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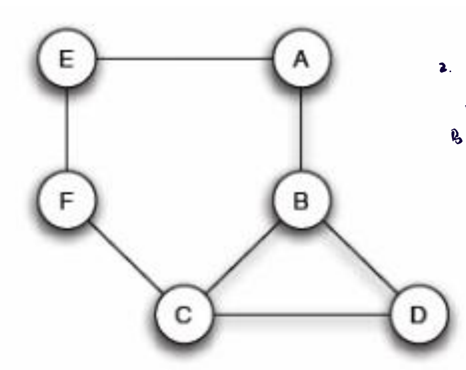
You need to give the reasoning of your answers for BOTH why you choose AND why you do not choose. Otherwise, there is no mark.

Question 1



Question 1: 1. The length of shortest path from D to E is 3

shortest paths:

① D-C-F-E

② D-B-A-E

B is not in ①. So B isn't pivotal.

2. A-C: The shortest path is: A-B-C

A-D: A-B-D

B is both in A-B-C, A-B-D So 2 is correct.

3. The shortest path from D to E:

D-B-A-E, D-C-F-E

There's no indentical node appearing in both path, so there's no pivotal in pair D-E.

So 3 is correct.

4. (B-C): B-C is ok, D is not pivotal

in the polygon ABCFE: except path (B-C) The rest only have pivotal node inside the polygon.

So: D isn't pivotal for any path.

4 is correct.

A shortest path between two nodes is a path of the minimum possible length. We say that a node X is pivotal for a pair of distinct nodes Y and Z if X lies on every shortest path between Y and Z (and X is not equal to either Y or Z).

According to the above figure, which of the following statement(s) are correct?

1. Node B is pivotal for node pair of D - E
2. Node B is pivotal for node pairs of A - C and A – D
3. There is no pivotal for node pair D – E
4. Node D is not a pivotal for any pairs of distinct nodes

Question 2

For ①, expelling path consisting of linked path A-B, B-C, C-D, A-D

Consider A-C: A-B-C, A-D-C there's no identical node in both paths.

Consider B-D: B-A-D, B-C-D there's no identical node in both paths.

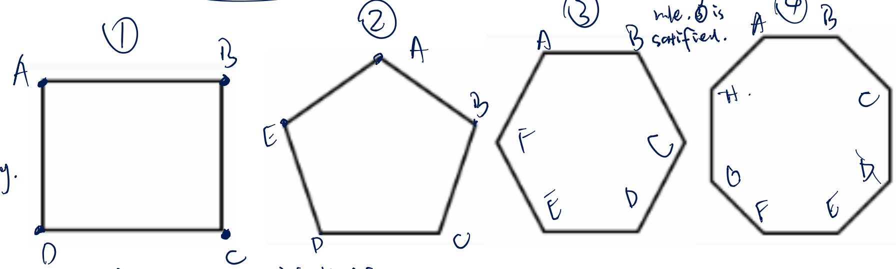
So: ① doesn't satisfy.

for ③: 1. A-C A-B-C, B is the pivotal.

The rule is found: for polygons who have more 4 edges, every node is the pivotal for a path whose length is 2.

Recall the definition of shortest path and pivotal stated in Question 1. Consider the following polygons where each vertex is a node. Which polygon satisfies that every node in this polygon is a pivotal of at least one node pair?

According to the rule, ③ is satisfied.



for ④:

A-C: A-B-C, B.

B-D: B-C-D, C

C-E: C-D-E, D

D-F: D-E-F: E.

E-G: E-F-G: F

F-H: F-G-H: G

G-A: G-H-A: H

H-B: H-A-B: A

Also, according to the rule found. ④ is satisfied.

for ②:

1. Consider A-C, the shortest path: A-B-C. So: node B is pivotal

2. Consider B-D B-C-D => C is pivotal

3. Consider C-E C-D-E, D is pivotal.

4. Consider D-A D-E-A E is pivotal.

5. Consider E-B E-A-B A is pivotal

So every node is the pivotal of some path, ② satisfies the requirement.

Question 3

Recall the definition of shortest path and pivotal stated in Question 1. Consider the following polygons where each vertex is a node. Which polygon satisfies that every node in this polygon is a pivotal of at least two node pairs?

for ①:

path (A-C) A-B-C, A-D-C

path (B-D) B-A-D, B-C-D

Both path has no pivotal node!

① doesn't satisfy.

for ②

Consider all shortest path.

(A-C) A-B-C

(B-D) B-C-D

(C-E) C-D-E

(D-A) D-E-A

(E-B) E-A-B

We've considered all possible paths. But every path has only one shortest path. ② doesn't satisfy.

for ③

Consider. (A-D) A-B-C-D, A-F-E-D

(B-E) B-C-D-E, B-A-F-E

Both path even has no pivotal nodes.

Consider (A-C) A-B-C

(B-D) B-C-D

(C-E) C-D-E

(D-F) D-E-F

(E-A) E-F-A

(F-B) F-A-B

These paths has only one shortest path.

③ doesn't satisfy.

for ④

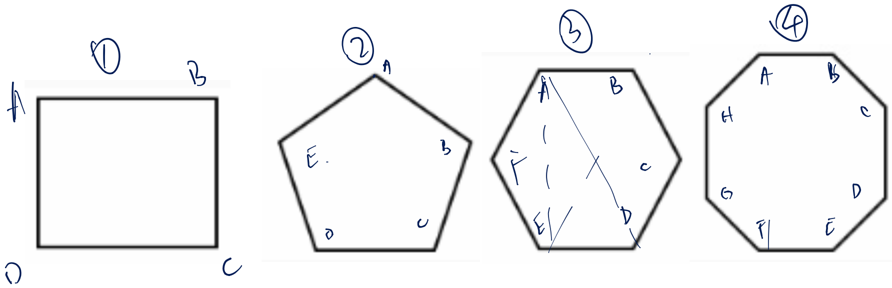
Consider path (A-D): A-B-C-D, (H-C): H-A-B-C So C is pivotal for 2 pairs.

Consider (A-B): G-H-A-B, (H-C) H-A-B-C So A is pivotal

(A-D): A-B-C-D, (C-H) C-B-A-H B is pivotal

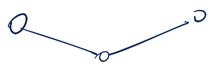
We find the regulation that: in ④, there'll always be two pairs of path, containing the same node.

So: ④ satisfies.



The following definitions are useful for question 4 and 5.

A node X is called a gatekeeper if for some other two nodes Y and Z, every path from Y to Z passes through X. A node X is called a local gatekeeper if there are two neighbors of X, say Y and Z, that are not connected by an edge. (That is, for X to be a local gatekeeper, there should be two nodes Y and Z so that Y and Z each have edges to X, but not to each other.)



Prove: A gatekeeper is a bridge.

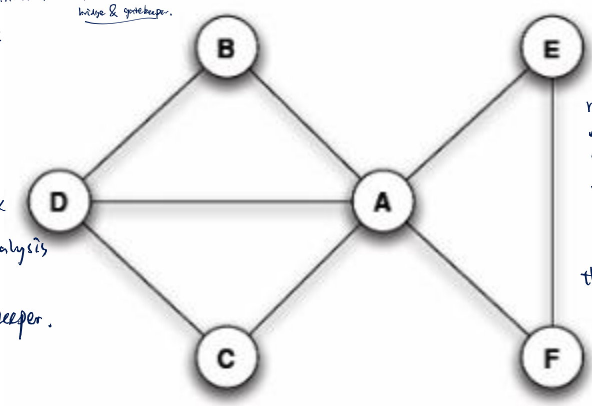
For 2 nodes, if every path has the gatekeeper, then there's only 1 connected component.
But when you remove the gatekeeper, there's no connection, so there must be 2 connected components.
⇒ Gatekeeper is a bridge.

Question 4

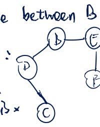
Based on the definition of "Gatekeeper" and "Local Gatekeeper", please consider the following statements and select the correct one(s) w.r.t the graph below.

In this question: if we remove A, there's 2 connected ..., so A is bridge & gatekeeper.

rm B → still 1, B ×
rm D → still 1, D ×
rm C → still 1, C ×
rm E → still 1, F ×
rm F → still 1, F ×
So after traversal analysis of nodes,
only A is the gatekeeper.



1. if we add an edge between B & E,
remove A, we get:



, only one connected component.
remove B: 1 connected component, B ×
rm C: 1 connected, C ×
rm D: 1 connected, D ×
rm E: 1 connected, E ×
rm F: 1 connected, F ×
So: there's no gatekeeper. The num of gatekeeper changed, 1 isn't correct.

2. According to analysis in the left, D isn't a gatekeeper. 2 isn't correct.

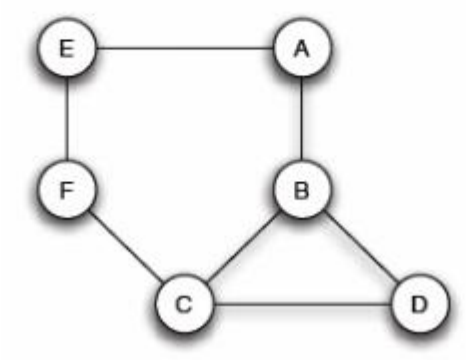
3. A is linked with B and C, and B, C are not connected,
So: A's a local gatekeeper.
3 is correct.

1. The number of local gatekeeper remains unchanged when an edge between node B and E is added
2. Node D is gatekeeper
3. Node A is local gatekeeper
4. There are 2 local gatekeepers in the above graph

4. for E: connected with A, F
But A, F are connected. E ×
for F, A, E are connected. F ×
for A, A is connected with B & C, B, C aren't connected, so A is local gatekeeper.
for D, it's connected with B, C
But B, C aren't connected.
So D is local gatekeeper.
for B, C. Both connected with A, D, But A, D connected. B, C ×
A, D are local gatekeepers.
So 4 is corrected.

Question 5

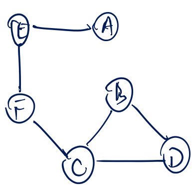
Based on the definition of "Gatekeeper" and "Local Gatekeeper", please consider the following statements and select the correct one(s) w.r.t the graph below.



1. If the edge between node A and B is removed, node C will be gatekeeper.
2. Over half of the number of nodes are local gatekeepers
3. If the edge between node B and C is removed, all of the nodes will be local gatekeepers.
4. If the edge between node C and D is removed, node B will be gatekeeper.

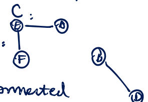
Question 5

1. if remove (A-B)



then there's only connected component.

if we remove C :
we'll have.

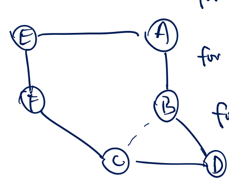


There's 2 connected components, increased.
So : C is a gatebeeper.

1 is correct.

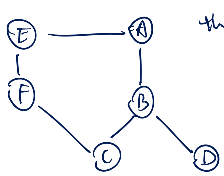
2. There's 6 nodes.
for A. A's connected with B, E B E aren't connected, So A's local gatekeeper.
for B. B connect with A. C
A. C aren't connected. B's ... .
for C: F. B are connected with C
But they're not connected.
C √
for E : A. F . So E √
there's already 4 local gatekeepers.
over half num are ... .
So 2 is correct.

3. remove (B-C)

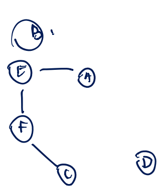


there's
for A. B, E not connected.
A √
for B A. D not connected.
B √
for C F, D not connected.
C √
for D B. C not connected
D √
for E
A. F not connected.
E √
for F
E. C not connected.
F √
So all nodes are local gatekeepers.
3 is correct !

4. remove (C-D)



there's only 1 connected component.
remove B.



There're 2 connected components !
So B is a gatekeeper.
4 is correct.