WEEK 4 SECTION PROBLEMS

Solve the following problems. If initial conditions are given, solve for all constants of integration. It is okay to leave answers in implicit form or with unsolved integrals.

1. For the following ODEs, give its type (i.e. linear/nonlinear, order, homogeneous/inhomogeneous) and, if possible, solve the ODE.
   a) $\frac{1}{2}y'' + y' + y = 0$
   b) $(y')^2 - y = 0$
   c) $xy' + y = x^2$
2. **Existence and uniqueness:** For the following ODE, give the region R_1 where the solution exists, and the region R_2 where the solution is unique.

$$y' + \sqrt{y}ln(x) = 5$$

$$y(5) = 2$$

3. **Numerical Stability:** Give the amplification factor and maximum step size for the following numerical methods for the model ODE $y' = \lambda y$, assuming that $\lambda < 0$.
   a) Forward Euler
   b) Backward Euler
4. **ODEs and Eigenvalues:** Transform the following second order ODEs into a system of first order ODEs, solve for the eigenvalues of the resulting matrices, and relate these to the solution to the ODE.
   a) $y'' - y = 0$
   b) $y'' + y = 0$
   c) $y'' + 2y' + y = 0$