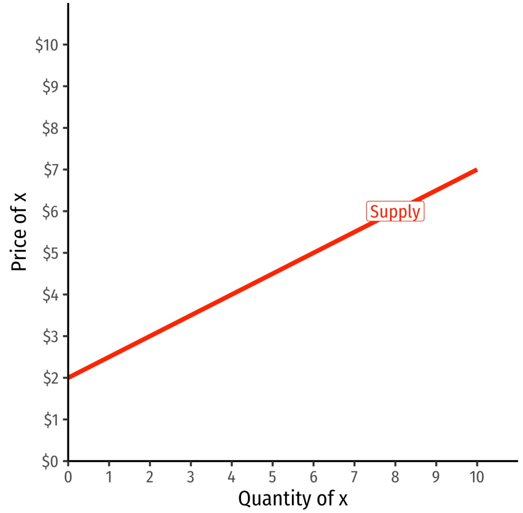
OLD DOMINION UNIVERSITY

Theory of the Firm

Tim Komarek
Department of Economics
Dragas Center for Economic Analysis and Policy
Strome College of Business

Producer Behavior

- How do **producers** decide:
  - which products to produce
  - in what quantity
  - using which resources
  - and for what price?
- Building blocks for **supply curves**
- Field of economics called **Industrial Organization**



Modeling firm decisions

What Do Firms Do?

- 1st Stage: **firm's profit maximization problem:**

1) Choose: < output >

2) In order to maximize: < profits >

What Do Firms Do?

- 1st Stage: **firm's profit maximization problem:**

1) Choose: < output >

2) In order to maximize: < profits >

- 2nd Stage: **firm's cost minimization problem:**

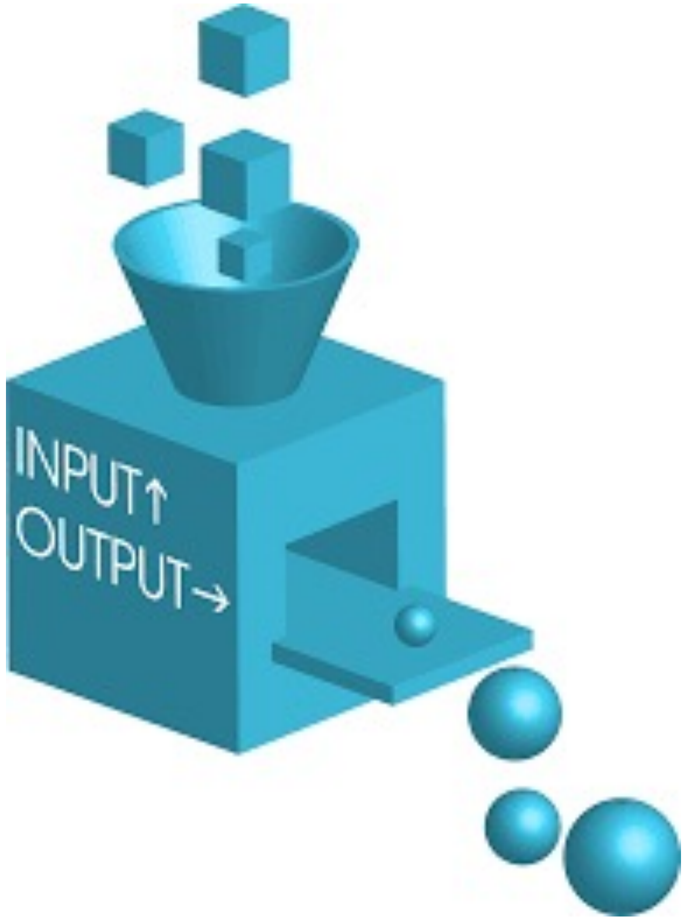
1) Choose: < inputs >

2) In order to *minimize*: < cost >

3) Subject to: < producing the optimal output >

What Do Firms Do?

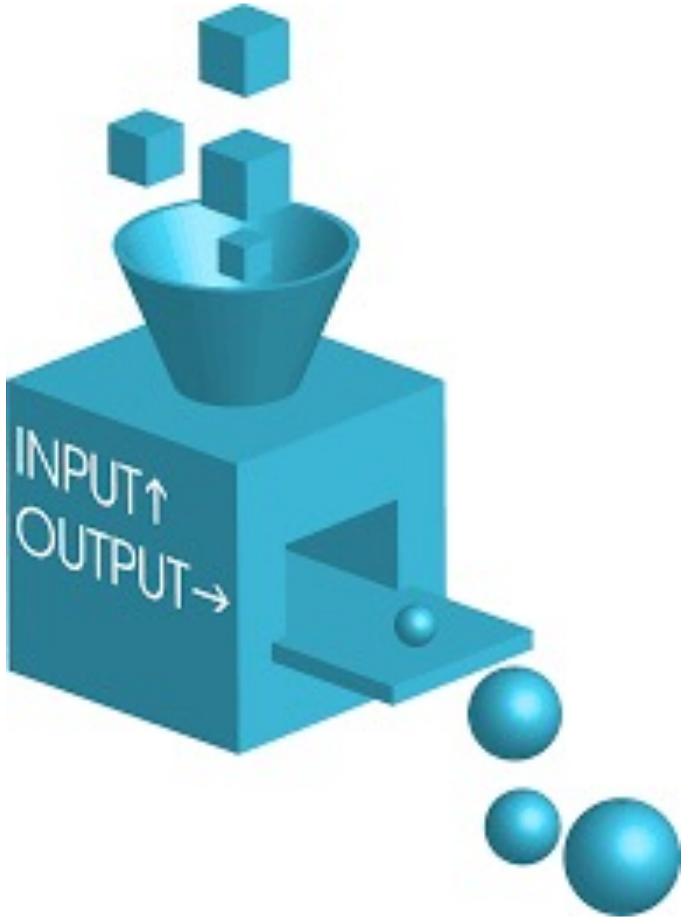
- Firms convert some goods to other goods:



What Do Firms Do?

- Firms convert some goods to other goods:

- **Inputs**: $x_1, x_2, \cdots, x_n$
  - **Examples**: worker efforts, warehouse space, electricity, loans, oil, cardboard, fertilizer, computers, software programs, etc.

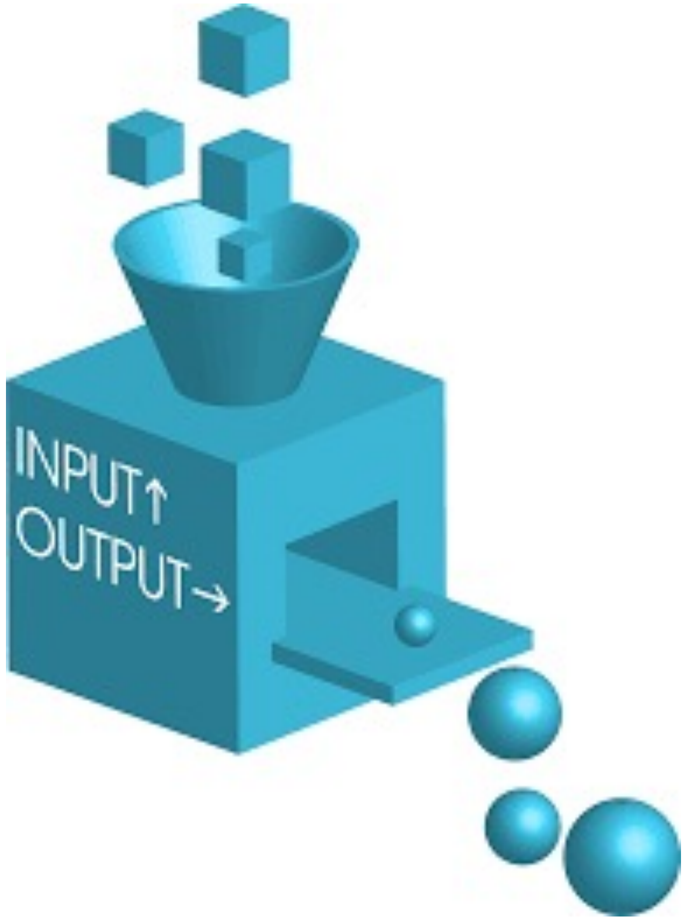


What Do Firms Do?

- Firms convert some goods to other goods:

- **Inputs**: $x_1, x_2, \cdots, x_n$
  - **Examples**: worker efforts, warehouse space, electricity, loans, oil, cardboard, fertilizer, computers, software programs, etc.

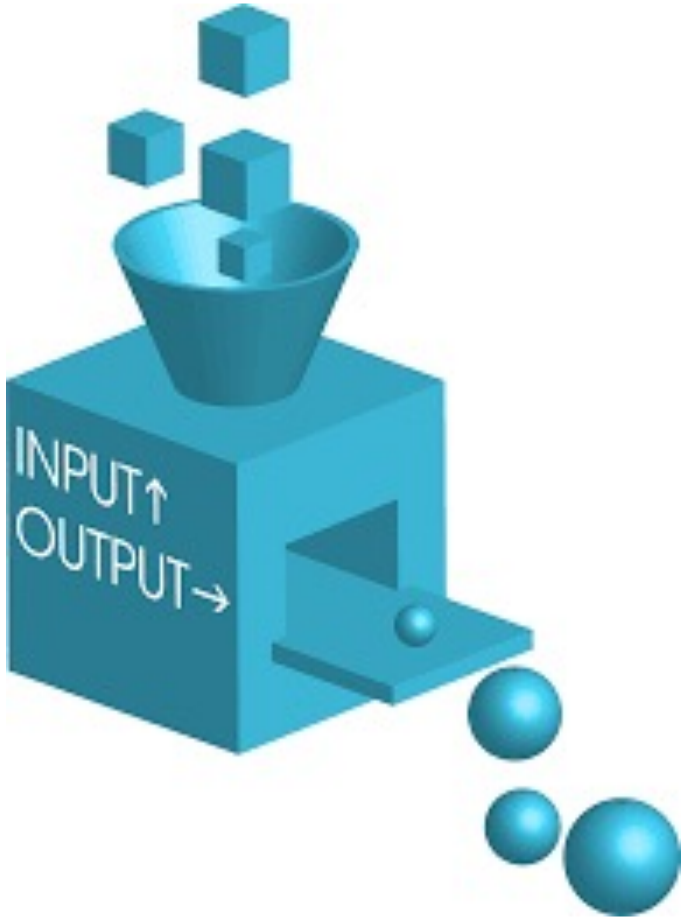
- **Output**: q
  - **Examples**: gas, cars, legal services, mobile apps, vegetables, consulting advice, financial reports, etc.



What Do Firms Do?

- **Technology** or a **production function**: rate at which firm can convert specified inputs ($x_1, x_2, \cdots, x_n$) into output (q)

$$q = f(x_1, x_2, \ldots, x_n)$$



Production Function

- The production function

READY IN: 1hr 20mins

YIELD: 2 loaves | UNITS: US

INGREDIENTS

Nutrition

5	cups all-purpose white flour
2	tablespoons yeast (or 2 x 7g pkts)
2	teaspoons sugar
1	teaspoon salt
2	cups warm-hot water
¼	cup cooking oil

- The production algorithm

DIRECTIONS

Put 4 cups of the flour, yeast, sugar and salt into large bowl.

Pour in hot water and oil and mix until combined- it will be sticky.

Add the remaining flour in increments until dough is no longer sticky.

Knead for about 5 minutes until dough is elastic and smooth.

Place dough back into bowl and cover with a damp teatowel and let it rise until double its size- about 1/2 hour.

Punch it down and divide dough into two pieces.

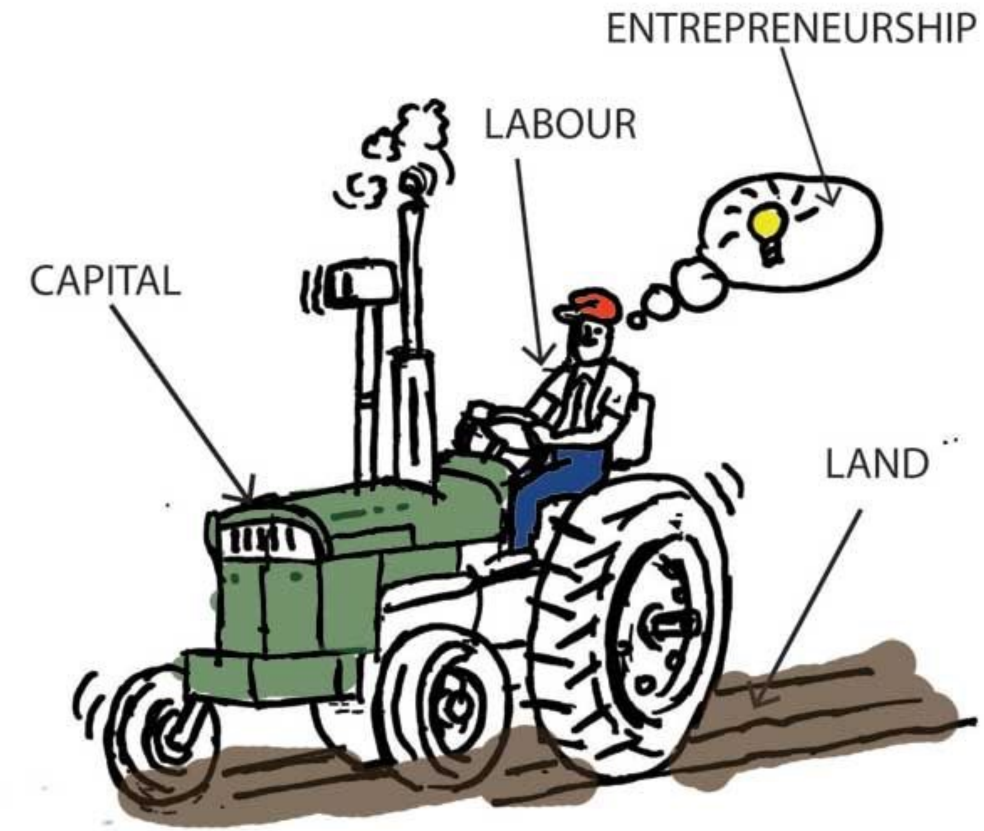
Roll pieces long enough to fill two well oiled loaf pans and leave to rise until dough has reached the rim of the pan.

Bake at 400F for 40 minutes.

Rub hot breads with water and wrap in a teatowel to'sweat' to soften the crust.

Production Function

$$q = Af(t, l, k)$$



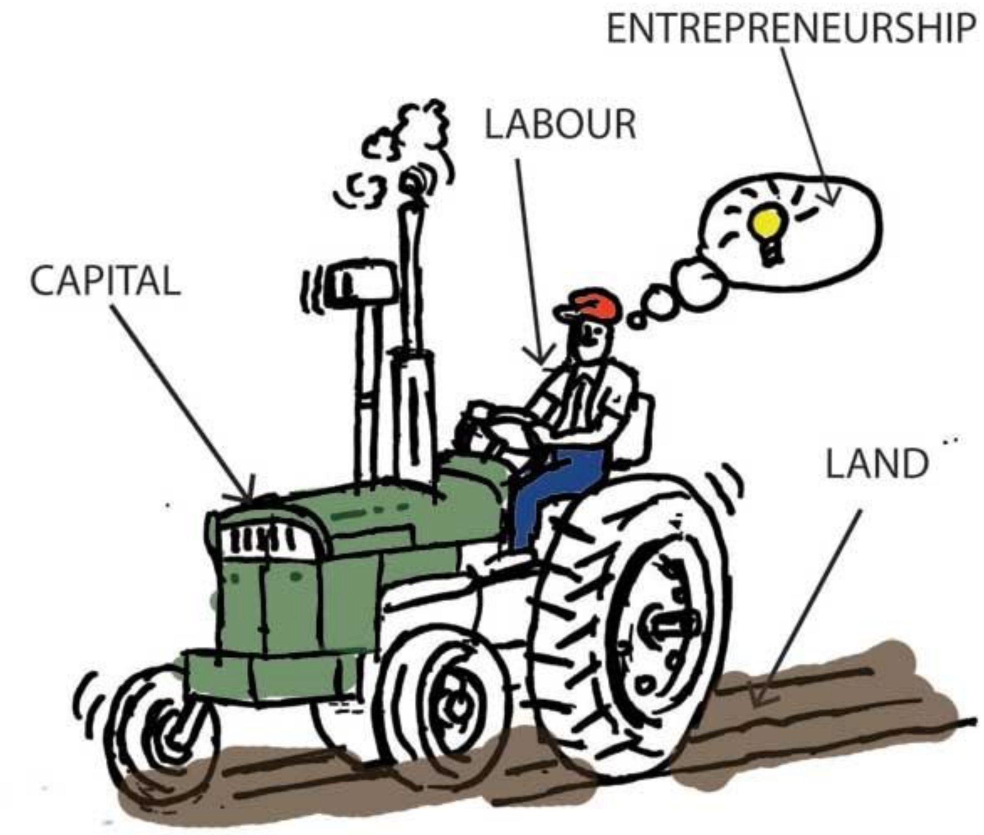
Production Function

$$q = Af(t, l, k)$$

- Economists typically classify inputs, called the **"factors of production" (FOP)**:

Factor	Owned By	Earns
Land (t)	Landowners	Rent
Labor (l)	Laborers	Wages
Capital (k)	Capitalists	Interest

- A: **"total factor productivity"** (ideas/knowledge/institutions)
- and Entrepreneurs/Owners who earn **Profit**



Production Function

Ounces of gold (pay dirt)= $Af(t, l, k)$

Land

Capital

Labor

Returns to Scale

- The **returns to scale** of production refers to the change in output when all inputs are increased *at the same rate*
- **Constant returns to scale**
- **Increasing returns to scale**
- **Decreasing returns to scale**

What does a firm maximize?

- 1st Stage: **firm's profit maximization problem:**

1) Choose: < output >

2) In order to maximize: < profits >

- Not true for all firms

- Even profit-seeking firms may also want to maximize additional things

Who gets the Profit?

- $\pi = pq \; - (wl + rk)$

- Profits are the **residual value** leftover after paying all factors

- Profits are income for the **residual claimant(s)**
  - i.e. **owner(s)** of a firm

The Firm's Problem(s)

1st Stage: **firm's profit maximization problem:**

- **Choose: < output >**
- **In order to maximize: < profits >**

Profit maximization

- $\pi = pq - (wl + rk)$
- $\max_{q} \pi = pq - cost(q)$

Profit maximization

- Prices?
  - Depends on the market the firm is operating in!
  - Study of **industrial organization**
- Essential question: how competitive is a market? This will influence what firms (can) do

Profit maximization

- $\pi = pq - (wl + rk)$
- $\max_{q} \pi = p(q)q - cost(q)$

The Firm's Problem(s)

- 2nd Stage: **firm's cost minimization problem:**

1) Choose: < inputs >

2) In order to *minimize*: < cost >

3) Subject to: < producing the optimal output >

- Minimizing costs ⟺ maximizing profits

The “Runs” of Production

- “Time”-frame usefully divided between short vs. long run analysis
- **Short run**: at least one factor of production is **fixed** (too costly to change)
- $q = f(\bar{k}, l)$
- Assume **capital** is fixed (i.e. number of factories, storefronts, etc)
- Short-run decisions only about using **labor**

The “Runs” of Production

- "Time"-frame usefully divided between short vs. long run analysis

- **Long run**: all factors of production are **variable** (can be changed)

- $q = f(k, l)$

Long Run Production

$q = \sqrt{lk}$

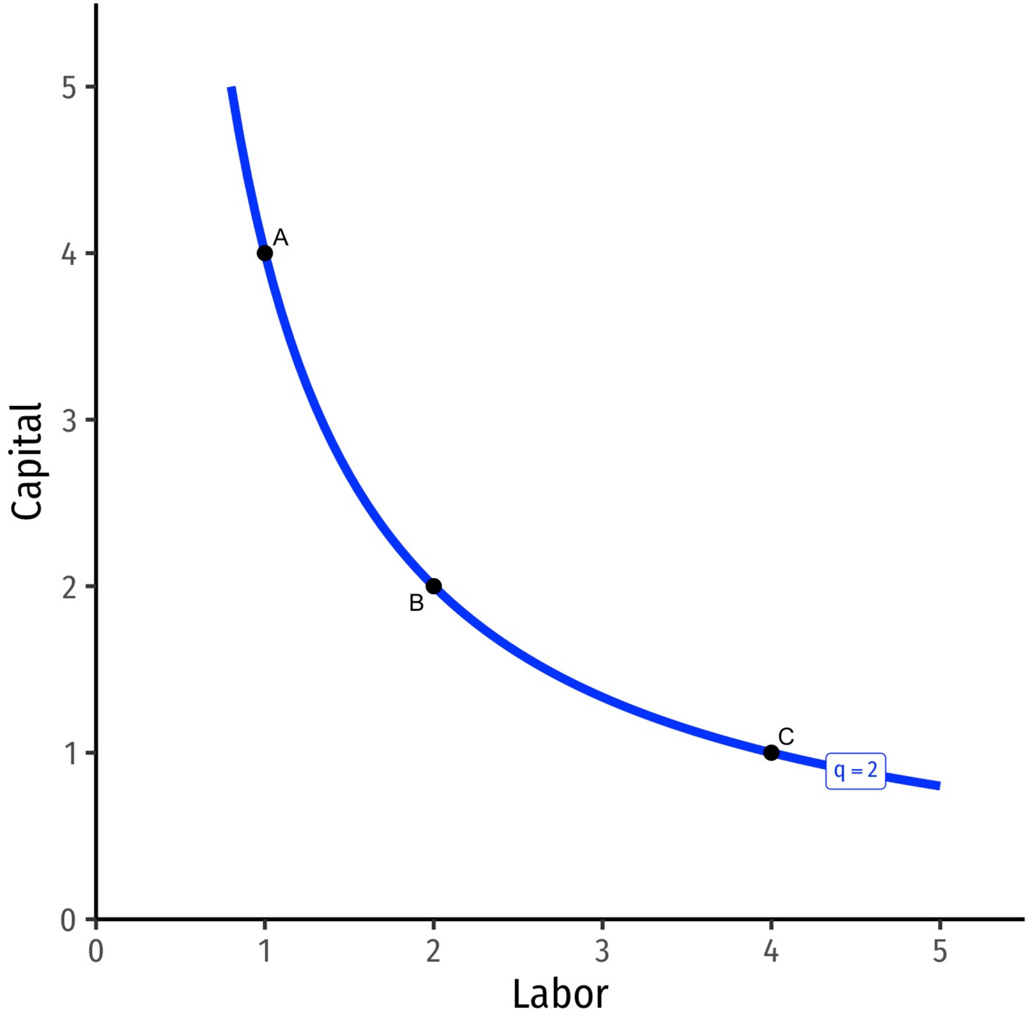
	k					
l	0	1	2	3	4	5
0	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	1.00	1.41	1.73	2.00	2.24
2	0.00	1.41	2.00	2.45	2.83	3.16
3	0.00	1.73	2.45	3.00	3.46	3.87
4	0.00	2.00	2.83	3.46	4.00	4.47
5	0.00	2.24	3.16	3.87	4.47	5.00

- Many input-combinations yield the same output!
- So how does the firm choose the *optimal* combination?

Isoquants and MRTS

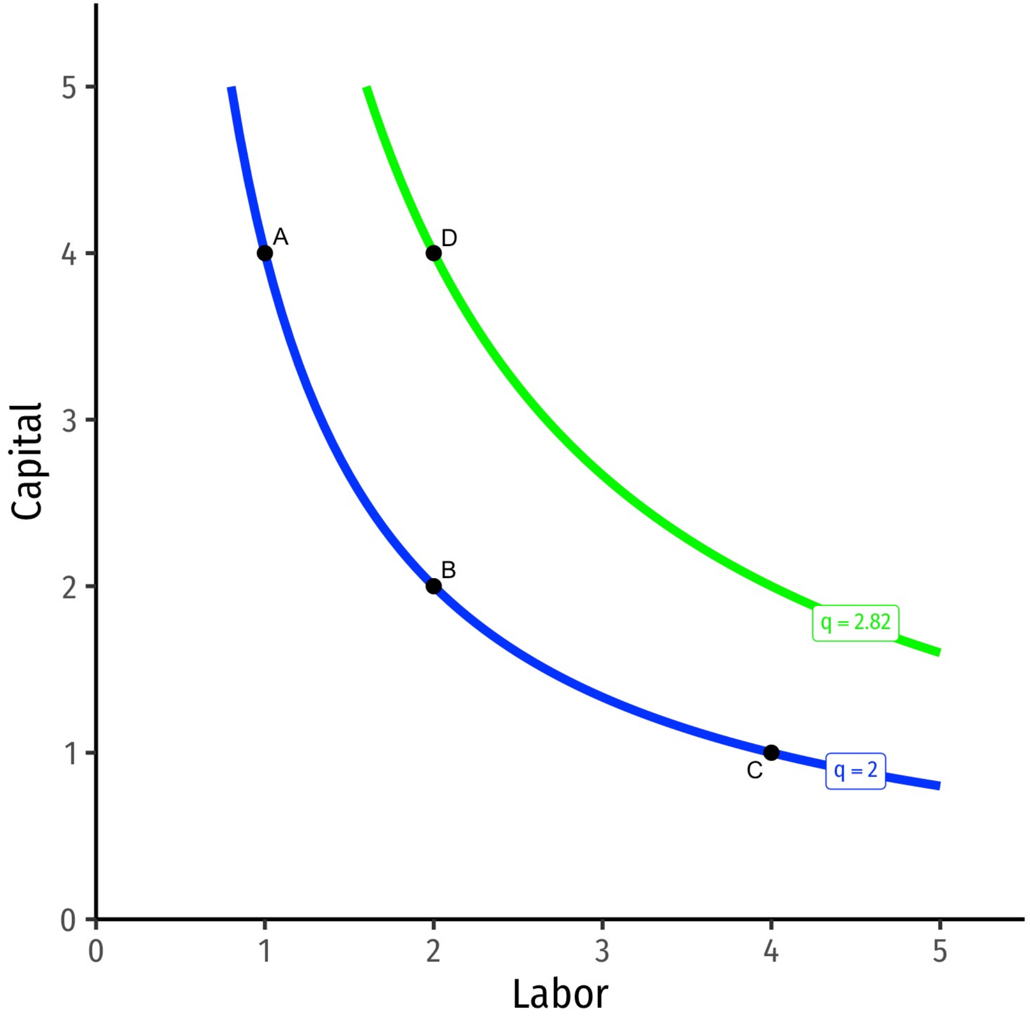
Isoquant Curves

- We can draw an **isoquant** indicating all combinations of l and k that yield the same q



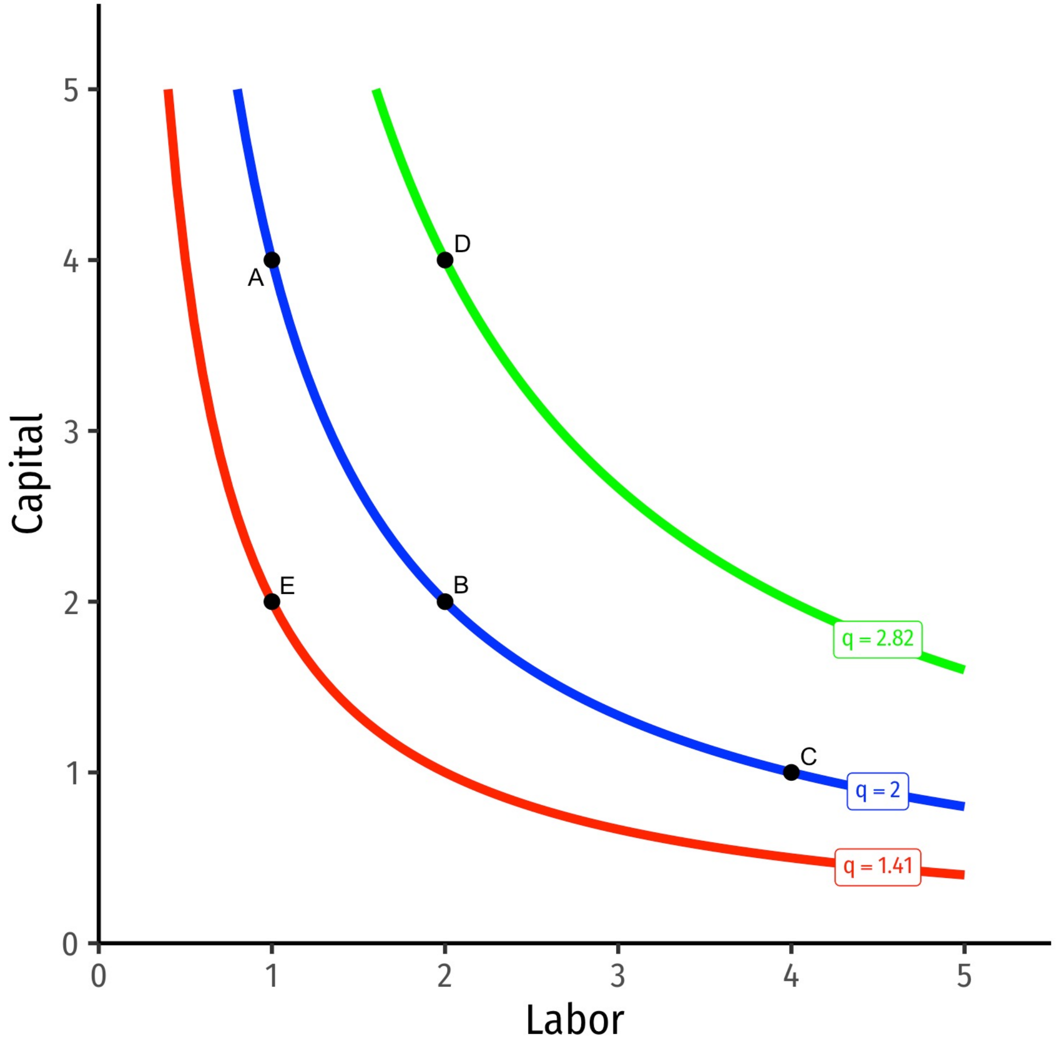
Isoquant Curves

- We can draw an **isoquant** indicating all combinations of l and k that yield the same q



Isoquant Curves

- We can draw an **isoquant** indicating all combinations of l and k that yield the same q



Marginal Rate of *Technical* Substitution

- If your firm uses fewer workers, how much more capital would it need to produce the same amount?

- **Marginal Rate of Technical Substitution (MRTS)**: rate at which firm trades off one input for another to *yield the same output*

Marginal Rate of *Technical* Substitution

- Think of this as the **opportunity cost**: # of units of k you need to give up to acquire 1 more l

- **MRTS** measures **firm's** tradeoff between l and k based on its **technology**

Marginal Rate of *Technical* Substitution

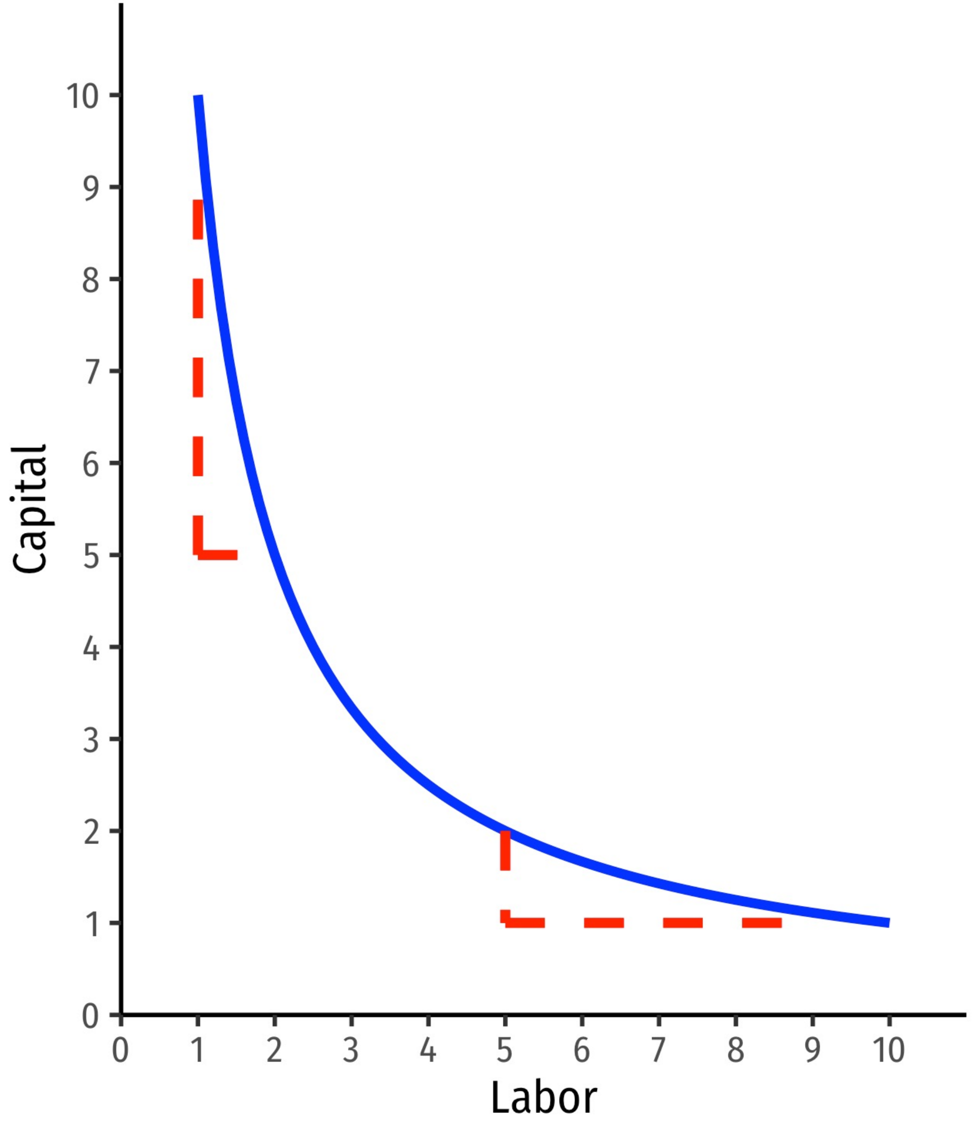
- MRTS is the slope of the isoquant

$$MRTS_{l,k} = -\frac{\Delta k}{\Delta l} = \frac{rise}{run}$$

- Amount of k given up for 1 more l

- Note: slope (MRTS) changes along the curve!

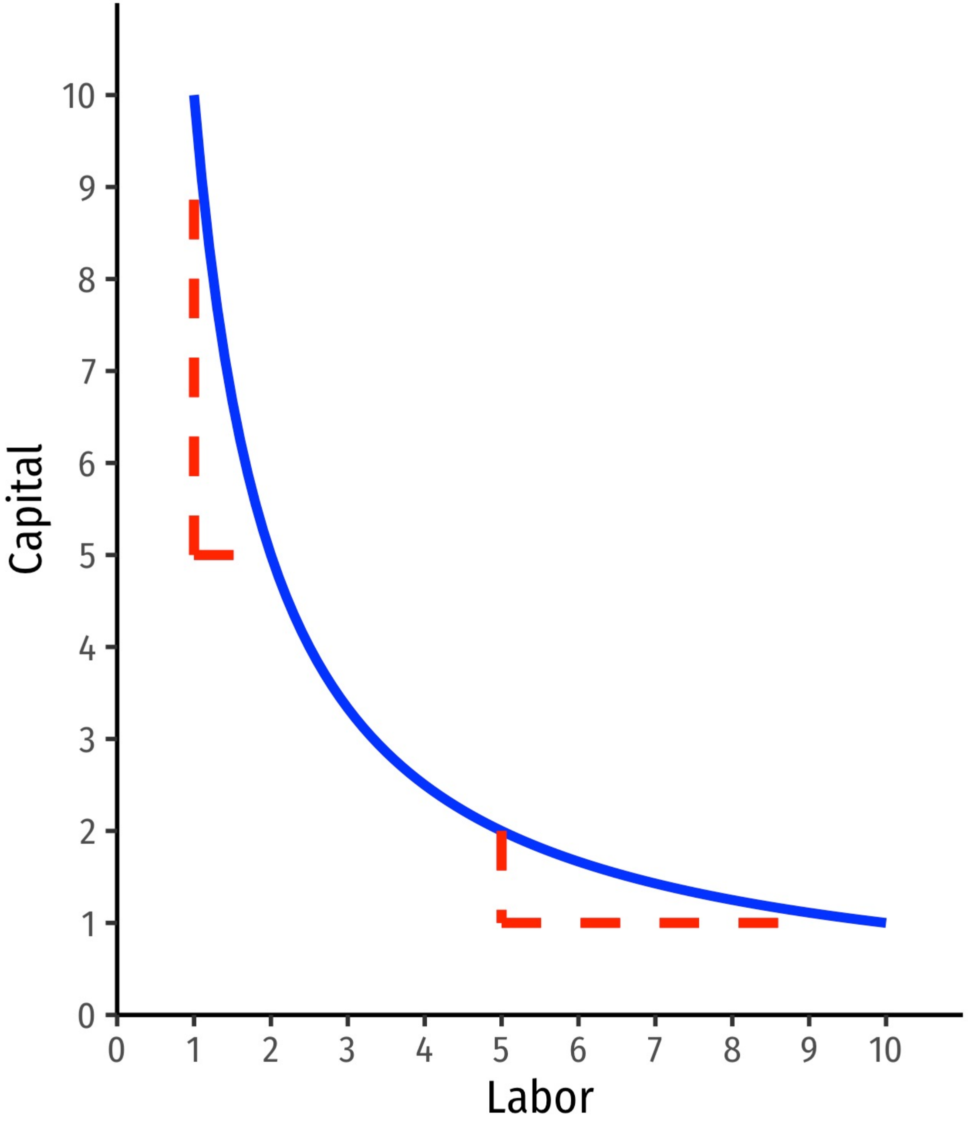
- **Law of diminishing returns**!



Marginal Rate of *Technical* Substitution

- Relationship between MP and MRTS:

$$\frac{\Delta k}{\Delta l} = \frac{rise}{run}$$

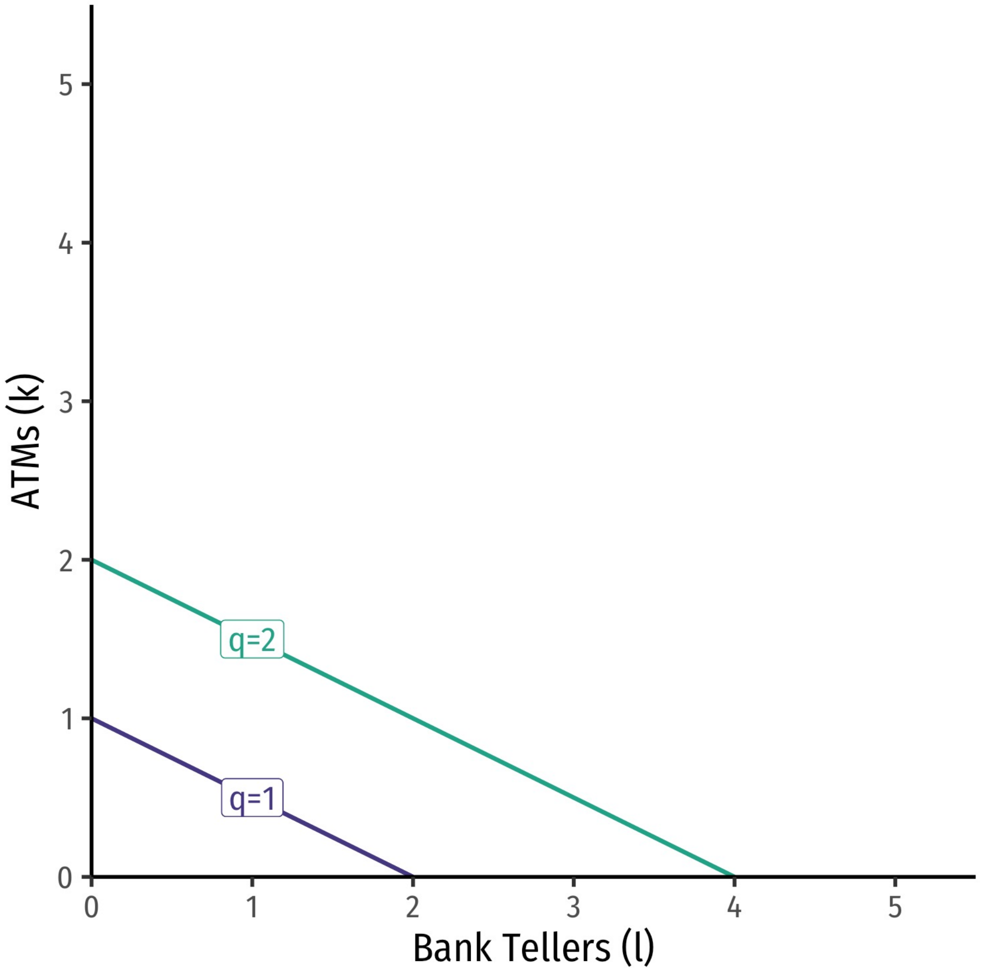


Special Case I: Perfect Substitutes

- One ATM can do the work of 2 bank tellers

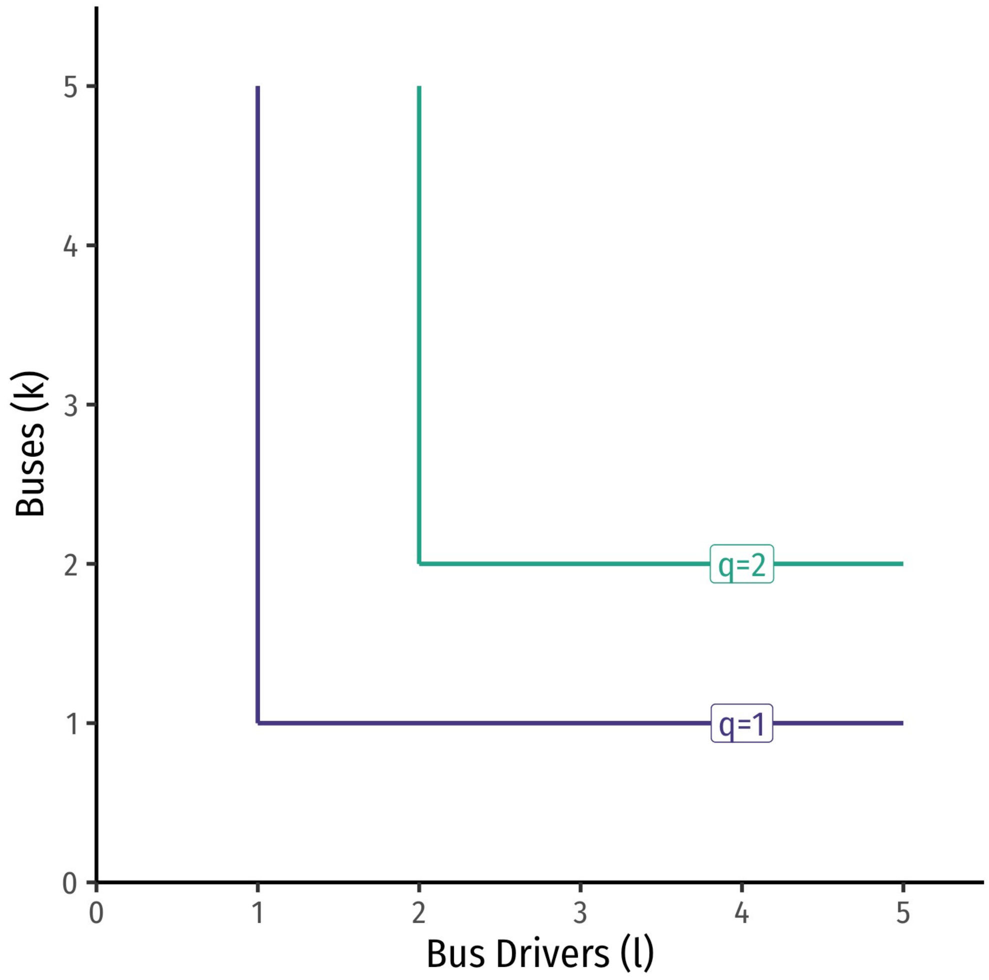
- **Perfect substitutes**: inputs that can be substituted at same fixed rate and yield same output

- $MRTS_{l,k} = -0.5$ (a constant!)

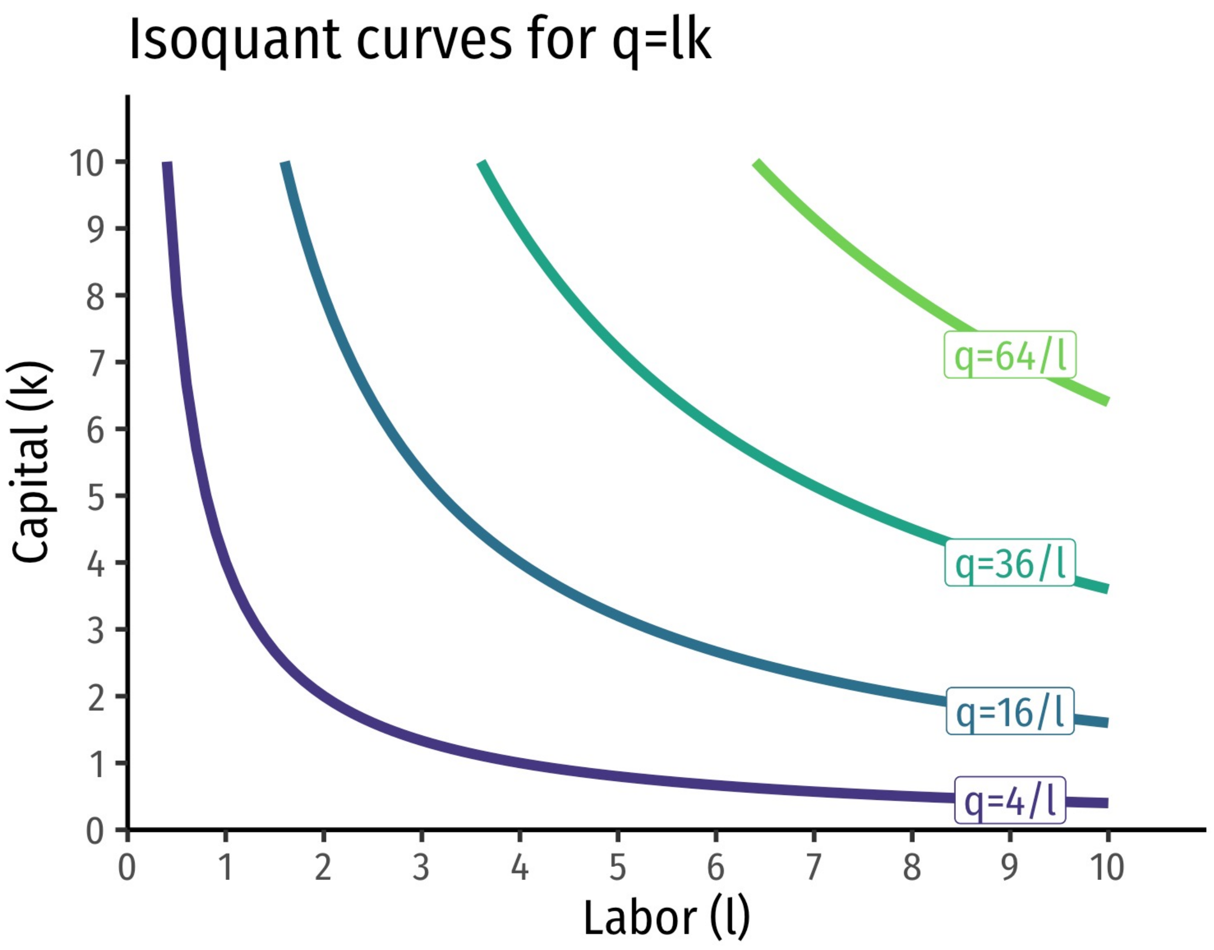


Special Case I: Perfect Complements

- Consider busses (k) and bus drivers (l)
- Must combine together in fixed proportions (1:1)
- **Perfect complements**: inputs must be used together in same fixed proportion to produce output



Cobb-Douglas Production Functions



Isocost Lines

Isocosts Lines

- What combination is the **cheapest?**

- Denote prices of each input as:
  - w: price of labor (wage)
  - *r*: price of capital

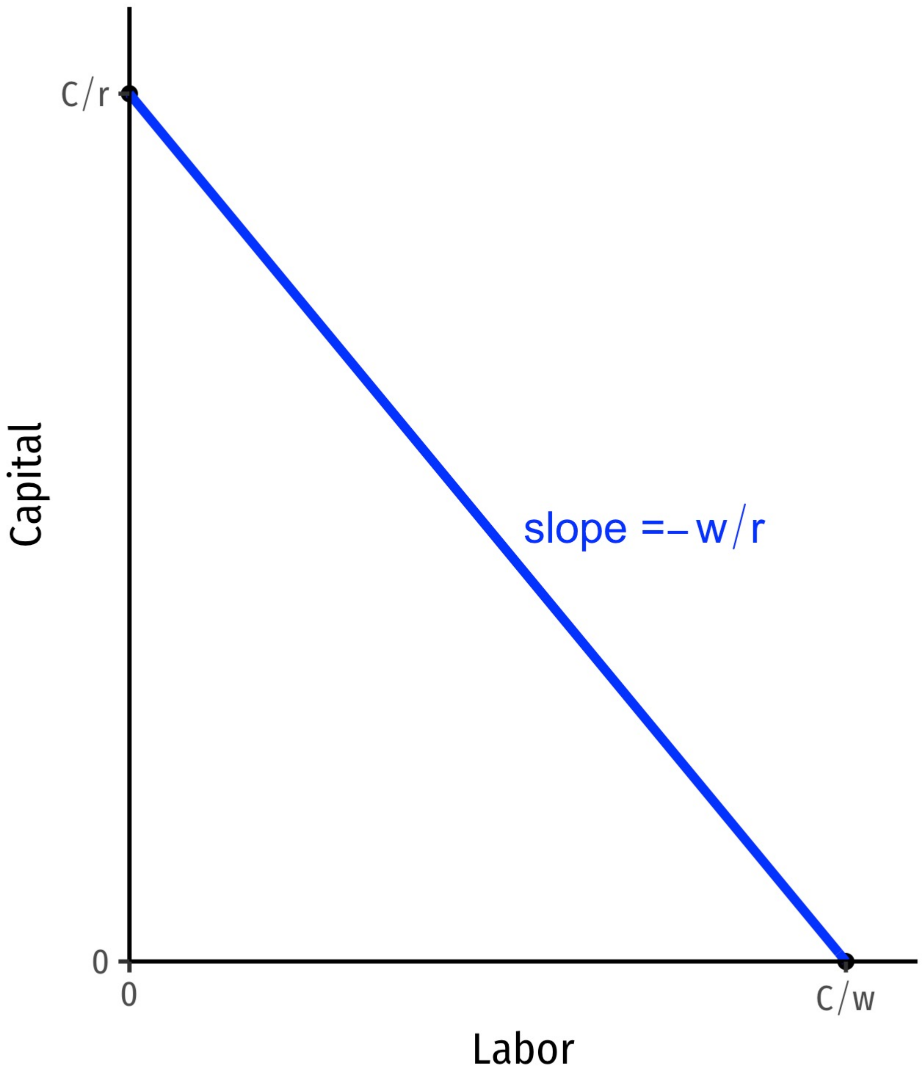
- C is the **total cost** of using inputs (l,k) at current input prices (w,r) to produce q units of output

Isocosts Lines

$$\text{wl} + \text{rl} = \text{C}$$

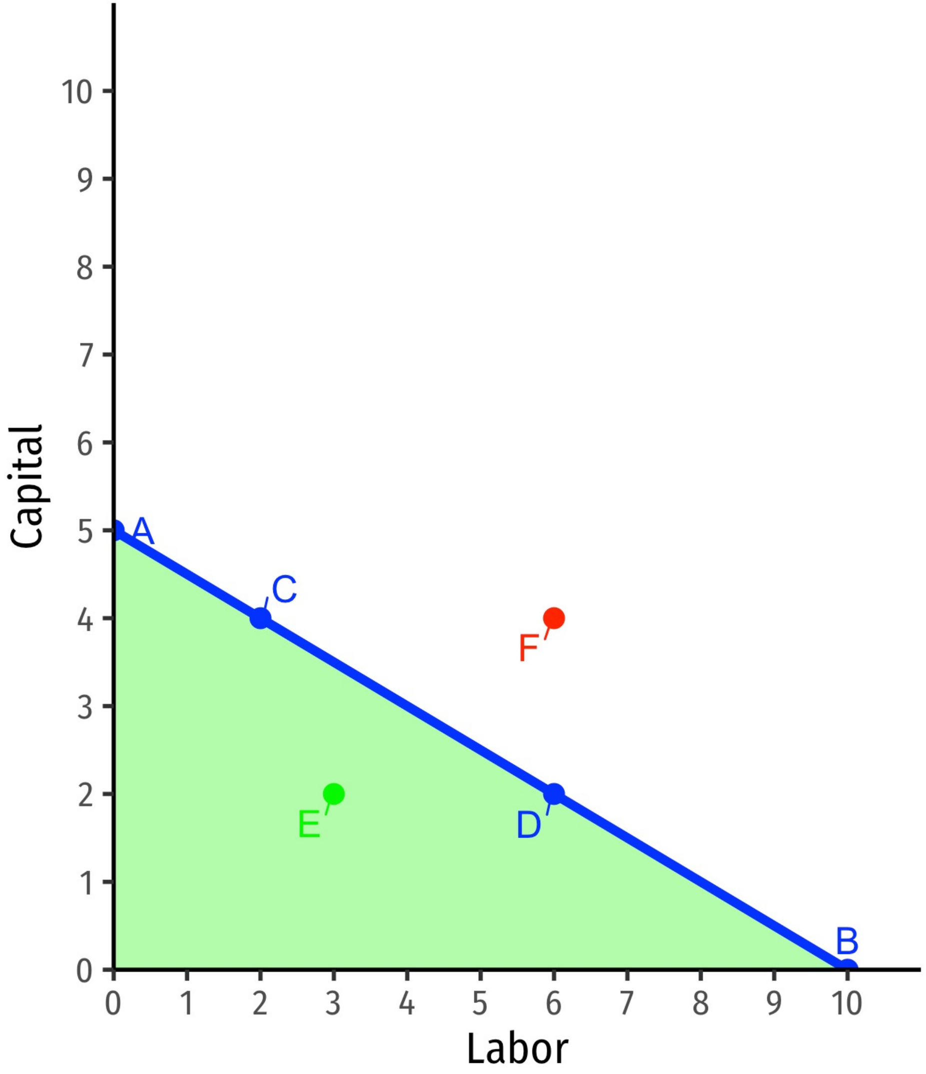
Solve for k to graph

$$k = \frac{C}{r} - \frac{w}{r} l$$



Interpreting the Isocost Line

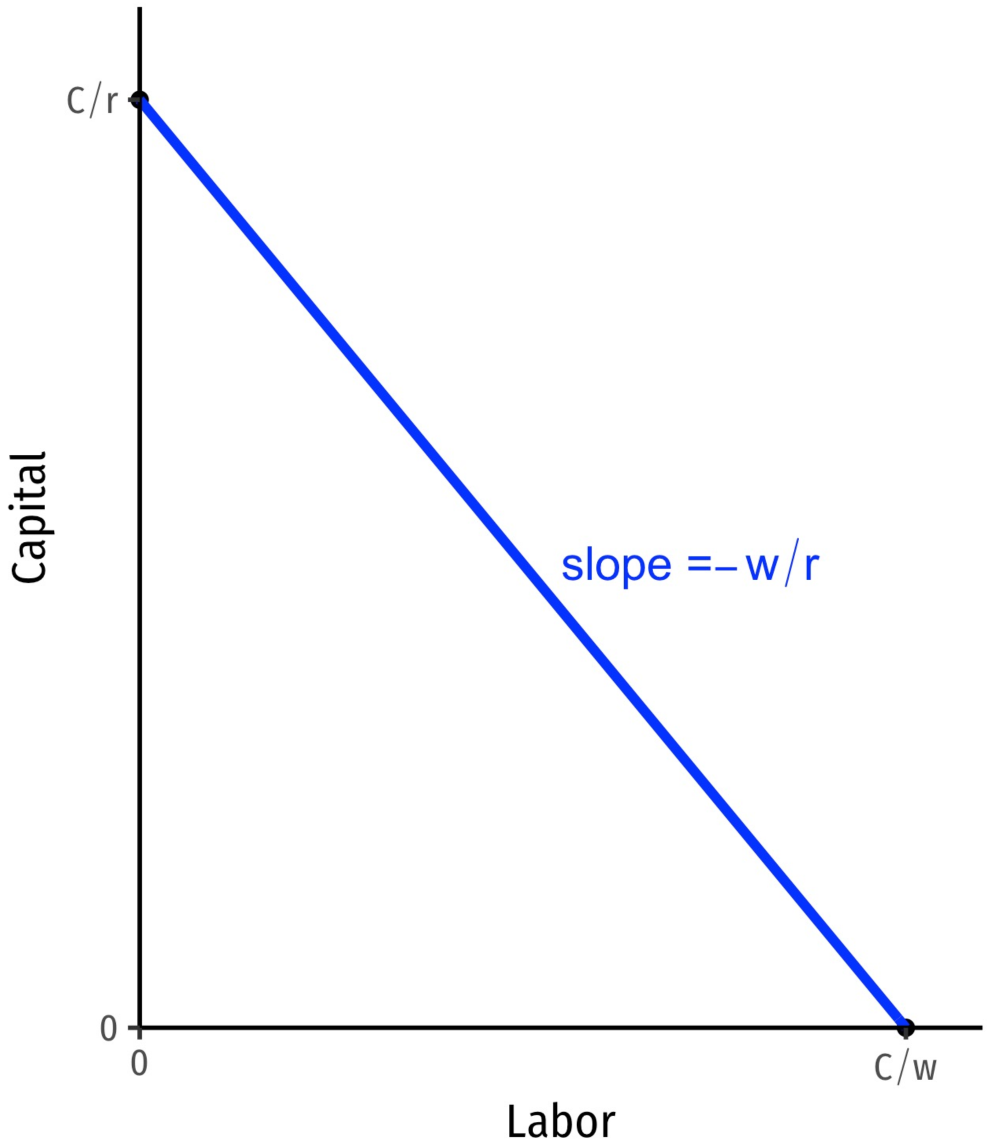
- Points on the line are same total cost
- Points beneath the line are cheaper (but may produce less)
- Points above the line are more expensive (and may produce more)



Interpreting the Slope

- **Slope**: market-rate of **tradeoff** between l and k

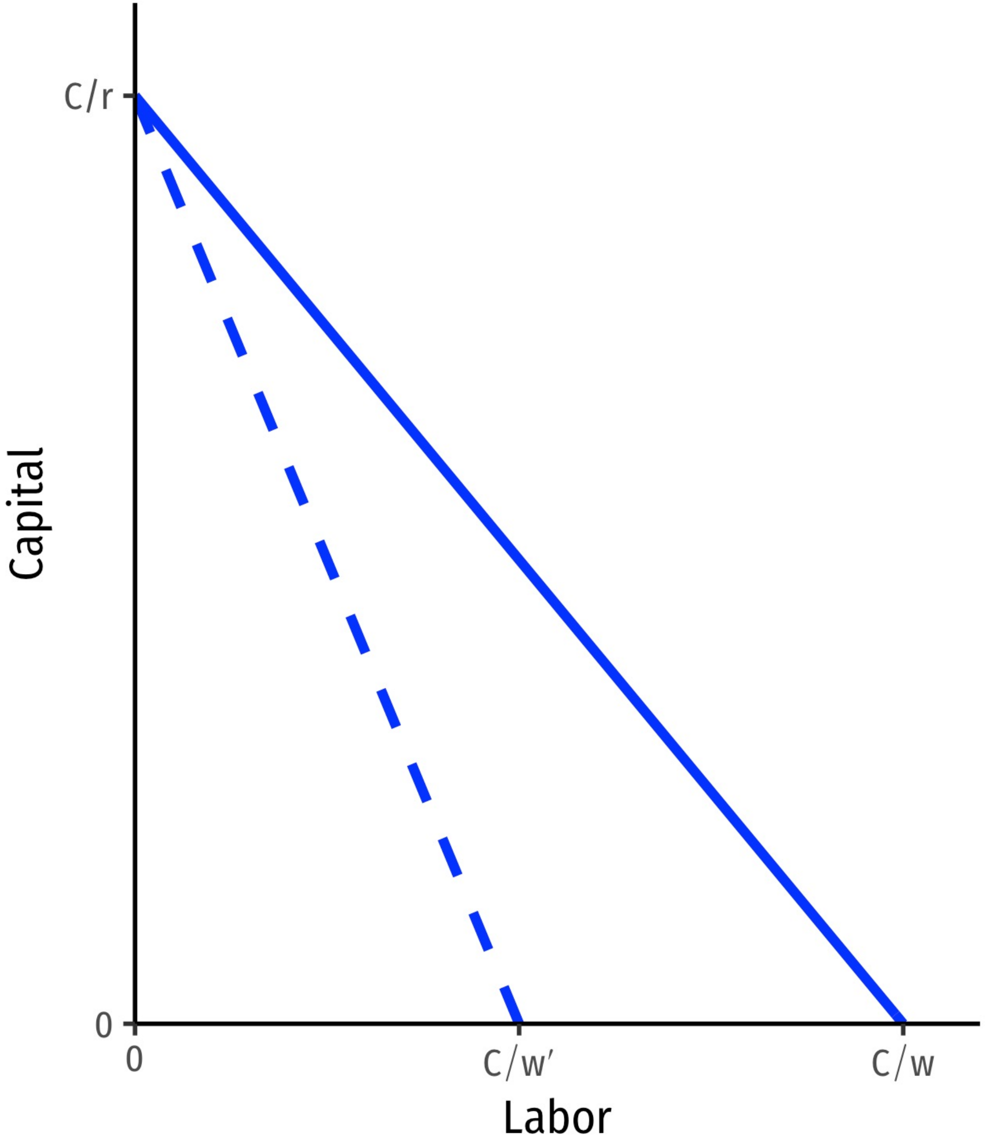
- **Relative price** of l or **opportunity cost** of l:
  - Using 1 more unit of l requires giving up $\frac{w}{r}$ units of k



Changes in Relative Factor Prices

- Changes in **relative factor prices**: *rotate* the line

- **Example**: An increase in the price of l
  - Slope changes: $-\frac{w}{r}$

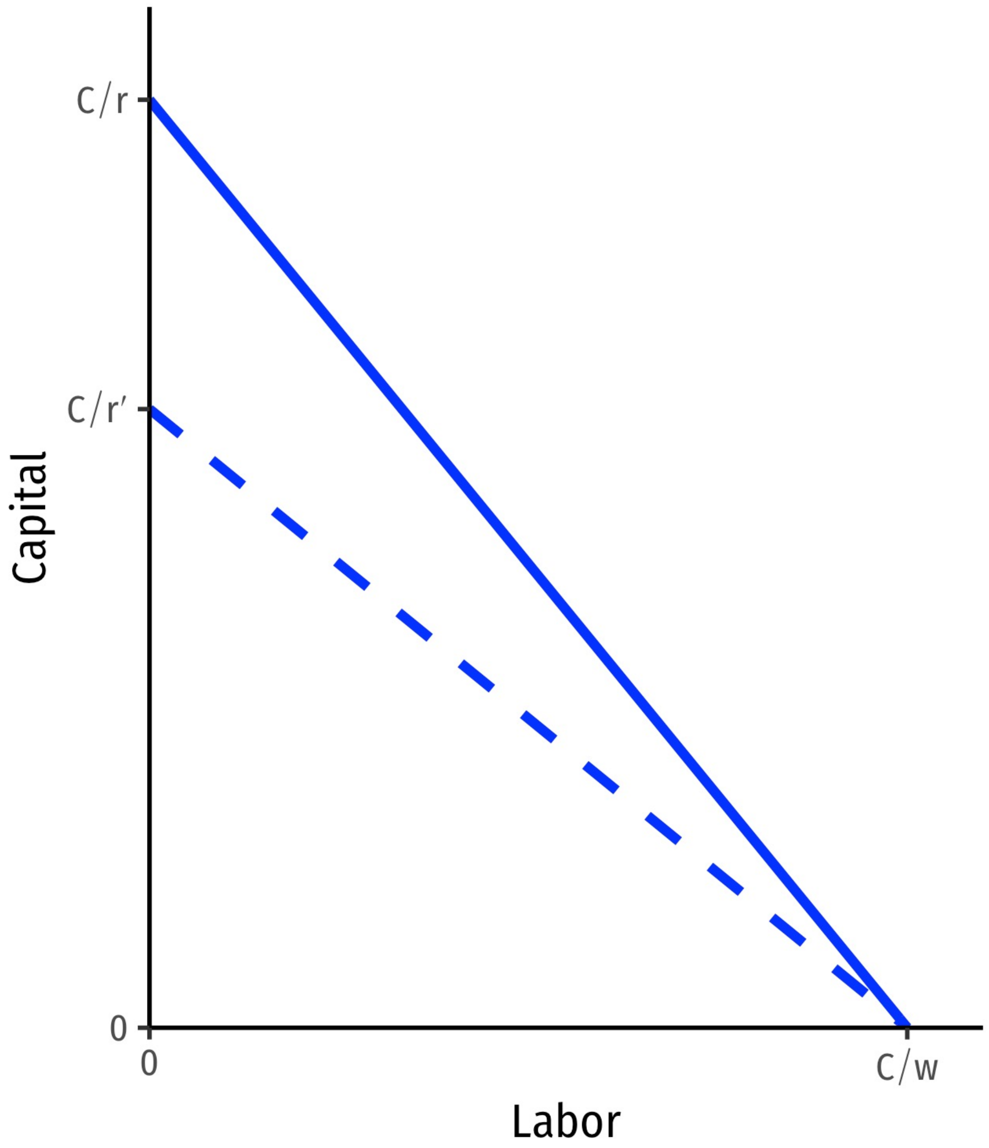


Changes in Relative Factor Prices

- Changes in **relative factor prices**: *rotate* the line

- **Example**: An increase in the price of k

  - Slope changes: $-\frac{w}{r}$



Changes in Relative Factor Prices

- Changes in **relative factor prices**: *rotate* the line

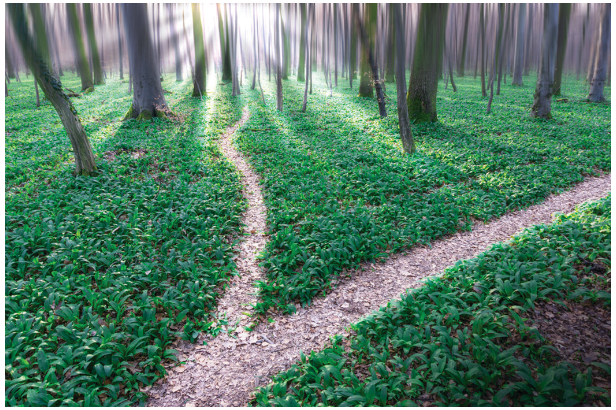
- **Example**: An increase in the price of k

  - Slope changes: $-\frac{w}{r}$

Solving the Cost Minimization Problem

The Firm's Cost Minimization Problem

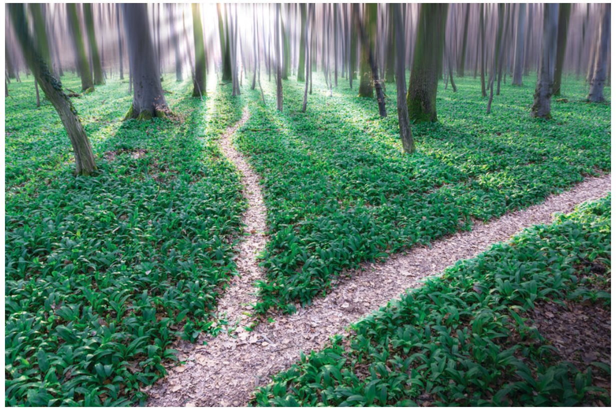
- The **firm's cost minimization problem** is:
- **Choose:** < inputs: l,k >
- **In order to minimize:** < total cost: $wl + rk$ >
- **Subject to:** < producing the optimal output: $q^* = f(l,k)$ >



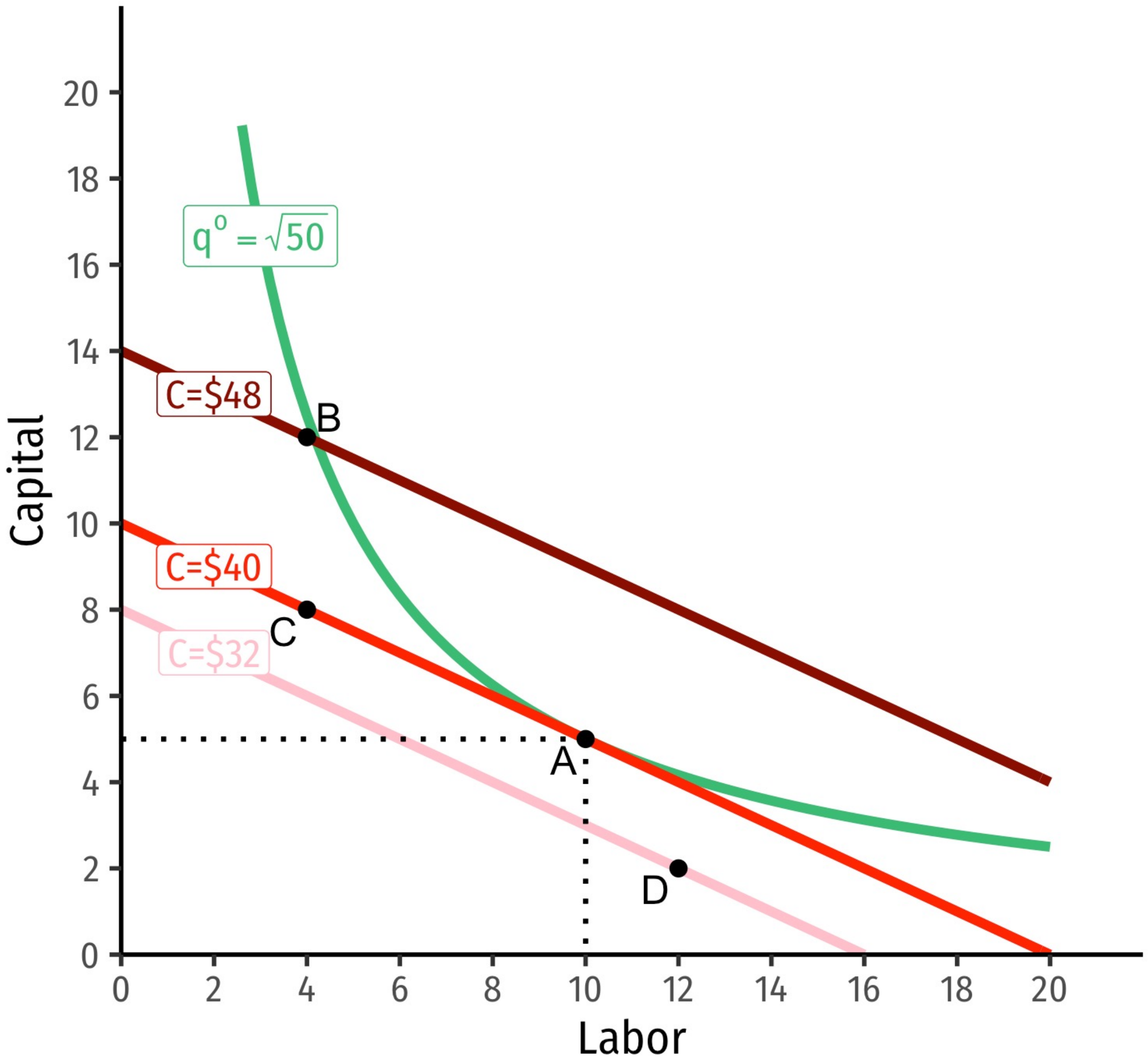
The Firm's Cost Minimization Problem

$$\min_{l,k} wl + rk$$

$$s.t.\, q^* = f(l, k)$$



The Firm's Cost Minimization Problem



$q = \sqrt{lk}$, $q^o = \sqrt{50}$, $w = \$2$, $r = \$4$

The Firm's Cost Minimization Problem

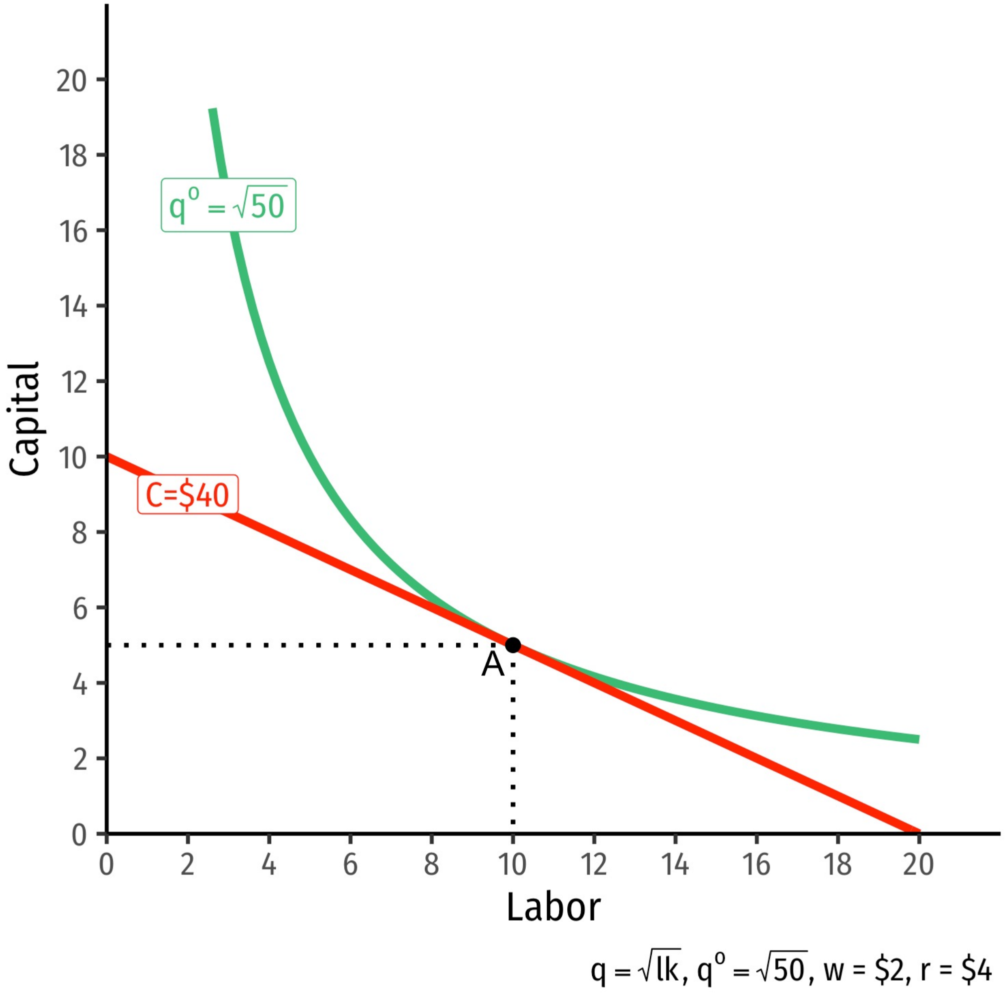
Isoquant slope = Isocost line slope

$$|MRTS_{l,k}| = \left|\frac{w}{r}\right|$$

$$\left|\frac{MP_l}{MP_k}\right| = \left|\frac{w}{r}\right|$$

$$|-0.5| = |-0.5|$$

- **Firm** would exchange at same rate as **market**
- **No other combination** of (l,k) **exists** at current prices & output that could lower cost to produce q∗!



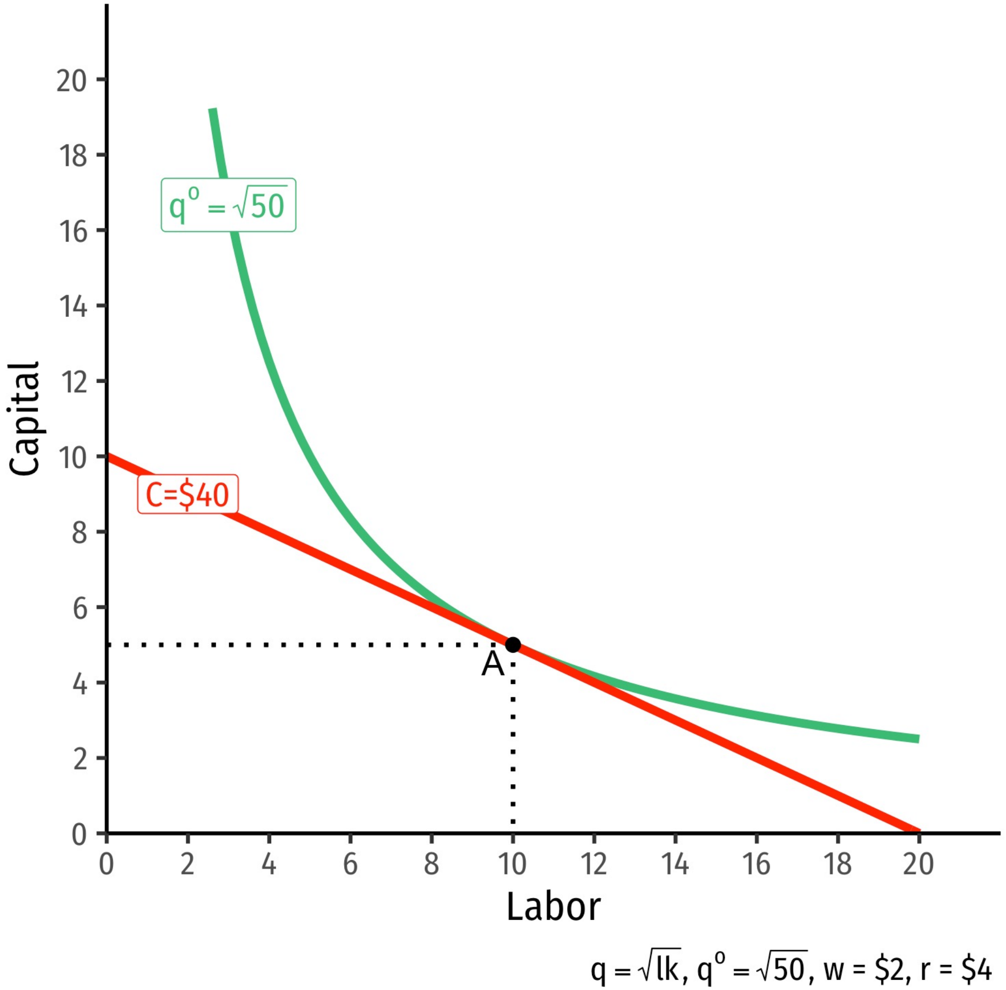
Two Equivalent Rules

Rule 1

$$\frac{MP_l}{MP_k} = \frac{w}{r}$$

Rule 2

$$\frac{MP_l}{w} = \frac{MP_k}{r}$$



Equimarginal rule

$$\frac{MP_l}{w} = \frac{MP_k}{r} = \cdots = \frac{MP_n}{n}$$

- Why is this the optimum?
- **Example:** suppose firm could get a higher marginal product per $1 spent on l than for k (i.e. "more bang for your buck"!)
  - Not minimizing costs!
  - Should use more l and less k!
    - This will raise MP_k and lower MP_l!
  - Continue until cost-adjusted marginal products are equalized

The Firm’s Problem

- The **firm's problem** is:

1) **Choose: < inputs and output >**
2) **In order to maximize: < profits >**
3) **Subject to: < technology >**

- It's actually much easier to break this into 2 *stages*, though it's possible to do it all at once.

The Firm's Problem

- 1st Stage: **firm's profit maximization problem:**

1) Choose: < output >

2) In order to maximize: < profits >

- 2nd Stage: **firm's cost minimization problem:**

1) Choose: < inputs >

2) In order to *minimize*: < cost >

3) Subject to: < producing the optimal output >

- Minimizing costs $\Leftrightarrow$ maximizing profits

Application in Urban Economics

Rosen-Roback Model

- How do firms decide where to locate and people where to reside?

- How do people's and firms' decisions jointly decide wages?

- How do these decisions effect housing prices in each city?

- What is the role of local amenities?

Rosen-Roback Model Assumptions

- Workers
  - 1 unit of labor and housing
  - Cares about local amenities

- Firms
  - Workers and land to produce a good
  - Pay and sell at market rates
  - Operating costs can be effected by local amenities

- Workers and firms are <u>perfectly mobile</u>

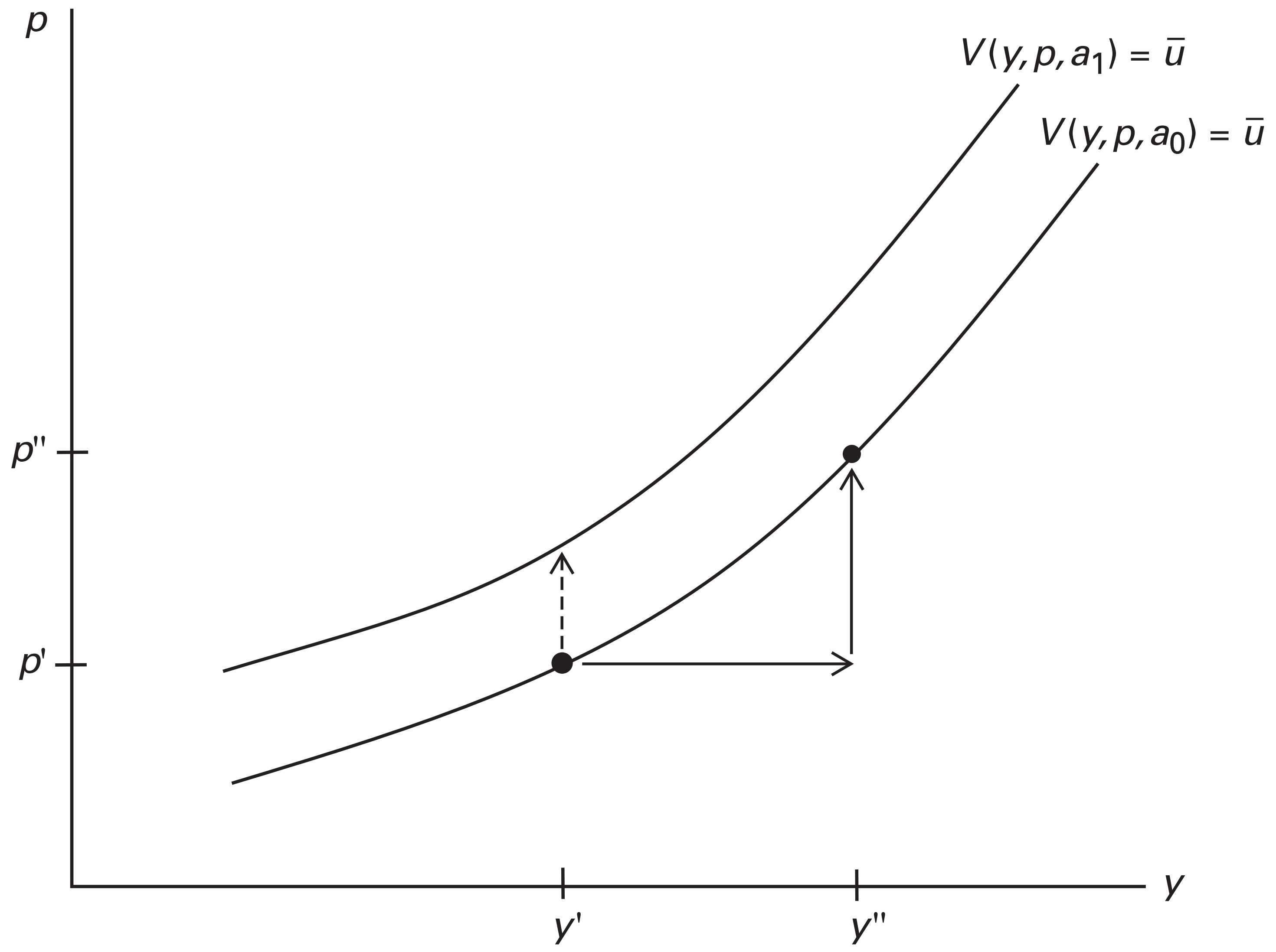
Rosen-Roback Model Assumptions

- Spatial equilibrium
  - Workers and firms have no incentive to move to another location
- Implications
  - Workers in different cities are all equally happy
  - Firms earn "zero profit"

Workers

- Indirect utility = V(y, p, a)
  - *y* is income from work in a city
  - *p* is the price for housing
  - *a* is the local amenity level

Worker's indifference curve



Worker’s indifference curve (notes)

- The indifference curve is upward sloping
  - if the worker can earn a higher income y in a city, to make equally happy, the city’s housing price p must increase.
  - when income in a city increases and housing price remains unchanged, workers from other cities will move here for the higher income, which drives up the housing price.
    - recall the free-mobility assumption.
  - similarly, when the housing price increases, income must also increase to make the worker equally happy.

Worker's indifference curve (notes)

- A higher amenity level a shifts the indifference curve up.
  - worker enjoys better amenity, for a given income level y, a worker is willing to put up with a higher housing price.
  - or alternatively, for a given housing price p, a worker is willing to take a lower wage if amenity is higher.

Firms

- The cost function C(y,p,a) = 1
  - *y* is income from work in a city
  - *p* is the price for housing
  - *a* is the local amenity level
    - Can impact firm's productivity

Firm's isocost curve (notes)

- The iso-cost (iso-profit) curve is downward-sloping.
  - when the firm has to pay higher wages for the worker (y is higher), land (housing) price p must decline to make the firm break even.
  - when the housing price p increases, the firm must suppress wages for the workers to break even.
- If amenity does not affect productivity, the iso-cost curve does not shift with changes in amenity levels.

Firm's isocost curve (notes)

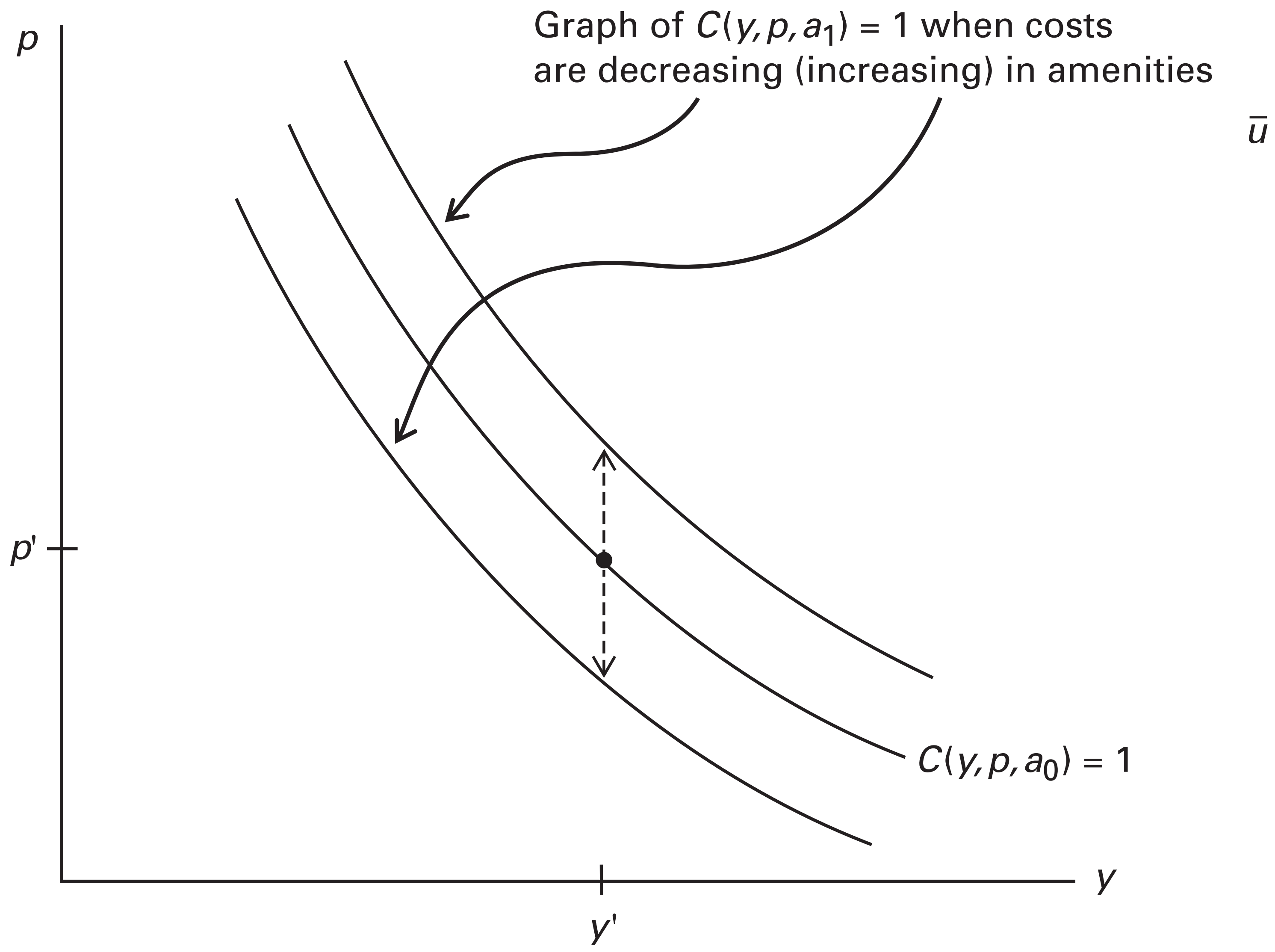
- If amenity is productive, when amenity level increases, cost reduces, the iso-cost curve shifts up
  - either housing price p or worker's wage y must increase to offset the productivity gain

- If amenity is counter-productive, when amenity level increases, cost increases, the iso-cost curve shifts down
  - either housing price p or worker's wage y must decrease to compensate for the productivity loss

Firm's isocost curve (notes)

- A higher amenity level a shifts the indifference curve up.
  - worker enjoys better amenity, for a given income level y, a worker is willing to put up with a higher housing price.
  - or alternatively, for a given housing price p, a worker is willing to take a lower wage if amenity is higher.

Firm's isocost curve



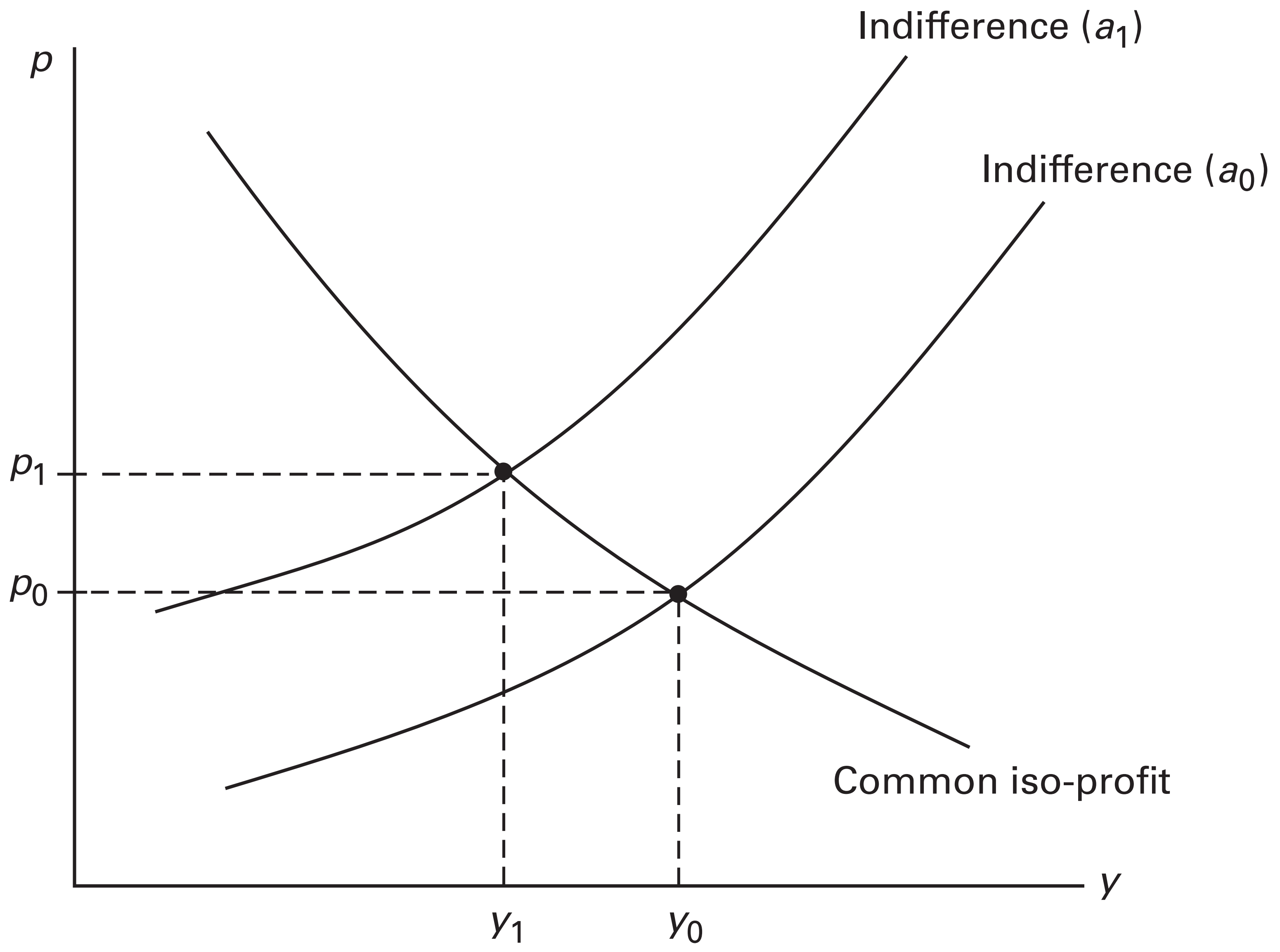
Spatial equilibrium

- Using the worker's indifference curve and firm's iso-cost curve
  - Analyze equilibrium wage y and housing price p
  - How amenity levels affect y and p
  - The relationship between y, p and a

- Three different cases
  - firm's cost is **independent** of amenities
  - firm's cost **decreases** with amenities
  - firm's cost **increases** with amenities

1. Firm's cost independent of amenities

1. Firm's cost independent of amenities



1. Firm’s cost independent of amenities: Intuition

- Other things equal, a higher amenity level makes a worker happier (V(y,p,a) is higher)

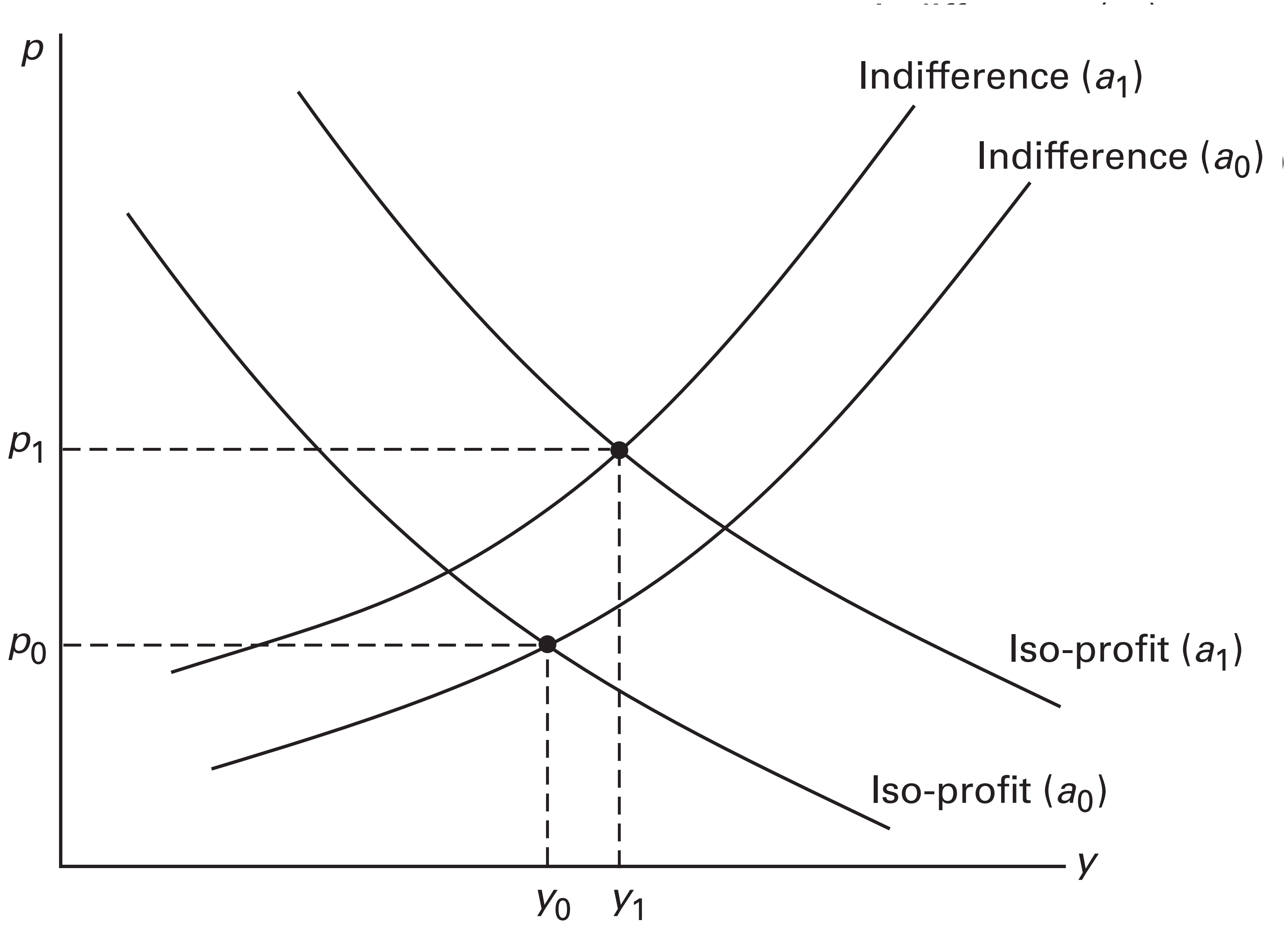
- More workers are willing to come to the city, driving up local housing prices

- To break even ($\pi = 0$), a competitive firm has to cut wages

- New equilibrium (when worker is indifferent between living in the city or some other city) with higher p and lower y.

2. Firm’s cost decrease with amenities

2. Firm's cost decrease with amenities

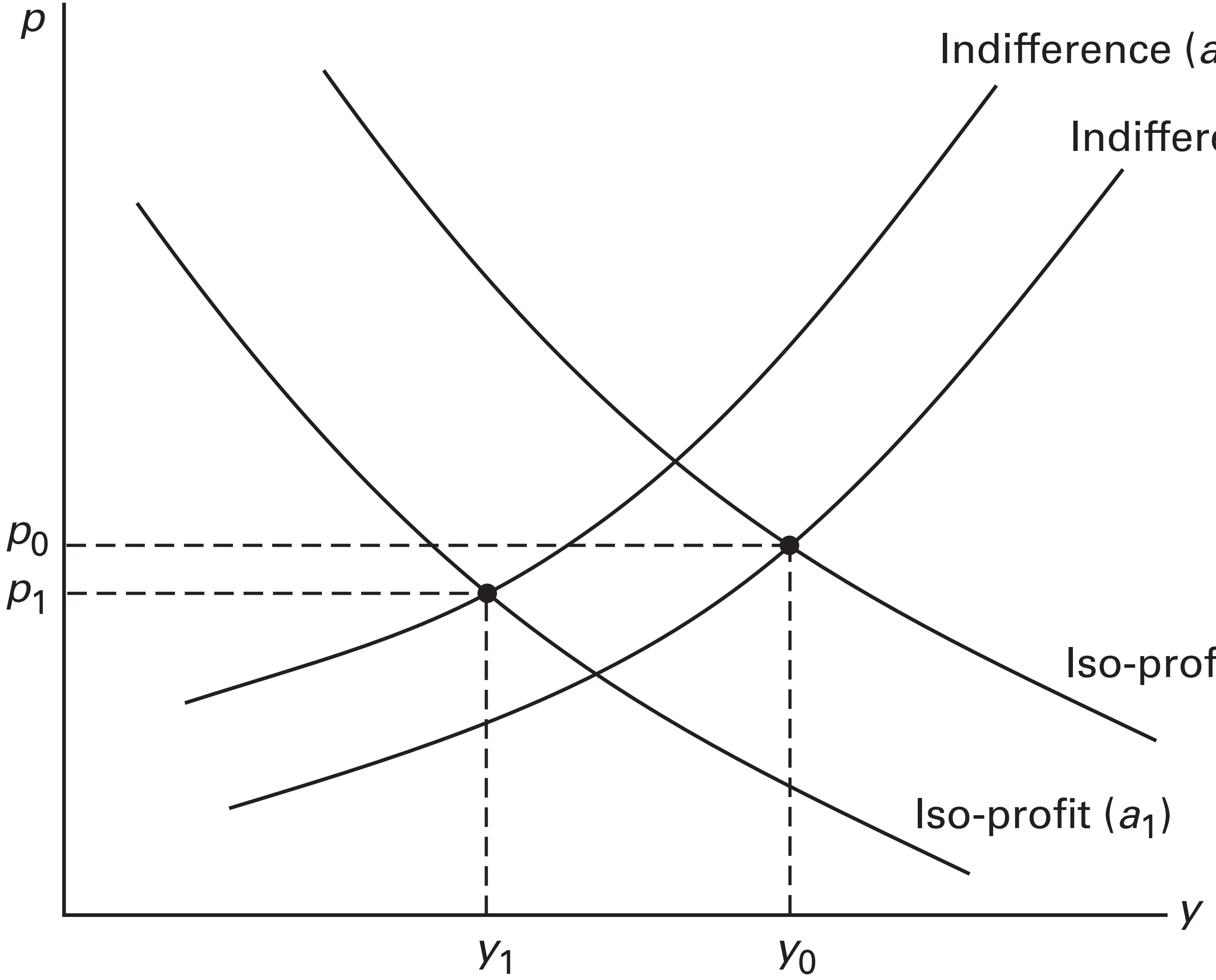


2. Firm's cost decrease with amenities: Intuition

- When amenities increase, a worker is willing to take a higher housing price and lower wage.
- Higher amenities also reduce firm's cost, tends further raise housing price, and increase worker's wage.
- Housing price will go up for sure; changes in worker's wage is ambiguous.

3. Firm's cost increase with amenities

3. Firm's cost increase with amenities



3. Firm's cost increase with amenities: Intuition

- When amenities increase, a worker is willing to take a higher housing price and lower wage
- Higher amenities also increases firm's cost, making it less productive, tends further decrease worker's wage and reduce the housing price
- Worker's wage will definitely decline, how housing price changes is ambiguous

Implications

- Three wages
  - nominal wage
  - real wage (adjusted for local cost of living, most importantly housing price)
  - amenity-adjusted real wage

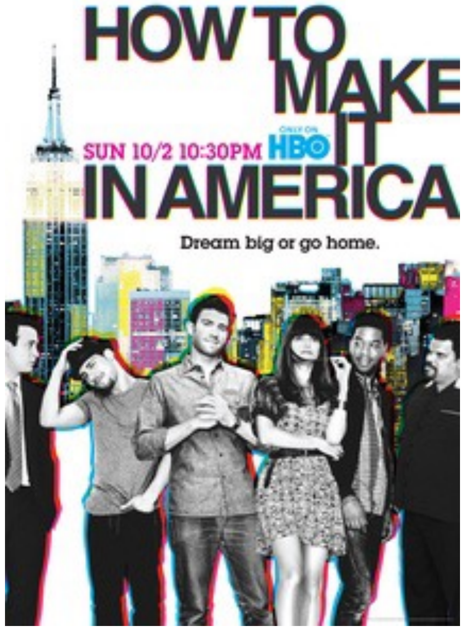
- Wage and housing price are usually observable, but value of amenities is not

- Provides a way to evaluate amenities

Loosening the assumption

- Perfect mobility
  - Migration costs
  - Location preferences

- Amenity-adjusted real wage can different across cities



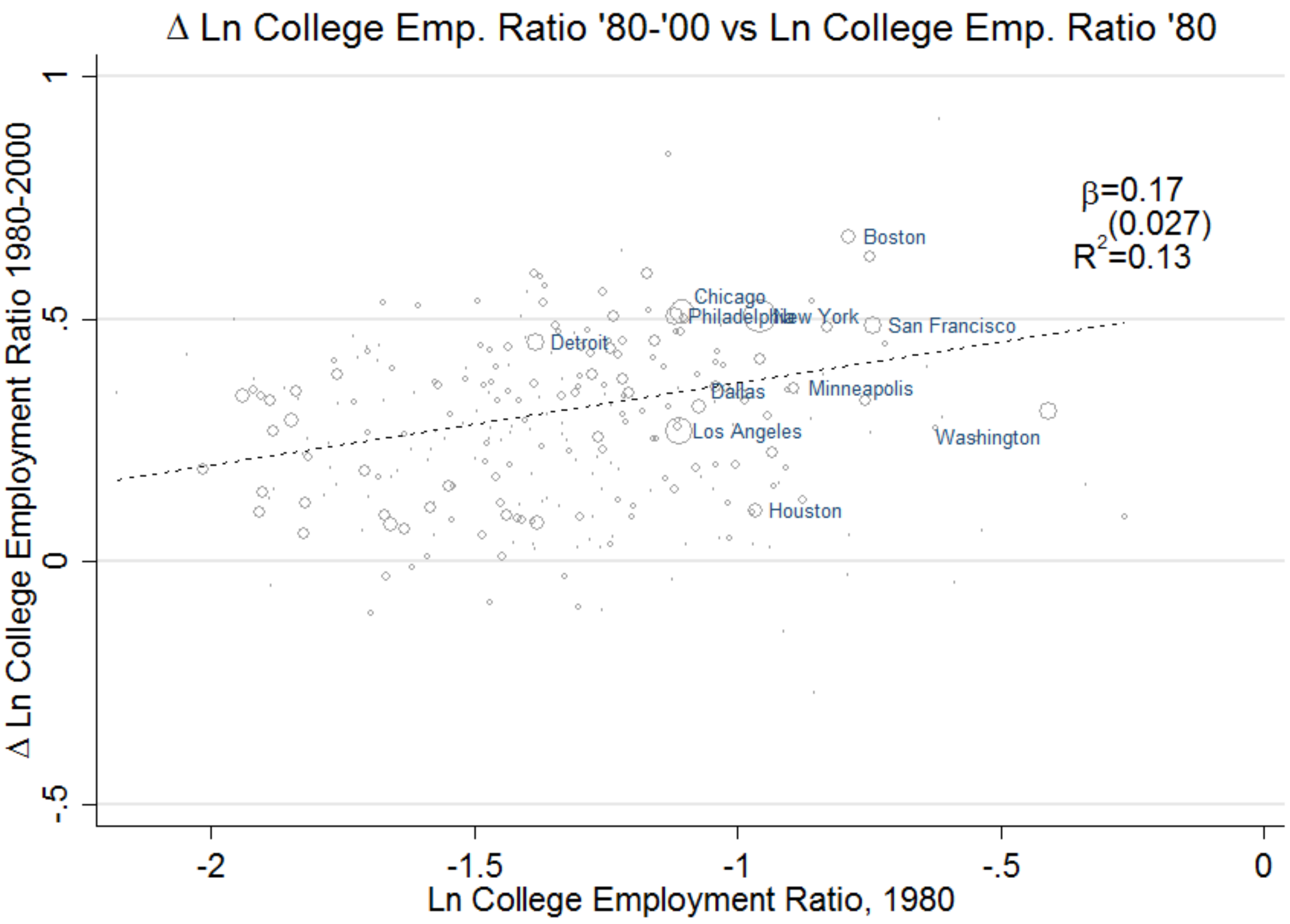
Loosening the assumption

- Homogenous workers

- Reality: Heterogenous workers
  - Many dimensions e.g. skills
  - College vs non-college educated

Loosening the assumption

- College vs non-college educated



Loosening the assumption

- Endogenous amenities