Homework 3: Answers

1.) An equilibrium in which each player is doing the best it can given the actions taken by it's competitors.

Thus, in a Nash equilibrium neither player has an incentive to change their decision unilaterally.

2.) Bertrand competition is when players compete with eachother based on the price of a product. In this case, each player has an incentive to under cut their competitor. The equilibrium under Bertrand competition, when products are identical, is for price to equal marginal cost.

3) $P = 100 - q_P - q_G$ marginal cost of production for each is \$20

A) Think about marginal revenue and marginal cost.
They are equal to each other in the optimal decision $MR = MC$

Grenada: $MR_G = 100 - q_P - 2q_G \overset{MC}{=} 20$

Reaction function $q_G(q_P) \Rightarrow q_G = 40 - 0.5q_P$

Penang $q_P = 40 - 0.5q_G$

B) Solving for equilibrium

Grenada $q_G = 40 - 0.5q_P = 40 - \underbrace{0.5(40 - 0.5q_G)}_{\text{embedded others reaction function}}$

$q_G = 20 + 0.25q_G$

$\boxed{q_G^* = 26.67}$

same idea for Penang $\boxed{q_P^* = 26.67}$

Market Price :

$$P = 100 - q_P - q_G$$

$$P = 100 - 26.67 - 26.67$$

$$\boxed{P^* = \$46.67}$$

Grenada

Profits : $TR_G - TC_G = (P - AC_G) * Q_G$

$$= (46.67 - 20) * 26.67 = \boxed{\$711.11}$$

Same thing for Penang $\quad \Pi_P = \$711.11$

c.) Stackelberg competition

Grenada is the first mover. They already know <u>Penguin's</u> optimal <u>reaction</u> $\boxed{q_P = 40 - 0.5q_G}$.

Grenada should use this in their decision.

i) $P = 100 - q_P - q_G = 100 - (40 - 0.5q_G) - q_G$

$= 60 - 0.5q_G$

ii) Output determined by $MR_G = MC_G$

$TR_G = (60 - 0.5q_G) * q_G$

$MR_G = 60 - q_G = 20$

$$\boxed{q_G^* = 40}$$

iii) $q_P = 40 - 0.5q_G = 40 - 0.5 * 40 = \boxed{20}$

iv.) $P = 100 - q_P - q_G = 100 - 20 - 40 = \$40$

The industry quantity is greater in stackleberg compared to ~~Cournot~~. The price is higher in Cournot.

v.) Grenada $\Pi = (40 - 20) * 40 = 800$

Penang $\Pi = (40 - 20) * 20 = 400$

Grenada has a first mover advantage in stackleber, which results in higher profits. Penang has considerably lower profits for stackleberg.

4) $P = 100 - 0.5Q$ $\quad Q = q_j + q_A$

$MC_j = MC_A = \$20$ $\quad$ (Total Cost_A = Total $\text{cost}_j = 20q$

A) Monopolist

$P \cdot Q - \text{cost}$

$\pi = (100 - 0.5Q) * Q - 20Q$

$\frac{\partial \pi}{\partial Q} = 100 - Q - 20 = 0$

$Q^* = 80$ $\quad$ price $= 100 - 0.5 * (80) = \$60$

b) i) $q_j = 40$

$P = 100 - 0.5(40 + q_A) = \boxed{80 - 0.5q_A}$

ii) Annie would max profit

$\pi_A = (80 - 0.5q_A)q_A - 20q_A$

$\frac{\partial \pi_A}{\partial q_A} = 80 - q_A - 20 = 0$ $\quad \boxed{q_A^* = 60}$

c) $Q = q_i + q_A = 40 + 60 = 100$

$\therefore \quad P = 100 - 0.5Q = 100 - 0.5 * 100 = \$50$

$\quad\quad P \quad - \quad AC_A \quad * \quad q_A$

$\pi_A = (\$50 - 20) * 60 = 1{,}800$

alternatively $50 \cdot 60 - 20 * 60$

$P \cdot q_A - \text{cost}$

$\pi_B \quad (50 - 20) \times 40 = 1{,}200$