WEEK 6 SECTION PROBLEMS

If not otherwise specified, solve the following problems. If initial conditions are given, solve for all constants of integration. It is okay to leave answers in implicit form or with unsolved integrals.

1. For the following ODEs, give the method we would use to solve them and why. *Note:* it is not necessary to solve these, simply give the steps you would hypothetically use to solve the ODE.
   a) $x^2 y'' + xy' + 2y = cos(x)$
   b) $(y'')^2 + y = cos(x)$
   c) $y'' + 2y' + y = cos(x)$
   d) $y'' + 2y' + y = cos^2(x)$
2. $y'' - 4y' + 4y = e^{2x}$
3. $y'' + 2y = sec(\sqrt{2}x)$
4. Improved Euler, also known as Heun's Method, has equations given by:

$$y' = f(t, y(t))$$

$$y_{i+1/2} = y_i + hf(t_i, y_i)$$

$$y_{i+1} = y_i + \frac{h}{2}[f(t_i, y_i) + f(t_{i+1}, y_{i+1/2})]$$

   Derive the amplification factor for the general equation $y' = \lambda y$. Using this, could you give the local accuracy for this method?

5. **Generalized RK2** The generalized formula for RK2 is given by:

$$k_1 = hf(t_n, y_n)$$

$$k_2 = hf(t_n + \alpha h, y_n + \alpha k_1)$$

$$y_{n+1} = y_n + (1 - \frac{1}{2\alpha})k_1 + \frac{1}{2\alpha}k_2$$

   where α is a free parameter and $\alpha \in (0, 1]$. For the model equation $y' = \lambda y$, derive the amplification factor $\sigma(h, \alpha)$ and maximum step size $h_{max}(\alpha)$. For what value of α is h_{max} maximized? (*Hint*: there may not be a maximum or minimum to this function.)