WEEK 2 SECTION PROBLEMS

Solve the following problems. If initial conditions are given, solve for all constants of integration. It is okay to leave answers in implicit form or with unsolved integrals.

1. **Linearity:** Show that the following ODEs are linear. If they are nonlinear, identify the nonlinear term and show that this term is nonlinear.

   a) $sin(x)y''' + y = e^x$

   b) $y' + y^2 = tan(x)$

   c) $y'' + 3sin(y) = 0$

   d) $y' + y = e^x$

   e) $y' + y = e^y$

2. **Separation of Variables:** Solve the following using separation of variables.

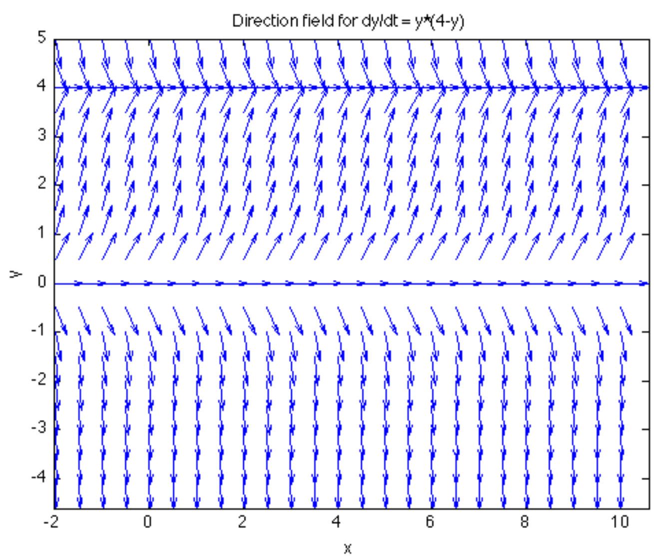
   a) $y' = cot(y)$

   b) $y' = y - y_0$

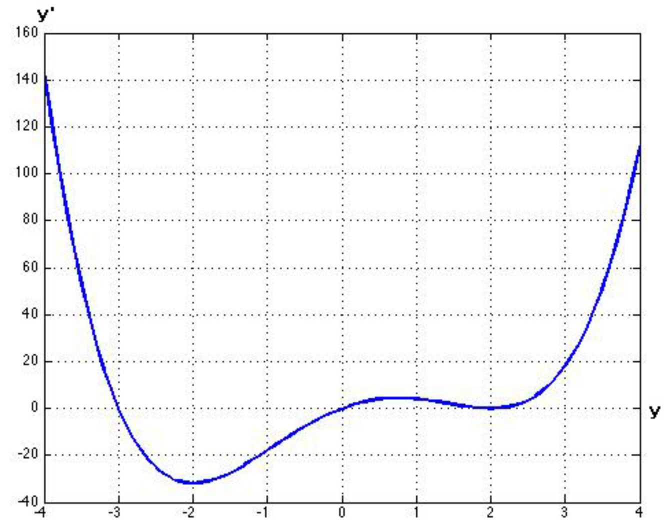
   c) $y' = 1 + e^x + e^y + e^{x+y}$

3. **Direction fields, phase space, and stability:** For an ODE of the form $y' = f(x, y)$ (where x and y can also be vectors), we can find the fixed points through one of two ways.

   a) One method it to look at the direction field in x-y space to find the fixed points and determine their stability. For the following figure, find the fixed points based on the direction field, and give their stability.



b) Another method is to examine function in what we call *phase space*. This is where we plot the function value against its derivatives. In this case, we will plot the function in y-y' space. For the following figure, find the fixed points by examine the function plotted against its derivative. Give the fixed points and their stability. *Hint:* recall that a fixed point is where $y' = 0$.



4. **MATLAB:** Write a *small* piece of MATLAB code for each of the following functions.

   a) Output every multiple of 5 from 0 to 100. *Hint:* the remainder of number a after division by b is given by mod(a,b).

   b) Plot the functions $f_1(y) = y^2 + 1$ and $f_2(y) = y + 1$ over the domain $y \in [0, 5]$ on the same graph in different colors with line thickness at 2pt. Pick a small enough increment in y that the resolution of f_1 and f_2 is good.

   c) Write a function that will take two numbers as arguments and output their sum plus five i.e. $f(a, b) = a + b + 5$.