Midterm #2 Review – Extra Problems

1. (Kreyszig 2.1 #4) $2xy'' = 2y'$
2. (Kreyszig 2.1 #6) $xy'' + 2y' + xy = 0, \quad y_1 = \cos(x)/x$
3. (Kreyszig 2.1 #7) $y'' + y'^3 \sin(y) = 0$
4. (Kreyszig 2.1 #10) $y'' + (1 + 1/y)y'^2 = 0$
5. (Kreyszig 2.2 #11) $4y'' - 4y' - 3y = 0$
6. (Kreyszig 2.2 #14) $y'' + 2k^2 y' + k^4 y = 0$
7. For an ODE of the form $y'' + ay' + by = 0$, find a and b that gives a solution of the form:
    a) (Kreyszig 2.2 #16) $e^{2x}, \ e^{-4x}$
    b) (Kreyszig 2.2 #17) $e^{-\sqrt{5}x}, \ xe^{-\sqrt{5}x}$
8. (Kreyszig 2.5 #4) $xy'' + 2y' = 0$
9. (Kreyszig 2.5 #14) $x^2 y'' + xy' + 9y = 0, \ y(1) = 0, \ y'(1) = 2.5$
10. (Kreyszig 2.7 #3) $y'' + 3y' + 2y = 12x^2$
11. (Kreyszig 2.7 #6) $y'' + y' + (\pi^2 + 1/4)y = e^{-x/2} \sin(\pi x)$
12. (Kreyszig 2.10 #2) $y'' + 9y = \csc(3x)$
13. (Kreyszig 2.10 #3) $x^2 y'' - 2xy' + 2y = x^3 \sin(x)$
14. For the following, compute the numerical solution for 3 steps using improved Euler and Runge-Kutta 4. Solve for the analytical solution and calculate the global error over the domain for which we have computed a numerical solution.
    a) (Kreyszig 21.1 #6) $y' = 2(1 + y^2), \quad y(0) = 0, \ h = 0.05$
    b) (Kreyszig 21.1 #7) $y' - xy^2 = 0, \quad y(0) = 1, \ h = 0.1$
15. Suppose I solve an ODE numerically using RK3 and find a global error of ϵ_1 with a step size of $h_1 = 0.5$. What step size h_2 should I pick so that the error ϵ_2 is $10^3\times$ smaller? (i.e. $\epsilon_2 = 10^{-3}\epsilon_1$)