Week 4 WeBWorK

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Problem 1. **(1 point)**
Perform the stated elementary row operation to each of the matrices below.

- Enter the result of applying the row operation $R_2 \leftrightarrow R_1$ to the given matrix.

$$\begin{bmatrix} 1 & -4 & 1 \\ 3 & -5 & 4 \\ -3 & -2 & 4 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

- Enter the result of applying the row operations $R_3 - 4R_1 \to R_3$ to the given matrix.

$$\begin{bmatrix} 5 & 2 & -4 \\ 3 & -5 & -1 \\ 3 & 3 & -4 \end{bmatrix} \xrightarrow{R_3 - 4R_1 \to R_3} \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

Correct Answers:

- $\begin{bmatrix} 3 & -5 & 4 \\ 1 & -4 & 1 \\ -3 & -2 & 4 \end{bmatrix}$
- $\begin{bmatrix} 5 & 2 & -4 \\ 3 & -5 & -1 \\ -17 & -5 & 12 \end{bmatrix}$

Problem 2. **(2 points)**
Let $\vec{v}_1 = \begin{bmatrix} -1 \\ -4 \\ -2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, and $\vec{b} = \begin{bmatrix} -2 \\ -14 \\ -10 \end{bmatrix}$.

Is $\vec{b}$ in the span of $\vec{v}_1$ and $\vec{v}_2$?

- choose
- Yes, it is in the span.
- No, it is not in the span.
- We cannot tell if it is in the span.

Either fill in the coefficients of the vector equation below, or leave blank if no solution is possible.

$$\vec{b} = \_\_\_\ \vec{v}_1 + \_\_\_\ \vec{v}_2$$

Correct Answers:

- Yes, it is in the span.
- 4
- 2

Problem 3. **(1 point)**
Let $\mathbf{a}_1 = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 25 \\ -20 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 60 \\ -51 \end{bmatrix}$.

Is $\mathbf{b}$ a linear combination of $\mathbf{a}_1$ and $\mathbf{a}_2$?

- A. $\mathbf{b}$ is not a linear combination.
- B. Yes $\mathbf{b}$ is a linear combination.
- C. We cannot tell if $\mathbf{b}$ is a linear combination.

Either fill in the coefficients of the vector equation, or enter "NONE" if no solution is possible.

$\mathbf{b} = \_\_\_\ \mathbf{a}_1 + \_\_\_\ \mathbf{a}_2$

Solution:

SOLUTION

$\mathbf{b}$ is a linear combination of $\mathbf{a}_1$ and $\mathbf{a}_2$.

Using row reduction, we see

$$\begin{bmatrix} -5 & 25 & 60 \\ 3 & -20 & -51 \end{bmatrix} \sim \begin{bmatrix} 1 & -5 & -12 \\ 0 & -5 & -15 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \end{bmatrix}$$

so

$$\mathbf{b} = 3\mathbf{a}_1 + 3\mathbf{a}_2$$

Correct Answers:

- B
- 3
- 3

Problem 4. **(2 points)**

Write $\begin{bmatrix} -16 \\ 11 \\ 6 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ -3 \\ -2 \end{bmatrix}$.

$\begin{bmatrix} -16 \\ 11 \\ 6 \end{bmatrix} = \_\_\_\_ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} + \_\_\_\_ \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix} + \_\_\_\_ \begin{bmatrix} 5 \\ -3 \\ -2 \end{bmatrix}$.

Correct Answers:

- -3; 1; -4

Problem 5. **(1 point)**

Let $\mathbf{u} = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} -8 \\ 4 \\ -5 \end{bmatrix}$.

We want to determine by inspection (with minimal computation) if $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly dependent or independent.

Choose the best answer.

- A. The set is linearly dependent because one of the vectors is a scalar multiple of another vector.
- B. The set is linearly dependent because two of the vectors are the same.
- C. The set is linearly dependent because the number of vectors in the set is greater than the dimension of the vector space.
- D. The set is linearly independent because we only have two vectors and they are not scalar multiples of each other.
- E. The set is linearly dependent because one of the vectors is the zero vector.
- F. We cannot easily tell if the set is linearly dependent or linearly independent.

Solution:

SOLUTION

The set is linearly dependent because the second vector is the zero vector.

Correct Answers:

- E

Problem 6. **(2 points)**
Let

$$B = \begin{bmatrix} 5 & 5 & 5 & 5 \\ -1 & -2 & -3 & -4 \end{bmatrix}.$$

(a) Find the reduced row echelon form of the matrix B.
$\text{rref}(B) = \begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}$

(b) How many pivot columns does B have?

____

(c) Do the vectors in the set $\left\{ \begin{bmatrix} 5 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -3 \end{bmatrix}, \begin{bmatrix} 5 \\ -4 \end{bmatrix} \right\}$ span $\mathbb{R}^2$? Be sure you can explain and justify your answer.

- choose
- `the vectors span R^2`
- `the vectors do not span R^2`

(d) Are the vectors in the set $\left\{ \begin{bmatrix} 5 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -3 \end{bmatrix}, \begin{bmatrix} 5 \\ -4 \end{bmatrix} \right\}$ linearly independent? Be sure you can explain and justify your answer.

- choose
- linearly dependent
- linearly independent

(d) Are the vectors in the set $\left\{ \begin{bmatrix} 5 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -3 \end{bmatrix}, \begin{bmatrix} 5 \\ -4 \end{bmatrix} \right\}$ a basis of $\mathbb{R}^2$? Be sure you can explain and justify your answer.
[choose/basis/not a basis]

Correct Answers:

- $\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \end{bmatrix}$
- 2
- `the vectors span R^2`
- linearly dependent
- not a basis

Problem 7. **(2 points)**
Let

$$B = \begin{bmatrix} 2 & 5 & 7 \\ 12 & 9 & 20 \\ -7 & -5 & -11 \end{bmatrix}.$$

(a) Find the reduced row echelon form of the matrix B.

$\text{rref}(B) = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$

(b) How many pivot columns does B have?

(c) Do the vectors in the set $\left\{ \begin{bmatrix} 2 \\ 12 \\ -7 \end{bmatrix}, \begin{bmatrix} 5 \\ 9 \\ -5 \end{bmatrix}, \begin{bmatrix} 7 \\ 20 \\ -11 \end{bmatrix} \right\}$ span $\mathbb{R}^3$? Be sure you can explain and justify your answer.

- choose
- `the vectors span R^3`
- `the vectors do not span R^3`

(d) Are the vectors in the set $\left\{ \begin{bmatrix} 2 \\ 12 \\ -7 \end{bmatrix}, \begin{bmatrix} 5 \\ 9 \\ -5 \end{bmatrix}, \begin{bmatrix} 7 \\ 20 \\ -11 \end{bmatrix} \right\}$ linearly independent? Be sure you can explain and justify your answer.

- choose
- linearly dependent
- linearly independent

(d) Are the vectors in the set $\left\{ \begin{bmatrix} 2 \\ 12 \\ -7 \end{bmatrix}, \begin{bmatrix} 5 \\ 9 \\ -5 \end{bmatrix}, \begin{bmatrix} 7 \\ 20 \\ -11 \end{bmatrix} \right\}$ a basis of $\mathbb{R}^3$? Be sure you can explain and justify your answer.
[choose/basis/not a basis]

Correct Answers:

- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- 3
- `the vectors span R^3`
- linearly independent
- basis

Problem 8. **(1 point)**
Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ be a linearly independent set of vectors.

Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is always a linearly independent set of vectors.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is never a linearly independent set of vectors.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ could be a linearly independent or linearly dependent set of vectors depending on the vectors chosen.
- D. none of the above

Solution:

SOLUTION

If the zero vector cannot be written as a nontrivial linear combination of a vectors in a smaller set, then it is also not a nontrivial combination of vectors in a proper subset of those vectors.

$\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is always a linearly independent set of vectors.

Correct Answers:

- A

Problem 9. **(1 point)**
Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be a linearly dependent set of vectors.

Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is a linearly independent set of vectors unless $\mathbf{u}_4$ is a linear combination of other vectors in the set.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly independent set of vectors.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ could be a linearly independent or linearly dependent set of vectors depending on the vectors chosen.
- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly dependent set of vectors.
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is a linearly independent set of vectors unless $\mathbf{u}_4 = \mathbf{0}$.
- F. none of the above

Solution:

SOLUTION

If the zero vector is a nontrivial linear combination of a vectors in a smaller set, then it is also a nontrivial combination of vectors in a bigger set containing those vectors.

$\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly dependent set of vectors.

Correct Answers:

- D

Problem 10. **(2 points)**
Represent each of the vectors $-15-15x-12x^2$, $6-3x-12x^2$ and $-14-8x$ with respect to the basis $\mathcal{B}=\{1,x,x^2\}$ of $\mathcal{P}_2$.

$$\mathrm{Rep}_{\mathcal{B}}(-15-15x-12x^2)=\begin{bmatrix}\_\_\_\_\\\_\_\_\_\\\_\_\_\_\end{bmatrix}.$$

$$\mathrm{Rep}_{\mathcal{B}}(6-3x-12x^2)=\begin{bmatrix}\_\_\_\_\\\_\_\_\_\\\_\_\_\_\end{bmatrix}.$$

$$\mathrm{Rep}_{\mathcal{B}}(-14-8x)=\begin{bmatrix}\_\_\_\_\\\_\_\_\_\\\_\_\_\_\end{bmatrix}.$$

Use these representations to determine whether the polynomials are linearly independent, span $\mathcal{P}_2$, and/or form a basis for $\mathcal{P}_2$.

1. Are the vectors $-15-15x-12x^2$, $6-3x-12x^2$ and $-14-8x$ linearly independent?

- Choose
- Linearly dependent
- Linearly independent

If the vectors are independent, enter zero in every answer blank since zeros are only the values that make the equation below true. If they are dependent, find numbers, not all zero, that make the equation below true. You should be able to explain and justify your answer.

$0=$ ___ $(-15-15x-12x^2)+$ ___ $(6-3x-12x^2)+$ ___ $(-14-8x)$.

2. Do the vectors $-15-15x-12x^2$, $6-3x-12x^2$ and $-14-8x$ span $\mathcal{P}_2$?

[Choose/Yes/No]

3. Do the vectors $-15-15x-12x^2$, $6-3x-12x^2$ and $-14-8x$ form a basis for $\mathcal{P}_2$?

[Choose/Basis/Not a basis]

Correct Answers:

- $\begin{bmatrix}-15\\-15\\-12\end{bmatrix}$
- $\begin{bmatrix}6\\-3\\-12\end{bmatrix}$
- $\begin{bmatrix}-14\\-8\\0\end{bmatrix}$
- Linearly dependent
- 0.666667; −0.666667; −1
- No
- Not a basis

Problem 11. **(1 point)**

Let W_1 be the set: $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}1\\1\\0\end{bmatrix},\begin{bmatrix}1\\1\\1\end{bmatrix}\right\}$

Determine if W_1 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. W_1 is a basis.
- B. W_1 is not a basis because it is linearly dependent.
- C. W_1 is not a basis because it does not span $\mathbb{R}^3$.

Let W_2 be the set: $\left\{\begin{bmatrix}1\\0\\1\end{bmatrix},\begin{bmatrix}0\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix}\right\}$

Determine if W_2 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. W_2 is a basis.
- B. W_2 is not a basis because it does not span $\mathbb{R}^3$.
- C. W_2 is not a basis because it is linearly dependent.

Correct Answers:

- A
- BC

Problem 12. **(1 point)**

Let W_1 be the set: $\begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 9 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 5 \end{bmatrix}$.

Determine if W_1 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. W_1 is not a basis because it does not span $\mathbb{R}^3$.
- B. W_1 is a basis.
- C. W_1 is not a basis because it is linearly dependent.

Let W_2 be the set: $\begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix}$.

Determine if W_2 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. W_2 is not a basis because it is linearly dependent.
- B. W_2 is not a basis because it does not span $\mathbb{R}^3$.
- C. W_2 is a basis.

Correct Answers:

- C
- B

Problem 13. **(2 points)**
I recommend that you use software to answer this problem.

Let A be the matrix

$$A = \begin{bmatrix} -6.3 & -9.72 & 7.3 & -1.1 \\ -1.6 & -2.07 & -0.3 & 2.7 \\ -2 & 0.03 & -7.4 & 8.7 \\ 9.9 & 16.41 & -8.7 & -7.6 \end{bmatrix}.$$

What is the reduced row echelon form of the matrix A?

$$\begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}.$$

Use your answer to determine whether the columns of A are linearly independent, span $\mathbb{R}^4$, and/or form a basis for $\mathbb{R}^4$.

1. Are the columns of A linearly independent?

- Choose
- Linearly dependent
- Linearly independent

If the vectors are independent, enter zero in every answer blank since zeros are only the values that make the equation below true. If they are dependent, find numbers, not all zero, that make the equation below true. You should be able to explain and justify your answer.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \_\_ \begin{bmatrix} -6.3 \\ -1.6 \\ -2 \\ 9.9 \end{bmatrix} + \_\_ \begin{bmatrix} -9.72 \\ -2.07 \\ 0.03 \\ 16.41 \end{bmatrix} + \_\_ \begin{bmatrix} 7.3 \\ -0.3 \\ -7.4 \\ -8.7 \end{bmatrix} + \_\_ \begin{bmatrix} -1.1 \\ 2.7 \\ 8.7 \\ -7.6 \end{bmatrix}.$$

2. Do the columns of A span $\mathbb{R}^4$?

[Choose/Yes/No]

3. Do the columns of A form a basis for $\mathbb{R}^4$?

[Choose/Basis/Not a basis]

Correct Answers:

- $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -3.33333 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
- Linearly dependent
- 0.9; −1; −0.6; −0.3
- No
- Not a basis

Problem 14. **(1 point)**
Answer the following questions on your own paper and upload your answers into Gradescope. Make sure to give clear explanations and show your work where appropriate.

1. One basis for the solution set to the linear equation $x - 2y + z = 0$ is

$$\mathcal{B} = \left\langle \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} \right\rangle$$

(you don't have to verify this, you can accept it as true). The vector

$$\vec{v} = \begin{bmatrix} 5 \\ -1 \\ -7 \end{bmatrix}$$

is also a solution. Represent $\vec{v}$ with respect to $\mathcal{B}$. Show your work in your Gradescope submission. Your work should demonstrate how to find the representation, not just that the representation is correct.

$$\mathrm{Rep}_{\mathcal{B}}(\vec{v}) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

2. One basis for the space of polynomials of degree less than or equal to 2 is $\mathcal{B} = \{1 - x, x - x^2, x^2 + 1\}$. Represent $p(x) = 2 - 3x + x^2$ with respect to $\mathcal{B}$. Show your work in your Gradescope submission. Your work should demonstrate how to find the representation, not just that the representation is correct.

$$\mathrm{Rep}_{\mathcal{B}}(p(x)) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

Correct Answers:

- 2
- −3
- 2
- −1
- 0