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Exercise sheet 3
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Neurodynamics 2023

Homework: 2-D Models: FitzHugh-Nagumo.

In this weeks exercise, we begin exploring two dimensional models of spiking neurons. Our first candidate models is the FitzHugh-Nagumo model [1, 2] which was intended to show the excitability properties as demonstrated in the Hodgkin-Huxley model in a mathematically simpler, 2-D model. **Hand-in for this sheet is end of day on June, 9th**

Question 1. We are analysing the FitzHugh-Nagumo model in the following form in `brian2`:

$$\dot{V} = V - \frac{V^3}{3} - w + I$$
$$\dot{w} = \Phi(V + a - bw)$$

with parameters: a = 1.0, b = 0.5, $\Phi = 0.1$. Notice, that this model is dimensionless, i.e. we don't identify the individual variables with physical quantities and units[1].
First, find the nullclines for V and w in the system analytically. Use them to draw a phase portrait of the system for $I = 0$ and $I = 2$ by drawing the nullclines and indicating the system dynamics via its corresponding vector field[2]. You can get a particularly clean picture by drawing the tangent vectors with origins on the nullclines, alternatively you can draw a mesh of vectors evenly spaced. A good range of values to plot is between -3 and 3 for both V and w.
Can you get an intuition about the system dynamics already? Where are fixed points, how would trajectories evolve in phase space?

Question 2. Next, solve the system for $I = 0$ and $I = 2$ with starting value $(V = -2, w = 0.4)$ by implementing the FitzHugh-Nagumo model in `brian2`. Plot the trajectories $V(t), w(t)$ for both cases as part of the phase portraits you drew in the previous question. Next to these plots, plot the evolution of the system state as a function of time. Describe the dynamics and compare to your predictions from the previous question.

Question 3. Lastly, and similar to the last homework, plot the f-I curve of the model for I values in the range of 0.0 to 2.0. In this case, you cannot simply rely on the number of threshold detections. Instead, you will have to choose what to count as a spike in the dynamics and write an algorithm that counts the number of those. Think number of consecutive values above your threshold in the array of membrane voltages.
Compared to last weeks exercise, is the f-I curve of the FitzHugh-Nagumo model similar or different to the f-I curve of a leaky-integrate-and-fire model? Characterize one versus the other.

References

[1] Richard FitzHugh. Impulses and physiological states in theoretical models of nerve membrane. *Biophysical journal*, 1(6):445–466, 1961.

[2] Jinichi Nagumo, Suguru Arimoto, and Shuji Yoshizawa. An active pulse transmission line simulating nerve axon. *Proceedings of the IRE*, 50(10):2061–2070, 1962.

[1] t in brian is not dimensionless by default in `brian2`, so your implementation will have to take this into account as part of your equation.

[2] For a dynamical system $\dot{x} = f(x, y)$ and $\dot{y} = g(x, y)$, this vector field is the collection of tangent vectors $(f(x, y), g(x, y))$ for different states of the system (x, y) which are also the offset of the vectors from the origin.