$$y'' + \cos(x)\, y' + \sin(x)\, y = 1 - \sin(x)$$

$$x \in \left[0, \tfrac{\pi}{2}\right]$$

$$\begin{cases} 1 \cdot y(a) = A = 0, \\ 1 \cdot y(b) + 1 \cdot y'(b) = B = 1; \end{cases} \Rightarrow \begin{cases} u(0) = 0, \\ u'(0) = 0, \end{cases} \begin{cases} V(0) = 0, \\ V'(0) = -1. \end{cases}$$

Фикс. шаг $h = \dfrac{\pi}{4}$

1) $u'' + \cos(x)\,u' + \sin(x)\,u = 1 - \sin(x)$

$u_1 = u_0 + h \cdot F(x_0, u_0) =$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \frac{\pi}{4} \cdot \begin{pmatrix} 0 \\ 1 - \sin(0) \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{\pi}{4} \end{pmatrix} \approx \begin{pmatrix} 0 \\ 0{,}785 \end{pmatrix}$$

$$u_2 = \begin{pmatrix} 0 \\ \frac{\pi}{4} \end{pmatrix} + \frac{\pi}{4} \begin{pmatrix} \frac{\pi}{4} \\ 1 - \sin\frac{\pi}{4} - \frac{\pi}{4} \cdot \cos\frac{\pi}{4} \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{\pi^2}{16} \\ \dfrac{16\pi - 4\pi\sqrt{2} - \pi^2\sqrt{2}}{32} \end{pmatrix} \approx \begin{pmatrix} 0{,}617 \\ 0{,}579 \end{pmatrix}$$

2) $V'' + \cos(x)\,V' + \sin(x)\,V = 0$

$V_1 = V_0 + h \cdot F(x_0, V_0) =$

$$= \begin{pmatrix} 0 \\ -1 \end{pmatrix} + \frac{\pi}{4} \cdot \begin{pmatrix} -1 \\ -\cos(0) \cdot (-1) \end{pmatrix} = - \begin{pmatrix} \frac{\pi}{4} \\ 1 - \frac{\pi}{4} \end{pmatrix} = \begin{pmatrix} -\frac{\pi}{4} \\ -0{,}215 \end{pmatrix}$$

$$V_2 = V_1 + h \cdot F(x_1, V_1) = - \begin{pmatrix} \frac{\pi}{4} \\ 1 - \frac{\pi}{4} \end{pmatrix} + \frac{\pi}{4} \begin{pmatrix} -1 - \frac{\pi}{4} \\ p(x)u' \\ + q(x)u \end{pmatrix} =$$

$$= - \begin{pmatrix} \frac{\pi}{4} \\ 1 - \frac{\pi}{4} \end{pmatrix} + \frac{\pi}{4} \begin{pmatrix} -1 + \frac{\pi}{4} \\ -\cos\left(\frac{\pi}{4}\right) \cdot \left(1 - \frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right) \cdot \frac{\pi}{4} \end{pmatrix} \ominus$$

$$\left(\dfrac{\dfrac{-8\pi + \pi^2}{16}}{-1 + \dfrac{\pi}{4} + \dfrac{2\pi\sqrt{2} - \pi^2\sqrt{2}}{16}} \right) = \left(\begin{array}{c} -0,954 \\ -0,532 \end{array} \right)$$

3) $C = \dfrac{B - 0,617 - 0,579}{-0,954} \approx 0,205$

4) $y_0 = u_0 + v_0 \cdot C = 0$ (здесь v_i есть первый элемент вектора)

$y_1 = u_1 + v_1 \cdot C = -0,785 \cdot 0,205 = -0,161$

$y_2 = u_2 + v_2 C = 0,617 \cancel{\cdots} + (-0,954) \cdot 0,205 = 0,421$

5) $x^h = \left\{ 0; \dfrac{\pi}{4}; \dfrac{\pi}{2} \right\}$

$y^h = \left\{ 0; -0,161; 0,421 \right\}$

Полученная ошибка:

$err_0 = 0$

$err_1 = \dfrac{\sqrt{2}}{2} + 0,161 = 0,868$

$err_2 = 1 - 0,421 = 0,579$

Получена ошибка порядка 1