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Discrete II

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Assignment 2

Problem 1: NFA Properties (20)

Show by giving an example that if *M* is an NFA that recognizes language *C*, swapping the accept and non-accept states in *M* **doesn’t** necessarily yield a new NFA that recognizes the complement of *C*.  Is the class of language recognized by NFAs closed by complement?  Explain your answer.  *Hint: Remember equivalence and its power, and implication and its limitations.*

A picture containing wall, watch

Description automatically generated



These two NFAs have inverted accept and non-accept states. both would be acceptable to have any number of a’s followed by two b’s. “aaabb” would be accepted in both NFAs. The second NFA accepts a sequence which is not in the complement of the first NFA, we can conclude that swapping the accept and non-accept states of an NFA doesn’t necessarily yield a new NFA that recognizes its complement and the class of language is not closed by complement.

### Problem 2: Regular Expression (15)

In certain programming languages, comments appear between delimiters such as **/#** and **#/**.  Let C be the language of all valid delimted comment strings.  A member of C must begin with **/#** and end with **#/** but have no intervening **#/**.  For simplicity, assume the alphabet Σ for C is Σ = { **a**, **b**, **/**, **#** }.  Give a regular expression that generates C.

(/#(a|b)\*#/)

### Problem 3: The Regular Property (30)

With A and B as regular languages, show that the following languages **are** regular:

* PSHUFFLE(A, B) = { s | s = a1b1a2b2…akbk, a1a2…ak ∈ A, b1b2…bk ∈ B, and each ai, bi∈ Σ }
* SHUFFLE(A, B) = { s | s = a1b1a2b2…akbk, a1a2…ak ∈ A, b1b2…bk ∈ B, and each ai, bi∈ Σ\* }

𝑀 = (𝑄, Σ, 𝑞0, 𝐹, 𝛿)

𝑄 = (𝑄A ∗ 𝑄b ) ∪ 𝑞0

Σ = Σa ∗ Σb

𝐹 = (Fa ∗ 𝐹b) ∪ 𝑞­0

𝛿 = (­­­­­, ) ( (𝑥, 𝑎), 𝑦) ∈ 𝛿((𝑥, 𝑦), 𝑎) (𝑥, ­­(𝑦, 𝑎)) ∈ 𝛿((𝑥, 𝑦), 𝑎)

### Problem 4: Parameterized Regularity (20)

Let Σ = { **a**, **b** }.  For each positive integer k, let Ck be the language consisting of all strings that contain an **a** exactly k places from the right-hand end; in other words, Ck is all strings Σ\***a**Σk-1.  Describe an NFA with k + 1 states that recognizes Ck in terms of **both**a state diagram **and** a formal description.

A pair of glasses

Description automatically generated with low confidence

### Problem 5: Irregularity (40)

Show that the following languages are not regular:

1. { sss | s is any string over Σ } with Σ = { **a**, **b** }
2. { x = y + z | x, y and z are binary integers, and x is in fact the sum of y and z } with Σ = { **0**, **1**, **+**, **=** }
3. { **a**n**b**m**a**n | m and n are non-negative integers } with Σ = { **a**, **b** }
4. { s | s is a string over Σ that is not a palindrome } with Σ = { **a**, **b** }

### Problem 6: Extra Credit (10)

**This problem is difficult.  Don’t attempt it until you’ve finished the rest of the assignment.**

Let language A’s rotation ROT(A) = { yx | xy ∈ A }.  In other words, it is comprised of all strings composed of a concatenated prefix and suffix, such that the concatenated suffix and prefix are in A.

* Show that ROT(ROT(A)) = ROT(A).
* Show that the class of regular languages is closed under rotation.