MIDTERM #1 REVIEW PROBLEMS

If not otherwise specified, solve the following problems. If initial conditions are given, solve for all constants of integration. It is okay to leave answers in implicit form or with unsolved integrals if it is not possible to reduce the solution further.

1. **Separation of variables**

    (a) $y' = \sec^2 y$

    (b) $y' = (y + 4x)^2$

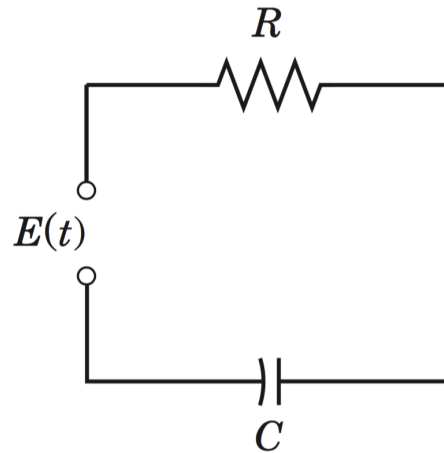   (c) $xy' = x + y$

2. **Equilibrium solutions**

    (a) Find and characterize the equilibria for $y' = (y^2 - 7y + 10)(y^2 + 4y + 4)$.

3. **Linear 1st order ODE and related topics**

    (a) $y = 2y - 4x$

    (b) $y' = (y - 2)\cot(x)$

    (c) Derive the equation for the following circuit and find the current $I(t)$ where $E(t) = e^{-t}$ i.e. a constant voltage, $R = C = 1$, and $I(0) = 0$:

    (d) Solve: $y' + y = -x/y$

4. **Eigenvalues and eigenvectors**

    (a) Solve the following system of ODEs:

$$\begin{aligned} x' &= 3x - y \\ y' &= x + 3y \end{aligned}$$

5. **Numerical solutions**

   (a) Solve $y' = xy - 10y$, $y(0) = 2$ using forward Euler for 2 steps using $h = 0.5$.

   (b) Repeat (a) using backward Euler. What do you notice about the two solutions?

   (c) Why does `ode45()` have its name? What order accuracy is it? Does it take uniform steps when solving an ODE?

   (d) Write a short piece of MATLAB code to solve and plot the solution for the following system:
   $$\begin{aligned} x' &= \frac{x^2(y-5)}{y^2(x+3)}, \quad x(0) = 2, \ y(0) = 3 \\ y' & \end{aligned}$$
   with `ode45()` over the range $0 \le t \le 10$. (This should only take 3 or 4 lines of code.)