MIDTERM #2 REVIEW PROBLEMS

If not otherwise specified, solve the following problems. If initial conditions are given, solve for all constants of integration. It is okay to leave answers in implicit form or with unsolved integrals if it is not possible to reduce the solution further.

1. **Eigenvalue solutions of ODEs** Solve the following system of ODEs.

   (a)
   $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

   (b)
   $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2. **Nonlinear Second Order ODEs**

   (a) $y'' + y'^3 \sin(y) = 0$

   (b) $yy'' = 3y'^2$

   (c) $y'' = 1 + y'^2$

3. **Second Order Linear ODEs**

   (a)
   $$t^2 y'' - t(t+2)y' + (t+2)y = 2t^4, \quad y_1 = t$$

   (b)
   $$(x-1)y'' - xy' + y = (x-1)^2,\ x > 1; \quad y_1 = e^x$$

   (c)
   $$y'' - y' - 2y = -2t + 4t^2$$

   (d)
   $$y'' - 2y' + y = e^x$$

   (e)
   $$y'' + 4y = 4\csc(2t)$$

   (f)
   $$t^2 y'' - 2y = 3t^2 - 1$$

   (g)
   $$x^2 y'' + 4xy' - 4y = \ln(x)$$

4. **Higher Order ODEs and MATLAB**

a) A system of couple pendulums could be modeled by the following system of equations:

$$\theta_a'' + \theta_b'' + \frac{g}{\ell}\theta_a + \frac{K}{M}(\theta_a - \theta_b) = 0 \tag{1}$$

$$\theta_a'' + 2\theta_b'' + \frac{g}{\ell}\theta_b + \frac{K}{M}(\theta_b - \theta_a) = 0 \tag{2}$$

(i) Rewrite this as a coupled system of first-order ODEs.

(ii) Write a MATLAB function to evaluate the derivative such that the system could be solved with `ode45()`. Pass in g, ℓ, K, and M as parameters.

Note that there are many correct ways to write this function, this is just one example.

(iii) Write a script to call `ode45()` to solve this ODE over the domain $0 \le t \le 10$ with initial conditions:

$$\theta_a = \pi,\ \theta_b = 0,\ \theta_a'(0) = 0,\ \theta_b'(0) = 0$$

Include code to plot the solution for θ_b as a function of t. Use the values $g = \ell = K = M = 1$ and pass these in as parameters to your `yp()` function.

b) For the following initial value problem

$$y'' + x^2 y' + y = 1, \quad y(0) = 1, \quad y'(0) = 1$$

write a piece of MATLAB code to numerically solve the system using Backward Euler method. Use step size $h = 0.1$, solve over interval $x \in [0, 1]$, and include code to plot your solution.