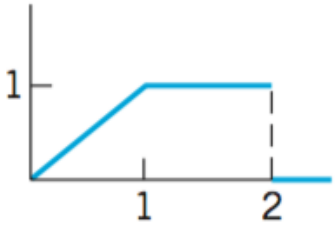
LAPLACE TRANSFORM REVIEW PROBLEMS

1. Find the Laplace transform for the following functions. If an image is given, first write out the function and then take the transform.
   a) $e^{-t}\sinh(4t)$
   b) $1.5\sin(3t-\pi/2)$
   c) Function given in the following figure:



2. Solve the following initial value problems:
   a) $y''+9y=10e^{-t}, \quad y(0)=y'(0)=0$
   b) $y''+0.04y=0.02t^2, \quad y(0)=-25, \quad y'(0)=0$

3. Find the inverse Laplace transform:
   a) $Y(s)=\frac{2(e^{-s}-e^{-3s})}{s^2-4}$
   b) $Y(s)=\frac{1+e^{2\pi(s+1)}(s+1)}{(s+1)^2+1}$

4. Solve the initial value problem:

$$y''+9y=f(t), \quad y(0)=0, \quad y'(0)=4$$

   where $f(t)=8\sin(t)$ for $0<t<\pi$ and 0 for $t>\pi$.

5. Solve the initial value problem:

$$y''+4y'+5y=\delta(t-1), \quad y(0)=0, \quad y'(0)=3$$

6. a) Find the Laplace transform: $f(t)=\frac{1}{2}te^{-3t}$
   b) Find the inverse Laplace transform: $F(s)=\cot^{-1}\left(\frac{s}{\pi}\right)$. *Hint:* $\frac{d}{dx}\left(\cot^{-1}(x)\right)=\frac{-1}{1+x^2}$.

7. Solve $y''+4y=f(t), \quad y(0)=y'(0)=0$ with $f(t)$ defined by the following figure: