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Question 1

$(1+i)^5 z^4 = 2-2i$

assume $w = z^4$

$2-2i = 2\sqrt{2}\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right) = 2\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+\sin\left(-\frac{\pi}{4}\right)i\right) = 2\sqrt{2}e^{-\frac{\pi}{4}i}$

$1+i = \sqrt{2}\left(\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\cdot i\right) = \sqrt{2}e^{\frac{\pi}{4}i}$

$\therefore (1+i)^5 = 2^{\frac{5}{2}} e^{\frac{5}{4}\pi i}$

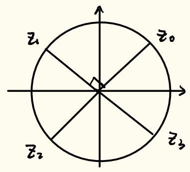
$$\therefore w = \frac{2-2i}{(1+i)^5} = \frac{2^{\frac{3}{2}} e^{-\frac{\pi}{4}i}}{2^{\frac{5}{2}} e^{\frac{5}{4}\pi i}} = \frac{1}{2} e^{-\frac{3}{2}\pi i} = \frac{1}{2} e^{\frac{1}{2}\pi i + 2k\pi i}$$

$$\therefore z_k = 2^{-\frac{1}{4}} e^{\frac{1}{8}\pi i + \frac{1}{2}k\pi i}, \quad k = 0,1,2,3$$

$\therefore z_0 = 2^{-\frac{1}{4}} e^{\frac{1}{8}\pi i}$ $\quad z_1 = 2^{-\frac{1}{4}} e^{\frac{5}{8}\pi i}$

$z_2 = 2^{-\frac{1}{4}} e^{\frac{9}{8}\pi i}$ $\quad z_3 = 2^{-\frac{1}{4}} e^{\frac{13}{8}\pi i}$

Draw in the x-y coordination $\longrightarrow$



Question 2

We have the augmented matrix:

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 2 & 3 & 4 & 5 & 3 \\ 3 & 4 & a & 6 & 5 \\ 4 & 5 & 7 & b & 4 \end{array}\right] \xrightarrow[\substack{r_3-3r_1 \to r_3 \\ r_4-4r_1 \to r_4}]{r_2-2r_1 \to r_2} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 & -1 \\ 0 & 1 & a-3 & 3 & -1 \\ 0 & 1 & 3 & b-4 & -4 \end{array}\right] \xrightarrow[r_4-r_2 \to r_4]{r_3-r_2 \to r_3} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 & -1 \\ 0 & 0 & a-5 & 0 & 0 \\ 0 & 0 & 1 & b-7 & -3 \end{array}\right]$$

① if $a-5=0$

$\therefore$ The system becomes.

$$\begin{cases} x_1+x_2+x_3+x_4=2 \\ x_2+2x_3+3x_4=-1 \\ 0x_3=0 \\ x_3+(b-7)x_4=-3 \end{cases}$$

i. $b-7=0$

we solve $x_3=-3$

we still have 3 unknown to solve

$\rightarrow$ limitless solutions

ii. $b-7\neq 0$

we have 3 equations, 4 unknown.

$\rightarrow$ limitless solutions.

② if $a-5\neq 0$.

i. if $b-7=0$

The system becomes:

$$\begin{cases} x_1+x_2+x_3+x_4=2 \\ x_2+2x_3+3x_4=-1 \\ (a-5)x_3=0 \\ x_3=-3. \end{cases}$$

$\because a-5\neq 0$.

we solve $x_3=0$.

But through 4^{th} row,

$x_3=-3$.

$\therefore$ no solution this case.

ii. if $b-7\neq 0$

$$\begin{cases} x_1+x_2+x_3+x_4=2 \\ x_2+2x_3+3x_4=-1 \\ (a-5)x_3=0 \\ x_3+(b-7)x_4=-3 \end{cases}$$

we can solve $x_3=0$.

$x_4 = \frac{3}{7-b}$

$x_1 = 3-\frac{6}{b-7}$

$x_2 = -1+\frac{9}{b-7}$

The unique solution.

Summary:

In unique solution case,

we have $\begin{cases} x_1 = 3-\frac{6}{b-7} \\ x_2 = -1+\frac{9}{b-7} \\ x_3 = 0 \\ x_4 = \frac{3}{7-b} \end{cases}$

Consider limitless solutions case.

if we assume $x_4=t$,

we can solve $x_3 = (7-b)t-3$

$x_2 = 5+(2b-17)t$

$x_1 = (9-b)t$

$$\therefore \begin{cases} x_1 = (9-b)t \\ x_2 = 5+(2b-17)t \\ x_3 = (7-b)t-3 \\ x_4 = t \quad t\in\mathbb{R}. \text{ (free parameter)} \end{cases}$$

Question 3

(a) we have:

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 1 & 0 & 1 & -1 \\ -1 & 3 & 2 & 0 \end{bmatrix} \xrightarrow{r_2-r_1 \to r_2} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & -2 & 1 & -2 \\ -1 & 3 & 2 & 0 \end{bmatrix} \xrightarrow{r_3+r_1 \to r_3} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & -2 & 1 & -2 \\ 0 & 5 & 2 & 1 \end{bmatrix} \xrightarrow{2r_3 \to r_3} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & -2 & 1 & -2 \\ 0 & 10 & 4 & 2 \end{bmatrix}$$

$$\xrightarrow{r_3+5r_2 \to r_3} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & -2 & 1 & -2 \\ 0 & 0 & 9 & -8 \end{bmatrix}$$

We see that the Rank of matrix $=3$.

So the matrix can span R^3.

(b) Assume: $x_1\vec{v_1}+x_2\vec{v_2}+x_3\vec{v_3}+x_4\vec{v_4}=0$

$$\Rightarrow \begin{cases} 9x_3-8x_4=0 \\ -2x_2+x_3-2x_4=0 \\ x_1+2x_2+x_4=0 \end{cases} \Rightarrow \begin{array}{l} \text{assume } x_4=1, \quad x_3=\frac{8}{9} \\ x_2=\frac{1}{2}x_3-x_4=\frac{4}{9}-1=-\frac{5}{9} \\ x_1=-2x_2-x_4=\frac{10}{9}-1=\frac{1}{9} \end{array}$$

$$\Rightarrow \frac{1}{9}\vec{v_1}-\frac{5}{9}\vec{v_2}+\frac{8}{9}\vec{v_3}+\vec{v_4}=0$$

$$\boxed{\therefore \vec{v_4} = -\frac{1}{9}\vec{v_1}+\frac{5}{9}\vec{v_2}-\frac{8}{9}\vec{v_3}}$$

Question 4

Solve the eigenvalue of A first. Assume the eigenvalue is λ.

$$\det\left(\begin{bmatrix} 4-\lambda & 1 & 1 \\ 1 & 4-\lambda & 1 \\ 1 & 1 & 4-\lambda \end{bmatrix}\right) = 0 \qquad \begin{vmatrix} 4-\lambda & 1 & 1 \\ 1 & 4-\lambda & 1 \\ 1 & 1 & 4-\lambda \end{vmatrix} = 0$$

$$\det(A) = \sum_{k=1}^{3} a_{1k} \cdot C_{1k} \qquad C_{1k} = (-1)^{1+k} \cdot M_{1k}$$

$$= (4-\lambda)\begin{vmatrix} 4-\lambda & 1 \\ 1 & 4-\lambda \end{vmatrix} - 1 \cdot \begin{vmatrix} 1 & 1 \\ 1 & 4-\lambda \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & 4-\lambda \\ 1 & 1 \end{vmatrix}$$

$$= (4-\lambda)\left[(4-\lambda)^2 - 1\right] - \left[\cancel{4-\lambda-1}\ 3-\lambda\right] + \left[1 - 4 + \lambda\right]$$

$$= (4-\lambda)\cdot(5-\lambda)(3-\lambda) - 2(3-\lambda)$$

$$= (\lambda^2 - 9\lambda + 18)(3-\lambda) = (\lambda-3)(\lambda-6)(3-\lambda)$$

we solve eigenvalues: $\lambda_1 = 3$, $\lambda_2 = 3$, $\lambda_3 = 6$

We next use $(A-\lambda I)\vec{v} = \vec{0}$ to solve eigenvectors $\vec{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

i. $\lambda_1 = \lambda_2 = 3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \vec{0}$$

$\therefore$ $a + b + c = 0$

$$\Rightarrow \vec{v_1} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \quad \vec{v_2} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

ii. $\lambda_3 = 6$

$$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \vec{0}$$

$$\begin{cases} -2a + b + c = 0 & ① \\ a - 2b + c = 0 & ② \\ a + b - 2c = 0 & ③ \end{cases}$$

③ − ②

$3b - 3c = 0$

$b = c$

①: $b + c = 2a$

$\Rightarrow b = c = a$

$$\therefore \vec{v_3} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

We now have eigenvalues $\lambda_1 = 3$, $\lambda_2 = 3$, $\lambda_3 = 6$ and eigenvectors $\vec{v_1} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ $\vec{v_2} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ $\vec{v_3} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

D is the diagonal matrix

$$D = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{pmatrix} \qquad P = (\vec{v_1}, \vec{v_2}, \vec{v_3}) = \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

$$P^{-1} = \frac{1}{\det(P)} C^T, \quad C = \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix} \quad C_{mk} = (-1)^{m+k} M_{mk}$$

To calculate $\det(P)$: Span along 3^{rd} row.

$$\det(P) = \sum_{k=1}^{3} a_{3k} m_{3k} = (-1)\cdot(-1)^{3+1}\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} + 0 + 1\cdot(-1)^{3+3}\begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix}$$

$$= -2 - 1 = -3$$

$$C_{11} = (-1)^2\begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} = -1 \qquad C_{21} = (-1)^3\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = -1 \qquad C_{31} = (-1)^4\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 2$$

$$C_{12} = (-1)^3\begin{vmatrix} 0 & 1 \\ -1 & 1 \end{vmatrix} = -1 \qquad C_{22} = (-1)^4\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 2 \qquad C_{32} = (-1)^5\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = -2$$

$$C_{13} = (-1)^4\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} = -1 \qquad C_{23} = (-1)^5\begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix} = -1 \qquad C_{33} = (-1)^6\begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} = -1$$

$$\therefore P^{-1} = \frac{1}{-3} C^T, \quad C = \begin{pmatrix} -1 & -1 & -1 \\ -1 & 2 & -1 \\ 2 & -2 & -1 \end{pmatrix} \quad C^T = \begin{pmatrix} -1 & -1 & 2 \\ -1 & 2 & -2 \\ -1 & -1 & -1 \end{pmatrix}$$

$$\therefore P^{-1} = \frac{1}{-3}\begin{pmatrix} -1 & -1 & 2 \\ -1 & 2 & 2 \\ -1 & -1 & -1 \end{pmatrix}$$

$$A^7 = P D^7 P^{-1} = \frac{1}{-3}\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 3^7 & 0 & 0 \\ 0 & 3^7 & 0 \\ 0 & 0 & 6^7 \end{pmatrix}\begin{pmatrix} -1 & -1 & 2 \\ -1 & 2 & -2 \\ -1 & -1 & -1 \end{pmatrix}$$

$$= \frac{1}{-3}\begin{pmatrix} 3^7 & 3^7 & 6^7 \\ 0 & -3^7 & 6^7 \\ -3^7 & 0 & 6^7 \end{pmatrix}\begin{pmatrix} -1 & -1 & 2 \\ -1 & 2 & -2 \\ -1 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 94770 & 92583 & 92583 \\ 92583 & 94770 & 92583 \\ 92583 & 92583 & 94770 \end{pmatrix}$$

Question 5

(a) assume the eigenvalue: λ

$|A - I\lambda| = 0$.

$$\begin{vmatrix} 4-\lambda & 1 & 0 \\ 1 & 4-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0$$

Span along 3^{rd} row:

we have

$$(3-\lambda)(-1)^{3+3}\begin{vmatrix} 4-\lambda & 1 \\ 1 & 4-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)(3-\lambda)(5-\lambda) = 0$$

$\therefore$ Solve: $\lambda_1 = 3$, $\lambda_2 = 3$, $\lambda_3 = 5$

next we solve eigenvectors $\vec{v} = (a, b, c)$

i. $\lambda_1 = \lambda_2 = 3$

$(A - \lambda I)\vec{v} = \vec{0}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \vec{0}$$

$\therefore$ $a + b = 0$, c free.

We solve:

$$\vec{v_1} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \quad \vec{v_2} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

ii. $\lambda_3 = 5$

$$\begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \vec{0}$$

$$\Rightarrow \begin{cases} -a + b = 0 \\ a - b = 0 \\ -2c = 0 \end{cases}$$

we solve $c = 0$, $a = b$

$$\therefore \vec{v_3} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

(b) Find orthogonal matrix Q, diagonal matrix D.

We have the eigenvectors $\vec{v_1}, \vec{v_2}, \vec{v_3}$

And $\langle \vec{v_1}, \vec{v_2} \rangle = 0$ $\langle \vec{v_1}, \vec{v_3} \rangle = 0$

$\langle \vec{v_2}, \vec{v_3} \rangle = 0$

Assume $Q = (\vec{w_1}, \vec{w_2}, \vec{w_3})$

$\|\vec{v_1}\| = \sqrt{2}$ $\|\vec{v_2}\| = 1$. $\|\vec{v_3}\| = \sqrt{2}$.

$\therefore$ $\vec{w_1} = \frac{1}{\sqrt{2}}\vec{v_1}$ $\vec{w_2} = \vec{v_2}$ $\vec{w_3} = \frac{1}{\sqrt{2}}\vec{v_2}$ (normalization)

$$\therefore Q = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \end{bmatrix}, \quad Q^T = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

(c) From (b), we know that $A = QDQ^T$

$A^{-1} = (QDQ^T)^{-1} = QD^{-1}Q^T$

verify: $\underbrace{QDQ^T}_{A} \cdot \underbrace{QD^{-1}Q^T}_{A^{-1}?} = QD \cdot I \cdot D^{-1}Q^T$

$= Q(DD^{-1})Q^T = QIQ^T = QQ^T = I$

$\therefore A^{-1} = QD^{-1}Q^T$

$\therefore A - 9A^{-1} = QDQ^T - 9QD^{-1}Q^T = Q(D-9D^{-1})Q^T$

Next: Calculate $D - 9D^{-1}$.

$$D - 9D^{-1} = \begin{bmatrix} 3-9\times\frac{1}{3} & 0 & 0 \\ 0 & 3-9\times\frac{1}{3} & 0 \\ 0 & 0 & 5-9\times\frac{1}{5} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{16}{5} \end{bmatrix}$$

$$(A-9A^{-1})^3 = Q(D-9D^{-1})^3 \cdot Q^T$$

$$= Q \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{4096}{125} \end{bmatrix} Q^T$$

And we have:

$$D^{-1} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}.$$

$\therefore (A - 9A^{-1})^3$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{4096}{125} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$= \frac{2048}{125} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Question 6

$y' + xy = y(\cos x + 3)$, $y(0) = 1$

$y' = y(\cos x + 3) - xy$

$= y(\cos x + 3 - x)$

$\therefore \frac{dy}{dx} = y(\cos x + 3 - x)$

$\therefore \int \frac{1}{y}\,dy = \int (\cos x + 3 - x)\,dx$

$\ln|y| = \sin x - \frac{1}{2}x^2 + 3x + c$

$y = e^c \cdot e^{\sin x + 3x - \frac{1}{2}x^2}$

$= Ce^{\sin x + 3x - \frac{1}{2}x^2}$

$\because y(0) = 1$, $\therefore 1 = Ce^0$, $C = 1$

$\therefore y = e^{\sin x + 3x - \frac{1}{2}x^2}$

Question 7

$\frac{dy}{dx} + 3 \cdot y = e^{-x} \Rightarrow \frac{dy}{dx} + p(x) \cdot y = q(x)$

$\therefore p(x) = 3$, $q(x) = e^{-x}$ $\quad \mu(x) = e^{\int p(x)dx} = e^{\int 3\,dx} = e^{3x + c_1} = C_1e^{3x}$

$y = \frac{1}{\mu(x)}\left[\int q(x)\mu(x)\,dx + c_2\right]$

$= C_1e^{-3x}\int e^{2x}\,dx + e^{-3x} \cdot C_2$

$= \frac{1}{2}C_1e^{-x} + C_2e^{-3x}$

Question 8

$y' + 3xy = 3xy^3$, $y(0) = \frac{1}{2}$

$y' = 3x(y^3 - y) = \frac{dy}{dx}$

$\int 3x\,dx = \int \frac{1}{y^3 - y}\,dy$

For the RHS

$\frac{1}{y^3 - y} = \frac{A}{y} + \frac{B}{y+1} + \frac{C}{y-1}$

$= \frac{Ay + A + By}{y(y+1)} + \frac{C}{y-1}$

$= \frac{(y-1)[A(y+1) + By] + C[(y+1) \cdot y]}{y(y+1)(y-1)}$

$\therefore \cancel{Ay^2} - A + By^2 - \cancel{By} + C\cancel{y^2} + \cancel{Cy}$

$\Rightarrow \begin{cases} -A = 1 \\ A + B + C = 0 \\ B - C = 0 \end{cases} \Rightarrow \begin{matrix} A = -1 \\ B = \frac{1}{2} \\ C = \frac{1}{2} \end{matrix}$

$\therefore \int \frac{1}{y^3 - y}\,dy$

$= \int \left(-\frac{1}{y} + \frac{1}{y+1} + \frac{1}{y-1}\right)dy$

$= -\ln|y| + \ln|y+1| + \ln|y-1| + C_1$

$\int 3x\,dx = \frac{3}{2}x^2 + C_2$

$\therefore -\ln|y| + \frac{1}{2}\ln|y+1| \cdot |y-1| = \frac{3}{2}x^2 + C$

$\frac{\sqrt{|y+1||y-1|}}{|y|} = ke^{\frac{3}{2}x^2}$

we have $y(0) = \frac{1}{2}$

$\therefore \frac{\sqrt{\frac{3}{2} \times \frac{1}{2}}}{\frac{1}{2}} = \sqrt{3} = ke^0$

$\Rightarrow k = \sqrt{3}$.

we should check the product under root.

$(y^2 - 1$ or $1 - y^2$?)

as we'll take $y = \frac{1}{2}$, and the product ≥ 0.

$\therefore$ we take $1 - y^2$

$\frac{\sqrt{1-y^2}}{|y|} = \sqrt{3}e^{\frac{3}{2}x^2}$

$\Rightarrow \frac{1-y^2}{y^2} = 3e^{3x^2} \longrightarrow \frac{1}{y^2} - 1 = 3e^{3x^2}$

$\frac{1}{y^2} = 1 + 3e^{3x^2}$

$\therefore y = \pm\sqrt{\frac{1}{1+3e^{3x^2}}}$