

Automated collision detection using machine learning algorithms on sensor data

Project Report

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Abstract

This paper deals with the application of machine learning approaches on IoT sensor data for a specific showcase scenario. The feasibility of machine learning algorithms for detecting collisions on a skateboard, on the basis acceleration data, is explored. Machine learning methods, restricted to algorithms provided by the Apache Spark function library “MLlib”, are configured and their performance is compared using a f_2 -score to answer the question of an optimal method. The resulting, best performing, model utilizes a random forest method. A naiv bayes method yields the worst performance.

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1 Introduction

1.1 Motivation

The field of data analytics and machine learning is increasingly becoming the main differentiator of industry competition. This is especially true in combination with another growing field, the Internet of Things (IoT). This is what the McKinsey Global Institute's study 'The Age of Analytics: Competing In A Data-Driven World' found. [1] IBM has set a goal to tackle the challenges of IoT and Big Data with an array of new services, industry offerings and capabilities for enterprise clients, startups and developers. [2] These services are part of IBM Bluemix, a cloud platform as a service (PaaS).

An important step for IBM is getting clients interested and aware of the capabilities of IoT and machine learning so that collaborations and projects with clients can be established. One way this is done is by presenting showcases – small projects that show the possibilities and values of the technology in an exemplary way. It is essential to show technologies from the IBM Bluemix portfolio to present their capabilities and outline differentiating factors to competitors.

The Sensorboard is such a showcase. A physical skateboard is equipped with a Texas Instrument's SensorTag and able to connect wirelessly to the IBM's Bluemix cloud platform. From there one or more Sensorboards can be monitored and managed. As part of the showcase a the Sensorboard is demonstrated as a rentable and cloud-managed vehicle for urban mobility.

This showcase will be extended with machine learning applications to present the value machine learning algorithms can have on sensor data. The specific goal of that extension is to recognize collisions that happen to the Sensorboard and that indicate accidents. This recognition will be based solely on sensor data and patterns in the time series data. For this IBM Bluemix services are used to showcase the capabilities of the platform. There are services which are requested to be demonstrated and used in the showcase. This is the IBM IoT Foundation, used to connect the Sensorboard to the platform. Moreover the IBM Data Science Experience should be used as a development environment for an underlying Apache Spark cluster, on which machine learning models will be created.

1.2 Objective

A consumer grade skateboard is equipped with a Texas Instrument's SensorTag. The Texas Instrument's SensorTag collects acceleration data from the skateboard on three spacial axis. To process this data cloud services of IBM Bluemix are used. This especially includes the IBM Data Science Experience and an Apache Spark service. Consequently utilizing Apache Spark as a framework for cluster computing, machine learning models are trained using Apache Spark. This narrows down the range of machine learning methods to the ones available for processing on Apache Spark. Such methods are provided by the Apache Spark MLlib function library.

This paper investigates on the feasibility of machine learning algorithms for collision detection in the above described setting and identifies, if feasible, the best performing algorithm under the described conditions.

The scenario of impact detection that may indicate accidents requires a higher focus on detecting an accident than on preventing false alarms. A metric is found to represents this unequal relationship in the assessment of algorithm performance.

1.3 Limitations of the Research

The technologies described - the Texas Instrument's SensorTag, the IBM Bluemix services and function libraries - are a given for the project by IBM. Therefore the decisions for those and no other technologies will not be discussed and no evaluation of alternatives will be conducted in this paper.

The findings will only be acquired and applicable for the specific hardware and configuration described in the paper. Findings can not be transferred to other vehicles, sensors, configurations or any other setup than the one described.

The data which is used to come to decisions and to train models is generated in experiments, which are conducted solely for this reason. No other sensor data of skateboards or other vehicles are used in the considerations of this paper. The machine learning algorithms examined only include algorithms currently provided by the function library Apache Spark MLlib.

1.4 Organization of the Research

The chapter "Theory" introduces the topic of machine learning. Relevant machine learning theory is explained with sufficient theoretical and mathematical background information.

The goal of the chapter is to create a basic understanding for methods and metrics this paper investigates.

In the following chapter “Analysis of the pattern recognition problem”, the pattern recognition problem will be analysed in detail. The goal of the chapter is to define specific possible solutions for the identified problem.

The chapter “Algorithm configuration” brings together the theoretical knowledge about machine learning and possible solution strategies. It is shown how algorithms are best configured and applied for the problem. The goal of the chapter is to generate a comparable result for the different possible solutions.

In the chapter “Evaluation”, the results are then compared to each other. The goal of this chapter is to answer the question of whether machine learning algorithms are feasible for collision detection. If feasible, a winning solution that is best performing under the considered conditions is identified. A conclusions about the results is drawn.

In the end, the chapter “Reflection and outlook” presents a critical reflection of the results and a proposal on possible further research and development.

2 Theory

2.1 Machine Learning

Historically computers had to be programmed on a specific task using domain knowledge of humans. Machine learning challenges this traditional way of programming. Rather than through extensive domain knowledge, machine learning algorithms are used to autonomously learn from data and information. [7]

This intention is often represented by a “learning” *input-output function*. Assume there exists a function, f , which represents a true behaviour or correlation. The task of any machine learning algorithm is thereby to find a *hypothesis function* h that resembles the behaviour of f as closely as possible. A function h is also called *model*. Both f and h are function of a fixed length vector-valued input $X = x_1, x_2, \dots, x_n$ which has n components. Those might be n attributes of an observed object. An input X is often referred to as a *data point*. The components of a data point (x_1, \dots, x_n) are the *features* of X . The features of a data point span the *feature space*. The feature space refers to the n -dimensional space in which the features of a data point are represented.

The function h can be thought of as a device that has X as an input and $h(X)$ as an output. The hypothesis function, h , is selected based on a *trainings set*, Ξ , of m input vectors. The process of selecting a function h , based on a *trainings set* Ξ , is called *training* a model or *fitting* a model.

There are two major settings in machine learning.

The first setting is *unsupervised learning*. In this case the trainings set of vectors, Ξ , doesn't provide information about the value $f(X)$. The goal of the unsupervised learning algorithms usually is to partition the training set into subset, $\Xi_1, \Xi_2, \dots, \Xi_r$. An example for unsupervised machine learning learning is *clustering*, which finds clusters and discovers relationships in the data.

The second setting is *supervised learning*, in which the values of f are known for the samples in the training set Ξ . The value of f corresponding to a data point X is also called the *label* of X . A dataset with a label for every data point is referred to as a *labelled* dataset. The assumption is that a hypothesis h is a good guess for f when $h(x_i)$ agrees with the values of $f(X_i)$ or the given labels for the majority of samples X_i .

In supervised machine learning two main areas are distinguished, *regression* and *classification*.

For regression the hypothesis h has a continuous output. Regression could, for instance, be used to approximate a function for a stock price and predict a future stock price as a numerical value on a continuous range of possible prices.

For classification the hypothesis h has a categorical output. Classification could, for instance, be used to classify emails to either spam or no spam. It also finds application in pattern recognition. The case of an classification task that has a need to classify data into two distinguished classes is called *binary classification*. The alternative case is *multiclass classification*, which aims to classify data points into one of multiple different classes.

Further the setting of *supervised binary classification* is explored in more detail. This setting is important for the intend of this paper.

2.2 Evaluation of Classifiers

Assessing the quality of a model is essential to choosing the right method or configuration of an algorithm. This is why, in this section, different metrics are introduced to empirically measure the quality of a *binary classifiers*. Also a way of consistently checking for indicating metrics with an *validation scheme* is shown.

Evaluation metrics

Assume a binary classification algorithm is trained on a dataset, that has samples (data points) of the classes 1 and 0. In the following, the created model $h(X)$ predicts a class label for for a given sample X . Assume the samples of the class 1 are relevant. The model performance is examined in regards to the class 1.

Let the set *positives* (*pos*) be the set of all samples where the class label is 1. In contrast let the set *negatives* (*neg*) be the set of all samples where the class label is 0.

The model $h(X)$ predicts some samples as positives and some as negatives. The prediction doesn't always coincide with the true class labels.

The samples that the model identifies as positives and that are actually labelled as positives are called *true positives* (*tp*). In contrast the samples that the model identifies as positives but that are labelled as negatives are called *false positives* (*fp*).

Similarly the samples that the model identifies as negatives and that are actually labelled as negatives are called *true negatives* (*tn*). Again, in contrast the samples that the

model identifies as negatives but that are labelled as positives are called *false negatives* (fn).

The most basic metrics of a classifier are it's *precision* and *recall*.

Precision is defined as the ratio of true positives to all positives, that the model h identified in the training data (true positives + false positives):

$$\text{Precision} = \frac{tp}{tp + fp} \quad (2.1)$$

The precision can be interpreted as a measure of how good a model is at identifying only the positives as positive, while avoiding to misclassify negatives as positives.

Recall is defined as the ratio of true positives to all positives the model h identified in the training data:

$$\text{Recall} = \frac{tp}{tp + fp} \quad (2.2)$$

The recall can be interpreted as a measure of how good a model is at identifying all the positives, while missing as little positives as possible.

Intuitively it can be seen that precision and recall have a inverse relationship to each other. For instance, if generally more samples are identified as positives by a model, the true positive rate rises, hence the recall. But the chance of identifying negatives as positives also rises and consequently the precision decreases.

The goal of creating a model with any given method is to increase both, precision and recall, simultaneously. Nevertheless in some situations it may be of advantage to value a good recall higher than a good precision or vice versa. This might, for instance, be the case if it is very important to find all the positives, while not caring that much about false negatives ("false alarms").

A way to merge the two metrics in a joined metric, both for the case of equal and unequal importance, is the *f-score*, also called *f_β -score*.

The f_β -score takes into account both recall and precision and can be configured to weight one more important than the other. The f_β -score is defined as follows:

$$f_\beta = (1 + \beta^2) \cdot \frac{\text{precision} \cdot \text{recall}}{(\beta^2 \cdot \text{precision}) + \text{recall}} \quad (2.3)$$

In equation 2.3 the parameter β can be used to put more weight on either precision or recall.

If $\beta > 1$ more weight is put on the recall, if $\beta < 1$ more weight is put on the precision.

For instance a f_2 -score would put more weight on the recall of a classifier:

$$f_2 = (1 + 2^2) \cdot \frac{\text{precision} \cdot \text{recall}}{(2^2 \cdot \text{precision}) + \text{recall}} \quad (2.4)$$

On the other side a $f_{0.5}$ -score would put more weight on the precision of a classifier:

$$f_{0.5} = (1 + 0.5^2) \cdot \frac{\text{precision} \cdot \text{recall}}{(0.5^2 \cdot \text{precision}) + \text{recall}} \quad (2.5)$$

In the case of equal importance, the f_1 -score is used, which is the *harmonic mean* of precision and recall:

$$f_1 = 2 \cdot \frac{\text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}} \quad (2.6)$$

A, weighted or not weighted, f_β -score can be used to compare different approaches to a classification problem to each other. [8] [3]

Validation scheme

To measure the recall, precision or other performance metrics of a model, the model has to be tested. Testing a model is done with data, that the model hasn't "seen" yet, meaning data points that weren't used in training of the model. This is usually done by *randomly* selecting a small fraction of all the available data and setting it aside as *testing data*. This data is used to test a model and evaluate it's performance later on. The remaining data is used to train or fit the model. It is called *training data*.

Doing this random selection into training and testing data can lead to problems: Depending on the random split that is done, the testing data may, by chance, be very hard to classify in comparison to the training data. This would lead to a exaggerated bad performance of the classifier when testing. On the other side, if the testing data is easy to classify in comparison to the training data later evaluation would show a better performance than it might actually have.

To counteract this phenomenon *cross-validation* is performed for testing. Cross-validation involves partitioning the available data into training and testing data multiple times,

evaluation for a specific metric, and average the results. A cross-validation approach that is relevant for this paper is *k-fold validation*.

K-fold validation is a non-exhaustive cross validation method, meaning it does not compute all the possible ways of spitting the available data into subsets.

In k-fold validation the available data is split into k equal sized, non-overlapping subsets of data points. In each validation step one of the subsets is seen as the testing data and the rest is combined to act as training data. A model is now trained on the training data and tested for different metrics, like recall and precision, on the testing data. This process is repeated k times, each time leaving out one subset (*fold*) of the available data as testing set. The metrics are then averaged.

Cross-validation averages many tests with different test and training data to achieve a consistent evaluation of a classifier. [9] [1]

2.3 Algorithms

This section will introduce basic concepts and specific algorithms for the classification setting. The purpose of this section is to give a introduction into methods and tools used in this paper and give the reader a understanding of the underlying principles. Some topics may be oversimplified or not discussed in their whole complexity. The reason for this is, that it is beyond the scope of this theory section.

Before getting into specific algorithms, a few basic concepts, relevant for most classification algorithms, are discussed.

Hyper-parameters:

Hyper-parameters are parameters that configure a machine learning algorithm. Changing the hyper-parameters of an algorithm will change the way the algorithm constructs a model h . An example for this would be the maximal height of a decision tree in a decision tree model (see section “Decision tree”). The task of finding good hyper-parameter for a algorithm is called *hyper-parameter tuning* or *hyper-parameter optimization*.

Model linearity:

A classification model classifies data points according to their feature vector. Visualized in a n -dimensional feature space, this means classification depends on the position of a data

point in the feature space. The decision of a model to classify a region in feature space as one class or another can be represented by a *decision boundary* in the feature space. Data points laying on the one side of the decision boundary will be classified to one class while data points on the other side are classified to another class.

This decision boundary of a model can have any form. If the form of the decision boundary is linear, meaning the decision boundary can be represented by a linear function in the form

$$f(x) = \beta_0 + \beta_1 X_1 + \dots + \beta_n X_n \quad (2.7)$$

in a n -dimensional feature space, the model is *linear*.

If this isn't the case the model is *non-linear*. [5]

Variance:

Variance is an error that is caused because a model varies too much with the data used for training. If a algorithm is too sensitive and captures random patterns in the training dataset it has high variance. This leads to *overfitting*. Overfitting on the training data means the model doesn't make assumptions about the data that are general enough and just "memorizes" the training data points. Then it can be said the model doesn't *generalize* enough. A high variance leads to low generalization of a model.

In the following sections the basics of certain relevant classification algorithms will be explained.

2.3.1 Linear support vector classifier

In this chapter the support vector classifier will be introduced. The method was developed in the 1990's and is often used as one of the best performing "out of the box" classifiers. The goal of a linear support classifier is to find a function for a *hyperplane* that splits the data points of two classes as best as possible. The support vector classifier is a extension of a maximal margin classifier. First the concept of classification using a hyperplane and with a maximal margin classifier will be explained. Then the extension to a support vector classifier will be shown. A model based on a support vector classifier is linear.

Hyperplane classification

The goal of the *hyperplane classification* is to classify data by finding a hyperplane that optimally separates two classes in a dataset. Assume data points are either in class $y = f(X) = -1$ or $y = f(X) = 1$.

A hyperplane is a flat affine subspace of dimension $p - 1$, where p is the number of dimensions of the parameter space. In the case of a maximal margin classifier the number of dimensions which a hyperplane will be constructed in is the number dimension of the feature space. This stems from the intention to separate data points which have a feature vector of p dimensions. Accordingly a hyperplane is defined by

$$\beta_0 + \beta_1 X_1 + \dots + \beta_p X_p = 0 \quad (2.8)$$

for a feature space of dimension p . It can be said that β_0 to β_p “define” the hyperplane. If a point $X = (x_1, x_2, \dots, x_p)$ lies on the hyperplane, the point features x_1 to x_p would satisfy the equation 2.8.

If this doesn’t hold true, but

$$\beta_0 + \beta_1 X_1 + \dots + \beta_p X_p > 0 \quad (2.9)$$

is true, the point X lies on the one side of the plane, whereas if

$$\beta_0 + \beta_1 X_1 + \dots + \beta_p X_p < 0 \quad (2.10)$$

is true, the the point X lies on the other side.

If a hyperplane is defined in a way that the data points of on class lay on the one side of the hyperplane and the data points of the other class on the other side, any given data point can be classified accordingly as either one or the other class according to equations 2.9 and 2.10.

The *magnitude* of $h(X)$ can be used to derive a confidence score for the classification. A higher magnitude of $h(X)$, which can be interpreted as distance to the plane, corresponds to a higher confidence in the classification.

Maximal margin classifier

If a class separating hyperplane exist there exists an infinite number of such hyperplanes separating the classes (a hyperplane can always be moved or varied slightly while still remaining class separating). The question arises which hyperplane should be used.

A maximal margin classifier uses the hyperplane, where the distance to the next data point is the biggest under all possibilities. The distance to the next data point is called

the *margin*. A hyperplane that has the biggest possible margin is a *maximal margin hyperplane*. Data points, for which the distance to the maximal margin hyperplane is equal to the margin, are called *support vectors* (see figure 2.1). The maximal margin hyperplane depends on the support vectors because moving or removing the support vectors would change the hyperplane. Moreover the vectors that are further away from the hyperplane than the support vectors don't contribute to the definition of the hyperplane. This allows the hyperplane to be solely defined by the support vectors.

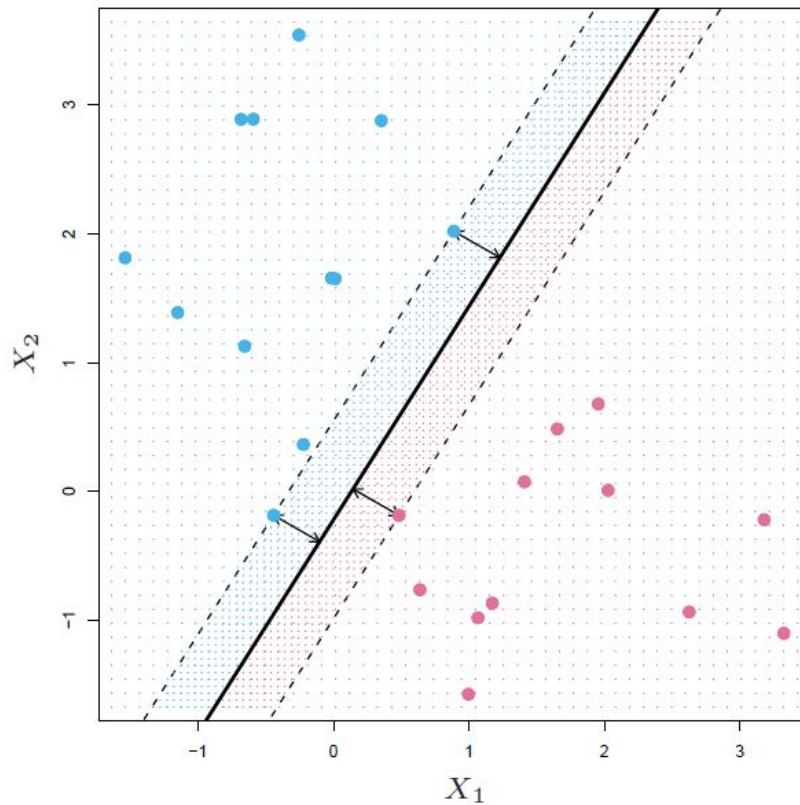


Figure 2.1: Maximal margin hyperplane separating two classes of data points with the margin borders visualized by dotted lines

To find a maximal margin hyperplane the parameters of the hyperplane have to be set in a way that maximises the size of the margin M and

$$y_i(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip}) \leq M \quad \forall i = 1, \dots, n \quad (2.11)$$

holds. y_i is the true class of the i th data point.

If all data points are classified correctly, which is the goal, the left side of the equation 2.11 is always positive. This is because either both y_i and $f(X_i)$ are negative or both are positive. The right side of the equation makes sure that every data point has a minimal distance of M to the hyperplane.

The maximal margin classifier has some faults that have to be considered. Because the data points of a class are strictly separated by a hyperplane of a linear form from the other class, it is not possible to find a hyperplane if the classes are not linear separable. Furthermore even when the classes are linear separable the hyperplane might be forced in a unfavourable spot by only one single data point that doesn't represent most of the other data points of the same class. The extension to a support vector classifier solves those problems.

Support vector classifier

To solve the above mentioned faults the *support vector classifier*, also called *soft margin classifier*, allows some data points of the training data to be not on the correct side of the margin or the hyperplane.

To find a hyperplane for a support vector classifier, the parameters of the hyperplane have to be set in a way, that maximises the size of the margin M and the equations

$$y_i(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip}) \leq M(1 - \epsilon_i) \forall i = 1, \dots, n \quad (2.12)$$

$$\epsilon_i \geq 0, \sum_i \epsilon_i \leq C \quad (2.13)$$

both hold.

The concept is similar to the one seen in equation 2.11. Differentiating here are the *slack variables* $\epsilon_1, \dots, \epsilon_n$. ϵ_i indicates where the i th data point is located relative to the hyperplane and it's margin.

If the i th data point is on the correct side of the margin $\epsilon_i = 0$. If the i th data point is on the correct side of the hyperplane but on the wrong side of the margin, it *violates* the margin and $\epsilon_i > 0$. If the i th data point is on the wrong side of the hyperplane then $\epsilon_i > 1$.

C is a hyper-parameter that limits the total sum of ϵ_i s.

A bigger C allows more datapoints to violate the margin or be on the wrong side of the hyperplane.

This way a hyperplane can be found even when the classes are not strictly linearly separable and isolated outlier data points that would distort the resulting hyperplane can be mostly ignored.

With a support vector classifier it's now possible to create a model h that can be trained on data with non linear separable classes. The model takes a data point and classifies it to

either one class ($h(x) < 0$) or the other class ($h(x) > 0$). The magnitude $|h(x)|$ can be interpreted as confidence score of the classification. [Burges1998] [james2013introduction]

2.3.2 Logistic regression

In this chapter the logistic regression will be introduced. The goal of logistic regression is to create a classifier which predicts the probability for a given data point to belong to a certain class. A model based on a support vector classifier is linear.

The probability of a data point X with p parameters belonging to a class c , can be written as the probability of a class c under the condition of X :

$$p(c|X) \in [0, 1] \quad (2.14)$$

The model $h(X) = p(c = 1|X)$ gives the probability of a data point X being of the class 1.

Logistic regression is based on a linear regression. Linear regression models a linear function. It tries to choose the function parameters β_0, \dots, β_p in a way, that the function approximately goes through every data point in the training data it was derived from.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n \quad (2.15)$$

How this can be done exactly is out scope for this paper. The focus is on the classification algorithm logistic regression.

Logistic regression tries to create a model predicting probability, hence a value $p \in [0, 1]$. The *standard logistic function*, as seen in 2.16, has the property of mapping an input to a range of 0 to 1. The codomain of the standard logistic function is 0 to 1.

$$f(x) = \frac{1}{1 + e^{-x}} = \frac{e^x}{1 + e^x} \quad (2.16)$$

Utilizing the linear regression in equation 2.15 the output is now transformed with the standard logistic function:

$$h(x) = \frac{e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n}}{1 + e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n}} \quad (2.17)$$

The function parameters β_1 to β_n are estimated based on the given data points. This way logistic regression creates a model which predicts the probability of a data point being in a certain class. [cox1958regression]

2.3.3 Decision tree

In this chapter the decision tree will be introduced. The goal of a decision tree is to create a classifier which splits the feature space into subspaces and classifies new data point according to which subspace it is in. A model based on a decision tree is non-linear.

The task, that a decision tree accomplishes, is splitting the feature space into meaningful slices, with which the create model can classify new data points.

This is done by splitting the feature space multiple times recursively. That means after a split, the two created subspaces are then further split. This recursive splitting can also be visualized in a binary tree. This binary tree is called a *decision tree*. Each node represents a split of the feature (sub-)space. Only the terminal nodes don't represent a split and rather an area of the feature space that corresponds to a specific class. First the decision making process is explained. Then the mapping of a terminal node to a class is explained.

The tree is constructed iteratively and top-down (starting at the root of the tree). This is often called *recursive binary splitting*. Construction follows a *greedy* approach, meaning a node is created only considering the benefit that comes with a certain splitting rule for this step, not planning for the future and assessing the benefit of a certain splitting rule for the later tree.

Splitting a space of any dimension into two subspaces requires two aspects to be considered: The axis on which the split is done and the place on the axis where the split is done. Accordingly a splitting rule can be written as follows:

$$a \leq c \quad (2.18)$$

where a represents one of p axis in an p -dimensional feature space and c represents a value in the domain of axis a . This decision rule splits the feature space orthogonal to the axis a , where the value of $a = c$.

The question arises, how this decision rule should be chosen. In the ideal case a split is placed in a way that separates most data point from one class from the data points of the other class and leaves two most "pure" subspaces. The *impurity* of a subspace is most commonly measured by the following metrics.

Gini impurity:

$$\sum_{i=1}^C f_i(1 - f_i) \quad (2.19)$$

C is the number of unique labels and f_i is the frequency of label i in a created feature subspace. For binary classification C is maximal 2.

Entropy:

$$\frac{1}{N} \sum_{i=1}^C -f_i \log(f_i) \quad (2.20)$$

C is the number of unique labels and f_i is the frequency of label i in a created feature subspace. For binary classification C is maximal 2.

To reach the highest “purity”, either one of the metrics has to be chosen and minimized.

Finding the optimal split requires finding a and c such that the splitting rule $a \leq c$ minimizes the sum of the impurity measures for both forming feature subspaces.

The impurity measures commonly used are Ginni impurity (equation 2.19) and Entropy (equation 2.19).

The iterative splitting or growing of the tree is stopped when either a certain height of the tree is reached, a minimal number of trainings data points in one terminal node is undercut or the purity of a subspace can’t be significantly improved by another split.

Now that a decision tree is constructed every terminal node is assigned a class prediction. The class prediction of a terminal node is simply the most common class of the data points in the feature subspace, that the terminal node represents.

For later ensemble methods based on a decision tree, it is important to note, that a terminal node t can not only predict but also give the probability $p(C_i|X)$ of a class C_i :

$$p_t(C_i|X) = \frac{\text{number of data labeles as class } C_i \text{ in terminal node } t}{\text{total number of data points in termin node } t} \quad (2.21)$$

A decision tree model can classify any new data point by passing it from the root of the decision tree to a terminal node, which holds the class prediction, by simply following the decision rules either to the right to the left child node (depending on how the tree is implemented) if the decision rule (as seen in equation 2.18) holds and to the other side if it doesn’t.

Decision trees have the advantage that the decision rules of a model are easy to understand and can be displayed graphically. [2] [10]

But a decision tree has also disadvantages, especially a high variance is often occurring, which leads to overfitting. This problem is counteracted by using *decision tree assemblies* like *random forest* or *boosted trees*. The two algorithms are explained in the following.

2.3.4 Random forest

In this chapter random forests will be introduced. The method utilizes decision trees and creates an assemble of trees improving, among others, on the high variance of a single decision tree. A model based on a random forest is non-linear.

To reduce variance, the method uses the fact that averaging a set of observations reduces variance in the resulting observation. *Bagging* (also called *bootstrap aggregation*) utilizes exactly that fact. From the single training set Ξ , subsets are being created. On every subset of Ξ a decision tree is constructed. The construction of a tree on a single subset is described below. The different models will then all vote in a *majority vote* to come to a conclusion.

Additional to bagging the random forest has another speciality when training models. Just bagging multiple decision trees would lead to an ensemble of very similar trees. Random forest prevents this during the tree construction phase.

A decision tree algorithm decides on a split, in particular on the axis or feature, and on the value where the split is done. A random forest algorithm prohibits the decision for some axis, each time a decision rule is created. In other words, for every decision rule there is a randomly chosen subset of all axis or features possible to choose from.

This way the construction algorithm is encouraged to make splits that are not always optimal at this step, but may lead to a better performing decision tree in the long run.

To make predictions on a new data point a majority vote across all decision trees is conducted and the winning class is predicted for the data point. The method reduces variance and minders the risk of overfitting. Another method that improves on the high variance is boosting and will be discussed in the next chapter. [Breiman2001]

2.3.5 Boosted trees

In this chapter Boosted trees will be introduced. The method also utilizes decision trees and tackles the problem of the high variance of a single decision tree. Similar to a random forest, boosted trees use an ensemble of decision trees. Boosting involves constructing multiple decision trees successively with every new decision tree encouraged to counteract the shortcomings of the previously constructed trees. This is done by fitting each decision tree on a modified version of the training set of the previous decision tree. A model based

on boosted trees is non-linear.

The way a boosted tree model is created decision trees are added one at a time.

In the beginning a decision tree is fitted to the entire training data. No bagging or other sampling method is performed here.

To enhance the existing tree or tree ensemble it firstly has to be assessed where the weaknesses of the previous decision tree or tree ensemble are. This is done by calculating the *residuals* for every data point. The residual of a data point is the differences of it's prediction to it's real label. The residual r_i for all i in the trainings set can be calculated as follows:

$$r_i = y_i - h(X_i) \quad (2.22)$$

Here y_i is the true label of the data point X_i and $h(X_i)$ is the prediction of the model. Note that this prediction $h(X_i)$ is the probability of the class C_i , as mentioned in the section "Decision tree".

The residuals that emerge from this process for every data point can be seen as a *weight* for all data points. The higher the residual of a data point, the worse the tree or ensemble of trees was at classifying it correctly. That means the model is weak at data points with high residual. In the next step more attention should be given to data points with high residual, to counteract these weak points.

The next tree will not be fitted to the the initial labels y_i of the training set, but to the residuals of the previous tree. This gives the data points a weight. Data points with higher weight are more relevant to construction of the next decision tree. This makes sense because a heigh residual means a weak performance of the previous decision tree or ensemble. This means for the following constructed decision tree y_i will be set to r_i ($y_i = r_i$).

Each created tree is added to the tree ensemble and the process is repeated until a stopping criterion is reached, like the maximal number of trees in the ensemble or a minimum threshold of enhancement in performance is undercut in one iteration.

Side note: In a gradient boosted tree algorithm, the weights are not only getting updated by the residuals, but a gradient descent procedure is used to minimize a loss function when adding trees. The details of this will not be discussed for the reason of being beyond the scope of the theory section of giving a introduction to various methods.

To make predictions on a new data point a majority vote across all decision trees is conducted and the winning class is predicted for the data point. The method reduces variance and minders the risk of overfitting similar to the previous tree ensemble method.[friedman2000additive] [james2013introduction]

2.3.6 Naive Bayes

In this chapter naive bayes classification will be introduced. The method based on Bayes' theorem. It is also assumed that the effect of an feature value of a data point on a given class is independent of the other feature values of the data point. The goal of a naive bayes classification is predicting a class by finding the highest probability of the all probabilities of a given data point belonging to a class. The method primarily used in text classification. A model based on naive bayes is non-linear.

The method is explained in the general setting for multiclass classification.

In the following we will regard data points A with n features A_1 to A_n . A concrete data point $X \in A$ with the features x_1 to x_n where n is the number of features of X , hence the length of the feature vector of X , is used. The i th class is denoted as C_i .

The task of finding the probability of an given data point X belonging to a specific class C_i can be expressed as maximising the following expression for every C_i :

$$p(C_i|X) \quad (2.23)$$

The equation 2.23 is the a posteriori probability of C_i conditioned on X .

The initial goal of maximising $p(C_i|X)$ can be rewrote using *Bayes' theorem*:

$$p(C_i|X) = \frac{p(X|C_i)P(C_i)}{P(X)} \quad (2.24)$$

$p(X)$ is the a priori probability of X . $p(C_i)$ is the a priori probability of C_i .

The a priori probability of X does not change when comparing probabilities of C_i conditioned on X . Therefore the factor can be dismissed in this comparison. This way the task is to maximise only the following term:

$$p(X|C_i)p(C_i) \quad (2.25)$$

The a priori probability of C_i $p(C_i)$ is also known. It can be approximated from the ratio of data points labelled as class C_i in the training data:

$$p(C_i) = \frac{\text{number of data points labelled as } C_i}{\text{number of total data points}} \quad (2.26)$$

This means this term doesn't has to be calculated again.

The term

$$p(X|C_i) \quad (2.27)$$

remains. Since X can consist of many features and represents a big feature vector it is computational expensive to calculate $p(X|C_i)$ for every X and C_i . To simplify the

computation involved a assumption is made that, in a sense, may be considered “naive”. The assumption made is *class conditional independence*. This means the effect of an feature value of a data point on a given class is independent of the values of the other feature values of the data point. Mathematically that assumption allows the following:

$$p(X|C_i) \approx \prod_{k=1}^n p(x_k|C_i) \quad (2.28)$$

The probabilities $p(x_1|C_i), \dots, p(x_n|C_i)$ can be more easily calculated from the training data: For A_k being continuous-valued the values are assumed to have a Gaussian distribution with a mean μ and standard deviation σ defined by

$$p(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (2.29)$$

so that for $p(X|C_i)$ holds:

$$p(X|C_i) = p(x, \mu, \sigma) \quad (2.30)$$

This way the mean μ_{C_i} and the standard derivation σ_{C_i} of values for features A_k for training samples of class C_i need to be calculated.

A data point is predicted as a class $C_a \in [C_1, \dots, C_k]$ if and only if C_a maximizes $(X|C_i)p(C_i)$.
[friedman2000additive] [russell1995modern] [rish2001empirical]

3 Analysis of the pattern recognition problem

3.1 Analysis of the time series data

This chapter will provide a closer examination of the time series data measured by the Texas Instrument's SensorTag. This data is prerequisite for the further work of this paper. The sensor data is measured and collected at a sampling rate of 10 measurements per second. To optimize accuracy, the maximal sampling rate of the Texas Instrument's SensorTag was chosen. The measurements include acceleration in 3 spacial axis. The axis each correspond to a direction on the Sensorboard and are orthogonal to each other as shown in figure 3.1. The acceleration is measured in g, which refers to the acceleration that is imparted by the gravity of earth. For g the following conversion to m/s^2 holds true: $1g = 9.80665m/s^2$.

The acceleration in any particular direction will be referred to as x, y or z acceleration.[4]

The measurements are taken simultaneously and handled together as one measurement collection. This collection is enriched with the time of the measurement in form of a Unix timestamp.

A measurement collection, as described above, will be referred to as a data point. The single measurements (for instance x acceleration) of a measurement collection are the features of a data point.

An exemplary data point in a key-value-pair format can be seen here in figure 3.2. The keys in figure 3.2 stand for acceleration in x direction, acceleration in y direction, acceleration in z direction and the Unix timestamp when the measurement took place.

Using the time of measurement, data can be plotted on a time axis plot as seen in figure 3.3.

Data from the Sensorboard has been collected in isolated experiments. During the experiments of the Sensorboard 69 collisions were simulated. Accordingly, 69 of the 6453 data points collected, were labelled as a collision, while the rest was labelled as no collision. The labelling was implemented by analysing video footage, recorded during the experiment. The exact time of a collision in the video was matched with the time of the recorded sensor data to label data points correctly.

The distribution of the data points labeled as collision or no collision can be seen in figure

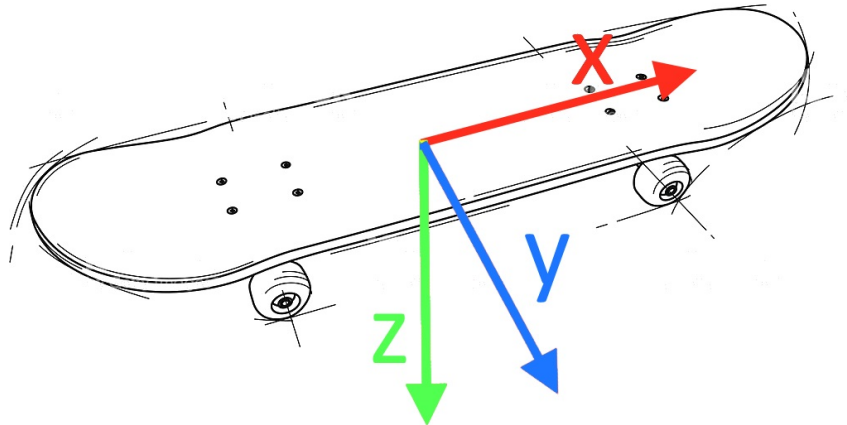


Figure 3.1: Illustration of Sensorboard and axis of measurement. The driving direction is to the right.

3.4. The scatter plot shows the relationship between the x and z acceleration and the label of the data point. The y acceleration is not visualized here. A blue data point represent a collision the red data points represent no collision.

3.2 Solution strategy

Combining the knowledge about the sensor data from the previous section and the objective as described in chapter 1, this chapter will narrow down on a specific problem type and solution strategy. In the process, the limitations described in Chapter 1 have to be considered.

A corpus of sensor data was collected as described above and labelled according to the event of a collision or no collision. In chapter 1 the necessity of using the Apache Spark library module “MLlib” is constituted.

Having a set of labeled sensor data allows for a supervised machine learning approach. Furthermore the existence of concrete data points, which represent the state of the Sensorboard at a given time makes it possible to base the recognition of collisions solely on single data points. The recognition of collisions can then in turn be formulated as the

```
{  
  "accx": -0.079590,  
  "accy": 0.001465,  
  "accz": -1.016602,  
  "time": 1502735960911  
}
```

Figure 3.2: Example data point in key-value-pair notation notation

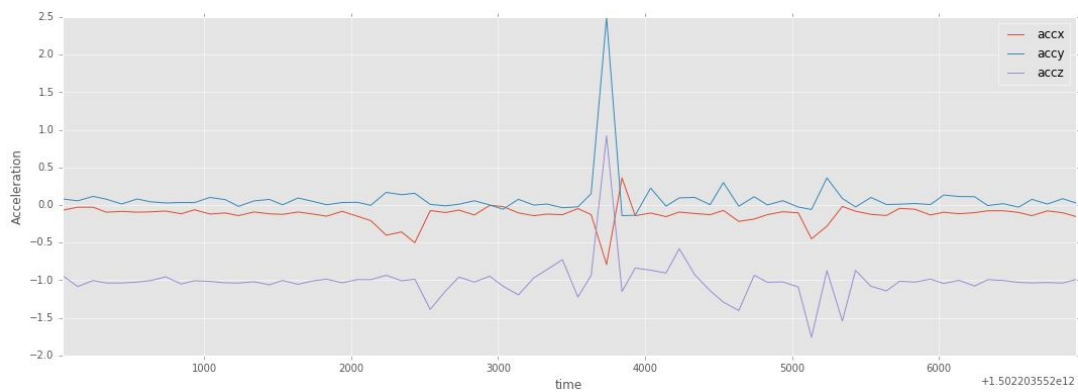


Figure 3.3: Example of measured accelerations on 3 Axis during a collision

problem of deciding whether a given data point corresponds to a collision or no collision. A problem of this kind can be solved by binary classification.

Having specified the solution method to a binary classification approach, specific algorithms can be considered. For the specific task of supervised binary classification the following algorithms are supported by the MLlib module:

- Linear support vector classifier ((Linear SVC))
- Logistic regression
- Decision tree
- Random forest
- Gradient-boosted trees
- Naive Bayes

Only the possible algorithms mentioned here will be parameterized and trained on the collected data in chapter 4.

To assess the performance of resulting models a clear metric will be used for comparison. This metric has to represent the unequal importance of detecting a collision and preventing false alarms. As mentioned in chapter 1 the act of detecting a collision that actually

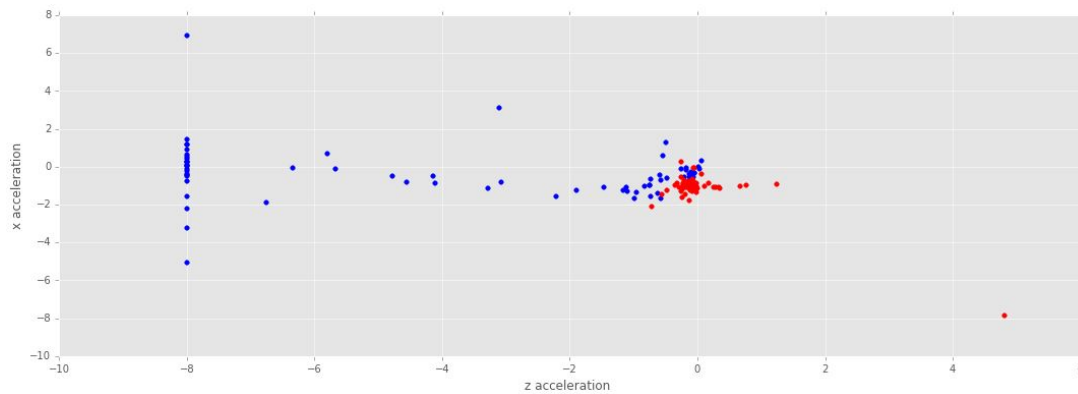


Figure 3.4: Distribution of data points labeled as collision (blue) or no collision (red)

happened (*true positives*) is more important than preventing the detection of a collision that didn't happen (*false positives*).

In a classification setting the effectiveness in detecting collision that actually happened is described by recall. The effectiveness of preventing the detection of collision that didn't happen is described by precision.

The metric that will be used to represent the unequal relationship between precision and recall is a f_2 -score. This metric puts more weight on the recall of a model. This way a later evaluation of trained classifiers can be done based on one correctly weighted metric.

4 Algorithm configuration

Many of the methods proposed in chapter 3 require *hyper-parameters*. A hyper-parameter is a setting parameter that varies aspects of the algorithm and might lead to better or worse results. To ensure that all the algorithms are comparable with each other this chapter goes through the process of finding the optimal hyper-parameters for every approach. In the process a f_2 -score is assigned to every algorithm under the condition of optimal hyper-parameters. []

Hyper-parameter tuning

The process of searching and finding good hyper-parameters for a given algorithm is called *hyper-parameter tuning*. Searching for good hyper parameters takes the following aspects to be considered:

- **an estimator**

The *estimator* in this case is a binary classifier. The 8 different binary classifiers have to be considered as mentioned in chapter 3.

- **a hyper-parameter space**

The *hyper-parameter space* defines a set of parameters or combination of hyper-parameters that will be tested. From this set one element will be chosen as most fitting. Because every algorithm requires different hyper-parameters, the hyper-parameter space is different for every algorithm. The hyper-parameter space will be defined in the sections below for every algorithm.

- **a method for searching or sampling candidates**

The method that is chosen is grid search. Grid search is a brute force search, which uses an exhaustive searching through a defined hyper-parameter space. Models are iteratively trained and evaluated by a score function. The model scores are then compared. The hyper-parameters that produced the best performing model will be chosen as optimal ones for the algorithm. [6]

- **a cross-validation scheme**

The *cross-validation scheme* used is a k-fold validation, with $k=10$.

- **a score function**

As mentioned in chapter 3 a f_2 -score will be used to evaluate the models in the end. Our goal for the hyper-parameter tuning is thereby also to maximise the f_2 -score. For that reason the f_2 -score is used as a score function.

For the hyper-parameter search no external functions or libraries are used. The search algorithm is implemented in the following way.

For the given estimator (one of the possible classification algorithms) the parameter space is defined manually for every hyper-parameter.

Models are now repeatedly trained by iterating through every combination of hyper-parameters in the parameter spaces.

For every trained model a k-fold validation, with $k=10$, is performed.

During the k-fold validation, the f_2 -score is calculated.

After this process is done the hyper-parameters that resulted in the highest f_2 -score are chosen.

In the following for each of the relevant algorithms a parameter space will be defined. The results of the hyper parameter search will then be presented.

Linear SVC

The tunable hyper-parameters for the the linear SVC classifier is the regularization. The regularization gives a weight to errors (like misclassifications) made on the training set. The model tries to minimize that error. The possible regularizations are:

L1 regularization:

$$\sum_{i=1}^n |y_i - f(x_i)|$$

y_i is the target value and x_i is the estimated value for y_i .

L1 regularization minimizes the sum of the absolute differences of target and estimated value.

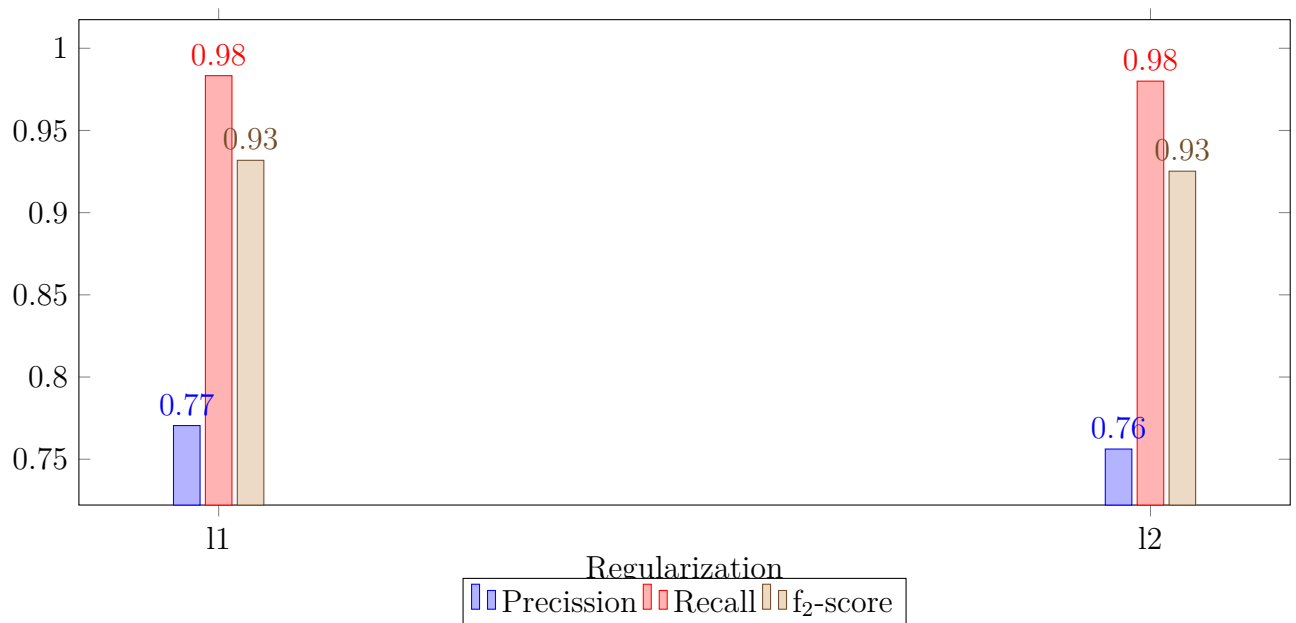
L2 regularization:

$$\sum_{i=1}^n (y_i - f(x_i))^2$$

y_i is the target value and x_i is the estimated value for y_i .

L2 regularization minimizes the square of the absolute differences of target and estimated value.

The hyper-parameter search reveals the following results for the two regularization approaches:



The best performing SVC model reaches a maximal f₂-score of **0.93** (precision: 0.77, recall: 0.98) with the following hyper-parameter:

Regularization: **l1 regularization**

Logistic regression

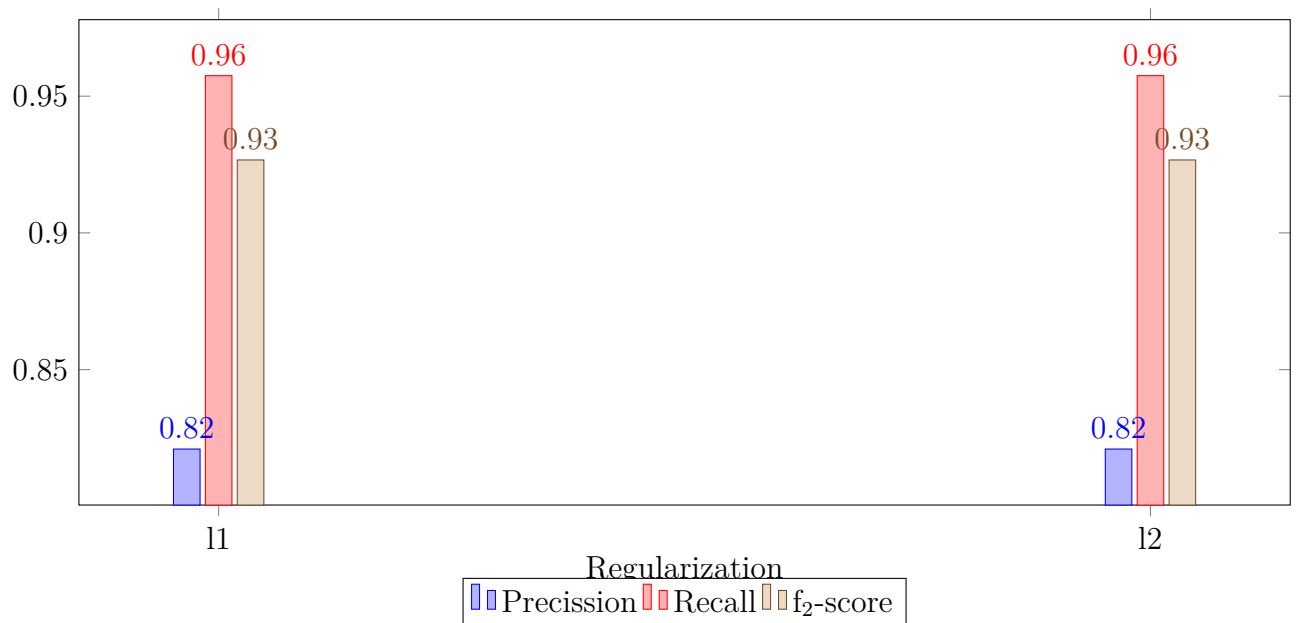
The tunable hyper-parameters for the the logistic regression classifier is the regularization.

The possible regularizations are:

L1 regularization

L2 regularization

The hyper-parameter search reveals the following results for the two regularization approaches:



The best performing logistic regression model reaches a maximal f_2 -score of **0.93** (precision: 0.82, recall: 0.95) independent of which regularization is used.

Decision tree

The tunable hyper-parameters for the decision tree algorithm are:

- **Impurity**

The impurity measure is used to decide on a splitting rule for each node in the decision tree building process. Two impurity measures for classification are provided (see chapter 2 for more detailed explanation). The possible impurity measures are:

Gini impurity

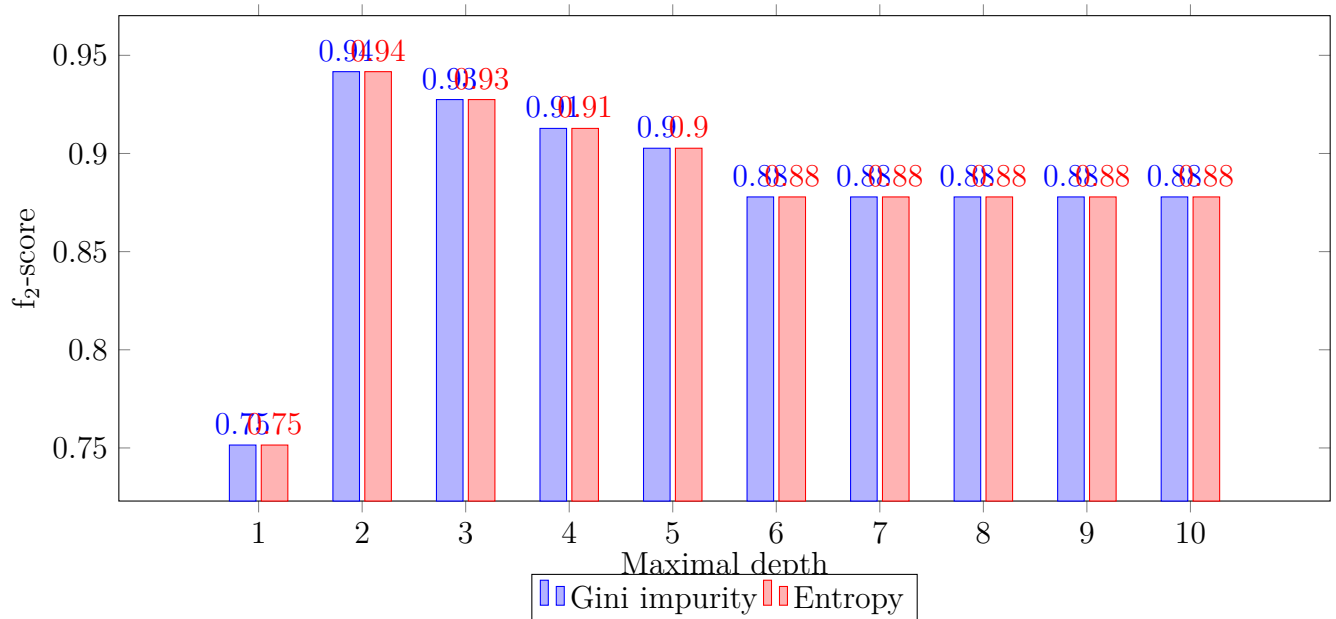
Entropy

- **Maximal depth**

The maximal depth determines a limit of growth for a decision tree. This consequently limits the number of nodes or decisions that are possible. It represents a stopping criteria.

The parameter space for the maximal depth is chosen **from 1 to 10** (inclusive).

The hyper-parameter search reveals the following results:



The best performing decision tree model reaches a maximal f_2 -score of **0.94** (precision: 0.92, recall: 0.96) with the following hyper-parameters:

Impurity: **Ginni Impurity** or **Entropy**

Maximal depth: **2**

Random forest

The tunable hyper-parameters for the random forest classifier are:

- **Number of trees**

The number of trees that are build during the training phase to act as an ensemble of (randomized) decision trees.

The parameter space for the number of threes is chosen **from 2 to 10** (inclusive).

- **Impurity**

The impurity measure is used to decide on a splitting rule for each node in the decision tree building process. Two impurity measures for classification are provided (see chapter 2 for more detailed explanation):

Gini impurity

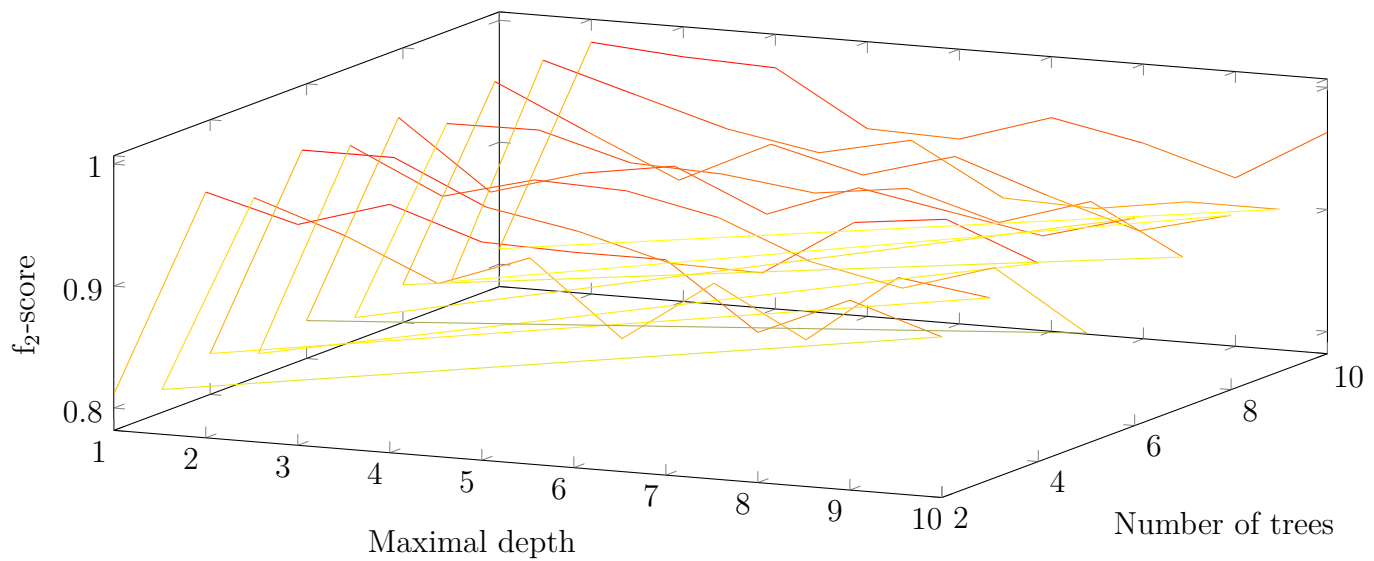
Entropy

- **Maximal depth**

The maximal depth determines a limit of growth for a decision tree. This consequentially limits the number of nodes or decisions that are possible. It represents a stopping criteria.

The parameter space for the maximal depth is chosen **from 1 to 10** (inclusive).

The hyper-parameter search reveals the following results:



The best performing decision tree model reaches a maximal f_2 -score of **0.99** (precision: 0.94, recall: 1.0) with the following hyper-parameters:

Number of trees: **4**

Impurity: **Entropy**

Maximal depth: **2 or 3**

or

Number of trees: **9 or 10**

Impurity: **Entropy**

Maximal depth: **2**

Gradient-boosted trees

The tunable hyper-parameters for the decision tree algorithm are:

- **Loss function**

The loss function provided are:

LogLoss

Least Squares Error

Least Absolute Error

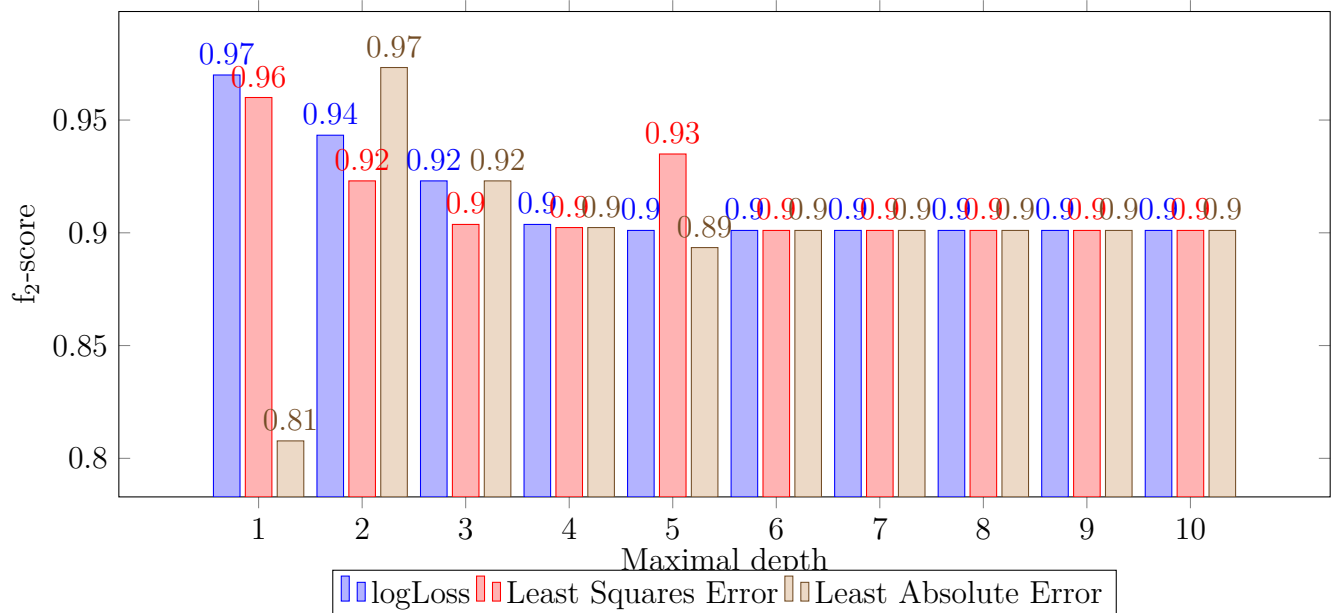
- **Maximal depth**

The maximal depth determines a limit of growth for a decision tree. This consequently limits the number of nodes or decisions that are possible. It represents a

stopping criteria.

The parameter space for the maximal depth is chosen **from 1 to 10** (inclusive).

The hyper-parameter search reveals the following results:



The best performing Gradient-boosted tree classifier reaches a maximal f_2 -score of **0.97** (precision: 0.96, recall: 0.98) with the following hyper-parameters:

Loss function: **Least Absolute Error**

Maximal depth: **2**

Naive Bayes

The tunable hyper-parameters for the the Naive Bayes classifier is a λ -value used for additive smoothing. Since the hyper-parameter search with $\lambda = [0.1, 10.0]$ didn't reveal any differences in performance the hyper-parameter is not going to be regarded and will not be discussed further in this paper.

The naive bayes classifier reaches a f_2 -score of **0.39** (precision: 1.00, recall: 0.34) .

5 Evalutation

The optimal configuration for the different algorithms has been worked out in chapter 4. Now this chapter is devoted to comparing and deciding on an optimal algorithm for the problem of collision detection.

To be able to compare the algorithms they are each used to train a model with the hyper-parameters found in chapter 3. The models are evaluated with a k-fold validation, with $k=10$. The results can be seen in figure 5.1.

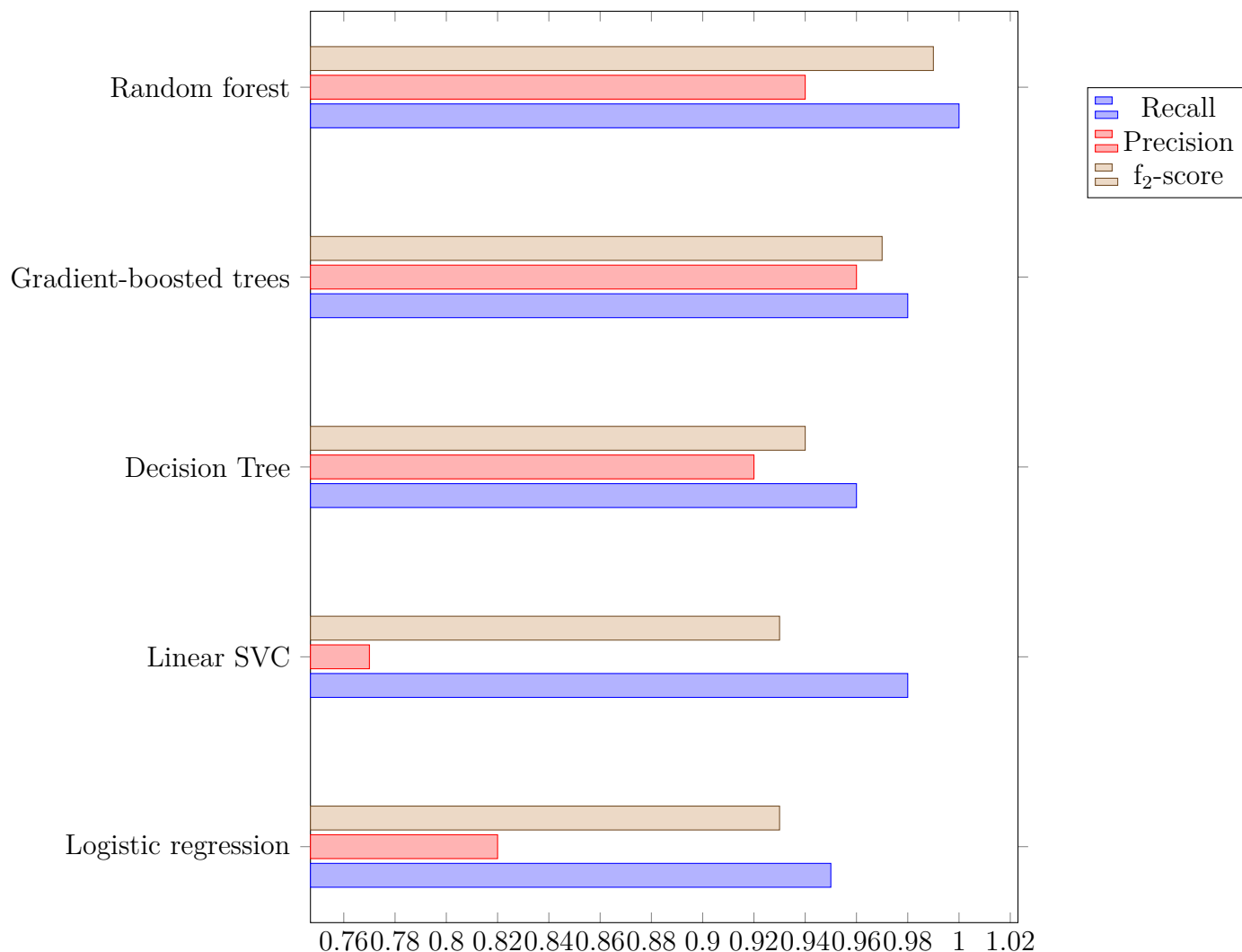


Figure 5.1: Ranking of classification algorithms

The naiv bayes method is not included in figure 5.1 for better depiction and comparison of the other methods. The best performing naiv bayes model ranks below all the other

classifiers with an f_2 -score of 0.34.

According to the f_2 -score the best performing method is a random forest classifier with 4 equally well performing hyper-parameter configurations:

Number of trees: 4	Impurity: Entropy	Maximal depth: 2
Number of trees: 4	Impurity: Entropy	Maximal depth: 3
Number of trees: 9	Impurity: Entropy	Maximal depth: 2
Number of trees: 10	Impurity: Entropy	Maximal depth: 2

The best performing random forest model reaches a maximal f_2 -score of **0.99** (precision: 0.94, recall: 1.0).

This means a collisions that occur will be classified correctly classified as a collision 100% of the time on the collected data and will be falsely classified as a collision 6% of the time when no actual collision happened.

This results indicate that the machine learning approach to recognizing collisions on a physical board is eligible and can perform well. Still it has to be considered that these results are achieved on data which is generated in experiment solely for that reason and results may differ when applied in real world situations.

In figure 5.1 the ranking of the classification methods shows that decision tree based approaches have a advantage over the remaining algorithms. Random forest and gradient-boosted tree models as decision tree assembles performed on average better than a single decision tree. The naive base algorithm performs worst under the possible algorithms.

6 Reflection and outlook

The previous chapter reached the conclusion that machine learning algorithms are, in fact, feasible for detecting collision detection in the case of the Sensorboard. This shows the potential that machine learning algorithms might have for detecting patterns corresponding to driving behaviours or accidents.

Regardless of the positive result, this study has to be viewed critically. The process of building models and evaluating them is based on datasets generated in experiments in isolated conditions. As no real driving data was recorded in the dataset, the outcome of the research might not be completely transferable to a real driving scenario. Conditions like rough or irregular surfaces and unexpected driving manoeuvres will influence the data quality and therefore the model consistency.

Furthermore the results of this study can not be applied to other driving scenarios as patterns in data will differ based on the vehicle and might not be recognized as easily or with the same methods as described in this paper.

Nevertheless the results of the research clearly indicate potential of this approach for similar use cases. The knowledge gained regarding the feasibility and best methods for a machine learning approach to this use case points further research in the right direction. In further research the results of this paper can be refined and checked by using sensor data originating from real driving of the Sensorboard as it resembles more realistic driving behaviour. This way the result can be applied to more realistic conditions.

In addition the experimental results of this study can be further examined. It can be studied why some algorithms perform better than others in the given scenario. The fact that tree based algorithms seem to work better here might be explained by the non linearity of the models created. In contrast the linear SVC and logistic regression represent generate linear models. The superiority of decision tree assemblies like the Random forest and gradient-boosted tree over a single decision tree might be explained by a reduction in variance of a classifier and thereby the prevention of overfitting and better generalization of the models. A definite explanation for those phenomenon can be explored in further research.

Furthermore the knowledge gained during this research can transposed on other similar problems. Other vehicles like bikes, motorbikes or cars might as well allow for a machine learning based detection of events like collisions.

Even when applied only to a showcase scenario as in this paper, it becomes clear that machine learning applied to IoT data has a lot of potential. Being able to showcase this fact to clients will lead them to a smarter way of making use of sensor data in the

information age. In this way the growing field of the Internet of Things in combination with machine learning might have an even bigger impact on industries and consumers. Rolling Window?

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