Problem Set 1

All numbered problems come from Stephen Boyd and Lieven Vandenberghe's Additional exercises for Convex Optimization[1]. The footnote has the link to the commit I used, which is also posted on the course website. Make sure that you do the correct problem (titles are included).

Warmup (not graded)

In this course, we often will visualize objects in two or three dimensions and then generalize the intuition we developed to higher dimensions. However, higher dimensional objects can behave quite differently than their lower dimensional counterparts. This question, adopted from Richard Hamming, illustrates this phenomenon.

0. Consider the n-dimensional ℓ_∞ ball with radius 2 (which is a cube with side length 4) $B = \{x \in \mathbb{R}^n : \|x\|_\infty \leq 2\}$. Draw a unit Euclidean (ℓ_2) ball at each of the points $\{(\pm 1, \pm 1)\}$. These balls lie entirely inside B and each touch B at n points. What is the radius of the largest Euclidean ball that fits inside (the union of) these 2^n balls? How does this compare to the radius of the ℓ_∞ ball? Check what happens when $n = 10$. The inscribed ball touches all 1024 unit balls on the inside, yet is reaches outside of the cube B. You may hear that n-dimensional are "spiky."

   It's helpful to consider the two and three dimensional cases before solving the problem. In two dimensions, draw a circle at each of the four points $\{(\pm 1, \pm 1)\}$. The largest circle that fits inside of these four circles has radius $\sqrt{2} - 1$.

Problems

1. 3.33 *DCP Rules.* (p. 16)

   Hint: try writing it in `Convex.jl` and recall the example from lecture. Also note that in `Convex.jl`, the functions do not have underscores, *e.g.,* `invpos(u)` instead of `inv_pos(u)`.

2. 4.3 *Formulating constraints in `Convex.jl`* (p. 25)

All data for the following problems are posted in the assignments section of the course website.

3. 16.6 *Planning an autonomous lane change* (p. 179)

4. 7.6 *Maximum likelihood estimation of an increasing nonnegative signal* (p. 94)

   Hint: The negative log-liklihood function has the form $\alpha + \beta\|\hat{y} - y\|_2$, where $\hat{y}$ is the noiseless estimate of y.

5. 15.13 *Bandlimited signal recovery from zero-crossings* (p. 172)

[1] `https://github.com/cvxgrp/cvxbook_additional_exercises/tree/a97b5858b6f9fc9449dda7356b0ec54fef8f05bc`