

You need to give reasoning of how you derive your answers. No mark if no reasoning is given.

### Question 1

Read the following payoff matrix. Select the correct statement(s) below.

		B	
		L	R
A	U	1, 1	4, 0
	D	4, 0	3, 3

For A, if choosing D, when B chooses R, choosing D is worse (2 < 4)  
 if choosing U, when B chooses L, choosing U is worse (1 < 4)  
 For B, if choosing R, when A chooses U, it's worse than L (0 < 1)  
 if B chooses L, when A chooses D, it's worse than choosing R (0 < 3)  
 So 1 is correct.  
 2, 3, 4 according to 1, 2 & 4 are incorrect.

1. Neither player A nor player B has a dominant strategy.
2. Player A has a dominant strategy D whereas player B has the dominant strategy R.
3. Only player B has a dominant strategy L. The best response of player A is strategy D.
4. Only player B has a dominant strategy L. The best response of player A is strategy U.

### Question 2

When multiple Nash Equilibria exist in a game, what should the players' action be?

1. Select the equilibrium with maximum payoff
  2. Need more external information to make decisions
  3. There is no rule to predict on the unique choice of the players
  4. Select the equilibrium with high payoff and low risk
- The description for this question is too general to choose the action.  
 There might have parameters which can influence the choice!

### Question 3

Select all Nash equilibrium under the following payoff matrix

		Player B	
		L (q)	R (1-q)
Player A	U (p)	1, 1	4, 2
	D (1-p)	3, 3	2, 2

1. (D, L)
  2. (1/3, 2/3)
  3. (1/2, 1/2)
  4. (1/4, 3/4)
  5. (U, R)
  6. (D, R)
- | A, B | initial   |
|------|---|
| U, L | 1, 1 (A wants to change to D, B wants to change to R) |
| U, R | 4, 2 ✓ Nash Eq  |
| D, L | 3, 3 ✓  |
| D, R | 2, 2 (B wants to change to L, A wants to change to U) |

So statements 1, 5 are correct.

Assume  $P(A \text{ choose } U) = p$   
 $P(B \text{ choose } L) = q$   
 if B choose L  
 $B \text{ get } = p + 3(1-p) = 3-2p$   
 if B choose R  
 $B \text{ get } = 2p + 2(1-p) = 2$   
 $3-2p = 2$   
 $p = \frac{1}{2}$   
 if A choose U:  
 $A \text{ get } = 1 \times q + 4(1-q) = 4-3q$   
 A choose D  
 $A \text{ get } = 3 \times q + 2(1-q) = 2+q$   
 $4-3q = 2+q$   
 $4q = 2, q = \frac{1}{2}$   
 $\therefore \text{Nash Eq} = (\frac{1}{2}, \frac{1}{2})$   
 3 is correct.

Final answer:

Statement 1, 3, 5

One correct!

#### Question 4

Select the Nash equilibrium for mixed strategy under the following payoff matrix.

		Player B	
		L $q$	R $1-q$
Player A	U $p$	5, 6	0, 10
	D $1-p$	4, 4	2, 2

- (1/2, 1/2)
  - (1/3, 2/3)
  - (1/4, 3/4)
  - (1, 0)
- Assume  $P(A \text{ choose } U) = p$   
 $P(B \text{ choose } L) = q$
- if A choose U  
 $A \text{ get} = 5q + 0(1-q) = 5q$   
 $5q = 5q$   
 $2 + 2q = 5q$   
 $3q = 2, q = \frac{2}{3}$
- if A choose D  
 $A \text{ get} = 4q + 2(1-q) = 2 + 2q$
- if B choose L  
 $B \text{ get} = 6p + 4(1-p) = 4 + 2p$   
 $4 + 2p = 4 + 2p$   
 $2 + 8p = 4 + 2p$   
 $6p = 2, p = \frac{1}{3}$
- if B choose R  
 $B \text{ get} = 10p + 2(1-p) = 2 + 8p$   
 $2 + 8p = 4 + 2p$   
 $6p = 2, p = \frac{1}{3}$
- $\therefore$  Nash Eq =  $(\frac{1}{3}, \frac{2}{3})$ , Statement 2 is correct.

A strategy is *weakly dominant* if, regardless of what any other players do, the strategy earns a player a payoff at least as high as any other strategy, and, the strategy earns a strictly higher payoff for some profile of other players' strategies. Hence, a strategy is weakly dominant if it is always at least as good as any other strategy, for any profile of other players' actions, and is strictly better for some profile of others' strategies.

Consider the following two questions with the following game

		Player B	
		L	R
Player A	U	1, 1	1, 1
	D	0, 0	2, 1

- There exists Nash equilibrium
- There exist Nash equilibrium using weak dominant strategy

Which of the following is correct in filling up the two questions?

- There exists, There is no
- There is no, There is no
- There exists, There exists

Normal Nash Eq  
 for A, if choosing U, while B chooses R.  
 A will change to D ( $2 > 1$ )  
 So (U, L) is Nash Eq.  
 if A chooses U, B chooses L, A'll not change to D.  
 While B'll not change to R (equal)

Weak dominant strategy  
 if A chooses D, B chooses R  
 A will not change to U ( $2 > 1$ ) B will not change to L  
 When B chooses R, R is weak dominant strategy for B.  
 So (D, R) is Nash Eq using weak dominant strategy.