This PDF is available for convenience. Assignments must be submitted within **WeBWorK** for credit.

Problem 1. **(1 point)**
The upper triangular form of an augmented matrix for a system of three linear equations on the variables x, y, and z is:

$$\left[\begin{array}{ccc|c} 1 & 7 & -3 & -9 \\ 0 & 1 & -5 & -4 \\ 0 & 0 & 0 & 7 \end{array}\right]$$

- Write this augmented matrix as a system of linear equations using the variables x, y, and z.

  ___ = ___

  ___ = ___

  ___ = ___

- Use the drop down menus to complete the following statement:

  Since the last equation is for all values of the variables, this system has

- zero
- exactly one
- exactly two
- infinitely many

  solutions.

Correct Answers:

- $$\begin{aligned} x + 7y - 3z &= -9 \\ y - 5z &= -4 \\ 0 &= 7 \end{aligned}$$
- False
- zero

Problem 2. **(1 point)**
The upper triangular form of an augmented matrix for a system of three linear equations on the variables x, y, and z is:

$$\left[\begin{array}{ccc|c} 1 & -8 & 3 & -3 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

- Write this augmented matrix as a system of linear equations using the variables x, y, and z.

  ___ = ___

  ___ = ___

  ___ = ___

- Use the drop down menu to complete the following statement:

  This system has

- zero
- exactly one
- exactly two
- infinitely many

  solutions.

- Express all of the solutions to this system in terms of the parameter t.

  $x =$ ___

  $y =$ ___

  $z =$ ___

Correct Answers:

- $$\begin{aligned} x - 8y + 3z &= -3 \\ y - z &= 1 \\ 0 &= 0 \end{aligned}$$
- infinitely many
- $$\begin{aligned} x &= 5 + 5t \\ y &= 1 + t \\ z &= t \end{aligned}$$

Problem 3. **(1 point)**
The reduced row-echelon forms of the augmented matrices of four systems are given below. How many solutions does each system have?

1. $\left[\begin{array}{ccc|c} 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 14 \end{array}\right]$

   - A. Unique solution
   - B. Infinitely many solutions
   - C. No solutions
   - D. None of the above

2. $\left[\begin{array}{cc|c} 1 & 0 & 11 \\ 0 & 1 & -3 \end{array}\right]$

   - A. No solutions
   - B. Unique solution
   - C. Infinitely many solutions
   - D. None of the above

3. $\left[\begin{array}{ccc|c} 1 & 0 & 15 & 0 \\ 0 & 1 & 19 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

   - A. No solutions
   - B. Unique solution
   - C. Infinitely many solutions
   - D. None of the above

4. $\left[\begin{array}{ccc|c} 1 & 0 & -9 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

   - A. Infinitely many solutions
   - B. No solutions
   - C. Unique solution
   - D. None of the above

Correct Answers:

- B
- B
- A
- B

Problem 4. **(2 points)**
Starting with the augmented matrix A, perform the given chain of elementary row operations in the order listed to reduce A to echelon form.

- First: $R_1 \leftrightarrow R_3$
- Second: $R_2 - 5R_1 \rightarrow R_2$
- Third: $R_3 + 4R_1 \rightarrow R_3$
- Fourth: $R_3 - 2R_2 \rightarrow R_3$

$$A = \left[\begin{array}{ccc|c} -4 & -10 & 4 & 28 \\ 5 & 16 & -6 & -43 \\ 1 & 3 & -3 & -10 \end{array}\right]$$

$$\xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}$$

$$\xrightarrow{R_2 - 5R_1 \rightarrow R_2} \begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}$$

$$\xrightarrow{R_3 + 4R_1 \rightarrow R_3} \begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}$$

$$\xrightarrow{R_3 - 2R_2 \rightarrow R_3} \begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}$$

Correct Answers:

- $\begin{bmatrix} 1 & 3 & -3 & -10 \\ 5 & 16 & -6 & -43 \\ -4 & -10 & 4 & 28 \end{bmatrix}$
- $\begin{bmatrix} 1 & 3 & -3 & -10 \\ 0 & 1 & 9 & 7 \\ -4 & -10 & 4 & 28 \end{bmatrix}$
- $\begin{bmatrix} 1 & 3 & -3 & -10 \\ 0 & 1 & 9 & 7 \\ 0 & 2 & -8 & -12 \end{bmatrix}$
- $\begin{bmatrix} 1 & 3 & -3 & -10 \\ 0 & 1 & 9 & 7 \\ 0 & 0 & -26 & -26 \end{bmatrix}$

Problem 5. **(2 points)**
Consider the following augmented matrix:

$$\left[\begin{array}{ccc|c} -4 & -9 & -4 & -24 \\ 3 & 10 & -3 & 55 \\ 1 & 3 & 2 & 5 \end{array}\right]$$

Use elementary row operations to reduce this matrix to echelon form. You do not need to reduce the matrix all the way to RREF, however you can if you want.

$$\begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}$$

Correct Answers:

- $\begin{bmatrix} 1 & 3 & 2 & 5 \\ 0 & 1 & -9 & 40 \\ 0 & 0 & 31 & -124 \end{bmatrix}$

Problem 6. **(2 points)**
Solve the following system using augmented matrix methods:

$$\begin{aligned} -3x + 6y &= -36 \\ -6x + 12y &= -72 \end{aligned}$$

(a) The initial matrix is:

$$\left[\begin{array}{cc|c} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{array}\right]$$

(b) First, perform the Row Operation $\frac{1}{-3}R_1 \to R_1$. The resulting matrix is:

$$\left[\begin{array}{cc|c} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{array}\right]$$

(c) Next, perform the operation $+6R_1 + R_2 \to R_2$. The resulting matrix is:

$$\left[\begin{array}{cc|c} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{array}\right]$$

(d) Finish simplifying the augmented matrix to reduced row echelon form. The reduced matrix is:

$$\left[\begin{array}{cc|c} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{array}\right]$$

(e) How many solutions does the system have? If infinitely many, enter ”Infinity”.

____

(f) What are the solutions to the system?

If there are no solutions, write ”No Solution” or ”None” for each answer. If there are infinitely many solutions let $y = t$ and solve for x in terms of t.

$x =$ __________
$y =$ __________

Correct Answers:

- -3
- 6
- -36
- -6
- 12
- -72
- 1
- -2
- 12
- -6
- 12
- -72
- 1
- -2
- 12
- 0
- 0
- 0
- 1
- -2
- 12
- 0
- 0
- 0
- ∞
- $12+2t$
- t

Problem 7. **(2 points)**
Solve the system by finding the reduced row-echelon form of the augmented matrix.

$$\begin{aligned} x-3y-z&=-1\\ -2x+9y-4z&=-1\\ 3x-8y-5z&=-4 \end{aligned}$$

Reduced row-echelon form:

$$\left[\begin{array}{ccc|c} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{array}\right]$$

How many solutions are there to this system?

- A. None
- B. Exactly 1
- C. Exactly 2
- D. Exactly 3
- E. Infinitely many
- F. None of the above

What is the set of solutions to the system? Use **s1**, **s2**, etc. for the free variables if necessary.

If there are no solutions, leave the answer blanks for x, y and z empty.

$x =$ ________________

$y =$ ________________

$z =$ ________________

Correct Answers:

- 1
- 0
- -7
- -4
- 0
- 1
- -2
- -1
- 0
- 0
- 0

- 0
- E
- $-4-(-7)\,s_1$
- $-1-(-2)\,s_1$
- s_1

Problem 8. **(2 points)**
Solve the linear system below by using Gauss-Jordan elimination.

$$\begin{array}{rcrcrcr} x & + & 2y & + & 3z & = & 9 \\ -3x & - & 2y & + & 4z & = & 11 \\ -6x & + & 2y & + & 3z & = & 30 \end{array}$$

- Write the above system as an augmented matrix:

$$\begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}$$

- Use elementary row operations to reduce the augmented matrix to echelon form.

$$\begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}$$

- Solve the original system of equations.

$x =$ ___

$y =$ ___

$z =$ ___

Correct Answers:

- $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ -3 & -2 & 4 & 11 \\ -6 & 2 & 3 & 30 \end{array}\right]$
- $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$
- -3
- 3
- 2

Problem 9. **(2 points)**
Consider the following system of linear equations:

$$\begin{array}{rcrcrcrcr} & & & & -z & + & w & = & -2 \\ x & - & y & + & z & + & w & = & 0 \\ & & y & + & 2z & + & w & = & 0 \\ -x & + & 2y & - & 3z & + & 5w & = & -9 \end{array}$$

Find the unique solution to this system.

$x =$ ___ $y =$ ___ $z =$ ___ $w =$ ___

Correct Answers:

- -1
- -1
- 1
- -1

Problem 10. **(2 points)**
Find the intersection of the lines $\mathbf{p}(t) = \langle -2-2t, 13+6t, 18+8t\rangle$ and $\mathbf{q}(s) = \langle 10+3s, -11-5s, 9+s\rangle$.

Write your answer as a point (a, b, c) where a, b, and c are numbers. If there is no intersection, write ”none”.

Answer: _________

Hint: set up a linear system that describes the intersection in terms of s and t.

Correct Answers:

- $(1, 4, 6)$

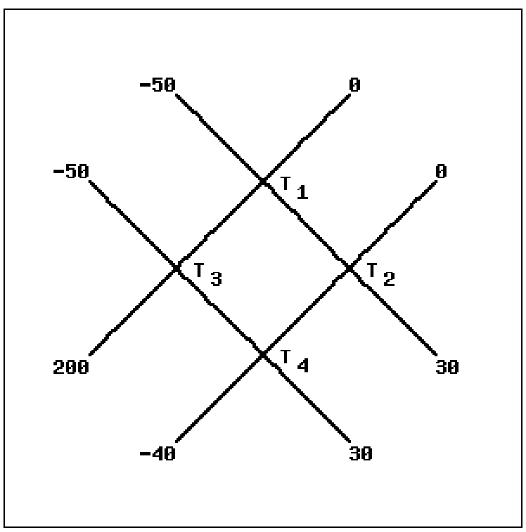
Problem 11. **(2 points)**
In a grid of wires, the temperature at exterior mesh points is maintained at the constant values shown in the figure. When the grid is in thermal equilibrium, the temperature at each interior mesh point is the average of the temperatures at the four adjacent points. For instance,

$$T_1 = \frac{T_2 + T_3 + 0 - 50}{4}.$$

Find the temperatures T_1, T_2, T_3, T_4, when the grid is in thermal equilibrium.

Hint: You already have one equation, find 3 more equations, one corresponding to each interior mesh point. Solve the resulting system. I recommend using software. If you have trouble viewing the image, click on it for a larger version.

$T_1 =$ ____
$T_2 =$ ____
$T_3 =$ ____
$T_4 =$ ____



Correct Answers:

- 0
- 10
- 40
- 10

Problem 12. **(1 point)**
IMPORTANT: Write down your answers to questions 1 and 2 and upload a PDF to Gradescope. Answer the remaining questions in WeBWorK.

A farmer has 3 different types of fertilizer, each of which contains different amounts of nitrogen, phosphorus, and potassium. The table below shows the amount (in pounds per bag) of each element in each type of fertilizer. For example, 1 bag of Fertilizer A contains 4 pounds of nitrogen.

	Nitrogen	Phosphorus	Potassium
Fertilizer A	4	6	4
Fertilizer B	4	7.5	6
Fertilizer C	5	8.5	7

The farmer needs to mix the fertilizers so that the mixture contains 303 pounds of nitrogen, 489.5 pounds of phosphorus, and 357 pounds of potassium. How many bags of each fertilizer does the farmer need?

1. In your Gradescope submission, write down a system of equations that will allow you to solve this problem. Make sure that you clearly indicate what real-world quantity each variable represents and what real-world information each equation conveys. The order of your equations and variables should be consistent with the order of fertilizers and elements in the chart, otherwise WeBWorK will not understand your answer below.

Enter the augmented matrix for this system below.

$$\left[\begin{array}{ccc|c} \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \end{array}\right]$$

2. In your Gradescope submission carry out the process of Gauss-Jordan elimination to solve the system. Make sure to show all steps, to indicate what row operation you are doing at each step, and to clearly indicate what the solution to the system is. State the answer in the context of the problem (in other words, answer the question "How many bags of each fertilizer does the farmer need?").

Verify that your answer is correct below:

How many bags of Fertilizer 1 are required?

____

How many bags of Fertilizer 2 are required?

____

How many bags of Fertilizer 3 are required?

____

Correct Answers:

- 4
- 4
- 5
- 303
- 6
- 7.5
- 8.5
- 489.5
- 4
- 6
- 7
- 357
- 46
- 16
- 11