

WEEK 3 SECTION SOLUTIONS

Solve the following problems. If initial conditions are given, solve for all constants of integration. It is okay to leave answers in implicit form or with unsolved integrals.

1. **Linear First Order ODEs:** Solve the following linear first order ODEs using either the formula derived in class or variation of parameters.

a) $xy' + y = x$, $y(1) = 1$

b) $xy' + y = \sin(x)$, $y(1) = 0$

c) $\frac{1}{2}y' + y = e^x$, $y(0) = 0$

2. **Numerical Accuracy:** Show the following.

a) Forward Euler is globally first-order accurate.

b) Backward Euler is locally second-order accurate.

Hint: The Taylor expansion for $\frac{1}{1-x}$ is given by:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + O(x^4)$$

3. **Systems of Linear Equations:** Put the following systems of equations into matrix-vector form. State whether each has a unique solution.

a)

$$4x + 5y + 6z = 2$$

$$x + 7z = 5$$

$$8y + 2z = 0$$

b)

$$3y_1 + 2y_2 + 5y_3 = 2$$

$$y_3 + y_2 = 7$$