WEEK 10 SECTION PROBLEMS

If not otherwise specified, solve the following problems. If initial conditions are given, solve for all constants of integration. It is okay to leave answers in implicit form or with unsolved integrals.

1. **Conceptual things:** For each of the following, give a short, snappy explanation or definition.
   a) Spanning vector/basis vector
   b) Basis function
   c) Power series (a.k.a. Taylor series)
   d) Series solution

2. **Example problems:** Solve the following using a power series solution.
   a) $y' + y = x$
   b) $y'' - xy = 0$
      This equation is known as the Airy Equation, and its solutions (known as Airy functions) have many important applications in optics and quantum mechanics.
   c) $y'' - y' + x^2 y = 0$

3. **Power series in action:** For the following ODE:

$$y'' + y = 0 \qquad y(0) = 1 \qquad y'(0) = 2$$

   a) Solve this ODE using methods from earlier in the quarter. What method did you pick?
   b) Solve this ODE using the method of power series.
   c) Recall the Taylor expansions for sine and cosine:

$$sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} \qquad cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

      Substitute these into your solution from part (a) and explain how this corresponds with your answer in part (b).