# LAPLACE TRANSFORM – EXTRA PROBLEMS

1. Find the Laplace transform of the following functions:
   a) (Kreyszig 6.1 #5) $f(t) = e^{2t}\sinh(t)$
   b) (Kreyszig 6.1 #6) $f(t) = e^{-t}\sinh(2t)$
   c) (Kreyszig 6.1 #8) $f(t) = 1.5\sin(3t - \pi/2)$
   d) (Kreyszig 6.2 #16) $f(t) = t\cos(4t)$
   e) (Kreyszig 6.2 #18) $f(t) = \cos^2(2t)$
   f) (Kreyszig 6.6 # 4) $f(t) = te^{-t}\cos(t)$
   g) (Kreyszig 6.6 # 9) $f(t) = \frac{1}{2}t^2\sin(\pi t)$
2. Find the inverse transform of the following functions:
   a) (Kreyszig 6.1 #30) $F(s) = \frac{4s+32}{s^2-16}$
   b) (Kreyszig 6.1 #31) $F(s) = \frac{s+10}{s^2-s-2}$
   c) (Kreyszig 6.2 #26) $F(s) = \frac{1}{s^4-s^2}$
   d) (Kreyszig 6.2 #29) $F(s) = \frac{1}{s^3+s}$
   e) (Kreyszig 6.6 #14) $F(s) = \frac{s}{(s^2+16)^2}$
   f) (Kreyszig 6.6 #17) $F(s) = \ln\frac{s}{s-1}$
   g) (Kreyszig 6.6 #18) $F(s) = \operatorname{arccot}\frac{s}{\pi}$. *Hint:* $\frac{d}{dx}\cot^{-1}(x) = \frac{-1}{1+x^2}$
3. (Kreyszig 6.2 # 4) Solve $y'' + 9y = 10e^{-t}, y(0) = 0, y'(0) = 0$ using both Laplace transform and another method from this class.
4. Solve the following using Laplace transform:
   a) (Kreyszig 6.3 # 22) $y'' + 3y' + 2y = f(t)$ with
   $$f(t) = \begin{cases} 4t & 0 \le t \le 1 \\ 8 & t > 1 \end{cases}$$
   $y(0) = 0, y'(0) = 0$
   b) (Kreyszig 6.3 # 24) $y'' + 3y' + 2y = f(t)$ with
   $$f(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & t > 1 \end{cases}$$
   $y(0) = 0, y'(0) = 0$
   c) (Kreyszig 6.4 # 10) $y'' + 5y' + 6y) = \delta(t - \pi/2) + u(t - \pi)\cos(t),\ y(0) = 0,\ y'(0) = 0$
   d) (Kreyszig 6.4 # 11) $y'' + 5y' + 6y) = \delta(t - 2) + u(t - 1),\ y(0) = 0,\ y'(0) = 1$