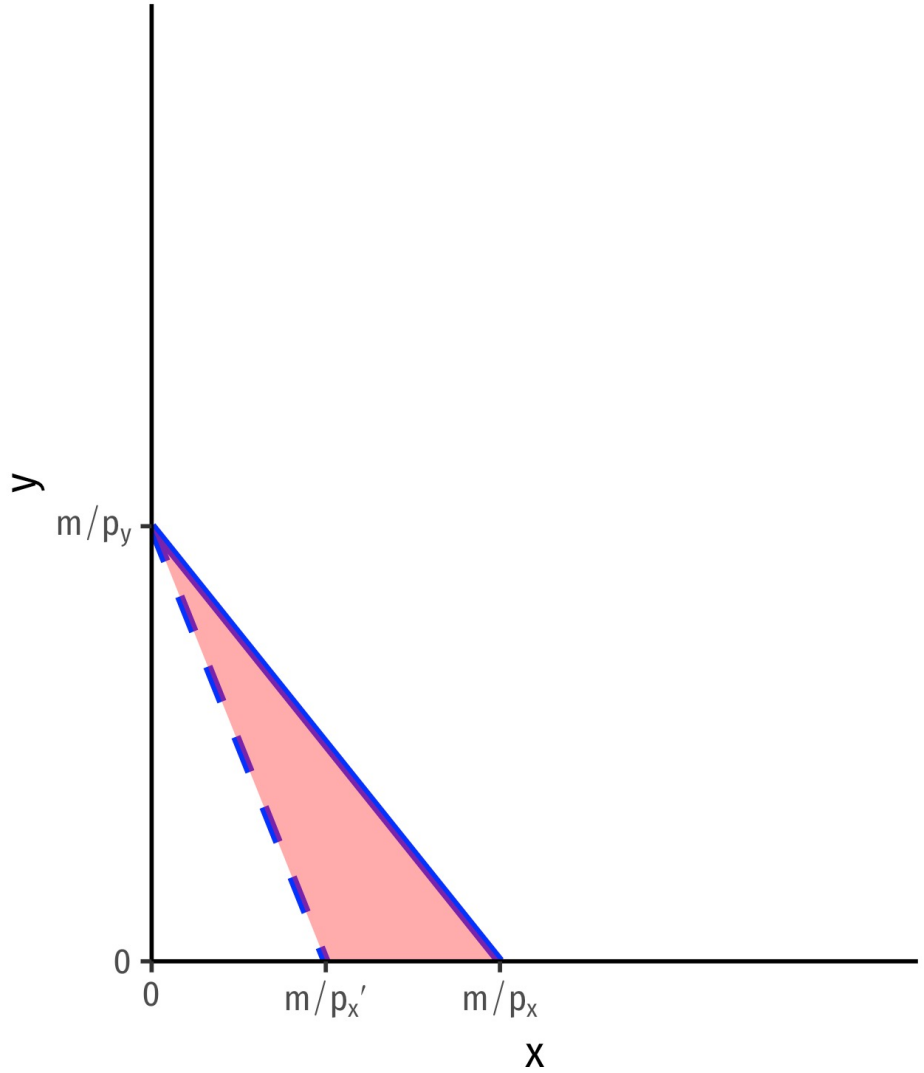
- **Example**: Continuing the food and beverages example, (income is $50, food is $5, beverages are $10), suppose your income doubles to $100.

  1) Find the equation of the new budget constraint (in graphable form).

  2) Graph the new budget constraint.

Changes in Relative Prices, p_x or p_y

- Changes in **relative prices**: *rotate* the budget constraint
- **Example**: An increase in the price of x
- Slope steepens: - $\frac{p_x}{p_y}$
- **Loss of affordable bundles**



Economics is About (Changes in) *Relative* Prices

- **Economics is about (changes in) *relative* prices**
- Budget constraint slope is $\frac{p_x}{p_y}$
- Only **"real"** changes in *relative* prices (from changes in market valuations) change consumer constraints

Economics is About (Changes in) *Relative* Prices

- **"Nominal"** prices are often meaningless!

- **Example**: Imagine yourself in a strange country. All you know is that the price of bread is "6"...

Changes in Relative Prices, p_x or p_y

- **Example**: Continuing the food and beverages example (income is $50, food is $5, beverages are $10).

  1) Suppose the price of food doubles from $5 to $10. Find the equation of the new budget constraint and graph it.

  2) Return to the original price of food ($5) and suppose the price of beverages falls from $10 to $5. Find the equation of the new budget constraint and graph it.