OLD DOMINION UNIVERSITY

Constrained Optimization: Utility and Preferences

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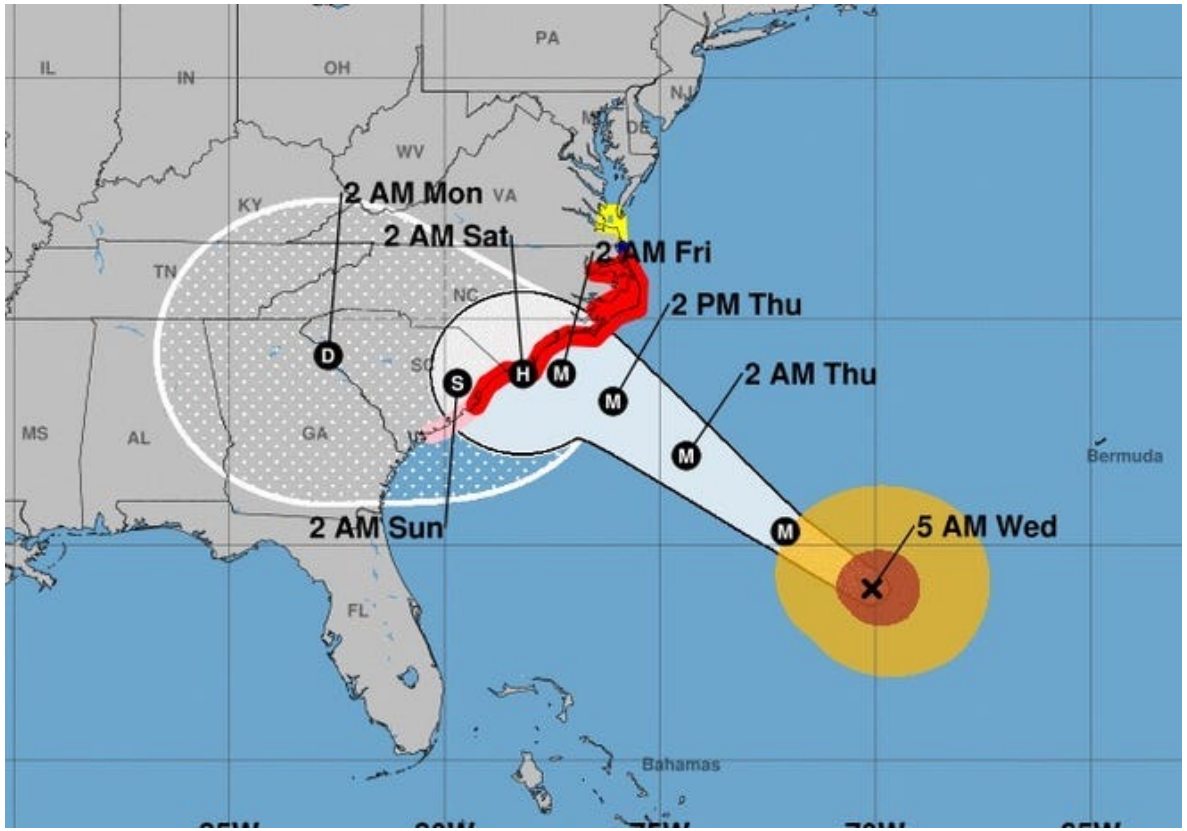
Models

Models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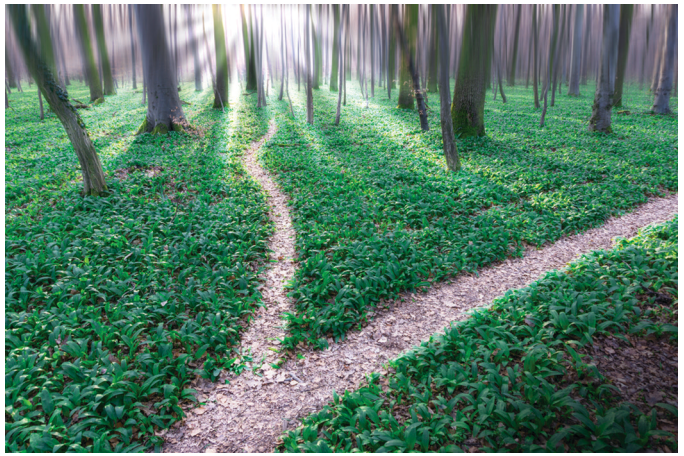
Models

Consumer's Objectives

- What do consumers want? What do they ***maximize***?
- We'll use the econ jargon:
  - **Utility**
  - Think, happiness, satisfaction, flourishing
- We'll assume people have **preferences**



Dog's Objectives

- What do dogs want? What do they ***maximize***?
- The econ jargon:
  - **Utility**
- Dogs tend to be pretty transparent
- Wag-o-meter

Preferences

Dog Preferences

Treats

Fetch

Love

Preferences

- Which bundles of (x,y) are **preferred** over others?

a = (x,y)

a = (12, 6)

b = (11, 8)

Preferences

- We will allow **three possible answers**:

  1) $a \succ b$: Strictly prefer a over b

  2) $a \prec b$: Strictly prefer b over a

  3) $a \sim b$: Indifferent between a and b

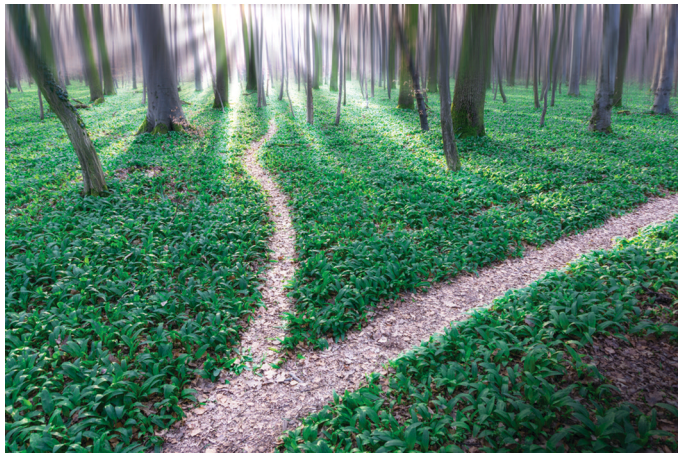
- ***Preferences* are a list of all such comparisons between all bundles**

Indifference Curves

Mapping Preferences

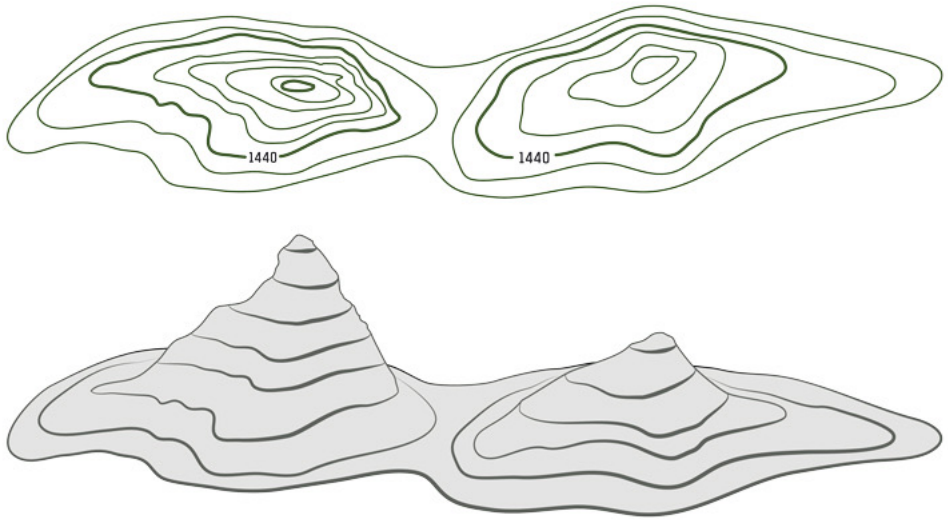
- For each bundle, we now have 3 pieces of information:
  - amount of x
  - amount of y
  - preference compared to other bundles

- How to represent this information graphically?



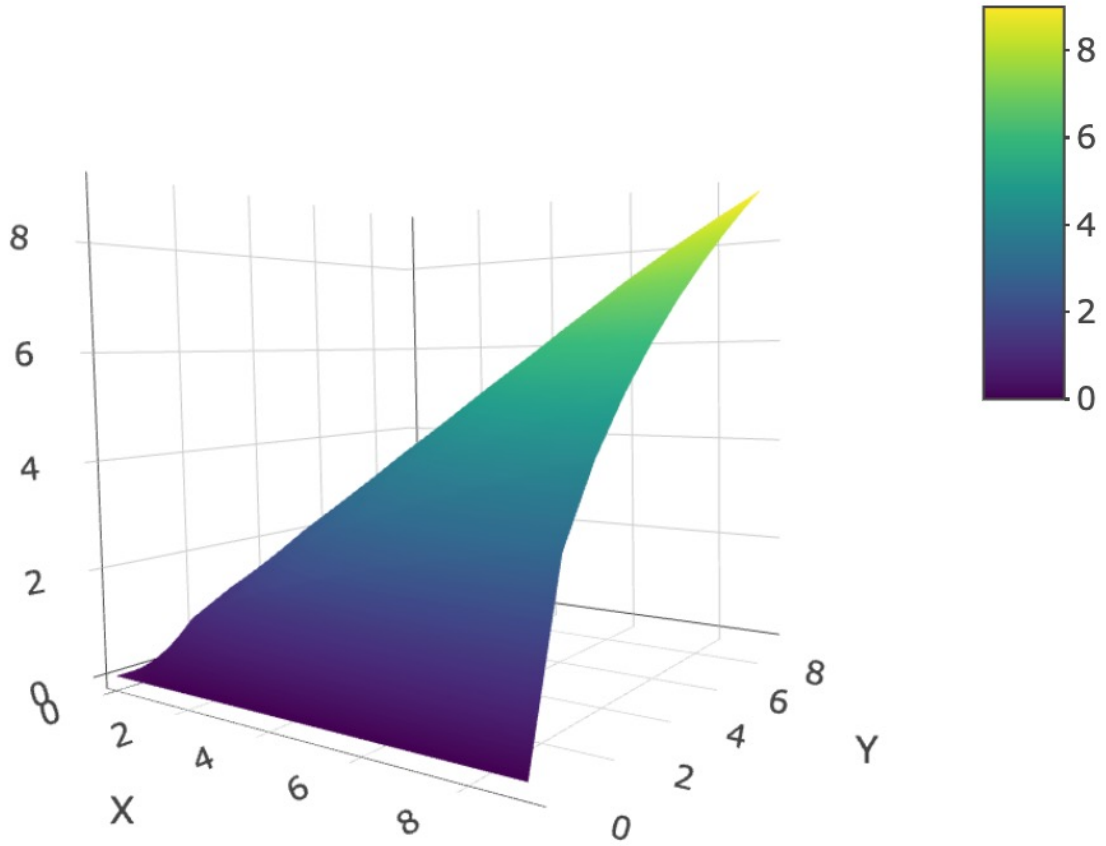
Mapping Preferences

- Cartographers have the answer for us
- On a map, **contour lines** link areas of **equal height**
- We will use **"indifference curves"** to link bundles of **equal preference**

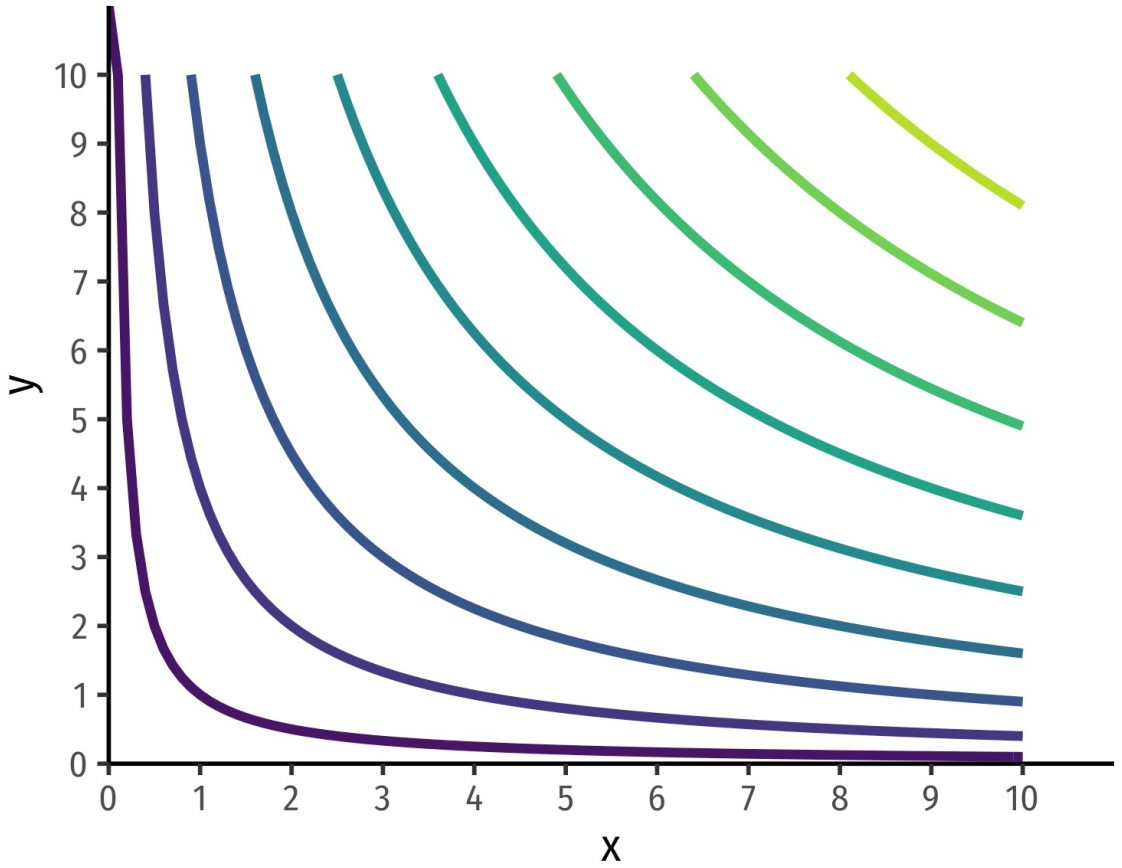


Mapping Preferences

- 3 - d

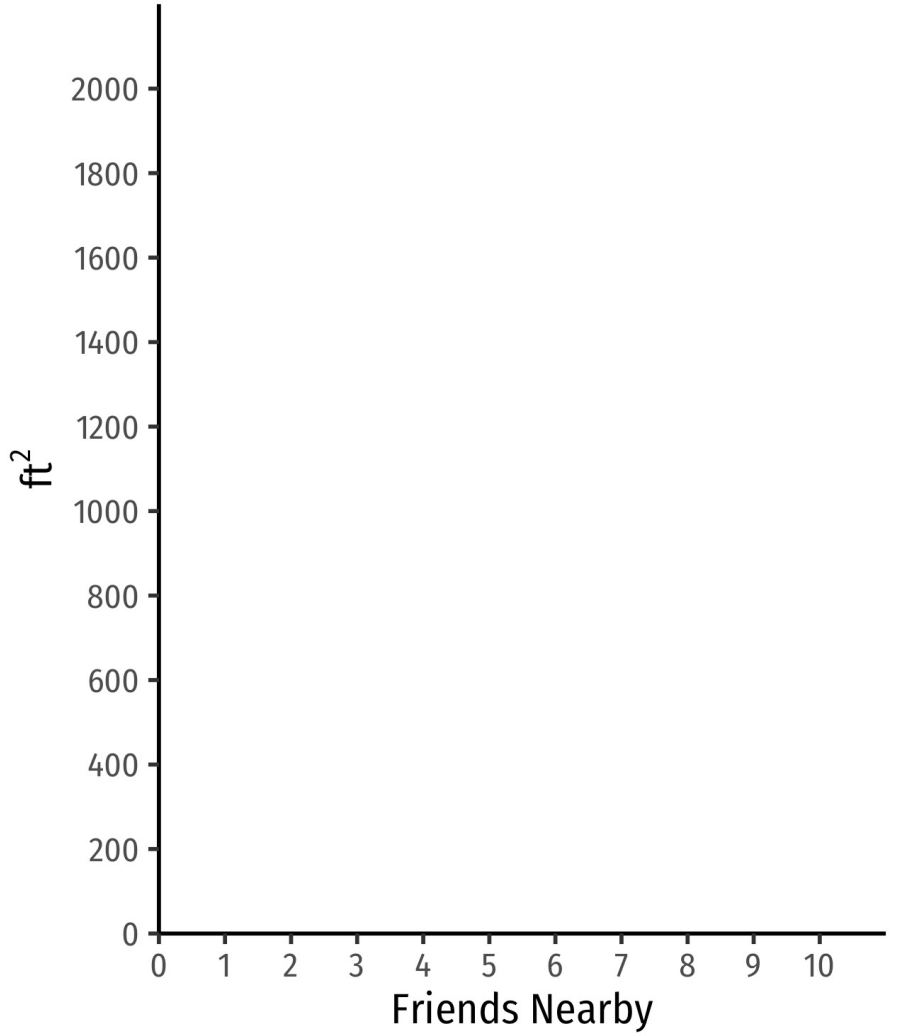


- 2 - d



Indifference Curves

- **Example**: Apartment search
  - Size of the apartment
  - Number of friends that live nearby
- Apt. *A* has 1 friend nearby and is 1,200 ft^2
  - Larger apartments and/or more friends $\succ A$
  - Smaller apartments and/or fewer friends $\prec A$



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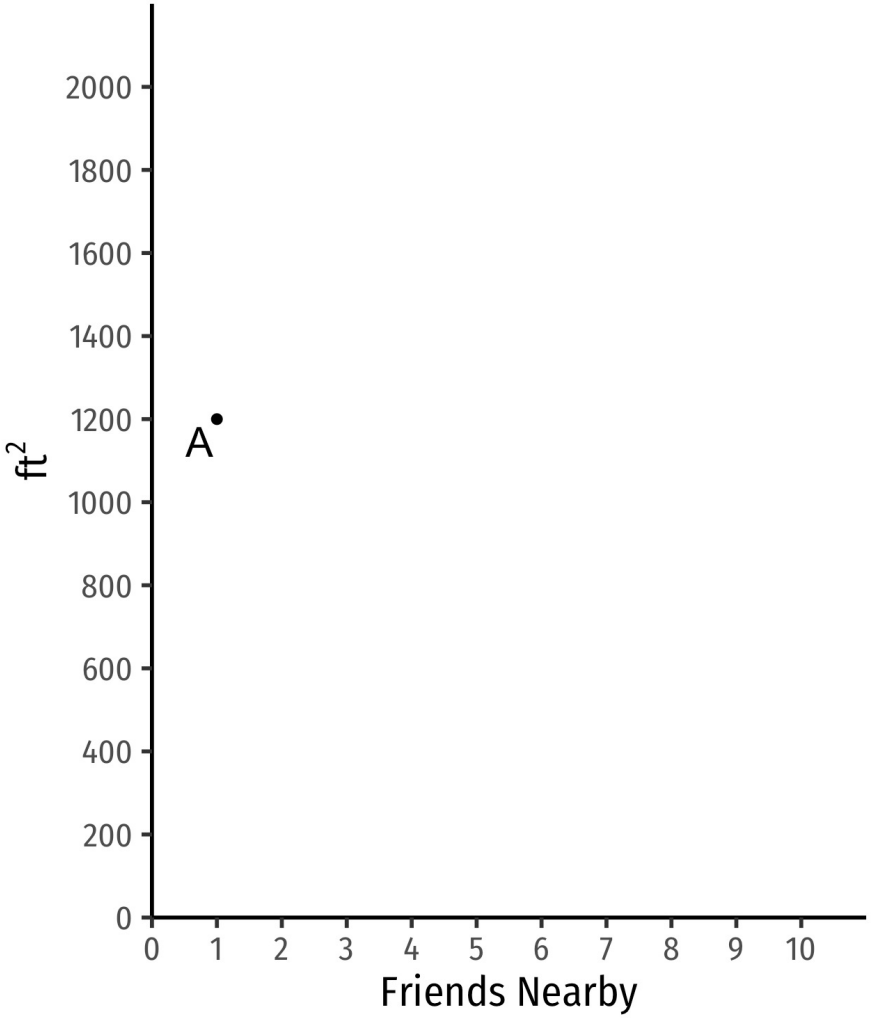

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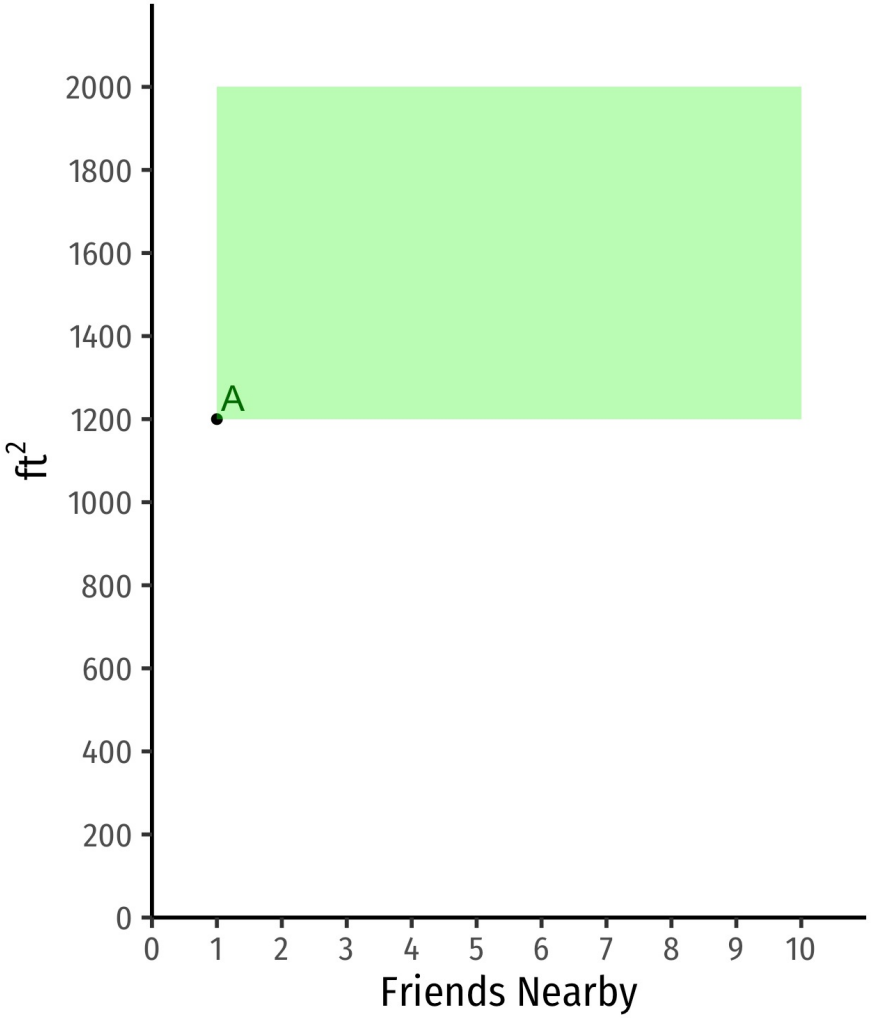

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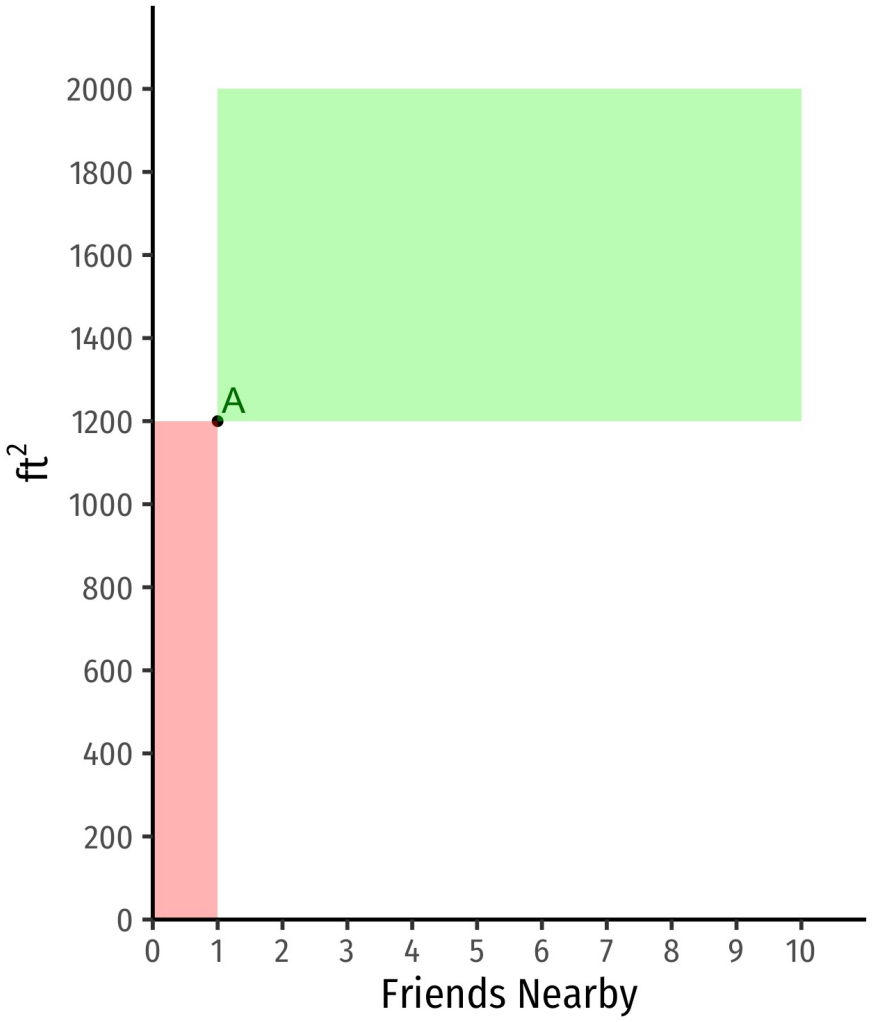
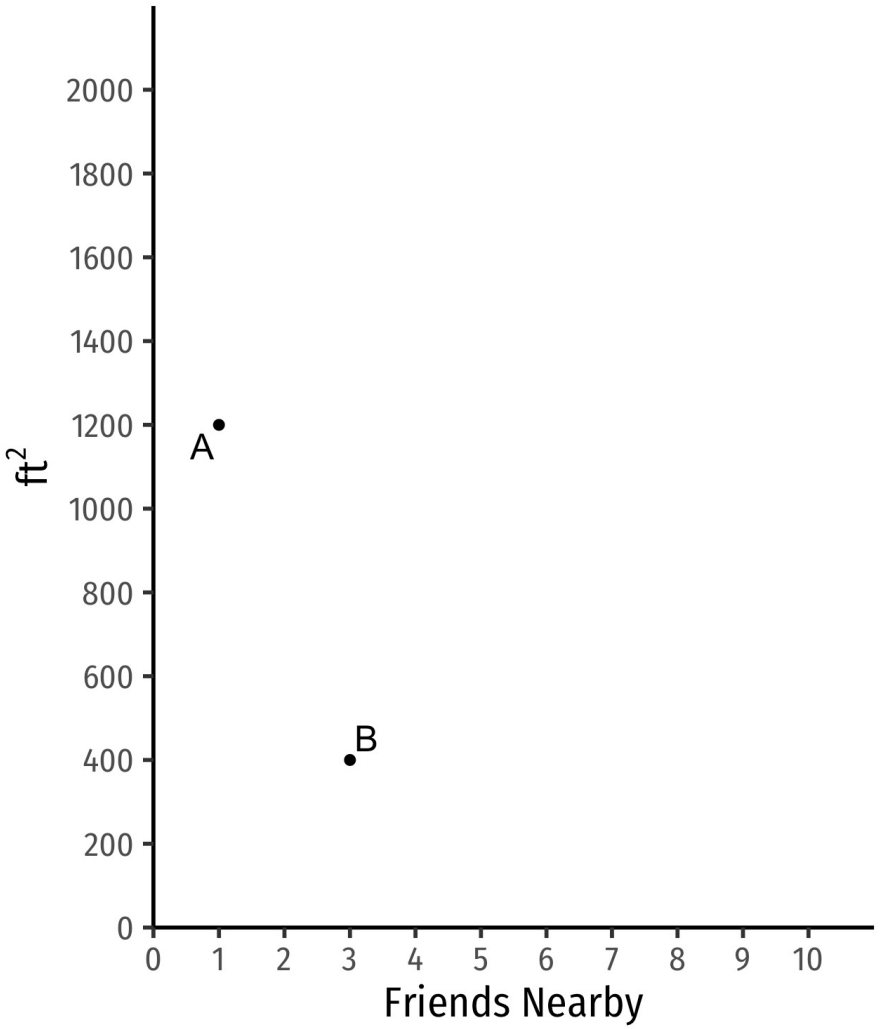


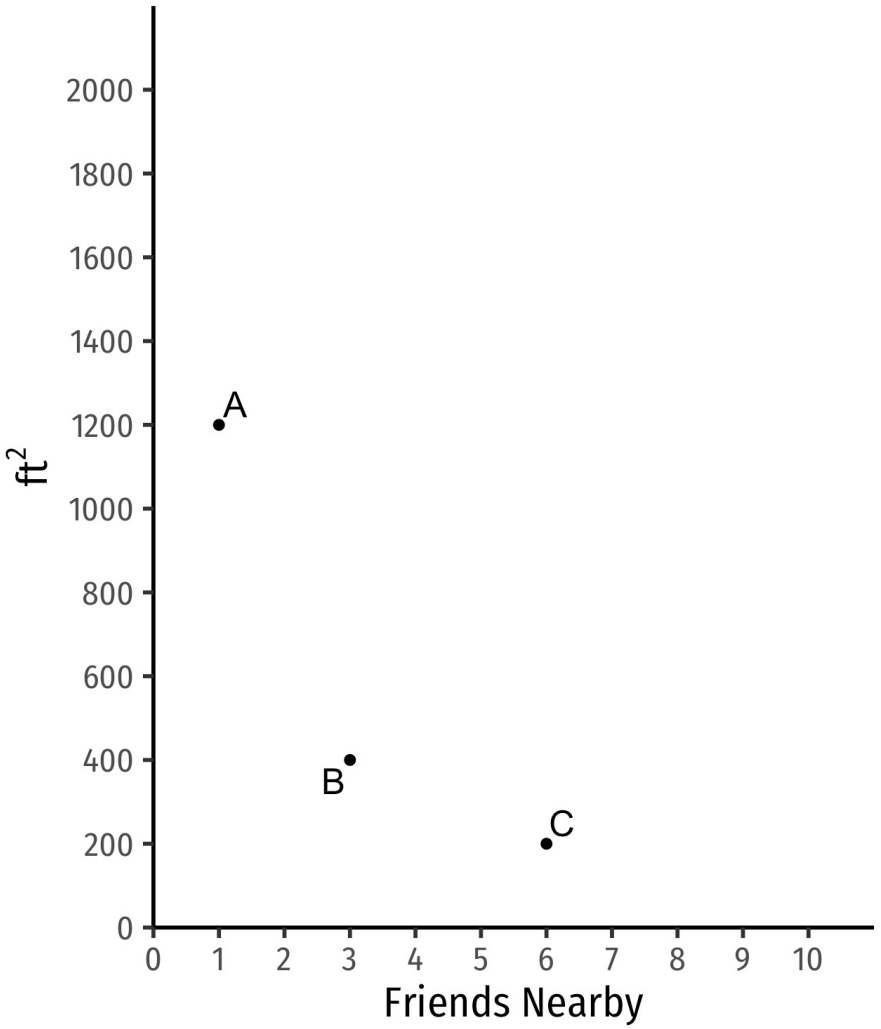
Indifference Curves

- Apt. *A* has 1 friend nearby and is 1,200 ft^2
- Apt. *B* has *more* friends but *less* ft^2



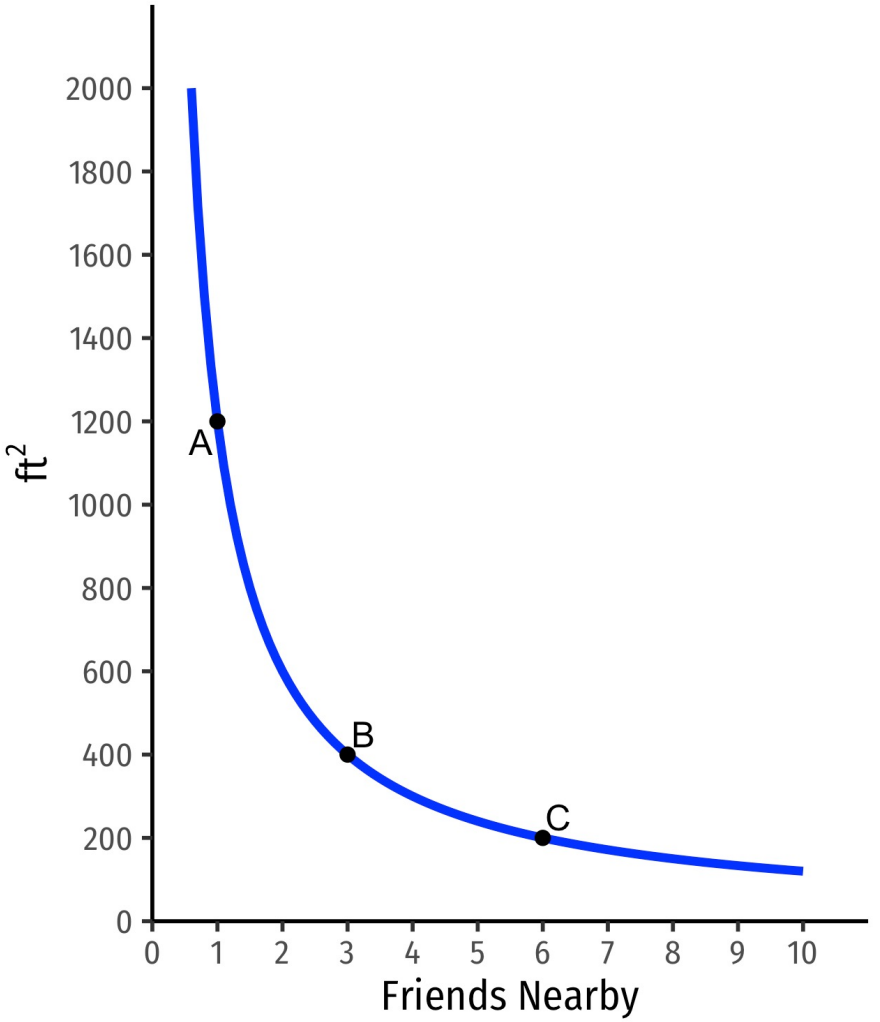
Indifference Curves

- Apt. *A* has 1 friend nearby and is 1,200 ft^2
- Apt. *B* has *more* friends but *less* ft^2
- Apt. *C* has *still more* friends but *less* ft^2



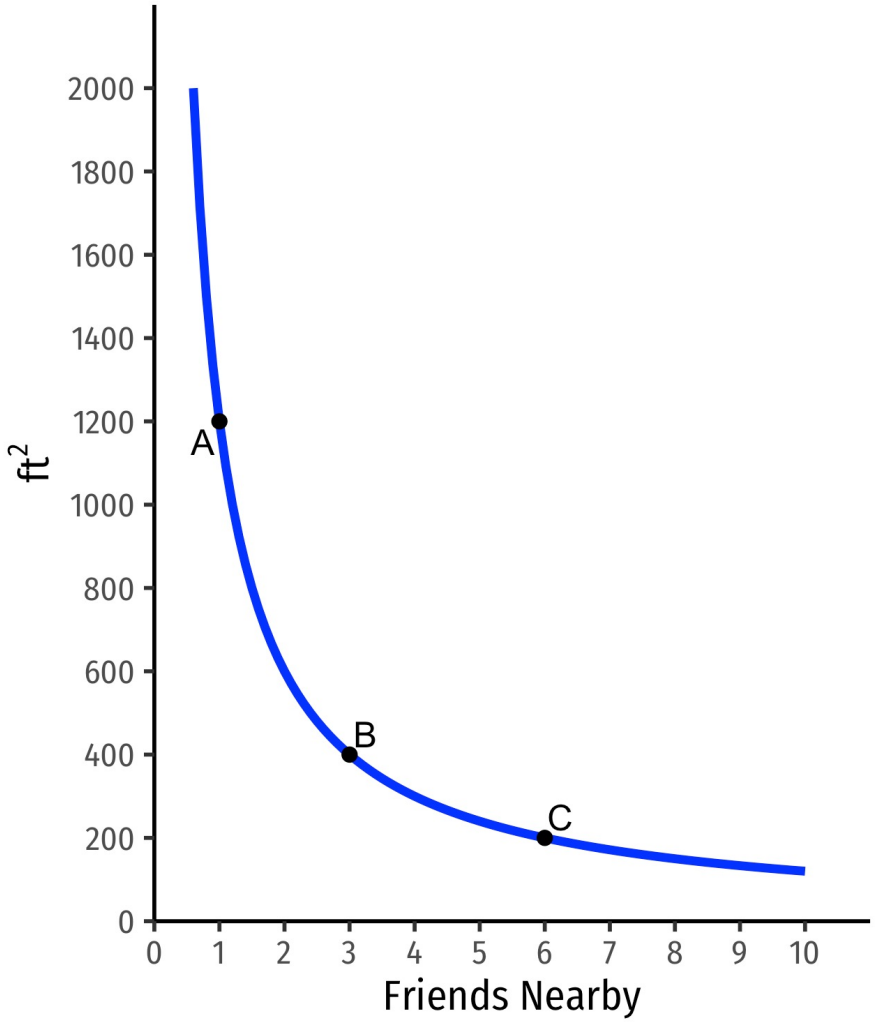
Indifference Curves

- Apt. *A* has 1 friend nearby and is 1,200 ft^2
- Apt. *B* has *more* friends but *less* ft^2
- Apt. *C* has *still more* friends but *less* ft^2
- If A~B~C, on same **indifference curve**



Indifference Curves

- **Indifferent** between all apartments on the **same** curve



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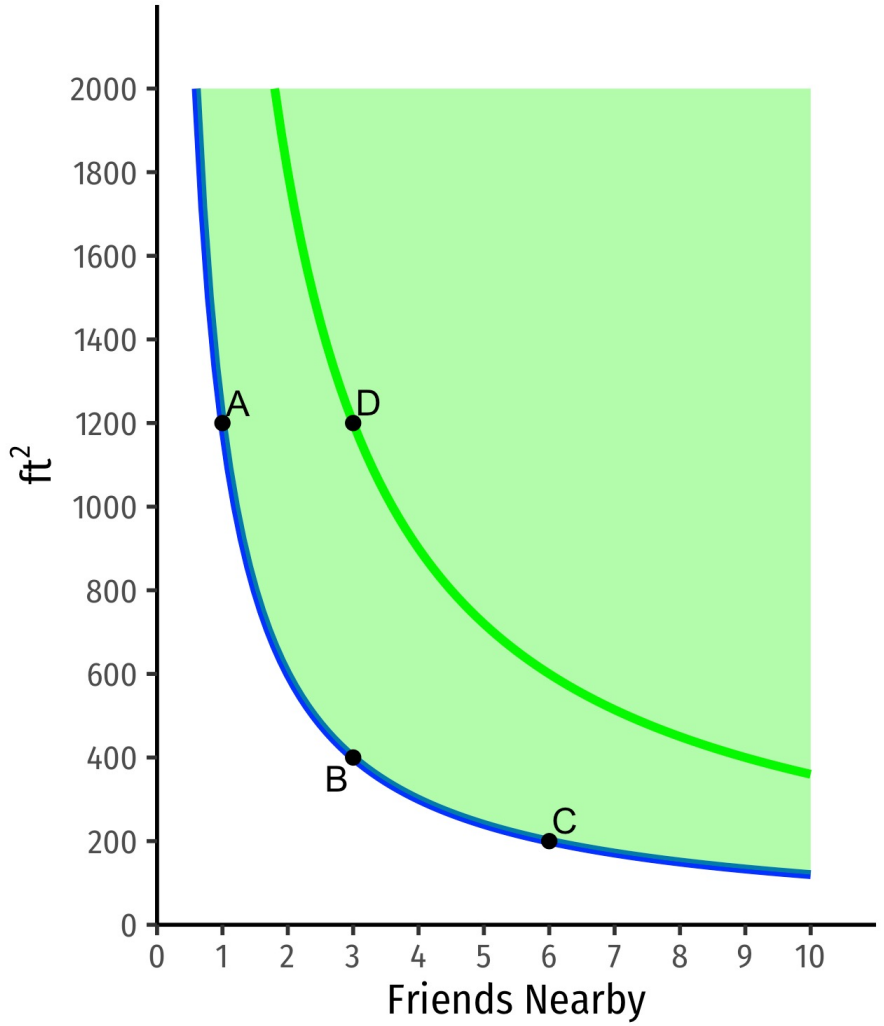
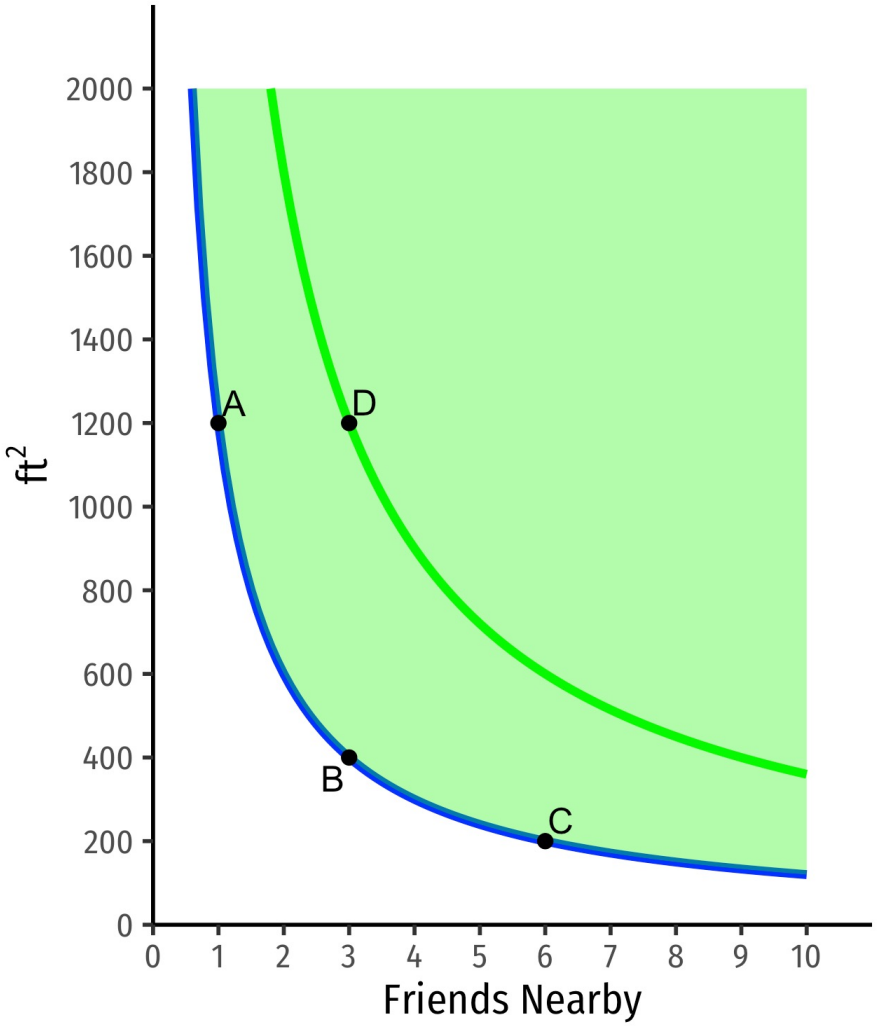


Indifference Curves

- **Indifferent** between all apartments on the **same** curve

- Apts **above** curve are **preferred over** apts on curve
  - D ≻ A ~ B ~ C
  - On a **higher curve**

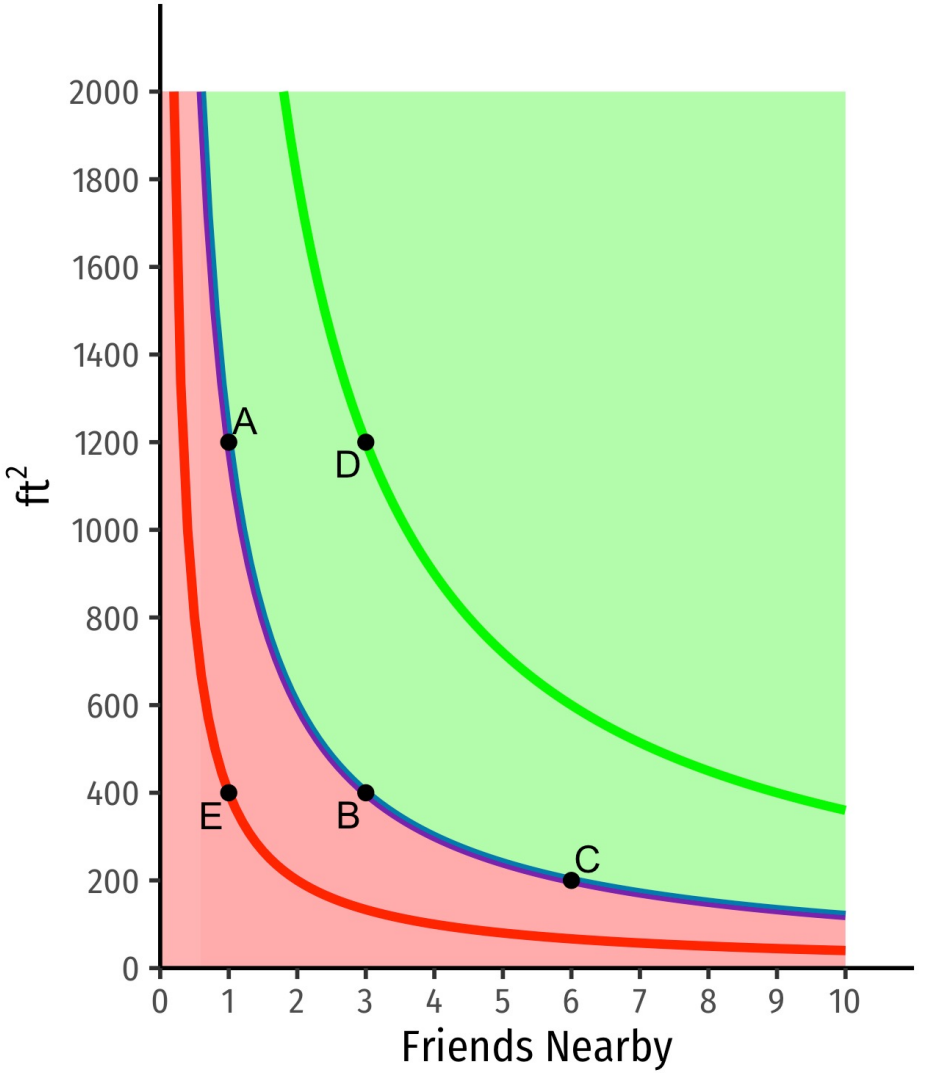


Indifference Curves

- **Indifferent** between all apartments on the **same** curve

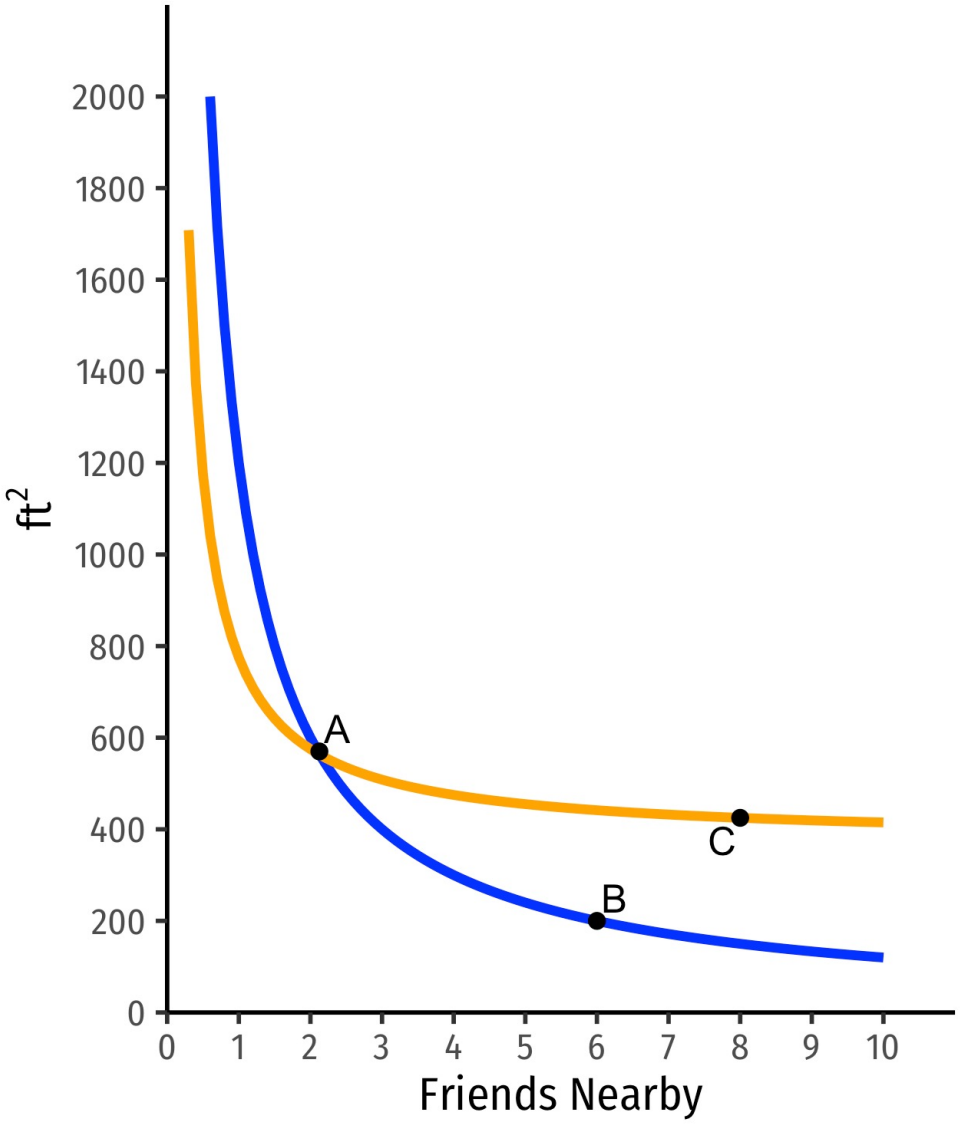
- Apts **above** curve are **preferred over** apts on curve
  - D ≻ A ~ B ~ C
  - On a **higher curve**

- Apts **below** curve are **less preferred** than apts on curve
  - E ≺ A ~ B ~ C
  - On a **lower curve**



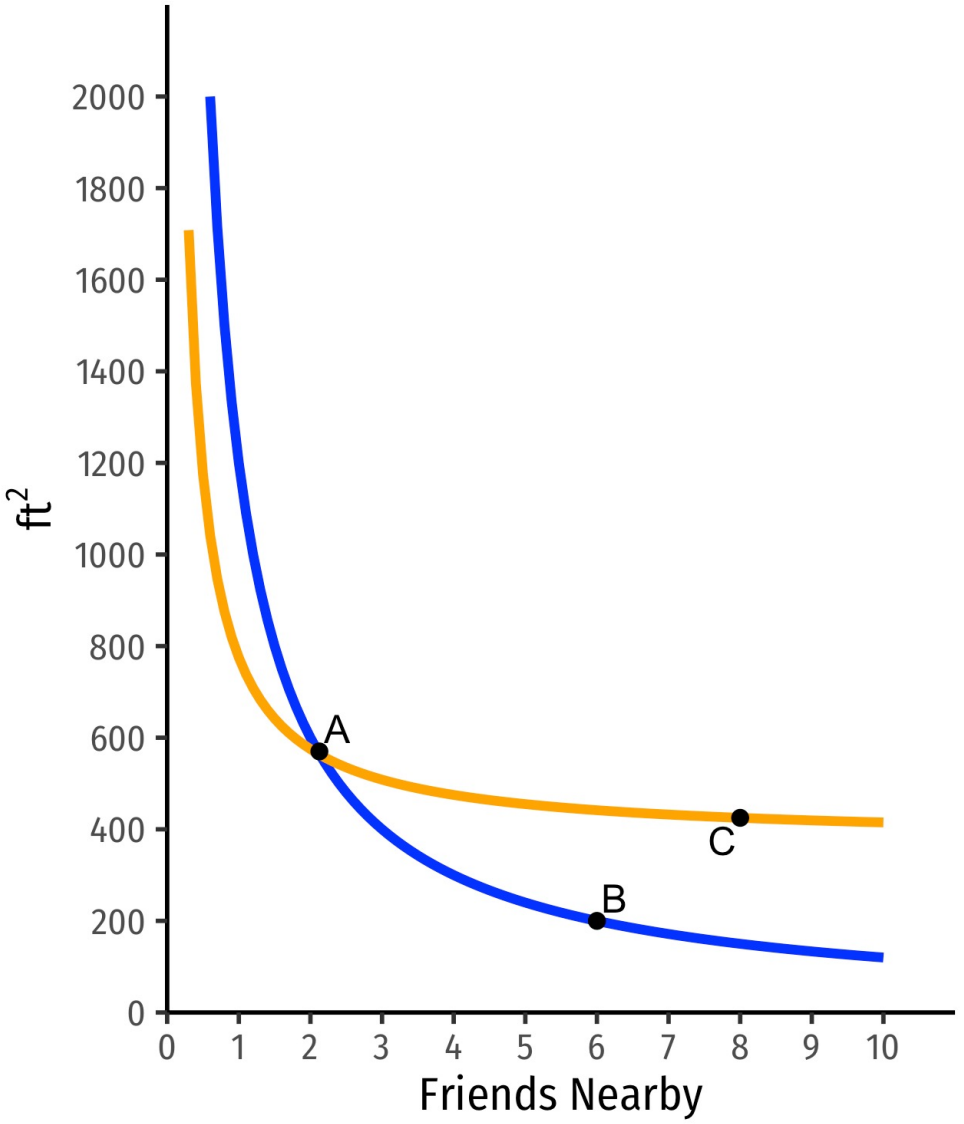
Indifference Curves

- **Indifference curves can never cross**: preferences are transitive
  - If I prefer A>B, and B>C
  - I must prefer A>C



Indifference Curves

- **Indifference curves can never cross**: preferences are transitive
  - If I prefer A≻B, and B≻C
  - I must prefer A≻C
- Suppose two curves crossed:
  - A~B
  - B~C
  - But C ≻ B!
  - Preferences are not transitive!



Marginal Rate of Substitution

- Take away one friend nearby, how many more ft^2 to stay **indifferent**?

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- **Marginal Rate of Substitution (MRS)**: rate at which you trade off one good for the other and remain *indifferent*

- Think of this as your **opportunity cost**: # of units of y you need to give up to acquire 1 more x

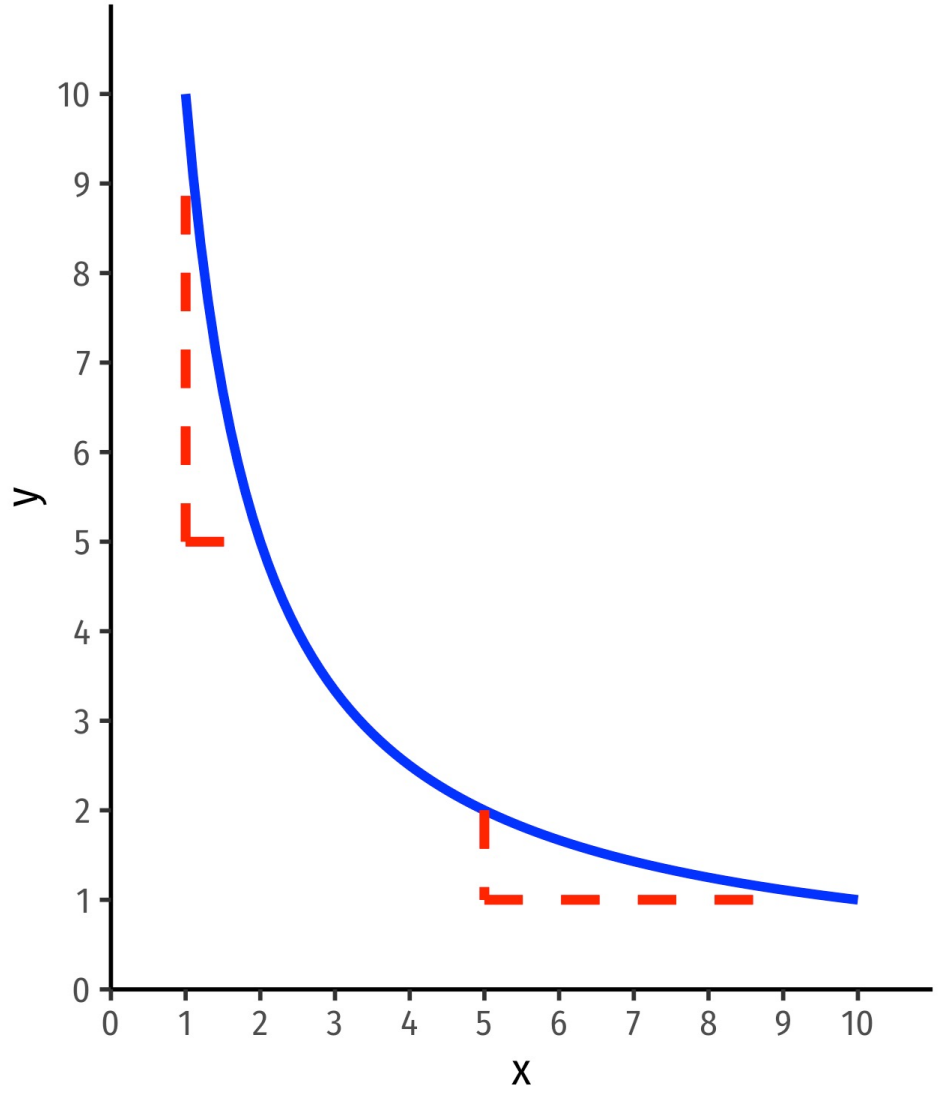
MRS vs. Budget Constraint Slope

- Budget constraint (slope) measured the **market's** tradeoff between x and y based on market prices

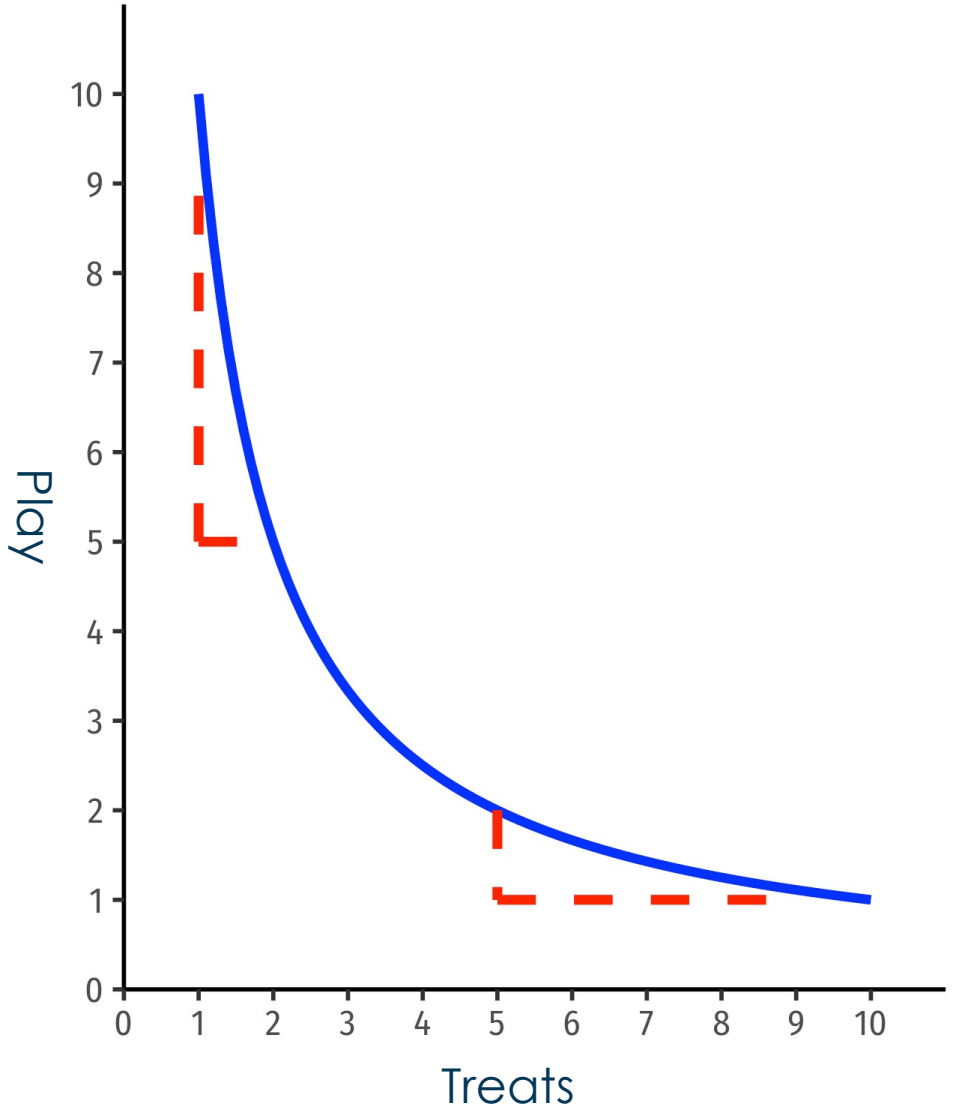
- **MRS** measures your **personal** evaluation of x vs. y based on your preferences

Marginal Rate of Substitution

- MRS is the slope of the indifference curve
- $MRS_{x,y} = -\frac{\Delta y}{\Delta x} = \frac{rise}{run}$
- Amount of y given up for 1 more x
- Note: slope (MRS) changes along the curve!



Marginal Rate of Substitution



Utility

Utility Functions?

- A utility function $u(\cdot)$ shows the preference $(\succ, \prec, \sim)$
- Assign utility numbers to bundles, such that, for any bundles a and b:
- $a \succ b \Leftrightarrow u(a) > u(b)$

Utility Functions?

- We can model "**as if**" the consumer is maximizing utility/preferences by **maximizing the utility function**:

- **"Maximizing preferences"**: choosing a such that a $\succ$ b for all available b

- **"Maximizing utility"**: choosing a such that u(a) > u(b) for all available b

Utility Functions, Plural

- Imagine three alternative bundles of (x,y):

- a = (1,2)
- b = (2,2)
- c = (4,3)

Utility Functions, Plural

- Imagine three alternative bundles of (x,y):

- a = (1,2)
- b = (2,2)
- c = (4,3)

- Create a utility function u(·) that assigns each bundle a utility level of

- u(a) = 1
- u(b) = 2
- u(c) = 3

- Does it mean that bundle c is 3 times the utility of a?

Utility Functions, Plural

- Imagine three alternative bundles of (x,y):

- a = (1,2)
- b = (2,2)
- c = (4,3)

- Create a utility function u(·) that assigns each bundle a utility level of

- u(a) = 1 v(a) = 3
- u(b) = 2 v(b) = 5
- u(c) = 3 v(c) = 7

Utility Functions, Plural

- Utility numbers have an **ordinal** meaning only, **not cardinal**

  - **Only the <u>ordering</u>** c>b>a **matters!**

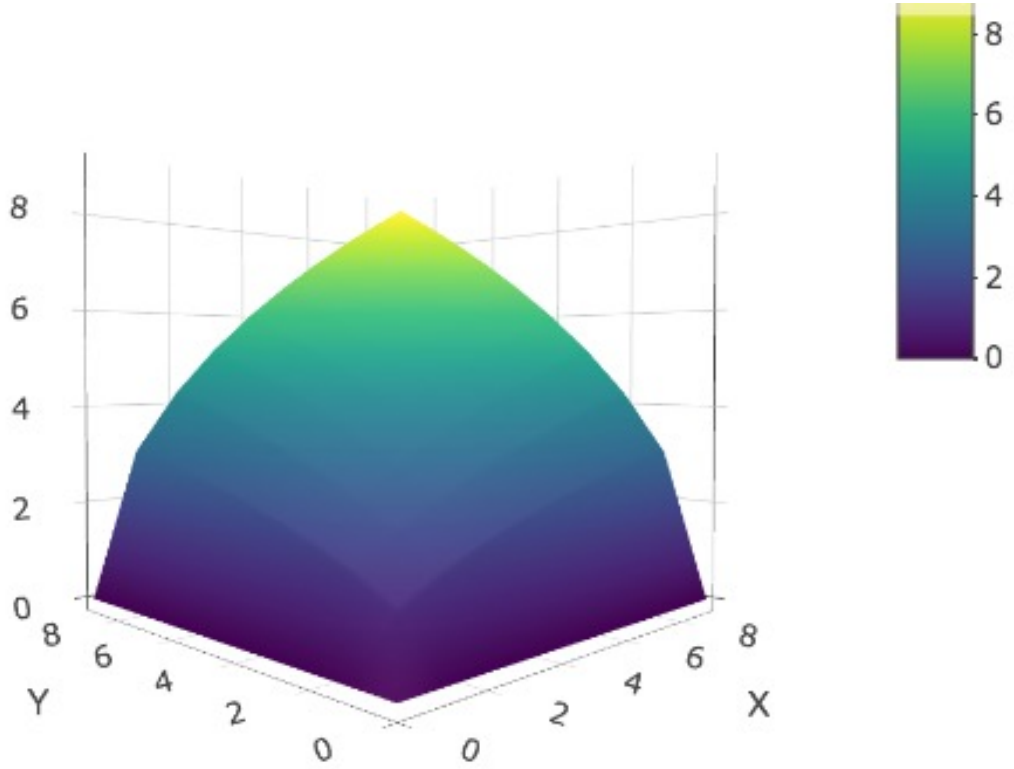
- Both are valid:
  - u(c)>u(b)>u(a)
  - v(c)>v(b)>v(a)

Utility Functions, Plural

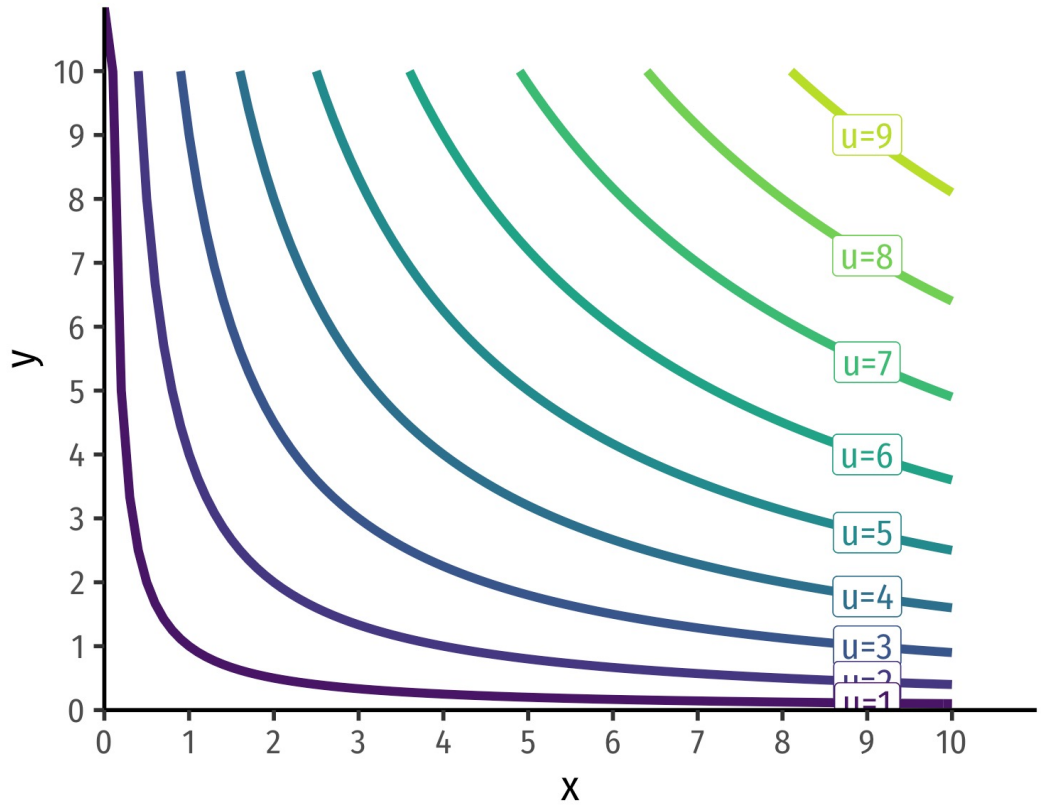
- Representing preferences:
  - **indifference curves**
  - **utility functions**
- Indifference curve: all **equally preferred** bundles ⇔ **same utility level**
- Each indifference curve represents one level (or contour) of utility surface (function)

The Budget Set & the Budget Constraint

- 3-D Utility Function:
- $u(x, y) = \sqrt{xy}$



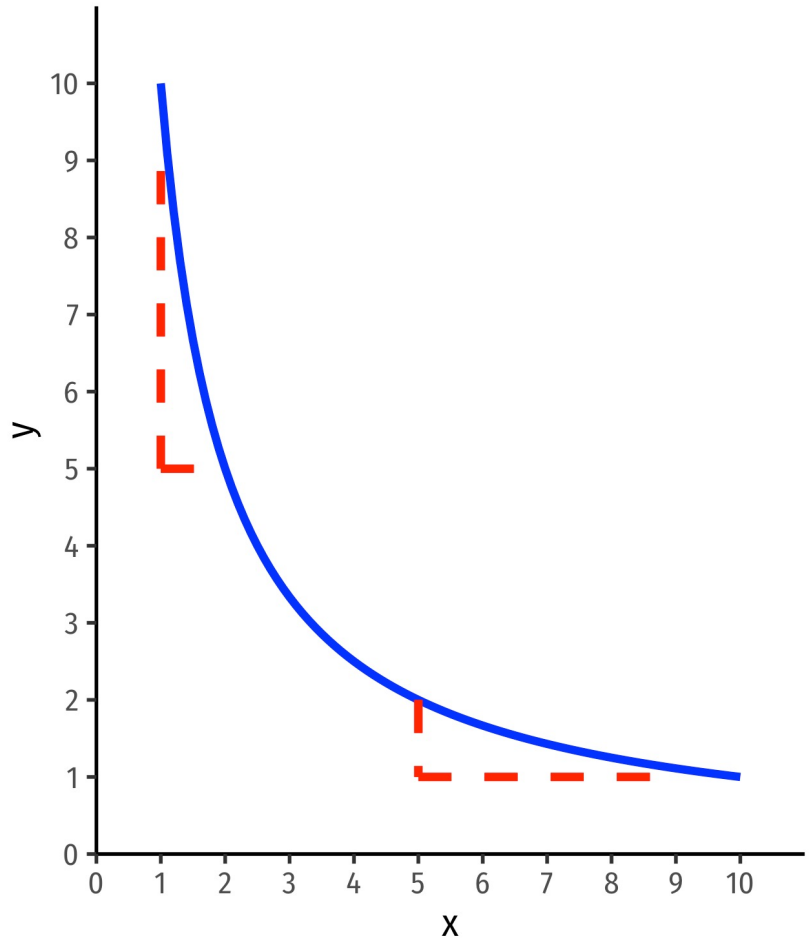
- 2-D Utility Function:
- $y = \frac{u^2}{x}$



Marginal Utility

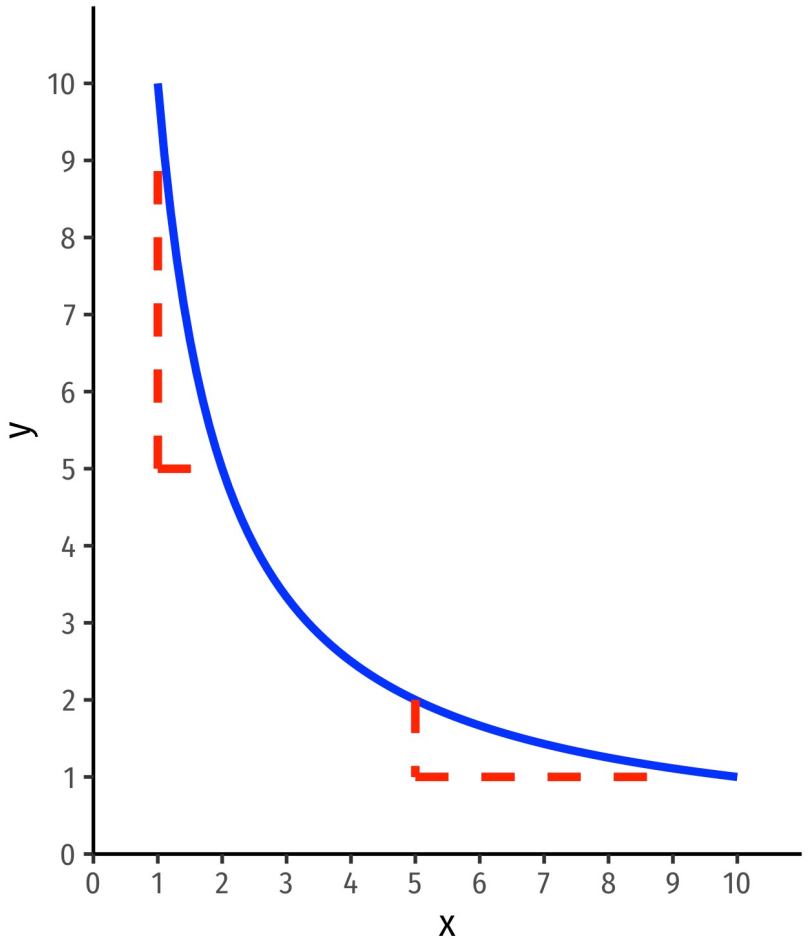
MRS and Marginal Utility

- Recall: **marginal rate of substitution** $MRS_{x,y}$ is slope of the indifference curve
  - Amount of y given up for 1 more x
- How to calculate MRS?
- Recall it changes (not a straight line)!
- We can calculate it using something from the **utility function**



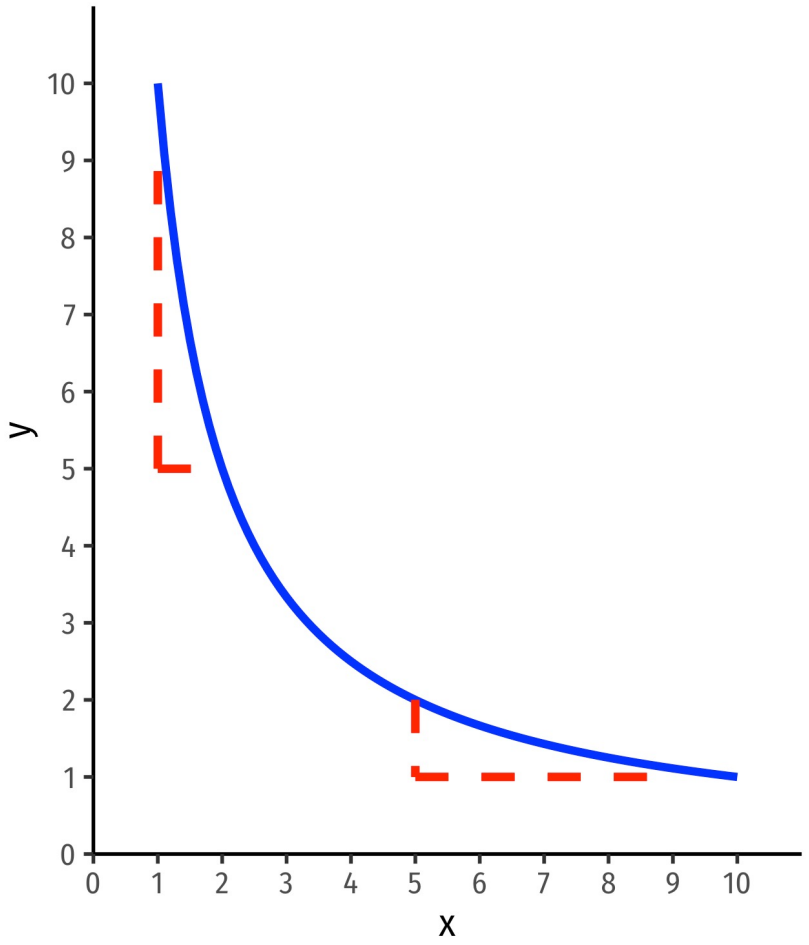
MRS and Marginal Utility

- **Marginal utility**: change in utility from a marginal increase in consumption



MRS and Marginal Utility

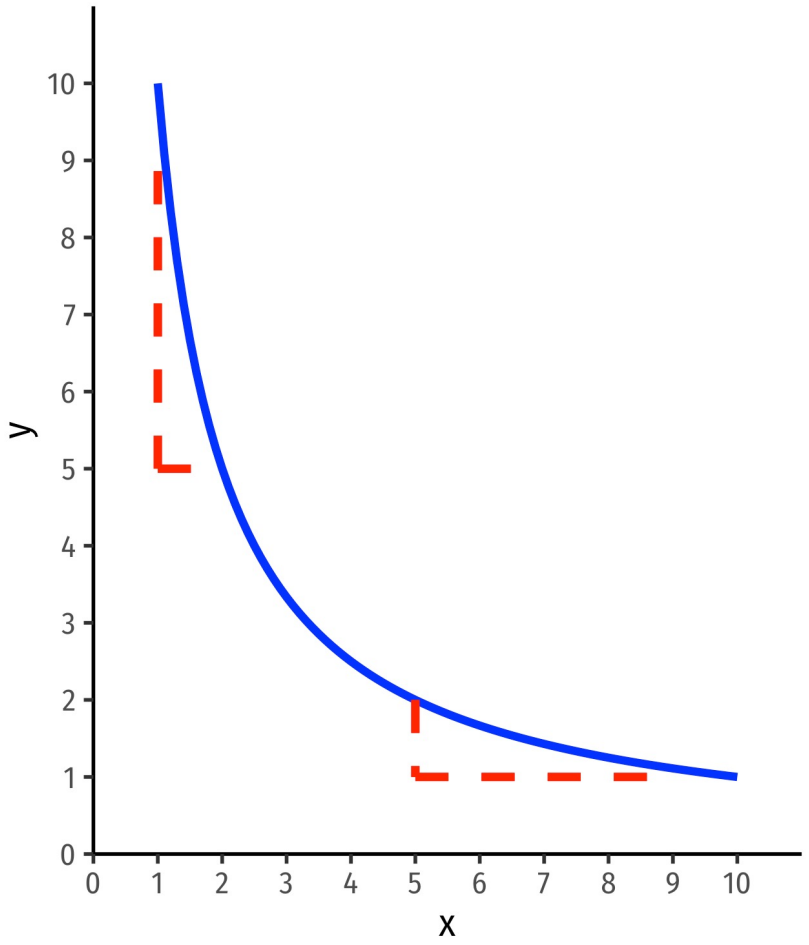
- **Marginal utility**: change in utility from a marginal increase in consumption
  - **Marginal utility of** x: $MU_x = \frac{\Delta u(x,y)}{\Delta x}$
  - **Marginal utility of** $MU_y = \frac{\Delta u(x,y)}{\Delta y}$



MRS and Marginal Utility

- **Marginal utility**: change in utility from a marginal increase in consumption

- Math (calculus): "*marginal*" means "*derivative with respect to*"



Marginal Utility

For example:

$$u(x, y) = x^2 + y^3$$

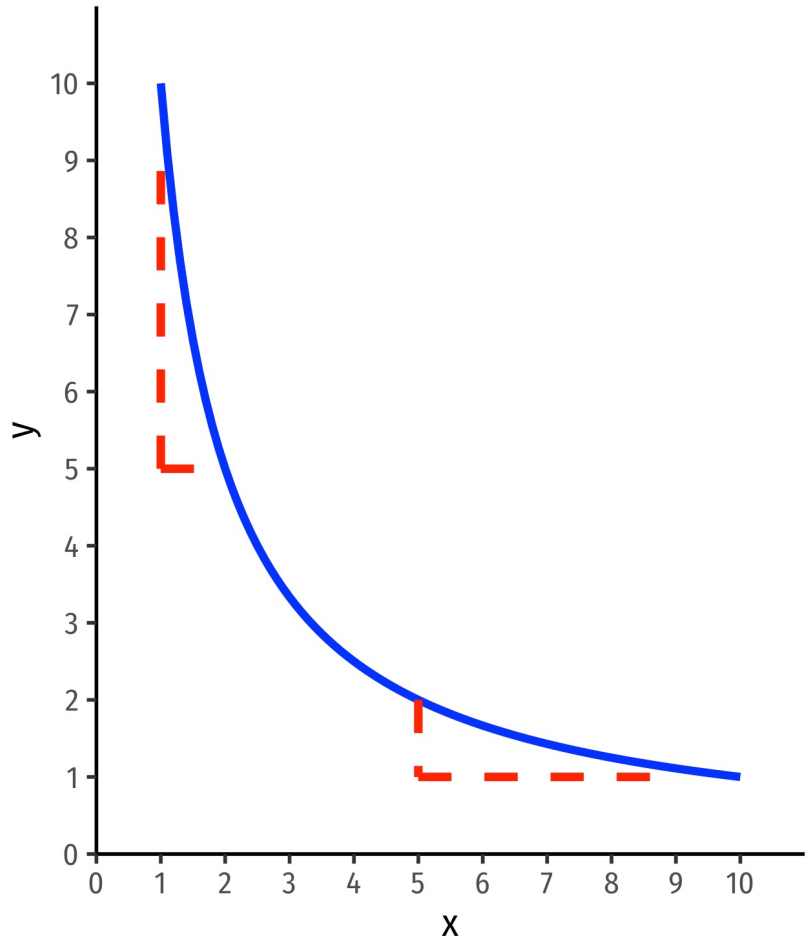
Marginal utility of x: $MU_x = 2x$

Marginal utility of y: $MU_y = 3y^2$

MRS and Marginal Utility

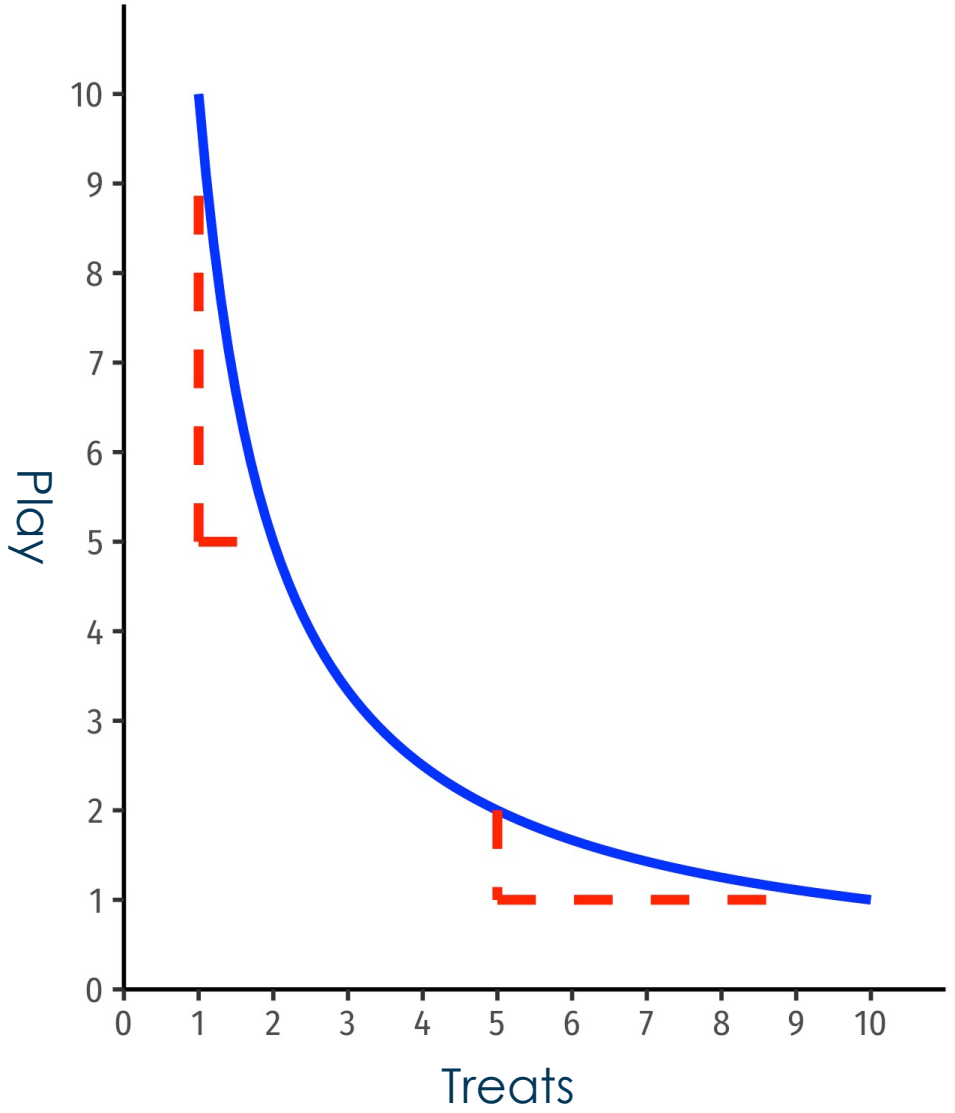
- Relationship between MU and MRS:

- $(x, y) = \frac{\Delta y}{\Delta x} = -\frac{MU_x}{MU_y}$

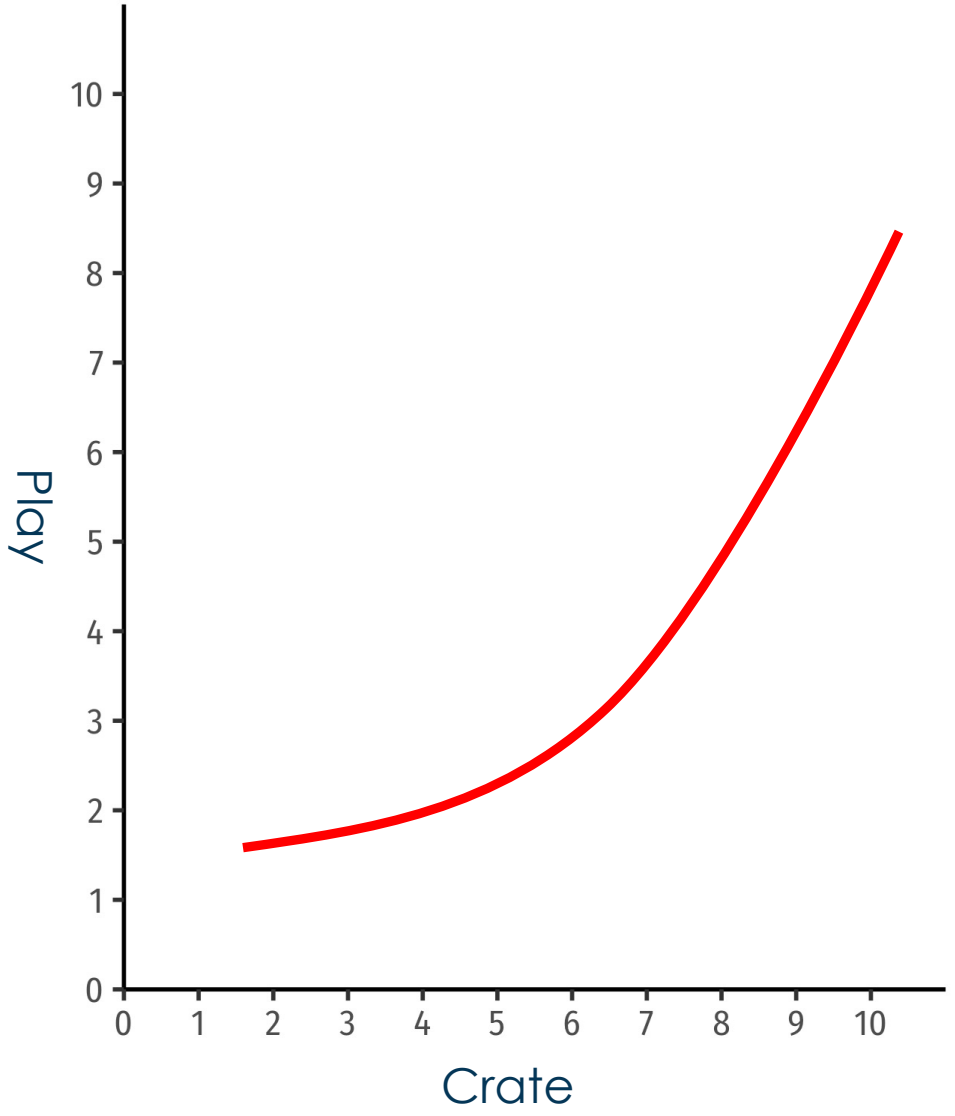


MRS and Preferences

MRS and Preferences: “Goods” and “Bads”

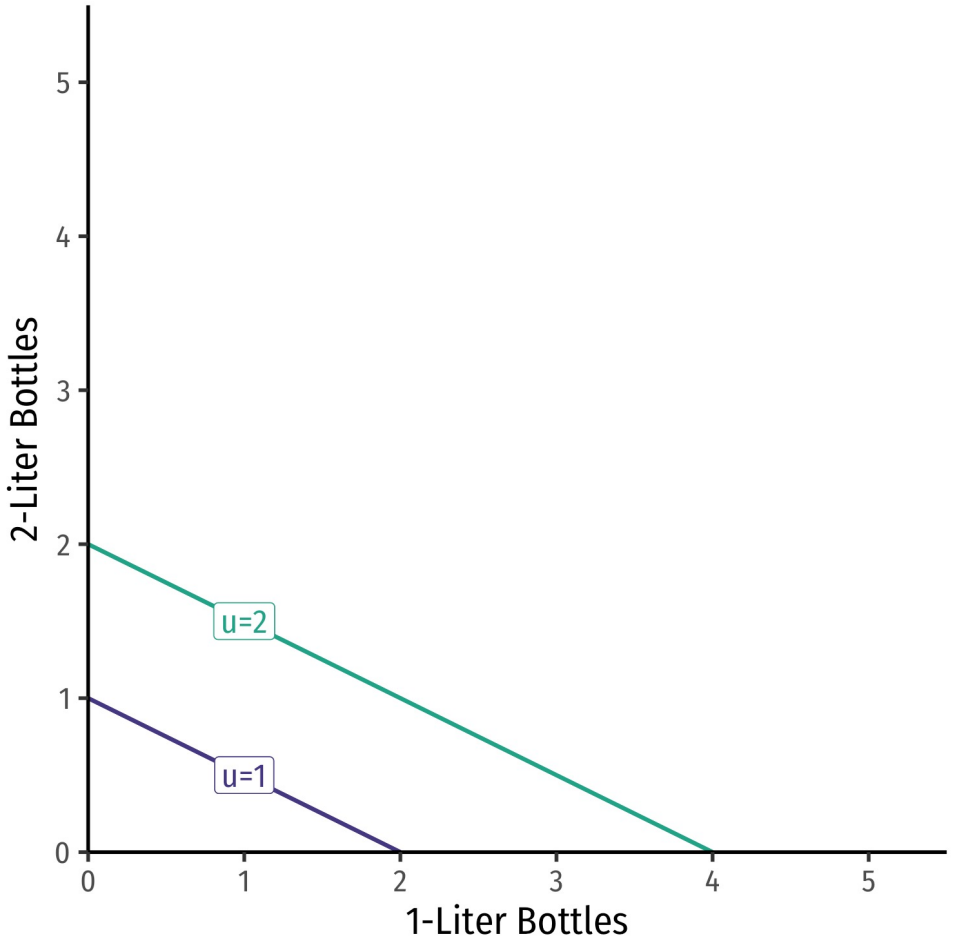


MRS and Preferences: “Goods” and “Bads”



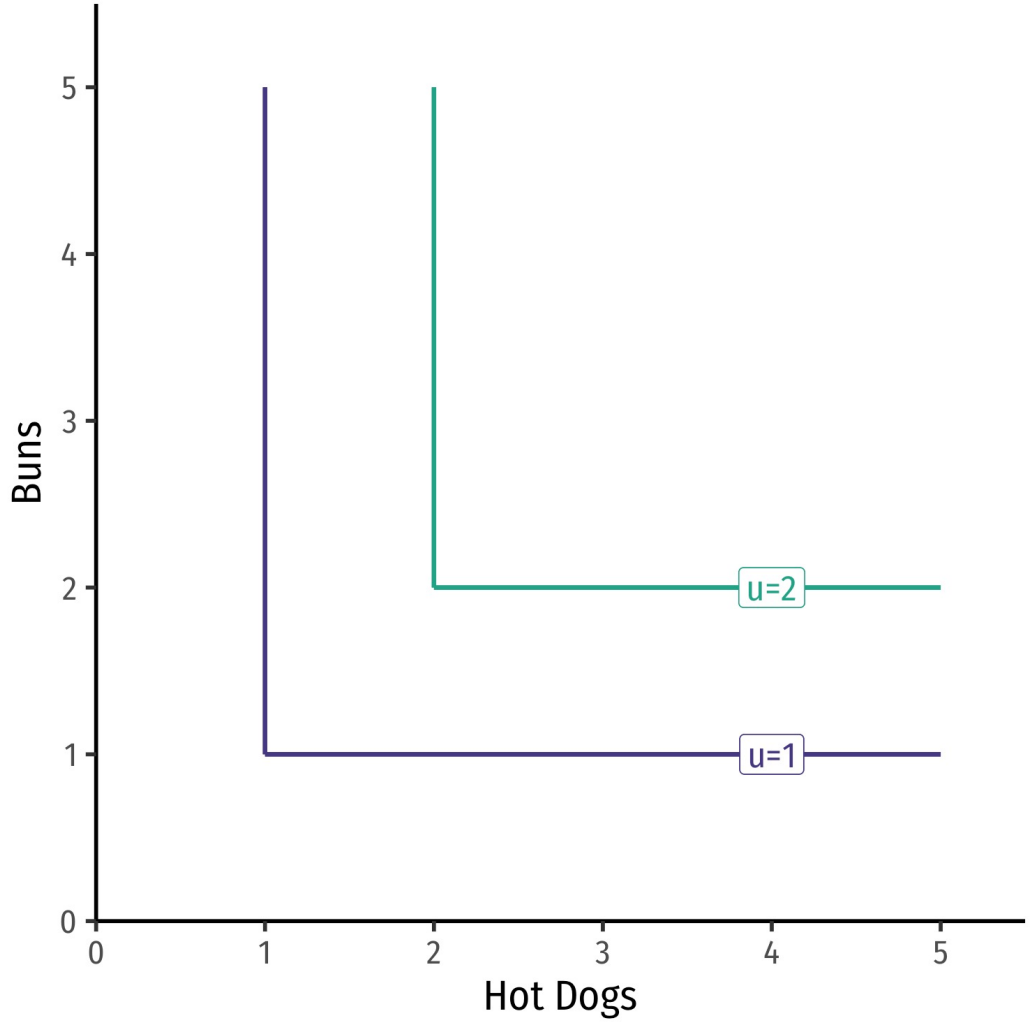
MRS and Preferences: Substitutes

- **Example**: Consider 1-Liter bottles of coke and 2-Liter bottles of coke
- Always willing to substitute between Two 1-L bottles for One 2-L bottle
- **Perfect substitutes**: goods that can be substituted at same fixed rate and yield same utility
- $MRS_{1L,2L} = -0.5$ (a constant!)



MRS and Preferences: Substitutes

- **Example**: Consider hot dogs and hot dog buns
- Always consume together in fixed proportions (in this case, 1 for 1)
- **Perfect complements**: goods that can be consumed together in same fixed proportion and yield same utility
- $MRS_{1L,2L} = ?$



Cobb-Douglas Utility Functions

- A very common functional form in economics is **Cobb-Douglas**

- $u(x, y) = x^a y^b$

- Where a, b > 0 (and very often a + b = 1