⚠ You can see the solution process in the next page.

AMA1500 - Foundation Mathematics for Accounting and Finance
Assignment 2
Due: 5pm, 27 March (Thursday) 2025, submission via Blackboard

1. (4 marks) Find the domain of $f(x) = 2x - \frac{1}{x^2}$ and find the intervals on which the function is increasing or decreasing.

   $(-\infty, -1) \cup (0, +\infty)$ increasing, $(-1, 0)$ decreasing

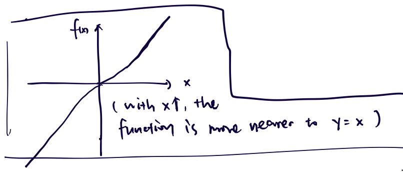
2. (4 marks) For $f(x) = \frac{x^3}{x+1}$, find the local maximum and minimum.

   no local max. local min $= \frac{27}{4}$

3. (4 marks) Find the absolute maximum and minimum of the function $f(x) = \frac{x}{1+x^2}$ on the closed interval $[-1, 2]$.

   $f(1) = \frac{1}{2}$ $\quad f(-1) = -\frac{1}{2}$

4. (10 marks) Sketch the graph of the curve $f(x) = \frac{x^3}{x^2+1}$. The analysis is in the next page.



5. (12 marks) Consider the function

$$g(x) = \frac{x^2 - 16}{x - 5},$$

where $x \neq 5$.

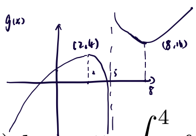
(a) Find all critical points of the function. Determine the intervals in which $g(x)$ is increasing and/or decreasing, and hence find all the local (relative) maxima and local (relative) minima of the function. increasing: $(-\infty, 2) \cup (8, +\infty)$ decreasing $(2, 5) \cup (5, 8)$ local min: 16, local max = 4

Critical point: $(8, 16)$ $(2, 4)$

(b) Find the intervals in which $g(x)$ is concave up and/or concave down. Hence determine the points of inflection of the function. ∴ concave up $(5, +\infty)$ concave down $(-\infty, 5)$ And there's no inflection point

(c) Find all asymptotes of the function (including vertical, horizontal and inclined asymptotes). ∴ $y = x+5$ & $x = 5$

(d) Sketch the graph of $g(x)$.



6. (6 marks) Given that $\int_1^4 f(x)dx = 5$, $\int_3^4 f(x)dx = 7$ and $\int_1^8 f(x)dx = 11$, find the following definite integrals

$$(a) \int_4^8 f(x)dx; \quad (b) \int_4^3 f(x)dx; \quad (c) \int_1^3 f(x)dx.$$

(6) $\quad$ (−7) $\quad$ (−2)

7. (10 marks) Use the definite integral to compute the following limits.

$$\text{(a)} \quad \lim_{n\to\infty} \left[\frac{1}{n} \sum_{j=1}^{n} (\ln(n+j) - \ln n) \right]; \quad 2\ln 2 - 1$$

$$\text{(b)} \quad \lim_{n\to\infty} \frac{1}{\sqrt{n^3}} \left(\sqrt{n+1} + \sqrt{n+2} + \cdots + \sqrt{n+n} \right).$$

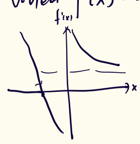
$\frac{4\sqrt{2} - 2}{3}$

1. Domain: $x^2 \neq 0$

$\{x \mid x \neq 0, x \in R\}$

$f'(x) = 2 + 2x^{-3}$

when $f'(x) > 0$, $f(x)$'s increasing



$\therefore (-\infty, -1) \cup (0, +\infty) \uparrow$

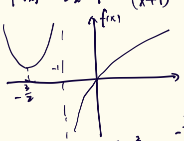
$(-1, 0) \downarrow$

2. $f(x) = \frac{x^3}{x+1}$ $(x \neq -1)$ $\lim_{x \to -1^+} f(x) = \frac{-1}{0^+} = -\infty$

$= x^2 - x + 1 - \frac{1}{x+1}$ $\lim_{x \to -1^-} \frac{-1}{0^-} = \infty$

$f'(x) = 2x - 1 + \frac{1}{(x+1)^2}$ $f'(x) = 0$ $x = -\frac{3}{2}$

$\lim_{x \to -\infty} f(x) = \frac{3x^2}{1} = \infty$



local min point $= \left(-\frac{3}{2}, \frac{27}{4}\right)$

$\therefore$ local min $= f\left(-\frac{3}{2}\right) = \frac{-\frac{27}{8}}{-\frac{1}{2}} = \frac{27}{4}$

3. $f(x) = \frac{x}{1+x^2}$

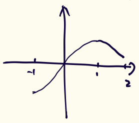
$f'(x) = \frac{1+x^2 - 2x \cdot x}{(1+x^2)^2}$

$= \frac{1-x^2}{(1+x^2)^2}$

$\therefore f'(x) > 0$ $1 - x^2 > 0$ $x^2 < 1$

$(-1, 1)$ $f(x) \uparrow$

$(1, 2)$ $f(x) \downarrow$



$\therefore$ absolute max $= f(1) = \frac{1}{2}$

absolute min $= f(-1) = -\frac{1}{2}$

6. (a) $\int_4^8 f(x)\,dx$

$= \int_1^8 f(x)\,dx + \int_4^1 f(x)\,dx$

$= \int_1^8 f(x)\,dx - \int_1^4 f(x)\,dx$

$= 11 - 5 = 6$

(b) $\int_4^3 f(x)\,dx$

$= -\int_3^4 f(x)\,dx$

$= -7$

(c) $\int_1^3 f(x)\,dx$

$= \int_1^4 f(x)\,dx + \int_4^3 f(x)\,dx$

$= \int_1^4 f(x)\,dx - \int_3^4 f(x)\,dx$

$= 5 - 7 = -2$

4. $f(x) = \frac{x^3}{x^2+1}$ $\lim_{x \to \infty} f(x) = \frac{3x^2}{2x}$ $(x \to \infty)$ $= \lim_{x \to \infty} \frac{6x}{2} = \infty$

$= x - \frac{x}{x^2+1}$

$f'(x) = 1 - \frac{x^2+1-2x^2}{(x^2+1)^2}$

$= 1 - \frac{1-x^2}{(x^2+1)^2}$

$\therefore f'(x) > 0$

$f'(x) > 0$

$1 > \frac{1-x^2}{(x^2+1)^2}$

$\Rightarrow x^4 + 2x^2 + 1 > 1 - x^2$

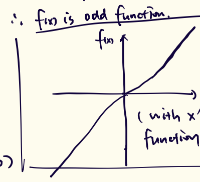
$x^4 + 3x^2 > 0$

$x^2(x^2+3) > 0$ $(x \neq 0)$

$\because f(x) = -f(-x)$

$= -\frac{-x^3}{x^2+1}$

$\therefore f(x)$ is odd function.



(with $x \uparrow$, the function is more nearer to $y = x$)

又 $\because f'(x) = 1 + \frac{x^2-1}{x^4+2x^2+1}$

$\lim_{x \to \infty} f'(x) = \lim_{x \to \infty} 1 + \frac{2x}{4x^3+4x}$

$= \lim_{x \to \infty} 1 + \frac{2}{12x^2+4}$

$= 1$

5. (a) $g(x) = \frac{x^2 - 16}{x - 5} = \frac{(x+4)(x-4) - (x+4) + (x+4)}{x-5}$

$= x + 4 + \frac{x+4}{x-5} = x + 4 + \frac{x-5+9}{x-5}$

$= x + 5 + \frac{9}{x-5}$

concave up $f''(x) > 0$

~ down $f''(x) < 0$

$\therefore$ when $x = 0$ $\boxed{g(x) \text{ passes } (0, \frac{16}{5})}$

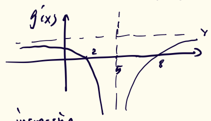
$g(x) = 0$ $x^2 - 25 = -9$ $x^2 = 16$, $x = \pm 4$

$\boxed{g(x) \text{ passes } (4, 0) \text{ and } (-4, 0)}$

$\boxed{\text{local min } (8, 16) \quad \text{local max } (2, 4)}$

$g'(x) = 1 - \frac{9}{(x-5)^2}$ $g'(x) = 0$ $\frac{9}{(x-5)^2} = 1$, $(x-5) = 3 / -3$

$x = 8$ or 2



increasing

$\boxed{(-\infty, 2) \cup (8, +\infty)}$

decreasing

$\boxed{(2, 5) \cup (5, 8)}$

(b) $g'(x) = 1 - \frac{9}{(x-5)^2}$

$g'(x) = 1 - \frac{9}{x^2 + 25 - 10x}$

$g''(x) = -\left(-\frac{(2x-10)9}{((x-5)^2)^2}\right) = \frac{18x - 90}{(x-5)^4}$

$g''(x) > 0$ $x > 5$

$g''(x) < 0$ $x < 5$

$\boxed{\begin{array}{l} \therefore \text{concave up } (5, +\infty) \\ \text{concave down } (-\infty, 5) \end{array}}$

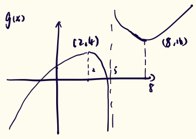
(c) Domain $x \neq 5$

$\lim_{x \to \infty} g(x) = \infty$ $g(x) = x + 5 + \frac{9}{x-5}$

$\therefore y = x + 5$ is $g(x)$'s sympotom

$\boxed{\therefore y = x+5 \ \& \ x = 5}$

(d)



6. (a) $\int_4^8 f(x)\,dx$

$= \int_1^8 f(x)\,dx + \int_4^1 f(x)\,dx$

$= \int_1^8 f(x)\,dx - \int_1^4 f(x)\,dx$

$= 11 - 5 = 6$

(b) $\int_4^3 f(x)\,dx$

$= -\int_3^4 f(x)\,dx = -7$

(c) $\int_1^3 f(x)\,dx$

$= \int_4^3 f(x)\,dx + \int_1^4 f(x)\,dx$

$= -7 + 5 = -2$

7. (a)

$expr = \lim_{n \to \infty} \left[\frac{1}{n} \sum_{j=1}^{n} \ln\left(1 + \frac{j}{n}\right)\right]$

let's assume that

$x_j = \frac{j}{n}$ $(j = 1, 2 \dots n)$

$\therefore f(x_j) = \ln\left(1 + \frac{j}{n}\right)$ $\frac{1}{n} = \Delta x$

$\therefore expr$

$= \lim_{\Delta x \to 0} \left[\sum_{j=1}^{n} f(x_j) \Delta x\right]$

$\because x_j \in (0, 1]$

$\therefore expr = \int_0^1 f(x)\,dx = \int_0^1 \ln(x+1)\,dx$

$= \left(\ln(x+1)(x+1) - x + c\right)\Big|_0^1$

$= 2\ln 2 - 1$

(b)

$expr = \lim_{n \to \infty} \left[\frac{1}{\sqrt{n^3}} \sum_{i=1}^{n} \sqrt{n+i}\right]$

$= \lim_{n \to \infty} \left[\frac{1}{n} \cdot \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \sqrt{n+i}\right]$

$= \lim_{n \to \infty} \left[\frac{1}{n} \sum_{i=1}^{n} \sqrt{1 + \frac{i}{n}}\right]$

Assume $x = \frac{i}{n}$ $(i = 1, 2, \cdots n)$ $\frac{1}{n} = \Delta x$

$\therefore expr = \lim_{\Delta x \to 0} \left[\Delta x \sum_{i=1}^{n} \sqrt{1 + x_i}\right]$ $x_i \in (0, 1]$

$= \int_0^1 f(x)\,dx$ $f(x) = \sqrt{1+x} = (x+1)^{\frac{1}{2}}$

$= \left(\frac{2}{3}(x+1)^{\frac{3}{2}} + c\right)\Big|_0^1$ $F(x) = k(x+1)^{\frac{3}{2}}$

$F'(x) = \frac{3}{2}k(x+1)^{\frac{1}{2}}$ $k = \frac{2}{3}$

$= \frac{2}{3} \cdot 2^{\frac{3}{2}} - \frac{2}{3}$

$= \frac{4\sqrt{2} - 2}{3}$