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Problem 1. **(1 point)**
Let $\vec{v}_1, \vec{v}_2$, and $\vec{v}_3$ be vectors in $\mathbb{R}^3$. Select all of the true statements below.

- A. If there is no nontrivial linear combination of $\vec{v}_1, \vec{v}_2$, and $\vec{v}_3$ that equals $\vec{0}$, then the set $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent.
- B. If $\vec{v}_1$ can be written as a linear combination of $\vec{v}_2$ and $\vec{v}_3$, then the set $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent.
- C. If $\vec{v}_1$ cannot be written as a linear combination of $\vec{v}_2$ and $\vec{v}_3$, then the set $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent.
- D. If there is a nontrivial linear combination of $\vec{v}_1, \vec{v}_2$, and $\vec{v}_3$ that equals $\vec{0}$, then the set $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent.
- E. None of the above are true.

Correct Answers:

- ABD

Problem 2. **(1 point)**
Notice that

$$\begin{bmatrix} 2 \\ -8 \\ 20 \end{bmatrix} = 3 \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} - 7 \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}.$$

Use this information to select all of the true statements below.

- A. The vector $\begin{bmatrix} 2 \\ -8 \\ 20 \end{bmatrix}$ is in the span of $\left\{ \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \right\}$.
- B. The set $\left\{ \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -8 \\ 20 \end{bmatrix} \right\}$ is linearly independent.
- C. The vector $\begin{bmatrix} 2 \\ -8 \\ 20 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$.
- D. The set $\left\{ \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -8 \\ 20 \end{bmatrix} \right\}$ is linearly dependent.
- E. The set $\left\{ \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -8 \\ 20 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$.
- F. The vector $\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$ is a linear combination of

$$\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \text{ and } \begin{bmatrix} 2 \\ -8 \\ 20 \end{bmatrix}.$$

- G. None of the above are true.

Correct Answers:

- ACDF

Problem 3. **(1 point)**
Let $\vec{v}_1, \vec{v}_2, \vec{v}_3$, and $\vec{v}_4$ be vectors in $\mathbb{R}^3$. Determine whether each of the following statements is true or false.

If $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent, then $\{\vec{v}_1, \vec{v}_2\}$ must also be linearly independent.

- A. True
- B. False

If $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ spans $\mathbb{R}^3$, then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ must also span $\mathbb{R}^3$.

- A. True
- B. False

If $\vec{v}_1$ is a linear combination of $\vec{v}_2, \vec{v}_3$, and $\vec{v}_4$, then $\vec{v}_4$ must be a linear combination of $\vec{v}_1, \vec{v}_2$, and $\vec{v}_3$.

- A. False
- B. True

If $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a basis of $\mathbb{R}^3$, then $\vec{v}_4$ must be a linear combination of $\vec{v}_1, \vec{v}_2$, and $\vec{v}_3$.

- A. True
- B. False

Correct Answers:

- A
- A
- A
- A

Problem 4. **(1 point)**
Which of the following are bases of $\mathbb{R}^2$? Select all that apply.

- A. $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$.
- B. $\left\{ \begin{bmatrix} -7 \\ -5 \end{bmatrix}, \begin{bmatrix} -21 \\ -15 \end{bmatrix} \right\}$.
- C. $\left\{ \begin{bmatrix} -4 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \end{bmatrix} \right\}$.
- D. $\left\{ \begin{bmatrix} 7 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ -1 \end{bmatrix} \right\}$.
- E. $\left\{ \begin{bmatrix} 8 \\ -6 \end{bmatrix}, \begin{bmatrix} 16 \\ -13 \end{bmatrix} \right\}$.
- F. None of the above are bases of $\mathbb{R}^3$.

Correct Answers:

- ADE

Problem 5. **(1 point)**
The set

$$\left\{ \begin{bmatrix} -4 \\ 4 \\ -12 \end{bmatrix}, \begin{bmatrix} -3 \\ 8 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ -8 \end{bmatrix} \right\}$$

is a basis for $\mathbb{R}^3$ (you don't have to show this, you can assume it is true). Use this information to select all of the true statements below. You shouldn't need to perform any computations.

- A. The set $\left\{ \begin{bmatrix} -4 \\ 4 \\ -12 \end{bmatrix}, \begin{bmatrix} -3 \\ 8 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ -8 \end{bmatrix}, \begin{bmatrix} -9 \\ 7 \\ 2 \end{bmatrix} \right\}$ is linearly dependent.
- B. The vector $\begin{bmatrix} 3 \\ -4 \\ -1 \end{bmatrix}$ is in the span of $\left\{ \begin{bmatrix} -4 \\ 4 \\ -12 \end{bmatrix}, \begin{bmatrix} -3 \\ 8 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ -8 \end{bmatrix} \right\}$.
- C. The vector $\begin{bmatrix} -5 \\ -4 \\ 4 \end{bmatrix}$ can be written as a linear combination of $\begin{bmatrix} -4 \\ 4 \\ -12 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 8 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ -1 \\ -8 \end{bmatrix}$.
- D. The vector $\begin{bmatrix} -4 \\ 4 \\ -12 \end{bmatrix}$ is in the span of $\left\{ \begin{bmatrix} -3 \\ 8 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ -8 \end{bmatrix} \right\}$.
- E. The set $\left\{ \begin{bmatrix} -4 \\ 4 \\ -12 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ -8 \end{bmatrix} \right\}$ spans $\mathbb{R}^3$.
- F. None of the above are true.

Correct Answers:

- ABC