Game Theory
Thinking Strategically

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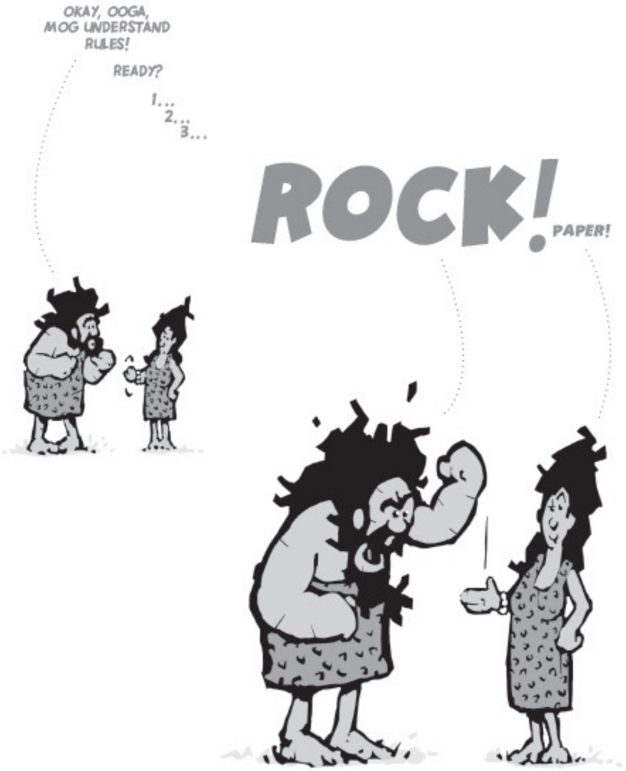
Game Theory Overview

- Overview of games and strategic thinking
- Simultaneous-move, one-shot games
- Infinitely repeated games
- Finitely repeated games
- Multistage games

Game Theory Overview

- *Game theory* is a general framework to aid decision making when agents' payoffs depends on the actions taken by other players.



Game Theory Overview

- Games consist of the following components:
  - **Players** or agents who make decisions.
  - **Payoff** of players under different strategy scenarios.
  - A description of the **order of play**.
  - A description of the **frequency of play or interaction.**
  - **Planned actions** of players, called strategies.

Game Theory Overview

- Check out this video for a brief overview

- Deal or No Deal: An entertaining example of game theory in action

Order of Decisions

- Simultaneous-move game
  - Game in which each player makes decisions without the knowledge of the other players' decisions.
- Sequential-move game
  - Game in which one player makes a move after observing the other player's move.

Frequency of Interaction

- One-shot game
  – Game in which players interact to make decisions only once.

- Repeated game
  – Game in which the same players interact to make decisions more than once.

One Shot Games

One Shot Game: Theory

- Normal-form game
  - A representation of a game indicating the players, their possible strategies, and the payoffs resulting from alternative strategies.

One Shot Game: Theory

	Rock	Paper	Scissors
Rock	Mog: $0 Ooga: $0	Mog: -$5 Ooga: $5	Mog: $5 Ooga: -$5
Paper	Mog: $5 Ooga: -$5	Mog: $0 Ooga: $0	Mog: -$5 Ooga: $5
Scissors	Mog: -$5 Ooga: $5	Mog: $5 Ooga: -$5	Mog: $0 Ooga: $0

One Shot Game: Theory

- Normal-form game
  - A representation of a game indicating the players, their possible strategies, and the payoffs resulting from alternative strategies.
- Strategy
  - A decision rule that describes the actions a player will take at each decision point
  - Plan of attack

Normal form game: One shot, simultaneous move

- Prisoner's Dilemma

- Caught on Naval Station Norfolk
  - Espionage or trespassing?

Normal form game: One shot, simultaneous move

Bob's strategies

		Bob	
	Strategy	Rat	Clam up
Al	Rat	-10, -10	0, -20
	Clam up	-20 , 0	-1, -1

Bob's possible payoffs from strategy "clam up"

Al's strategies

Player A's possible payoffs from strategy "clam up"

Payoffs: (Left, Top)	Payoffs: (Al, Bob)	Payoffs: Years in jail

Possible Strategies

- Strategy
  - A decision rule that describes the actions a player will take at each decision point
  - Plan of attack or “reaction”

Possible Strategies

- Strategy
  - A decision rule that describes the actions a player will take at each decision point
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- Nash equilibrium
  - A condition describing a set of strategies in which no player can improve her payoff by unilaterally changing her own strategy, given the other players’ strategies.
  - Your best move, considering other players best move
  - Predicted outcome

Al's Strategy

Al	Bob		
	Strategy	Rat	Clam up
	Rat	-10, -10	0, -20
	Clam up	-20 , 0	-1, -1

Al's strategy:

If Bob rats, then Al's highest payoff is?
If Bob clams up, then Al's highest payoff is?

Al’s Strategy

Al	Bob		
	Strategy	Rat	Clam up
	Rat	**-10**, -10	**0**, -20
	Clam up	-20 , 0	-1, -1

Al’s strategy:

If Bob **rats**, then Al’s highest payoff is? **Rat (-10 > -20)**
If Bob **clams up**, then Al’s highest payoff is **Rat (0 > -1)**

Possible Strategies

- Dominant strategy
  - A strategy that results in the highest payoff to a player regardless of the opponent's action.
  - Al is doing the same thing, **ratting**, regardless of what Bob does

Bob's Strategy

Al	Bob		
	Strategy	Rat	Clam up
	Rat	**-10**, **-10**	**0**, -20
	Clam up	-20 , **0**	-1, -1

Bob's has a **strategy**:

If Al **rats**, then Bob's highest payoff is? **Rat (-10 > -20)**
If Al **clams up**, then Bob's highest payoff is **Rat (0 > -1)**

Bob’s Strategy

		Bob	
Al	**Strategy**	Rat	Clam up
	Rat	**-10**, **-10**	**0**, -20
	Clam up	-20 , **0**	-1, -1

A Nash equilibrium results when Al rats and Bob rats

If either player changed their strategy unilaterally (by themselves) then they would be worse off.

Nash Equilibrium

- Named after John Nash
  - Life depicted in the movie “A Beautiful Mind”

  - Click to watch a Nash equilibrium depicted in the movie

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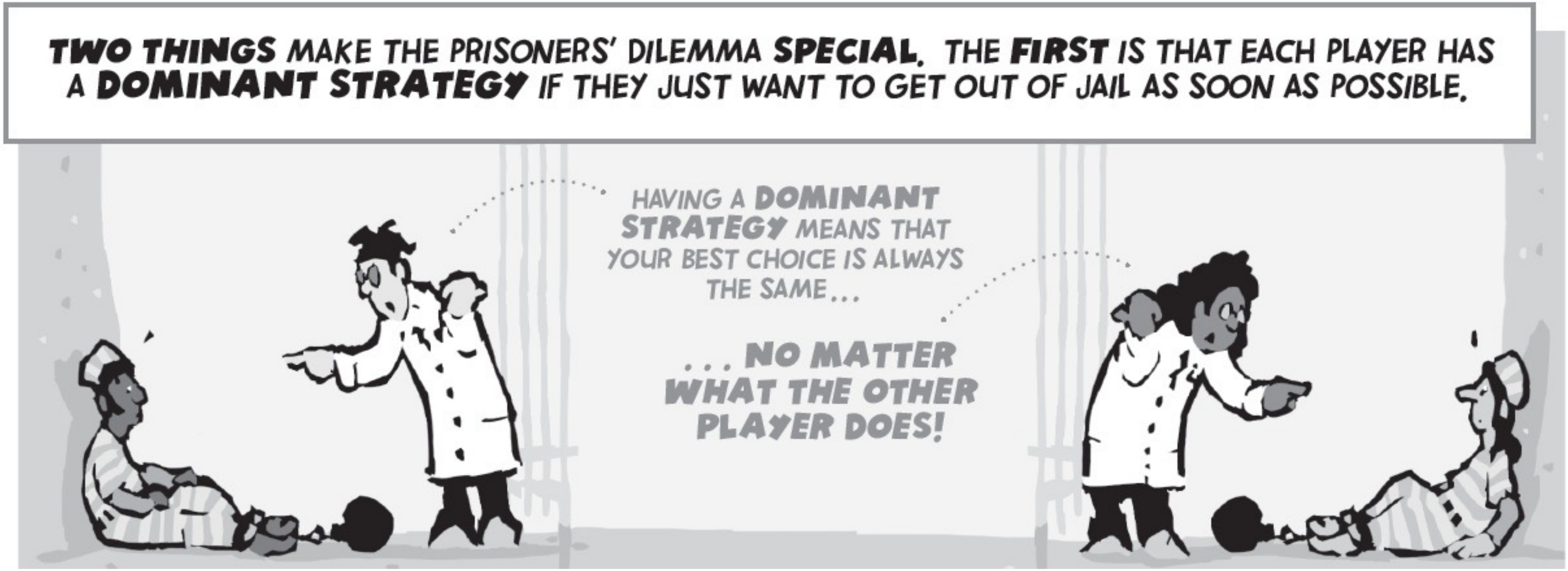
  - That is, do any of the players have an incentive change your move, unilaterally?

Nash Equilibrium

- Named after John Nash
  - Life depicted in the movie "A Beautiful Mind"
  - Click to watch a Nash equilibrium depicted in the movie
  - Question: Does this do a good job of explaining the concept of a Nash equilibrium?
  - That is, do any of the players have an incentive change your move, unilaterally?
  - Answer: No! Each guy has an incentive to change his move. Pretend he's going for one of the friends and then actually go for the "blonde"
  - It doesn't actually show Nash's key idea!

Nash Equilibrium and the Prisoner's Dilemma



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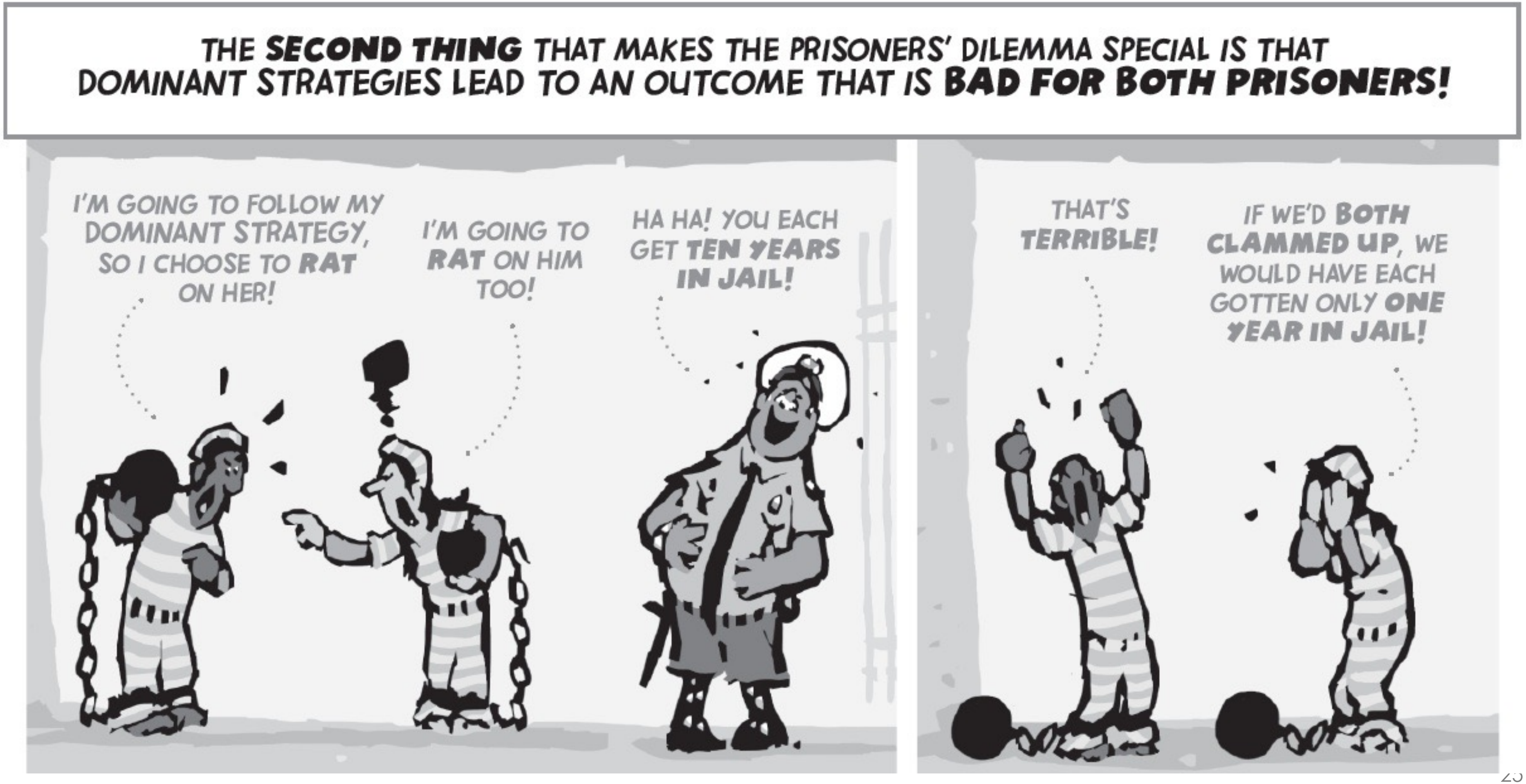

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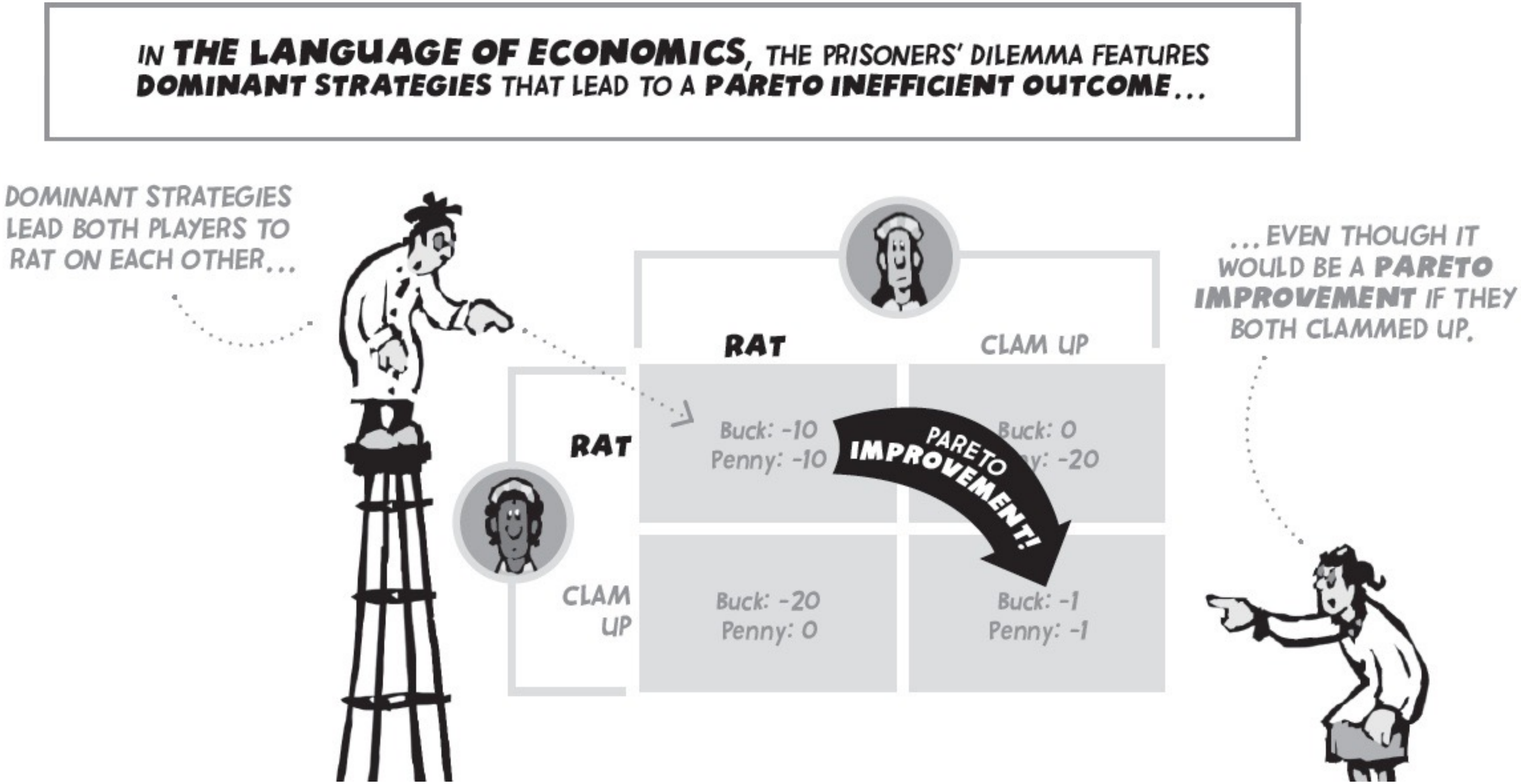
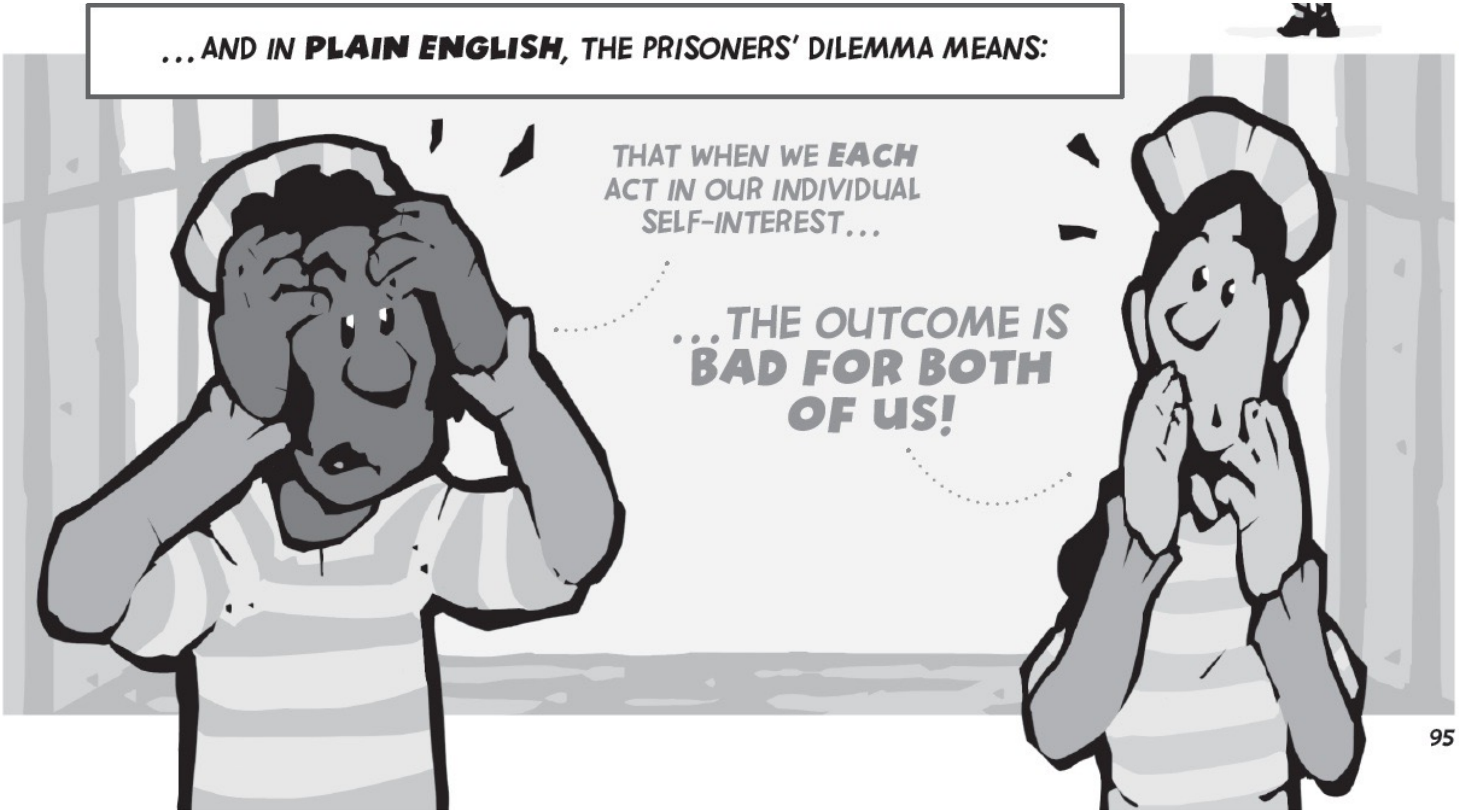


Changing their moves would make **both** players better off!

Nash Equilibrium and the Prisoner's Dilemma



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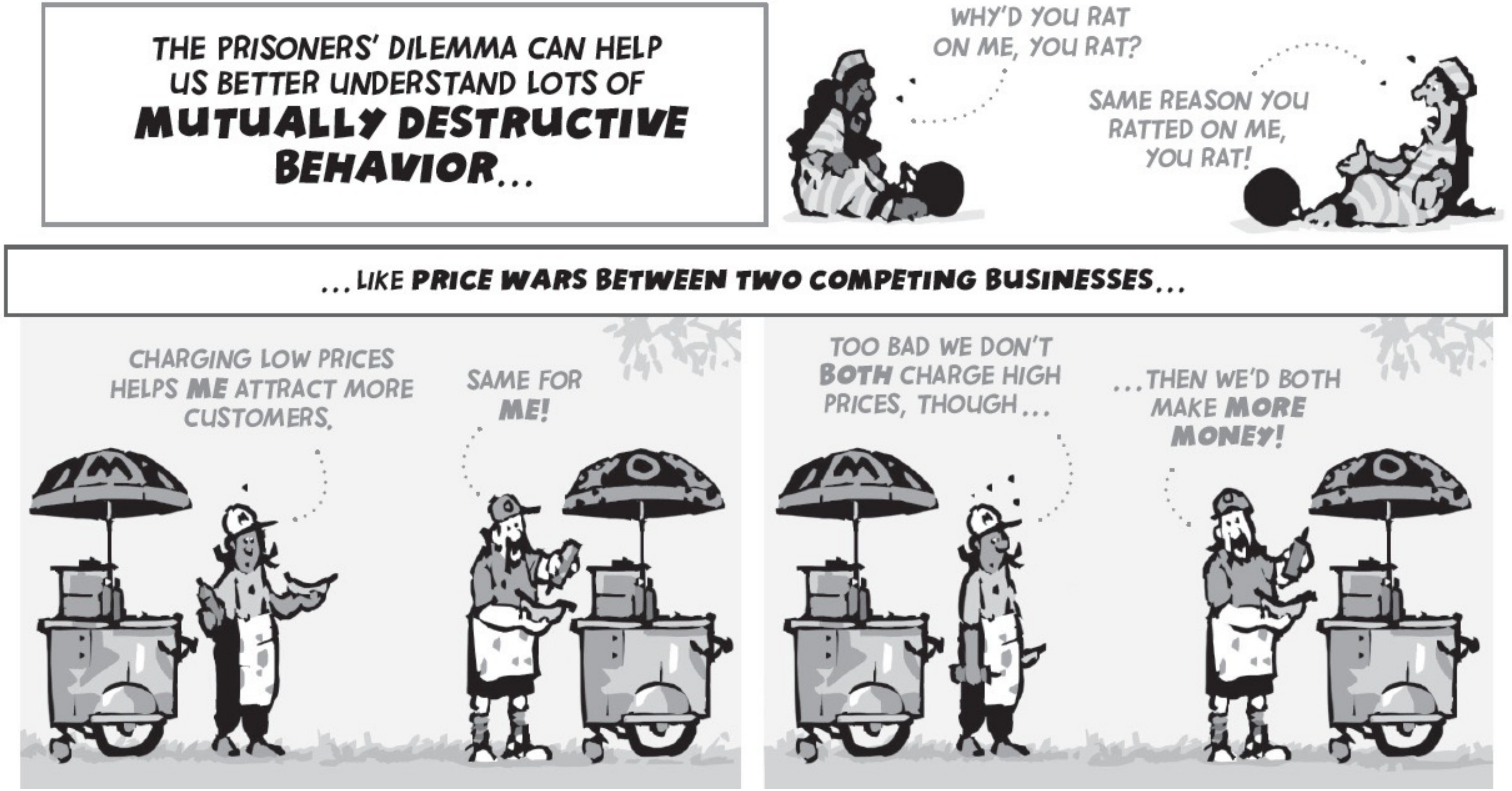

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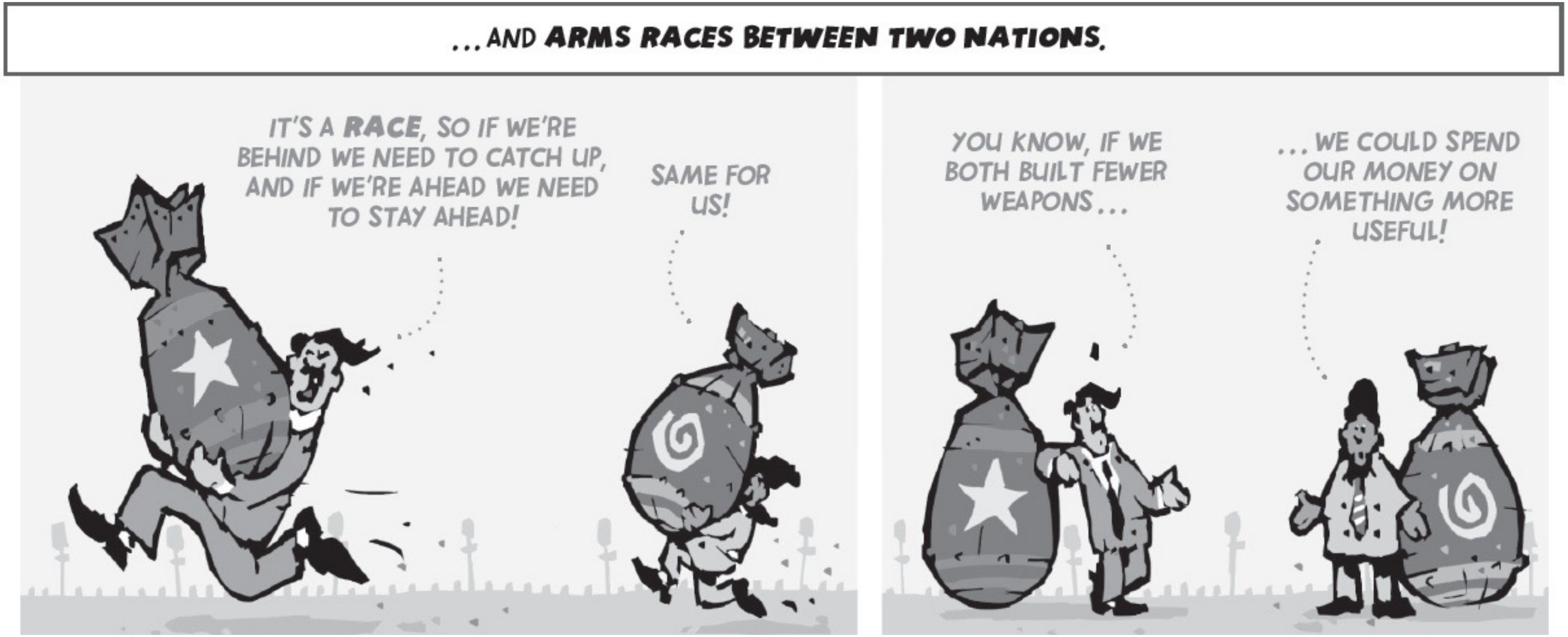

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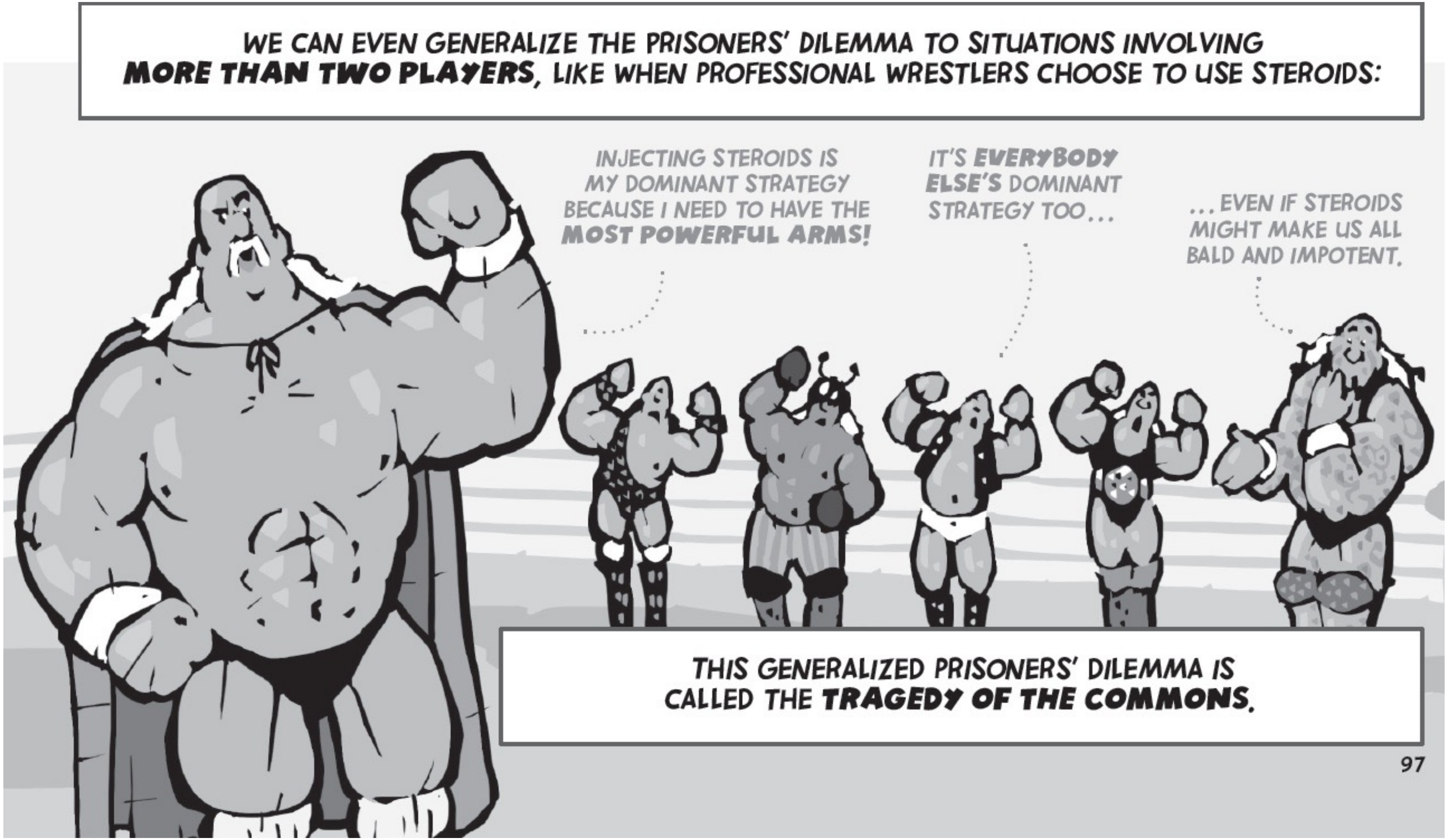
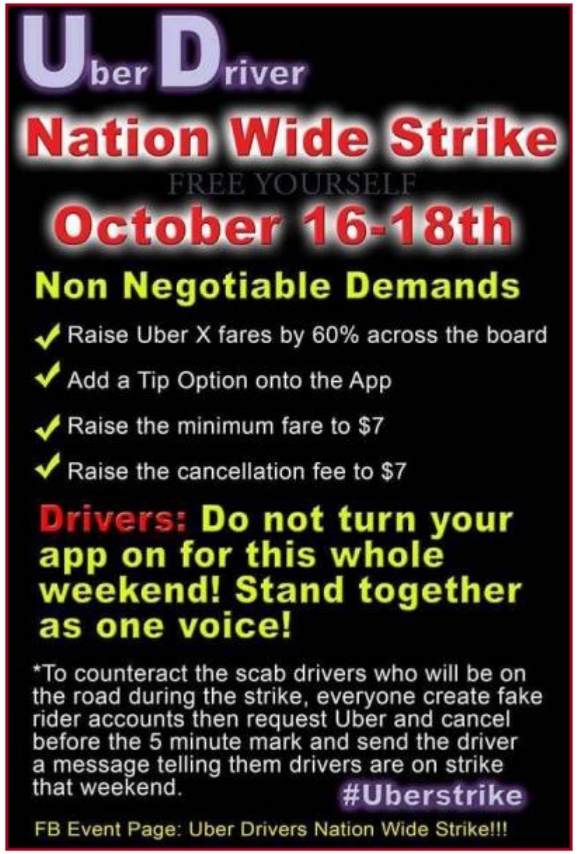


Nash Equilibrium and the Prisoner's Dilemma

- **<u>Uber and the prisoners dilemma</u>**
  - Check out this poster for Uber driver's attempt to strike for better working conditions



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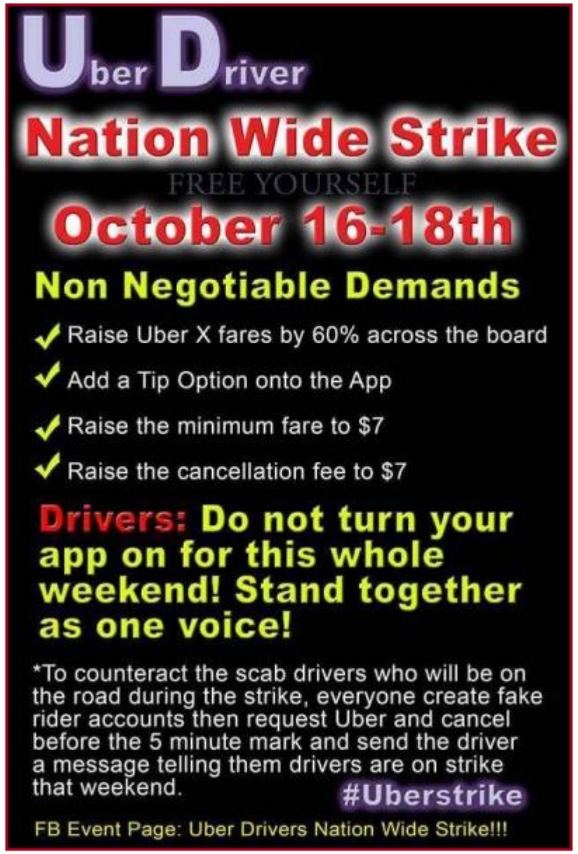


Because of surge pricing, the incentive for drivers to "defect" was huge. Opportunity to make tons of $$.

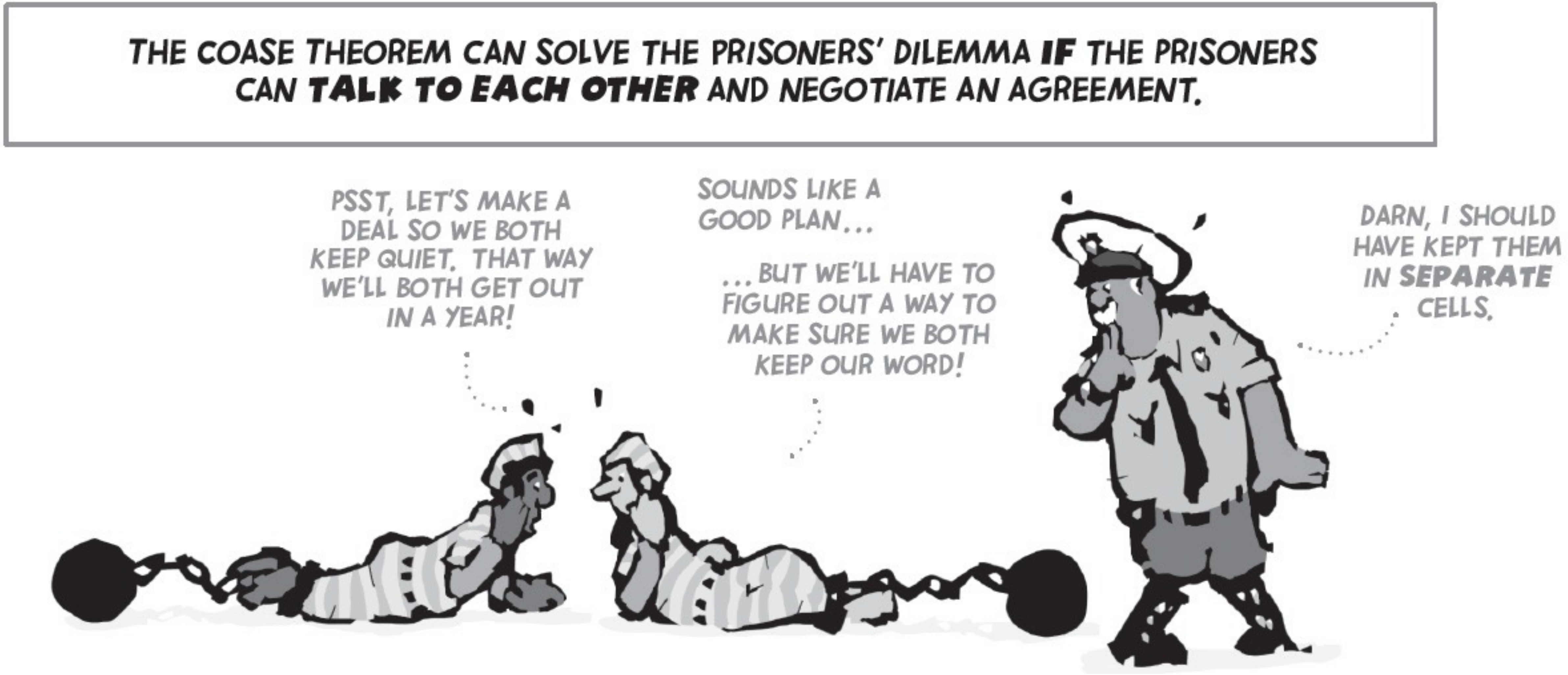
During the supposed strike, the number of Uber Drivers on the road was huge. Surge pricing was not activated because it was not necessary.

This was great for consumer!

Nash Equilibrium and the Prisoner's Dilemma

- Question
  - How might the Coase Theorem be helpful in a prisoner's dilemma?

Nash Equilibrium and the Prisoner’s Dilemma



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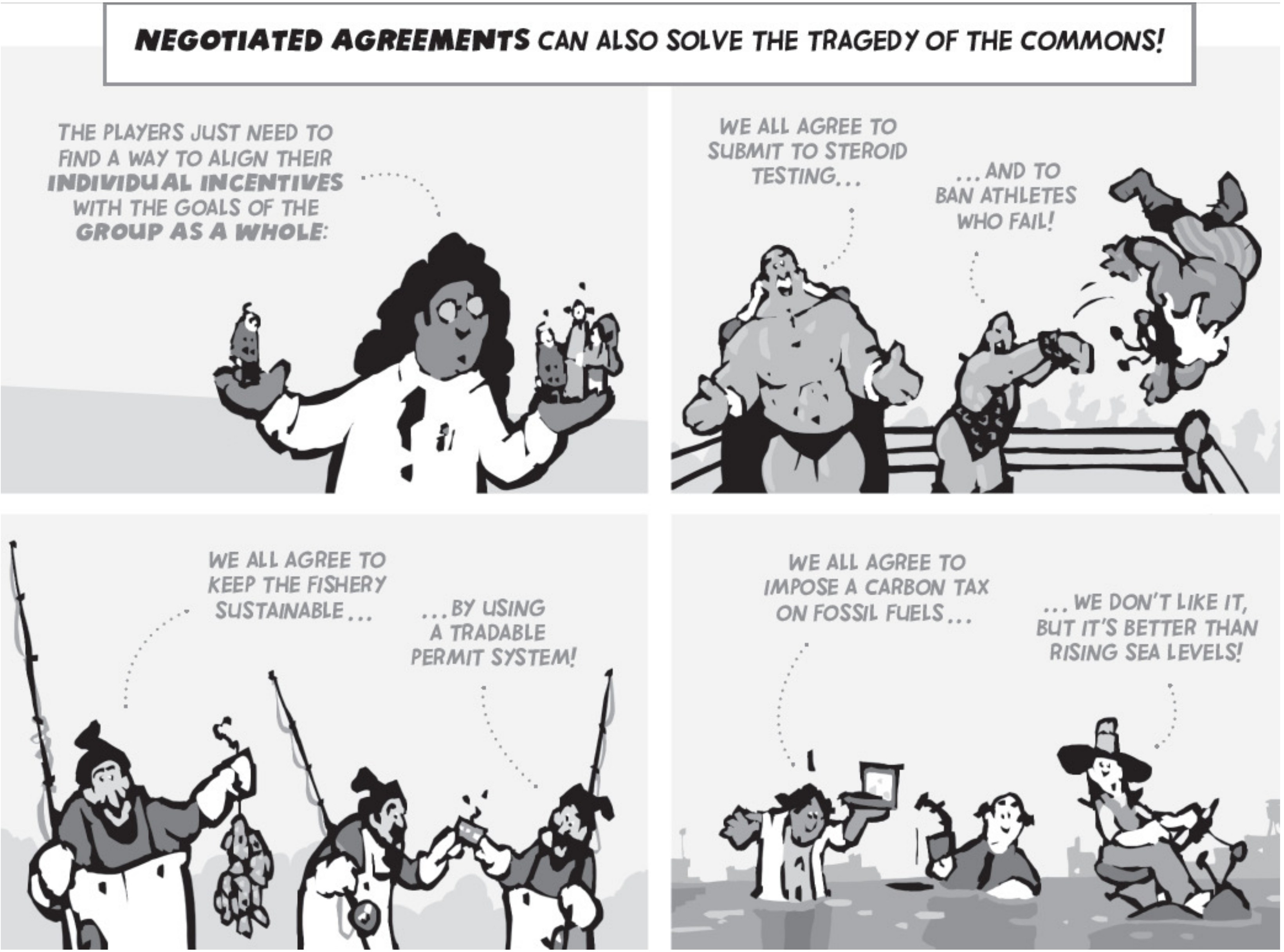

Nash Equilibrium and the Prisoner's Dilemma

- Question
  - How might the Coase Theorem be helpful in a prisoner's dilemma?

Don't forget about the transaction costs involved

- **I**nformation / search
- **C**ontracting / bargaining
- **E**nforcement / monitoring

Application of one shot pricing decisions

	Firm B		
Firm A	**Strategy**	Low price	High price
	Low price	0, 0	4, -1
	High price	-1, 5	3, 3

Application of one shot pricing decisions

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Firm A's strategy:
If Firm B goes low, then Firm A should go low (0 > -1)
If Firm B goes high, then Firm A should go low (4 > 3)

Application of one shot pricing decisions

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	Strategy	Low price	High price
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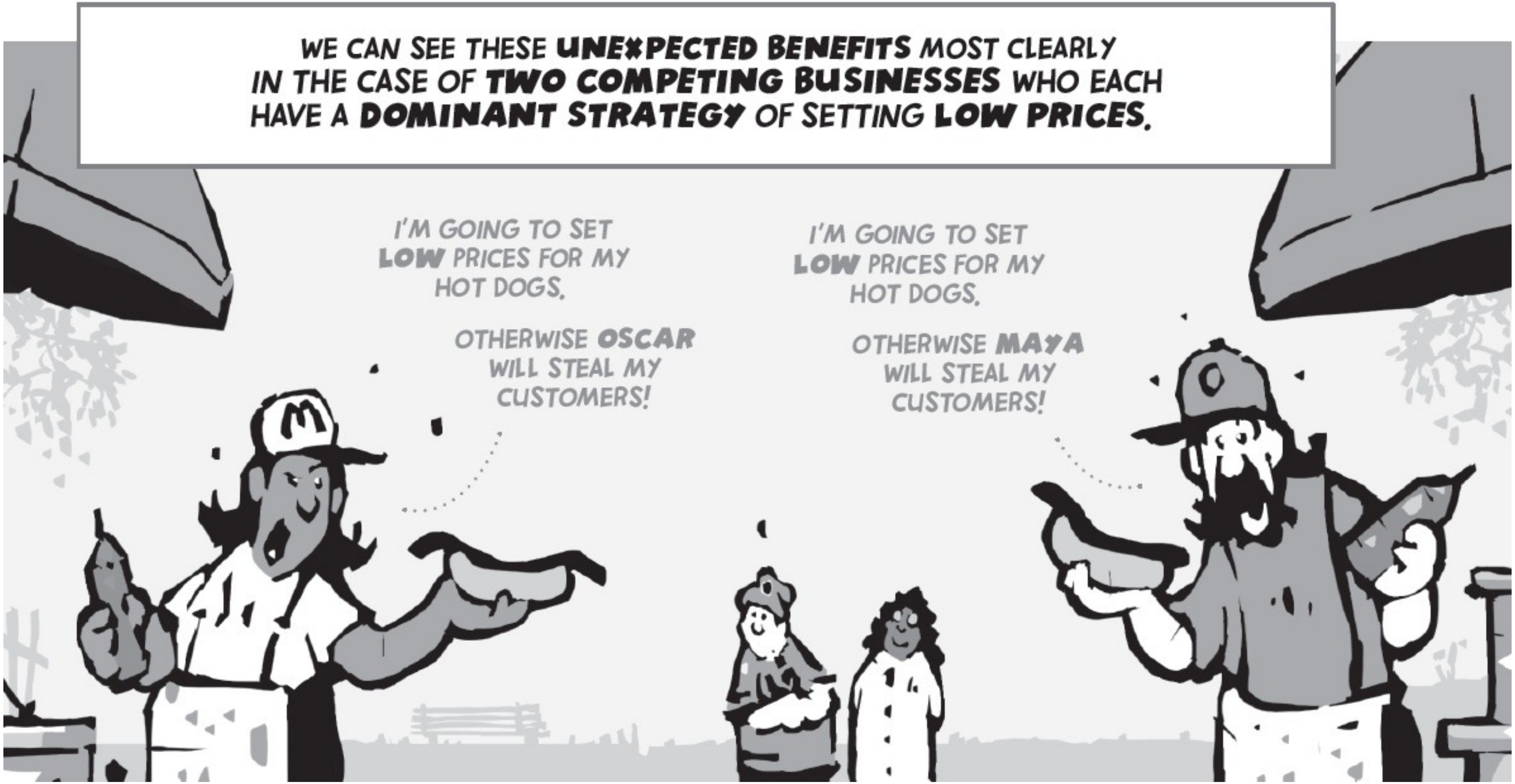
Firm A's strategy:
If Firm B goes low, then Firm A should go low (0 > -1)
If Firm B goes high, then Firm A should go low (4 > 3)

Firm Bs strategy:
If Firm A goes low, then Firm B should go low (0 > -1)
If Firm A goes high, then Firm B should go low (5 > 3)

Application of one shot pricing decisions



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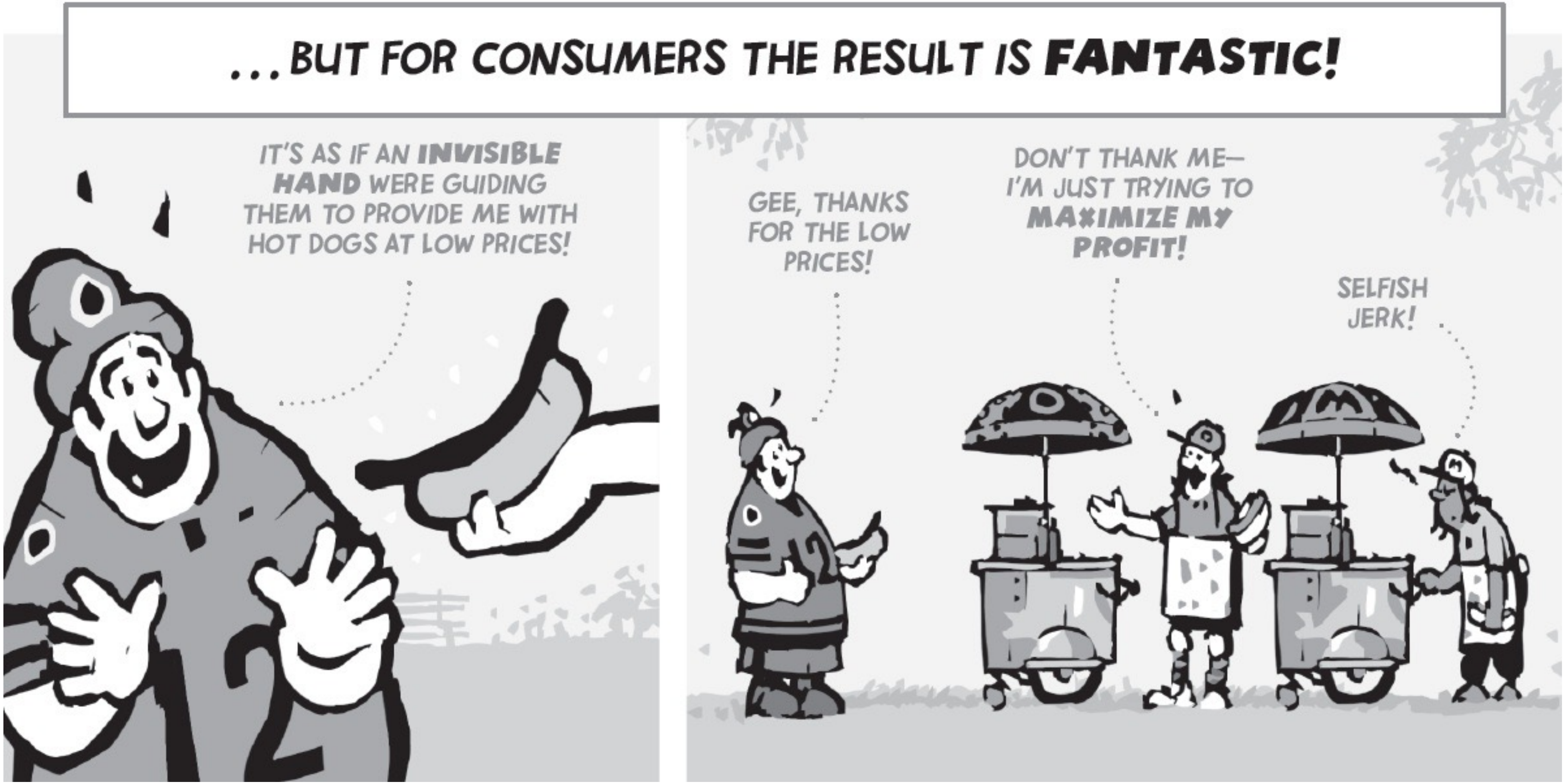

Nash Equilibrium and Coordination

- Sometimes games are more about coordinating with the other player

- E.g. consider this thought experiment from philosopher Jean-Jacques Rousseau

- Stag-Hunt Game

Nash Equilibrium and Coordination

VS

Application of one-shot decisions: Coordination

	Tim		
Brian	**Strategy**	Stag	Rabbit
	Stag	100, 100	0, 6
	Rabbit	6 , 0	3, 3

What are the strategies for each player?

Application of one-shot decisions: Coordination

Brian	Tim		
	Strategy	Stag	Rabbit
	Stag	100, 100	0, 6
	Rabbit	6 , 0	3, 3

Tim's strategy:

If Brian goes stag, then Tim should go stag (100 > 6)

If Brian goes rabbit, then Tim should go rabbit (3 > 0)

Application of one-shot decisions: Coordination

		Tim	
Brian	**Strategy**	Stag	Rabbit
	Stag	100, 100	0, 6
	Rabbit	6 , 0	3, 3

2 Nash equilibrium

What are some ways to coordinate on one equilibrium?

Strategic Interaction Penalty Kicks

Strategic Interaction Penalty Kicks

		Goalie	
Kicker	**Strategy**	Left	Right
	Left	-1, 1	1, -1
	Right	1, -1	-1, 1

Strategic Interaction Penalty Kicks

	Goalie		
Kicker	**Strategy**	Left	Right
	Left	-1, 1	1, -1
	Right	1, -1	-1, 1

Goalie's strategy:
If kicker goes left, then goalie should go left (1 > -1)
If kicker goes right, then goalie should go right (1 > -1)

Strategic Interaction Penalty Kicks

	Goalie		
Kicker	**Strategy**	Left	Right
	Left	-1, 1	1, -1
	Right	1, -1	-1, 1

Goalie's strategy:
If kicker goes left, then goalie should go left (1 > -1)
If kicker goes right, then goalie should go right (1 > -1)

Kicker's strategy:
If goalie goes left, then kicker should go right (1 > -1)
If goalie goes right, then kicker should go left (1 > -1)

Strategic Interaction Penalty Kicks

Kicker	Goalie		
	Strategy	Left	Right
	Left	-1, 1	1, -1
	Right	1, -1	-1, 1

There are no Nash equilibrium outcomes associated with this game.
Q: How should the players play this type of game?

A: Play a **mixed (randomized) strategy**, whereby a player randomizes over two or more available actions in order to keep rivals from being able to predict his or her actions.

Infinitely Repeated Games

Infinitely repeated games: Theory

- An *infinitely repeated game*
  - A game that is played over and over again … forever
  - Payoffs during each play of the game.

  - Disconnect between <u>current decisions</u> and <u>future payoffs</u> suggest that payoffs must be appropriately discounted.

Present Value: Review

When a firm earns the same profit π over an infinite time horizon

$$PV_{Firm} = \pi_0 + \frac{\pi_1}{1+i} + \frac{\pi_2}{(1+i)^2} + \cdots + \frac{\pi_T}{(1+i)^T} = \sum_{t=0}^{T} \frac{\pi_t}{(1+i)^t}$$

It simplifies to: $PV_{Firm} = \left(\frac{1+i}{i}\right)\pi$

i is the interest rate

This is the formula for the present value of discounting a payment for 'eternity'

Present Value: Review

For example π = \$15 and i = 5% over an infinite time horizon

$$PV = 15 + \frac{15}{1.05} + \frac{15}{(1.05)^2} + \frac{15}{(1.05)^3} + \cdots$$

$$PV = 15 + 14.29 + 12.95 + \cdots$$

$$PV = \frac{1.05}{0.05} 15 = \$315$$

Supporting collusion with trigger strategies

	Firm B		
Firm A	**Strategy**	Low price	High price
	Low price	0, 0	50, -40
	High price	-40 , 50	10, 10

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The Nash equilibrium to the one-shot, simultaneous-move pricing game is: Low, Low

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When this game is repeatedly played, it is possible for firms to collude without fear of being cheated on using ***trigger strategies***.

Supporting collusion with trigger strategies

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The Nash equilibrium to the one-shot, simultaneous-move pricing game is: Low, Low

When this game is repeatedly played, it is possible for firms to collude without fear of being cheated on using ***trigger strategies***.

Trigger strategy: a strategy that is contingent on the past play of a game and in which some particular past action “triggers” a different action by a player.

Supporting collusion with trigger strategies

		Firm B	
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	Low price	0, 0	50, -40
	High price	-40 , 50	10, 10

Trigger strategy example: Both firms charge the high price, provided neither of us has ever "cheated" in the past (charge low price).

Supporting collusion with trigger strategies

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Trigger strategy example: Both firms charge the high price, provided neither of us has ever "cheated" in the past (charge low price).
If one firm cheats by charging the low price, the other player will punish the deviator by charging the low price forever after.

When both firms adopt such a trigger strategy, there are conditions under which neither firm has an incentive to cheat on the collusive outcome.

Supporting collusion with trigger strategies

		Firm B	
Firm A	**Strategy**	Low price	High price
	Low price	0, 0	50, -40
	High price	-40 , 50	10, 10

Suppose firm A and B repeatedly play the game above, and the **interest rate is 40 percent**. Firms agree to charge a high price in each period, provided neither has cheated in the past.

Q: What are firm A's profits if it <u>cheats</u> on the collusive agreement?

Supporting collusion with trigger strategies

	Firm B		
Firm A	**Strategy**	Low price	High price
	Low price	0, 0	50, -40
	High price	-40 , 50	10, 10

Q: What are firm A's profits if it does not cheat on the collusive agreement?

Supporting collusion with trigger strategies

	Firm B		
Firm A	**Strategy**	Low price	High price
	Low price	0, 0	50, -40
	High price	-40 , 50	10, 10

Q: What are firm A's profits if it does not cheat on the collusive agreement?

A: $10 + \frac{10}{(1+0.4)} + \frac{10}{(1+0.4)^2} + \cdots = \boxed{\frac{10(1+0.4)}{0.4} = \$35}$

Supporting collusion with trigger strategies

	Firm B	
Firm A		
Strategy	Low price	High price
Low price	0, 0	50, -40
High price	-40 , 50	10, 10

Q: Does an equilibrium result where the firms charge the high price in each period?

A: Since $\$50 > \$35$, the present value of firm A's profits are higher if A cheats on the collusive agreement. In equilibrium both firms will charge low price and earn zero profit each period.

Supporting collusion with trigger strategies

	Firm B		
Firm A	**Strategy**	Low price	High price
	Low price	0, 0	50, -40
	High price	-40 , 50	10, 10

Q: What interest rate (value of future profit) would keep firm from cheating?

$$\$50 = \$10\left(\frac{1+i}{i}\right)$$

$$i = 0.25$$

Factors Effecting Collusion

Sustaining collusion via trigger strategies is easier when players know:

1) How to punish
   - Who their rivals are, who rivals customers are (to steal), when the other player cheats, etc.

2) Discount rate / time horizon
   - How much you and competitor values future payoffs

Tit-for-Tat Strategy

- A player mimics the opponents move in the previous period
  - E.g. cooperate this round if they cooperated last round

Tit-for-Tat Strategy

- A player mimics the opponents move in the previous period
  - E.g. cooperate this round if they cooperated last round
  - Yul Kwon on the podcast "[People I Mostly Admire]"
    - Strategy to win season 13 of Survivor
    - Game theory discussion starts 21 minutes in.

Tit-for-Tat Strategy

- Robert Axelrod
  - Political Scientist worked on the evolution of cooperation
  - Optimal way to play prisoner's dilemma?
  - 1980's organized a round robin tournament
    - People could submit a strategy to play: 200 rounds
    - Simplest strategy tit-for-tat (4 lines of code) won

Tit-for-Tat Strategy

- Strategy for life and to win survivor?

  1) Start out being kind, give people the benefit of the doubt

  2) Whenever someone betrays you, act fiercely and punitively. Make it clear it is not ok to betray you.

  3) Give them immediate forgiveness

  4) Repeat

Finitely Repeated Games

Finitely Repeated Game: Theory

- *Finitely repeated* games are one-shot game repeated a finite number of times.

- Variations of finitely repeated games where players
  1) do not know when the game will end;
  2) know when the game will end.

  – We'll discuss situations when you know the end point.

Repeated games with a known final period

Firm A	Firm B		
	Strategy	Low price	High price
	Low price	0, 0	50, -40
	High price	-40 , 50	10, 10

When this game is repeated some known, finite number of times and there is only one Nash equilibrium, then collusion cannot work.

The only equilibrium is the single-shot, simultaneous-move Nash equilibrium; in the game above, both firms charge low price.

Repeated games with a known final period

The only equilibrium is the single-shot, simultaneous-move Nash equilibrium, that is not cooperating in any time-period.

e

Repeated games with a known final period

The only equilibrium is the single-shot, simultaneous-move Nash equilibrium, that is not cooperating in any time-period.

- Why???
- Known as the *end of period problem*
- Neither player has an incentive to 'cooperate' in the last round
  - No ability to punish bad behavior
- If you'll cheat in the last period why should I cooperate in the 2nd to last period? Then 3rd to last period, etc.
- Becomes a vicious cycle . . .

Multi-Stage Games

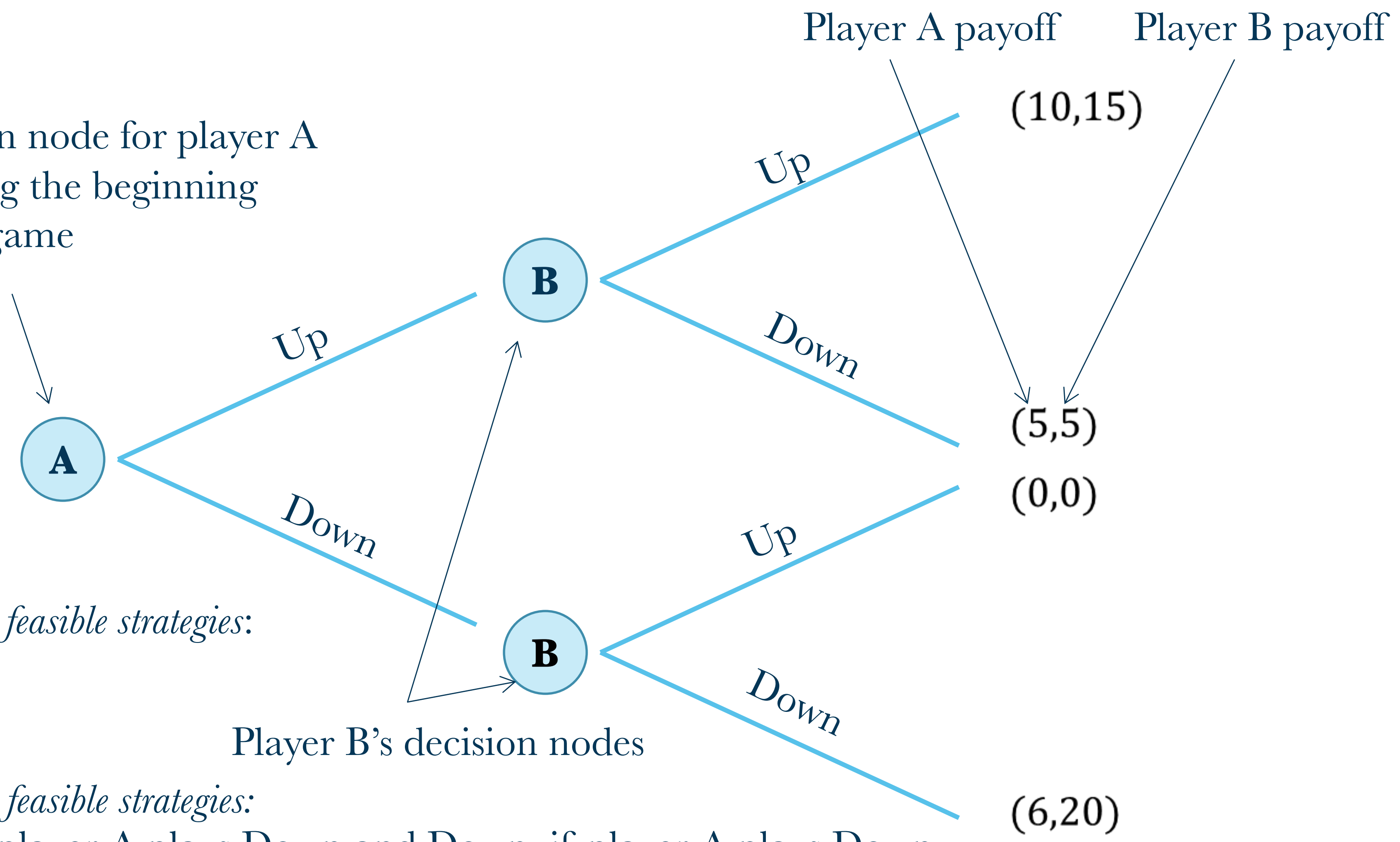
Multistage Games: Theory

- Multistage games differ from the previously examined games by examining the timing of decisions in games.

- Extensive form game (game tree)

Multistage Games: Theory

Decision node for player A denoting the beginning of the game



Player A feasible strategies:

Up

Down

Player B feasible strategies:

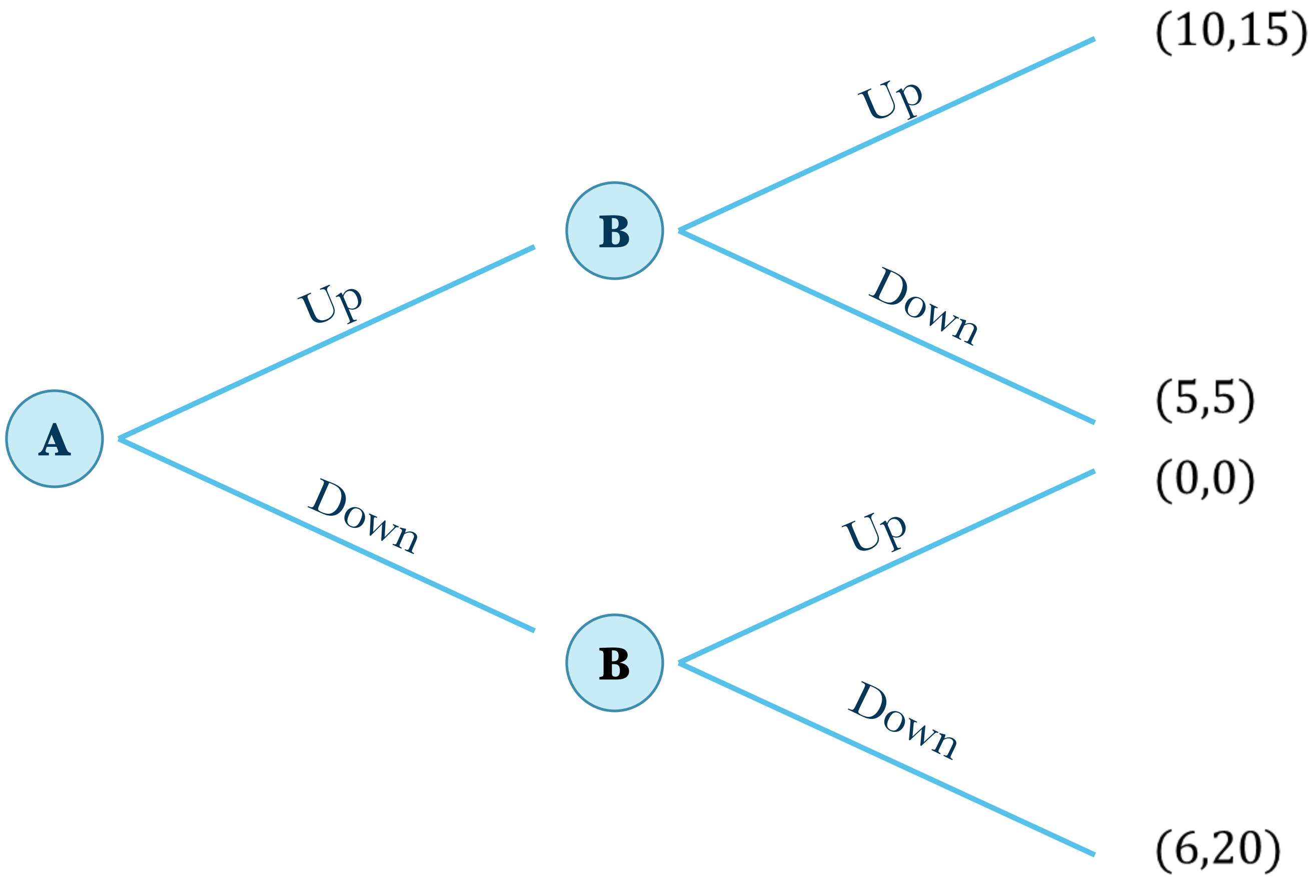
Up, if player A plays Down and Down, if player A plays Down

Up, if player A plays Up and Down, if player A plays Up

Theory: Sequential-move game in extensive form

- Solving game through backward induction
  - Fancy phrase to say, start at the end of the problem (second players choice)
  - Figure out how they would react
  - Then first player makes their choice that maximizes their (player 1s) payoff

Equilibrium



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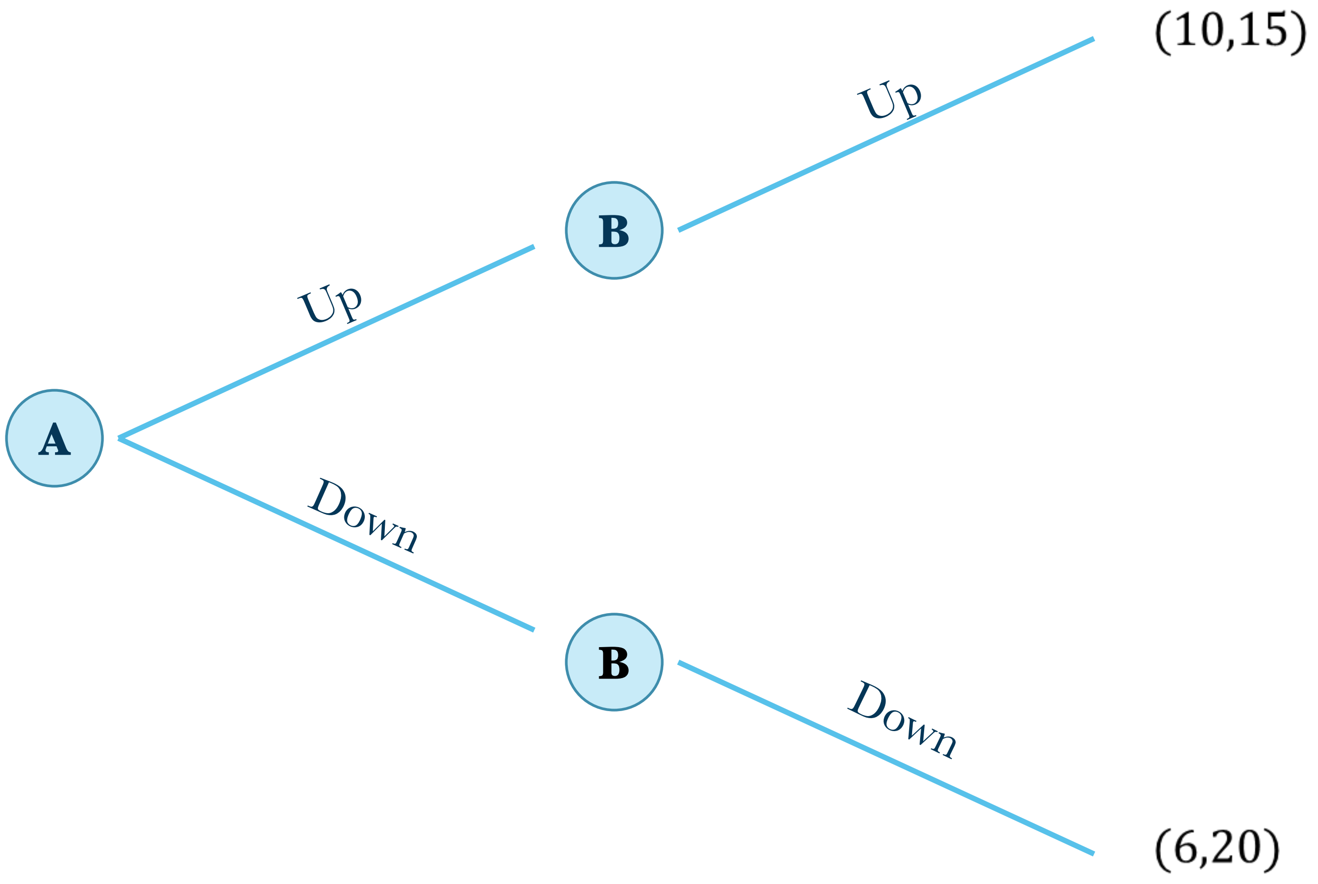

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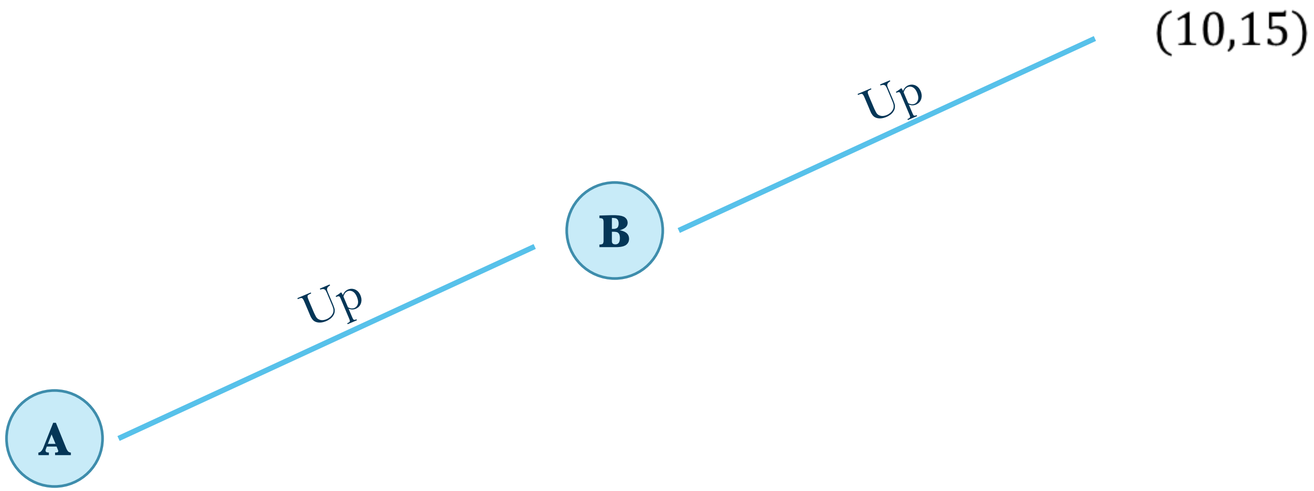

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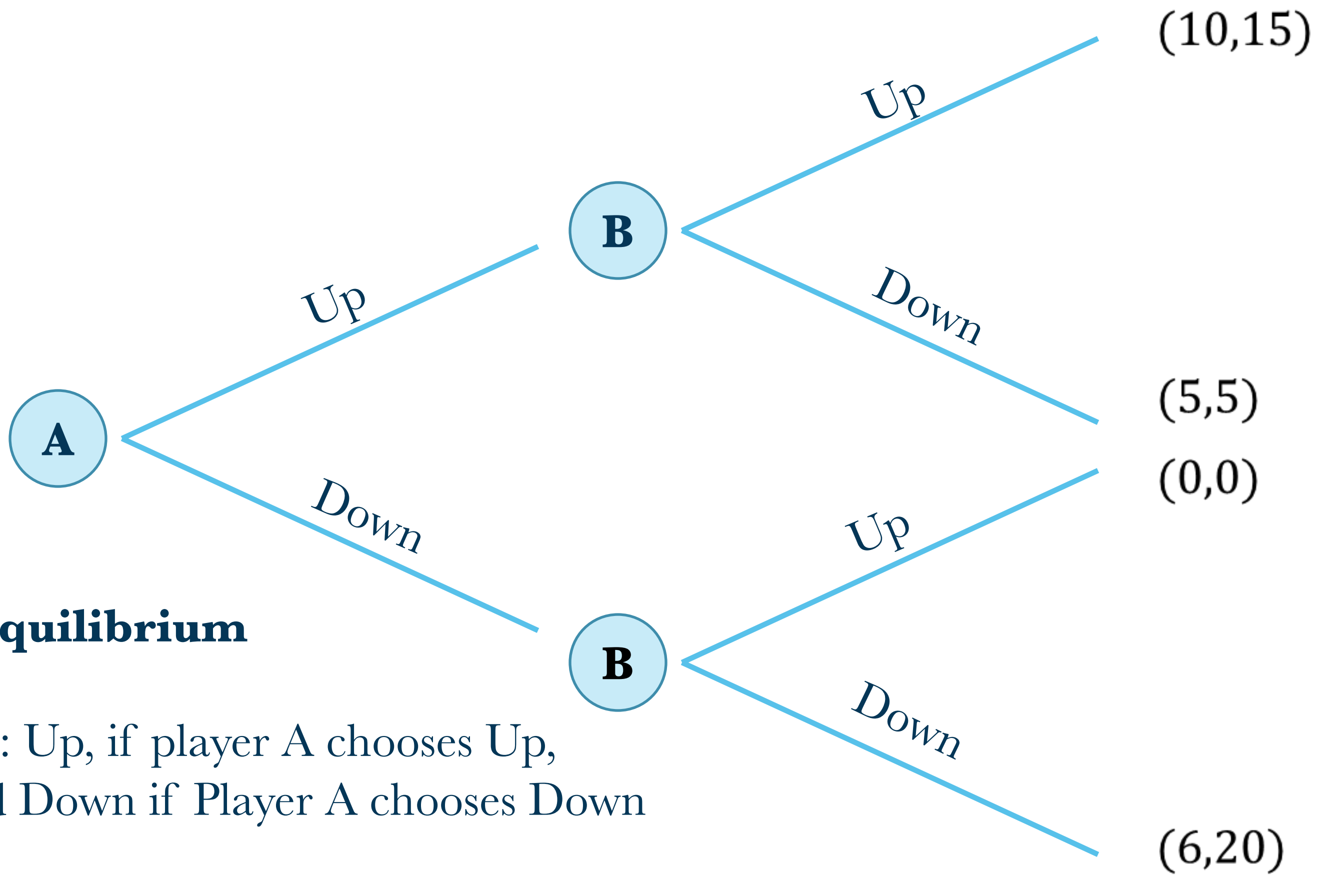

Nash Equilibrium

Player B: Up, if player A chooses Up,
and Down if Player A chooses Down

Player A: Up

Equilibrium

- Ultimatum Game
  - First person makes a “take-it-or-leave-it” offer.
  - Second person observes the first person’s offer and either accepts or rejects.
  - Strong parallel to bargaining situations observed in the real world.
  - Classical economic assumption: people care about maximizing own monetary payoffs.

Conclusion

- In one-shot games resulting payoffs are sometimes lower than they would be if players could collude
  - Players play according to their dominant strategy, which often corresponds to the “cheating” strategy

- In infinitely repeated games players can collude by using trigger strategies given a sufficiently low interest rate

- Tit-for-tat can be effective strategy as well

Conclusion

- In finitely repeated games
  - Collusion cannot be supported when players know when the game ends due to unraveling
  - In sequential, or multi-stage, games collusion can only be supported when threats are credible