

# Exercises for tomorrow

```
fmod NAT-ADD is  
  sort Nat .  
  op 0 :  $\rightarrow$  Nat [ctor] .  
  op s : Nat  $\rightarrow$  Nat [ctor] .  
  op _+_ : Nat Nat  $\rightarrow$  Nat .  
  
  vars M N : Nat .  
  
  eq 0 + M = M .  
  eq s(M) + N = s(M + N) .  
endfm
```

Make a new module which extends NAT-ADD and

1. define a function **op double** : Nat  $\rightarrow$  Nat . which doubles its argument. For example, double(0) should be 0 while double(s(s(s(0)))) should be s(s(s(s(s(0))))). Do not use +, only 0 and s.
2. define a function **op half** : Nat  $\rightarrow$  Nat . which divides a number by 2. For example, "half" of 0 is 0; "half" of 2 is 1; "half" of 3 is also 1; "half" of 4 is 2; "half" of 5 is 2. What is half of 86? of 87?
3. define a function **op minus** : Nat Nat  $\rightarrow$  Nat which computes "minus down to 0," i.e., max(m - n, 0).
4. define a function **op diff** : Nat Nat  $\rightarrow$  Nat . which computes the difference between two numbers. For example, the diff between 2 and 7 is 5 and the diff between 8 and 1 is 7.