MIDTERM #2 REVIEW PROBLEMS

If not otherwise specified, solve the following problems. If initial conditions are given, solve for all constants of integration. It is okay to leave answers in implicit form or with unsolved integrals if it is not possible to reduce the solution further.

1. **Second Order Linear ODEs:**

   (a)
   $$t^2y'' - t(t+2)y' + (t+2)y = 2t^4, \quad y_1 = t$$

   (b)
   $$(x-1)y'' - xy' + y = (x-1)^2,\ x > 1; \quad y_1 = e^x$$

   Hint: the solution of
   $$y'' = \frac{2-x}{x-1}y'$$
   is $y = c_1e^{-x}x + c_2$.

   (c)
   $$y'' - y' - 2y = -2t + 4t^2$$

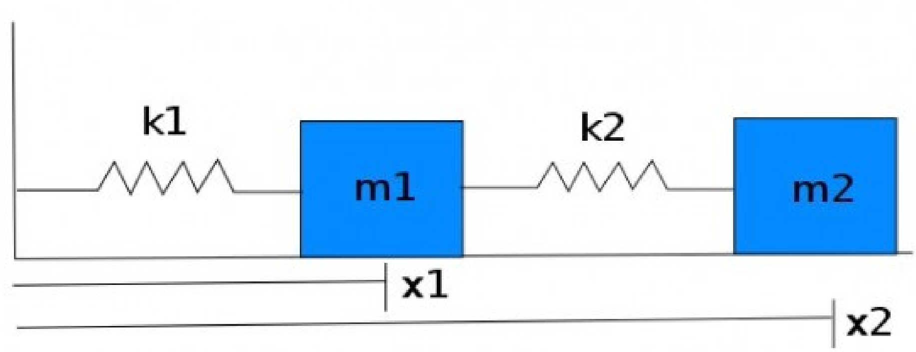
   (d)
   $$y'' - 2y' + y = e^x$$

   (e)
   $$y'' + 4y = 4\csc(2t)$$

   (f)
   $$t^2y'' - 2y = 3t^2 - 1$$

   (g)
   $$x^2y'' + 4xy' - 4y = \ln(x)$$

2. **Spring-mass systems:** For the system described by the image below, derive the system of ODEs governing the mass-spring system. Treat the masses as point masses and assume no damping or friction.



3. **Higher Order ODEs and MATLAB I:** A system of couple pendulums could be modeled by the following system of equations:

$$\theta_a'' + \theta_b'' + \frac{g}{\ell}\theta_a + \frac{K}{M}(\theta_a - \theta_b) = 0 \tag{1}$$

$$\theta_a'' + 2\theta_b'' + \frac{g}{\ell}\theta_b + \frac{K}{M}(\theta_b - \theta_a) = 0 \tag{2}$$

   (a) Rewrite this as a coupled system of first-order ODEs.

   (b) Write a MATLAB function to evaluate the derivative such that the system could be solved with `ode45()`. Assume $g = \ell = K = M = 1$.

4. **Higher Order ODEs and MATLAB II:** For the following initial value problem

$$y'' + x^2 y' + y = 1, \quad y(0) = 1, \quad y'(0) = 1$$

   write a piece of MATLAB code to numerically solve the system using backward Euler method. Use step size $h = 0.1$, solve over interval $x \in [0, 1]$, and include a line to plot your solution.