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Problem 1. (2 points)
We will usually write vectors as column vectors. Let

$$\vec{u} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}, \vec{v} = \begin{bmatrix} -8 \\ 1 \\ 9 \end{bmatrix}, \quad \text{and} \quad \vec{w} = \begin{bmatrix} -5 \\ -5 \\ 2 \end{bmatrix}.$$

Then $7\vec{u} + 3\vec{v} + 2\vec{w} = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$

Solution:

SOLUTION

$$7\vec{u} + 3\vec{v} + 2\vec{w} = \begin{bmatrix} 7*3+3*-8+2*-5 \\ 7*-2+3*1+2*-5 \\ 7*-1+3*-5+2*2 \end{bmatrix} = \begin{bmatrix} -13 \\ -21 \\ 24 \end{bmatrix}$$

Correct Answers:

- -13
- -21
- 24

Problem 2. (2 points)
Sometimes it is convenient to write vectors as row vectors. Let $\vec{x} = \begin{bmatrix} -5 & -2 \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} -3 & 2 \end{bmatrix}$. Find the following vectors.

$\vec{v} = 7\vec{x} = \begin{bmatrix} \_\_ & \_\_ \end{bmatrix}$.
$\vec{u} = 2\vec{y} = \begin{bmatrix} \_\_ & \_\_ \end{bmatrix}$.
$\vec{w} = 7\vec{x} + 2\vec{y} = \begin{bmatrix} \_\_ & \_\_ \end{bmatrix}$.

Correct Answers:

- $\begin{bmatrix} -35 & -14 \end{bmatrix}$
- $\begin{bmatrix} -6 & 4 \end{bmatrix}$
- $\begin{bmatrix} -41 & -10 \end{bmatrix}$

Problem 3. (2 points)
The general solution to a linear system is given. Express this as a linear combination of vectors.

$$x_1 = 7 + 7s_1 + 2s_2$$

$$x_2 = 7 + 3s_1 + 0s_2$$

$$x_3 = -5 - 9s_1 + 9s_2$$

$$x_4 = -5 - 1s_1 + 1s_2$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} + \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} s_1 + \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} s_2$$

Solution:

SOLUTION

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \\ -5 \\ -5 \end{bmatrix} + \begin{bmatrix} 7 \\ 3 \\ -9 \\ -1 \end{bmatrix} s_1 + \begin{bmatrix} 2 \\ 0 \\ 9 \\ 1 \end{bmatrix} s_2$$

Correct Answers:

- 7
- 7
- -5
- -5
- 7
- 3
- -9
- -1

- 2
- 0
- 9
- 1

Problem 4. (2 points)
Express the following vector equation as a system of linear equations.

$$x_1 \begin{bmatrix} 3 \\ -7 \end{bmatrix} + x_2 \begin{bmatrix} -9 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

(Keep the equations in order.)

$\_\_\_\; x_1 + \_\_\_\; x_2 = \_\_\_$

$\_\_\_\; x_1 + \_\_\_\; x_2 = \_\_\_$

Correct Answers:

- 3
- -9
- 1
- -7
- 4
- 5

Problem 5. (3 points)
Solve the system of equations. Your answers must be fractions (decimals are not allowed).

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 2 \\ 3x_1 - 2x_2 + 3x_3 &= 1 \\ -2x_1 - 2x_2 - x_3 &= -2 \end{aligned}$$

$$\vec{x} = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}.$$

Correct Answers:

- $\begin{bmatrix} \frac{7}{17} \\ \frac{8}{17} \\ \frac{4}{17} \end{bmatrix}$

Problem 6. (3 points)
Solve the vector equation

$$x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ -2 \\ -1 \end{bmatrix}.$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} + s \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$$

Correct Answers:

- $\begin{bmatrix} -1 \\ -1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$

Problem 7. (2 points)
Determine whether or not each set of vectors below forms a subspace of the given space.

Start by checking each of the three requirements to be a subspace.

- V_1 consists of all vectors in $\mathbb{R}^4$ of the form $\begin{bmatrix} a \\ 3a+b \\ a+2b \\ 4a-6b \end{bmatrix}$.

  Select all true statements:

- This set contains the zero vector.
- This set is closed under vector addition.
- This set is closed under scalar multiplications.
- This set is a subspace of $\mathbb{R}^4$.
- none of the above

- V_2 consists of all vectors in $\mathbb{R}^2$ of the form $\begin{bmatrix} a \\ b \end{bmatrix}$ such that $a \leq b$.

  Select all true statements:

- This set contains the zero vector.
- This set is closed under vector addition.
- This set is closed under scalar multiplications.
- This set is a subspace of $\mathbb{R}^2$.
- none of the above

Correct Answers:

- Choice 1, Choice 2, Choice 3, Choice 4
- Choice 1, Choice 2

Problem 8. (2 points)
Determine which of the following subsets of $\mathbb{R}^3$ are subspaces of $\mathbb{R}^3$.

Consider the three requirements for a subspace, as in the previous problem.

Select all that are subspaces.

- The set of all (b_1, b_2, b_3) with $b_1 = 0$
- The set of all (b_1, b_2, b_3) with $b_2 = 2b_3$
- The set of all (b_1, b_2, b_3) with $b_1 \leq b_2$
- The set of all (b_1, b_2, b_3) with $b_1 = 1$
- The set of all (b_1, b_2, b_3) with $b_3 = b_1 + b_2$
- The set of all (b_1, b_2, b_3) with $b_1 = b_2$
- None of the above

Correct Answers:

- Choice 1, Choice 2, Choice 5, Choice 6

Problem 9. (2 points)

Determine whether or not each set of vectors below forms a subspace of the given space.

Start by checking each of the three requirements to be a subspace.

(Recall that $\mathcal{P}_3$ is the set of all polynomials of degree less than or equal to 3.)

- V_1 consists of all odd polynomials in $\mathcal{P}_3$. In other words, polynomials of the form $ax^3 + bx$.

  Select all true statements:

- This set contains the zero polynomial.
- This set is closed under addition.
- This set is closed under scalar multiplications.
- This set is a subspace of $\mathcal{P}_3$.
- none of the above

- V_2 consists of all polynomials $f(x)$ in $\mathcal{P}_3$ with $f(1) \geq 0$.

  Select all true statements:

- This set contains the zero polynomial.
- This set is closed under addition.
- This set is closed under scalar multiplications.
- This set is a subspace of $\mathcal{P}_3$.
- none of the above

- V_3 consists of all polynomials $f(x)$ in $\mathcal{P}_3$ with $f(2) = 0$.

  Select all true statements:

- This set contains the zero polynomial.
- This set is closed under addition.
- This set is closed under scalar multiplications.
- This set is a subspace of $\mathcal{P}_3$.
- none of the above

Correct Answers:

- Choice 1, Choice 2, Choice 3, Choice 4
- Choice 1, Choice 2
- Choice 1, Choice 2, Choice 3, Choice 4

Problem 10. (0 points)

Answer the following questions on your own paper and upload your answers into Gradescope. Make sure to give clear explanations and show your work where appropriate.

1. Determine whether each of the following is a subspace of $\mathbb{R}^3$. Either show that the set satisfies the conditions necessary to be a subspace or give an example where it does not satisfy one of the conditions.

a. The set of vectors in $\mathbb{R}^3$ defined below.

$$\left\{ \begin{bmatrix} a \\ b \\ 2a-b \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; a, b \in \mathbb{R} \right\}.$$

b. The set of vectors in $\mathbb{R}^2$ defined below.

$$\left\{ \begin{bmatrix} a \\ b \end{bmatrix} \in \mathbb{R}^2 \;\middle|\; ab \geq 0 \right\}.$$