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Problem 1. **(2 points)**
By deleting linearly dependent vectors, find a basis of each subspace and give the dimension of the subspace.

A. The dimension of span $\left\{ \begin{bmatrix} 2 \\ -6 \end{bmatrix}, \begin{bmatrix} -3 \\ 9 \end{bmatrix} \right\}$ is ___.

B. The dimension of span $\left\{ \begin{bmatrix} 7 \\ -11 \end{bmatrix}, \begin{bmatrix} -15 \\ 22 \end{bmatrix} \right\}$ is ___.

C. The dimension of span $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ -3 \\ -3 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix} \right\}$ is ___.

D. The dimension of span $\left\{ \begin{bmatrix} 3 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} -12 \\ 12 \\ -12 \end{bmatrix}, \begin{bmatrix} 6 \\ 5 \\ 10 \end{bmatrix} \right\}$ is ___.

E. The dimension of span $\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 7 \\ 7 \end{bmatrix} \right\}$ is ___.

Answer(s) submitted:

- 1
- 2
- 1
- 2
- 3

submitted: (correct)
recorded: (correct)

Problem 2. **(2 points)**
Let $A = \begin{bmatrix} 1 & -3 & 1 \\ -2 & 5 & -7 \\ -5 & 11 & -25 \end{bmatrix}$.

- Let $\vec{b}_1 = \begin{bmatrix} 5 \\ 3 \\ 27 \end{bmatrix}$.

  Is $\vec{b}_1$ in col(A)?

- choose
- Yes, it is in col(A).
- No, it is not in col(A).
- We cannot tell if it is in col(A).

- Let $\vec{b}_2 = \begin{bmatrix} 4 \\ -5 \\ -7 \end{bmatrix}$.

  Is $\vec{b}_2$ in col(A)?

- choose
- Yes, it is in col(A).
- No, it is not in col(A).
- We cannot tell if it is in col(A).

Answer(s) submitted:

- Yes, it is in col(A).
- No, it is not in col(A).

submitted: (correct)
recorded: (correct)

Problem 3. **(2 points)**
Let $A = \begin{bmatrix} -2 & -1 & -6 \\ 4 & 8 & 0 \\ -2 & -5 & 2 \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ -1 \\ 6 \end{bmatrix}$, $w = \begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix}$ and $x = \begin{bmatrix} -5 \\ 5 \\ 3 \end{bmatrix}$.

Is v in null(A)? Type "yes" or "no". ____

Is w in null(A)? Type "yes" or "no". ____

Is x in null(A)? Type "yes" or "no". ____

Answer(s) submitted:

- NO
- YES
- NO

submitted: (correct)
recorded: (correct)

Problem 4. **(2 points)**
Suppose that A is a 5×7 matrix that has a reduced row-echelon form with two rows of zeros. Find the dimension of the row space of A, the dimension of the column space of A, and the dimension of the null space of A.

- The dimension of the row space of A: ___
- The dimension of the column space of A: ___
- The dimension of the null space of A: ___

Answer(s) submitted:

- 3
- 3
- 4

submitted: (correct)
recorded: (correct)

Problem 5. **(2 points)**
Find bases for the column space, the row space, and the null space of matrix A. You are given the RREF of A to make your work easier.

$$A = \begin{bmatrix} 4 & 26 & 5 \\ 3 & 1 & 0 \\ -5 & 1 & 3 \\ 3 & 5 & -2 \\ -5 & 8 & 4 \end{bmatrix} \qquad \mathrm{RREF}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A basis for the column space of A is

$$\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$$

A basis for the row space of A is

$\{ \langle \_\_, \_\_, \_\_ \rangle,$
$\langle \_\_, \_\_, \_\_ \rangle,$
$\langle \_\_, \_\_, \_\_ \rangle \}$

Note that since the only solution to $A\mathbf{x} = \mathbf{0}$ is the zero vector, there is no basis for the null space of A.

Answer(s) submitted:

- $\begin{bmatrix} 4 \\ 3 \\ -5 \\ 3 \\ -5 \end{bmatrix}; \begin{bmatrix} 26 \\ 1 \\ 1 \\ 5 \\ 8 \end{bmatrix}; \begin{bmatrix} 5 \\ 0 \\ 3 \\ -2 \\ 4 \end{bmatrix}$
- $\langle 4, 26, 5 \rangle; \langle 3, 1, 0 \rangle; \langle -5, 1, 3 \rangle$

submitted: (correct)
recorded: (correct)

Problem 6. (3 points)
Find bases for the column space, the row space, and the null space of the matrix

$$A = \begin{bmatrix} 1 & 4 & -1 & 1 \\ 3 & 14 & -1 & 6 \\ 2 & 12 & 2 & 8 \end{bmatrix}$$

Basis for the column space of $A = \left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$

Basis for the row space of $A = \left\{ [\ \_\_\ \ \_\_\ \ \_\_\ \ \_\_\ ], [\ \_\_\ \ \_\_\ \ \_\_\ \ \_\_\ ] \right\}$

Basis for the null space of $A = \left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$

Answer(s) submitted:

- $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 14 \\ 12 \end{bmatrix}$
- $[1, 4, -1, 1], [3, 14, -1, 6]$
- $\begin{bmatrix} 5 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ -\frac{3}{2} \\ 0 \\ 1 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 7. (3 points)

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ -2 & -4 & -3 \\ 0 & 1 & 1 \end{bmatrix}$.

Find bases for the following subspaces associated with A.

- Basis for the row space of A: $\{\ \_\_\ \}$
- Basis for the column space of A: $\{\ \_\_\ \}$
- Basis for the null space of A: $\{\ \_\_\ \}$

Answer(s) submitted:

- $\langle 1, 1, 1 \rangle, \langle 1, 2, 2 \rangle, \langle -2, -4, -3 \rangle$
- $\langle 1, 1, -2, 0 \rangle, \langle 1, 2, -4, 1 \rangle, \langle 1, 2, -3, 1 \rangle$
- empty

submitted: (correct)
recorded: (correct)

Problem 8. (2 points)
Find a basis for the subspace of $\mathbb{R}^4$ spanned by the vectors:

$$\begin{bmatrix} 1 \\ 2 \\ -4 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \\ -3 \end{bmatrix}, \begin{bmatrix} 7 \\ -2 \\ 0 \\ -8 \end{bmatrix}.$$

Basis for the span: $\{\ \_\_\ \}$

Answer(s) submitted:

- $\langle 1, 2, -4, -4 \rangle, \langle 1, -2, 3, 1 \rangle$

submitted: (correct)
recorded: (correct)

Problem 9. **(2 points)**
Find a basis for each of the subspaces of $\mathbb{R}^4$ below.

Hint: Start by finding a matrix so that the given subspace is either the column space or the null space of that matrix.

- V_1 is the set of all vectors whose coordinates sum to 0.

  Basis for V_1: $\{$ ___$\}$

- V_2 is the set of all vectors whose coordinates are all equal.

  Basis for V_2: $\{$ ___$\}$

Answer(s) submitted:

- $\langle -1,1,0,0\rangle, \langle -1,0,1,0\rangle, \langle -1,0,0,1\rangle$
- $\langle 1,1,1,1\rangle$

submitted: (correct)
recorded: (correct)

Problem 10. **(0 points)**
Answer the following questions on your own paper and upload your answers into Gradescope. Make sure to give clear explanations and show your work where appropriate.

Recall that

$$\mathcal{P}_3 = \{a_0 + a_1x + a_2x^2 + a_3x^3 \mid a_0, a_1, a_2, a_3 \in \mathbb{R}\}.$$

Consider the subset S of $\mathcal{P}_3$ defined by

$$S = \{p(x) \in \mathcal{P}_3 \mid p(2) = 0 \quad \text{and} \quad p(3) = 0\}.$$

In other words, S is the set of polynomials of degree 3 or less that have both 2 and 3 as roots.

1. Prove that S is a subspace of $\mathcal{P}_3$.

2. Write down a linear system, where the variables are the coefficients a_0, a_1, a_2, and a_3, whose solution set is exactly the set of coefficients corresponding to elements of S.

3. Solve the above linear system and use it to find a basis for S. You don't need to write down all of the steps, but you should write down the augmented matrix, the reduced augmented matrix, and the solution set.

4. Looking at the basis that you found, it is clear that there are no degree 1 polynomials in S. Briefy explain why this makes sense from the original definition of S.