1 Introduction

When analyzing economic data in practice, it is crucial to determine whether the data-generating process of the series under consideration is stationary because this has implications for the application of standard statistical tools. In this regard, the Augmented Dickey-Fuller test is often used to test for the existence of a unit root.
Also, usually the multivariate context is of interest. There, the dynamics between multiple time series can be modeled by vector autoregressive (VAR) models. In this regard, the idiosyncratic shocks can be used to assess the response of each system variables to a shock in one of the other system variables (impulse response function analysis).
In this paper, both theoretical concepts are applied to four time series of Swiss seasonally-adjusted macroeconomic data.

2 Data exploration

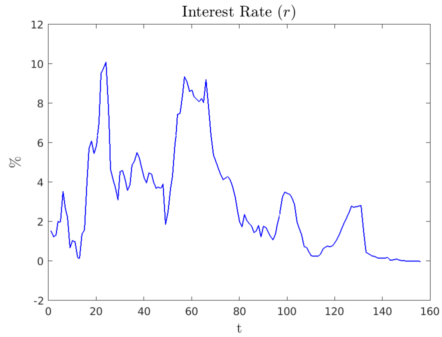
The following four time series are considered in the subsequent analysis: the quarterly interest rate (in percent) r, the quarterly Consumer Price Index p, the quarterly Gross Domestic Product (in Mio. CHF) g and the quarterly Money Stock M1 (in Mio. CHF) m. The period under examination ranges from the first quarter of 1976 to the last quarter of 2014 which results in $T = 156$ observations of the system variables. These four time series are plotted in Figure 1.

3 Unit root testing

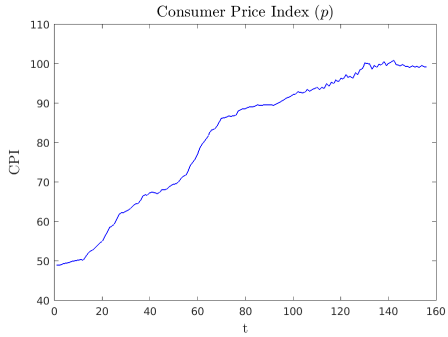
In this paper, the weapon of choice to test for the existence of a unit root in a time series is the Augmented Dickey-Fuller (ADF) test.

3.1 Theoretical background

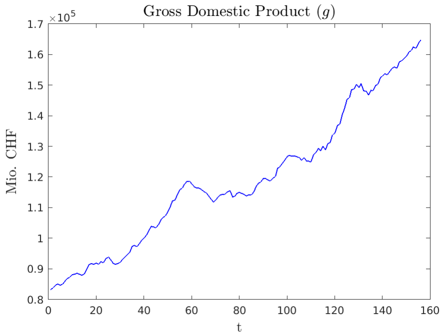
The ADF test is conducted based on OLS estimation and tests under the null hypothesis the existence of a unit root, i.e., $\rho = 1$. Accounting for different estimated models, in total four cases of the ADF test exist. However, cases 1, 2 and 4 generally provide the highest power when conducting such a test and therefore, it is focused on these cases. Equation 1 shows the estimated model for ADF case 4 including k lagged differences. The estimated model for case 2 is the same one without the time trend (δt) while the one for case 2 does not include a constant (α) and time trend



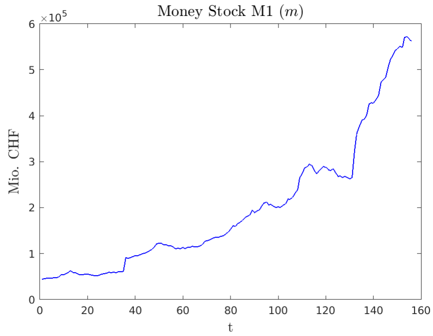
(a) Plot of r



(b) Plot of p



(c) Plot of g



(d) Plot of m

Figure 1: Plots of the four series under consideration

(δt). Hence, case 1 can be used for mean-zero series while cases 2 and 4 are more generally applicable.

$$y_t = \alpha + \delta t + \rho y_{t-1} + \zeta_1 \Delta y_{t-1} + ... + \zeta_k \Delta y_{t-k} + u_t \tag{1}$$

The parameters are estimated using the OLS estimator $\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$. Subsequently, the standard errors of the estimates can be computed by $\widehat{Var}(\mathbf{b}) = s^2(\mathbf{X}'\mathbf{X})^{-1}$ where $s^2 = \frac{1}{T-m}\sum(y_t - \hat{\alpha} - \hat{\delta}t - \hat{\rho}y_{t-1} - \hat{\zeta}_1\Delta y_{t-1} - ... - \hat{\zeta}_k\Delta y_{t-k})^2$ (again, for case 4). Here, m denotes the number of parameters used in the model. Subsequently, the t-statistic can be computed as usual by $t = \frac{(\hat{\rho}-1)}{s.e.(\hat{\rho})}$. Since it is tested under the null hypothesis of non-stationarity, the distribution of the t-statistic under the null is non-standard and the critical values have to be obtained by simulation.

3.2 ADF test for original series

Based on Figure 1, appropriate cases of the ADF test can be suggested. Due to the reverting structure of r, the ADF case 2 is considered. On the other hand, p, g, and m all exhibit a trending structure and therefore, ADF case 4 is proposed to include a time trend in the stationary alternative. While the ADF case is one important decision to make when applying the ADF test, another one is the number of lagged differences (k) to include in the model. Exemplary, the ADF test could be conducted with $k = 2$ lags leading to the following results (Table 1).

Series	ADF case	$\hat{\rho}$	s.e.($\hat{\rho}$)	t-statistic
r	2	0.9502	0.0207	-2.4035
p	4	1.0004	0.0089	0.0400
g	4	0.9715	0.0152	-1.8760
m	4	0.9934	0.0120	-0.5488

Table 1: Results of ADF tests for the four series with $k = 2$

As $k = 2$ was just one possibility, the (residual-based) Bayes-Schwarz information criterion (SBC) and the (residual-based) Akaike information criterion (AIC) are used to determine the optimal number of lagged differences (k).

$$SBC = \ln(s^2) + \frac{m}{T}\ln(T) \tag{2}$$

$$AIC = \ln(s^2) + \frac{2m}{T} \tag{3}$$

where s^2 is defined as above and $m = k + 3$ (again, for case 4)

Accordingly, the respective case of the ADF test is executed with $k = 0, 1, ..., 16$ for each time series. The corresponding results for r, p, g and m are shown in Appendices A-D. Summarising the findings, Table 2 displays the optimal number of lagged differences per time series and information criterion as well as the associated t-values based on the optimal k according to the SBC.

Series	ADF case	optimal k (SBC)	optimal k (AIC)	t-statistic (best SBC)
r	2	1	11	-2.3767
p	4	5	5	-0.3731
g	4	5	5	-1.3729
m	4	1	1	-0.5703

Table 2: Results of ADF tests for the four series and for $k = 0, 1, ..., 16$

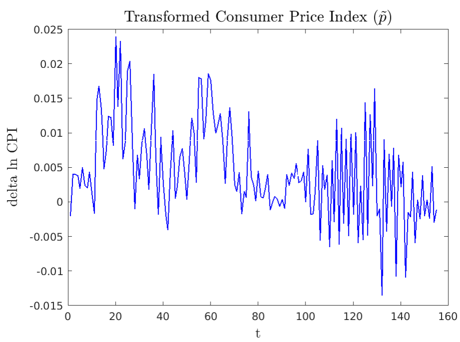
Here, it becomes apparent that AIC and SBC not always prefer the same number of lags. This is the case because they penalize model complexity (here represented by the number of parameters m) in a different way. Comparing Equations 2 and 3, it can be seen that the SBC results in a larger value than the AIC because

$\ln(T) = \ln(156) = 5.049 > 2$. Hence, the SBC penalizes larger k more strongly than the AIC and therefore, the SBC generally prefers a smaller k than the AIC (see r). In order to conduct the ADF test, the SBC-preferred model specifications are used. Accordingly, these Dickey-Fuller t-statistics (see Table 2) are compared to the respective case-specific critical values on a significance level of $\alpha = 0.05$. The following critical values are obtained using the entries for infinitely long series (Hamilton, 1994): $t_{crit} = -2.86$ (case 2) and $t_{crit} = -3.41$ (case 4). As the t-statistic is larger than the respective critical value for all time series, the null hypothesis of the existence of a unit root cannot be rejected for all time series for $\alpha = 0.05$. This seems plausible as Appendices A-D show estimates of ρ close to one.

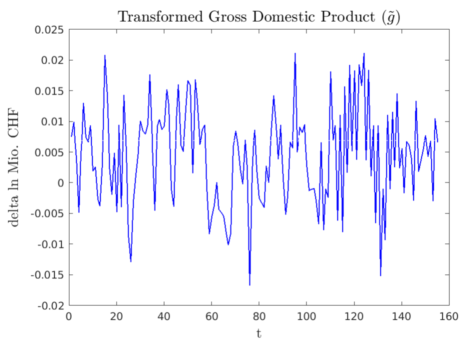
3.3 ADF test for (log) differences

In practice, the (log) differences of time series are often used for further statistical analysis because it is easier to defend stationarity for the processes that generated these transformed series. Accordingly, r is transformed by $\tilde{r}_t = \Delta r_t = r_t - r_{t-1}$ and m, p and g are transformed by $\tilde{y}_t = \Delta \ln(y_t) = \ln(y_t) - \ln(y_{t-1})$. This difference-taking leads to the loss of one observation such that $T = 155$. Again, $\tilde{r}$, $\tilde{p}$, $\tilde{g}$ and $\tilde{m}$ are plotted (Figure 2).

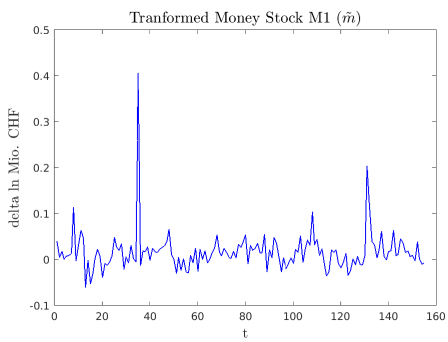
(a) Plot of $\tilde{r}$



(b) Plot of $\tilde{p}$



(c) Plot of $\tilde{g}$



(d) Plot of $\tilde{m}$

Figure 2: Plots of the four transformed series

Comparing the four transformed time series (Figure 2) to the original ones in levels (Figure 1), it can be seen that all of them now exhibit more fluctuations. In particular for p, g and m the originally trending behaviour vanishes due to taking the log differences. Based on these reverting fluctuations, the application of ADF case 2 for all series is suggested. As before, the optimal k (ranging from 0 to 16) is determined using the SBC (Formula 2) and AIC (Formula 3) and the results are displayed in Appendices E - H. The findings are summarized in Table 3.

Series	ADF case	optimal k (SBC)	optimal k (AIC)	t-statistic (best SBC)
$\tilde{r}$	2	0	8	-8.4707
$\tilde{p}$	2	3	4	-2.2024
$\tilde{g}$	2	4	4	-5.0691
$\tilde{m}$	2	0	0	-11.3069

Table 3: Results of ADF tests for the four transformed series and for $k = 0, 1, ..., 16$

Here, it becomes even clearer that the SBC often prefers a smaller k than the AIC, as explained before. Again, the t-statistics of the SBC-preferred model specifications are compared to the critical value of case 2 ($t_{crit} = -2.86$). Accordingly, the null hypothesis of the existence of a unit root can be rejected for $\tilde{r}$, $\tilde{g}$ and $\tilde{m}$ while it cannot be rejected for $\tilde{p}$ at $\alpha = 0.05$. Again, this seems reasonable considering the estimates for ρ in Appendices E - H.

Comparing the results of the ADF test between the original (in levels) and the transformed series (in differences), it can be summarized that the null hypothesis of a unit root cannot be rejected for any of the original series while it can be rejected for all but one of the transformed series. Also, the t-statistic of $\tilde{p}$ is close to the critical value. This is reasonable as it is easier to defend stationarity when taking (log) differences. These results are in fact in line with each other since unit-root processes are difference-stationary.

Finally, confidence intervals are constructed based on the ADF test results. For this purpose, the 95% confidence bounds of ρ for $\tilde{g}$ and $\tilde{m}$ are computed using the SBC-preferred model specifications with the corresponding parameter estimate, its standard error and the respective quantiles of the standard normal distribution:

Series	k	$\hat{\rho}$	s.e.($\hat{\rho}$)	$z_{0.025}$	95% Confidence Interval
$\tilde{g}$	4	0.4188	0.1147	1.96	$[0.4188 \pm 1.96 * 0.1147] = [0.1941, 0.6435]$
$\tilde{m}$	0	0.0863	0.0808	1.96	$[0.0863 \pm 1.96 * 0.0808] = [-0.0721, 0.2447]$

Table 4: 95% confidence intervals of ρ for $\tilde{g}$ and $\tilde{m}$

Conducting a t-test with any value inside the confidence interval as the hypothesized value would lead to a non-rejection of the null hypothesis for $\alpha = 0.05$. Also, the confidence intervals support the rejection of the null hypothesis of a unit root as they are rather far away from $\rho = 1$.

4 Estimation of Structural VAR

4.1 Theoretical considerations

To model the relationship between the variables associated with the Swiss monetary policy, a structural VAR is constructed based on the transformed series since it is easier to defend stationarity for the processes that generated those series.

$$\mathbf{B}_0\mathbf{y}_t = \mathbf{k} + \mathbf{B}_1\mathbf{y}_{t-1} + \mathbf{u}_t \tag{4}$$

where

$$\mathbf{B}_0 = \begin{bmatrix} 1 & b_{12}^{(0)} & b_{13}^{(0)} & b_{14}^{(0)} \\ b_{21}^{(0)} & 1 & b_{23}^{(0)} & b_{24}^{(0)} \\ b_{31}^{(0)} & b_{32}^{(0)} & 1 & b_{34}^{(0)} \\ b_{41}^{(0)} & b_{42}^{(0)} & b_{43}^{(0)} & 1 \end{bmatrix}, \quad \mathbf{y}_t = \begin{bmatrix} \tilde{m}_t \\ \tilde{r}_t \\ \tilde{p}_t \\ \tilde{g}_t \end{bmatrix}$$

and

$$\mathbf{k} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix}, \quad \mathbf{B}_1 = \begin{bmatrix} b_{11}^{(1)} & b_{12}^{(1)} & b_{13}^{(1)} & b_{14}^{(1)} \\ b_{21}^{(1)} & b_{22}^{(1)} & b_{23}^{(1)} & b_{24}^{(1)} \\ b_{31}^{(1)} & b_{32}^{(1)} & b_{33}^{(1)} & b_{34}^{(1)} \\ b_{41}^{(1)} & b_{42}^{(1)} & b_{43}^{(1)} & b_{44}^{(1)} \end{bmatrix}, \quad \mathbf{y}_{t-1} = \begin{bmatrix} \tilde{m}_{t-1} \\ \tilde{r}_{t-1} \\ \tilde{p}_{t-1} \\ \tilde{g}_{t-1} \end{bmatrix}, \quad \mathbf{u}_t = \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix}$$

Running regressions on the VAR in this structural form (Equation 4) violates one of the fundamental assumptions of OLS. Per construction, all endogenous variables are affected by all idiosyncratic shocks due to the contemporaneous effects of the system variables. Since these variables are also used as regressors, the regressors are correlated with the residuals (endogeneity). This cannot be avoided as soon as contemporaneous effects of the system variables are allowed. For OLS to deliver consistent estimates, the regressors need to be orthogonal, i.e. uncorrelated, to the residuals. For this purpose, the system (Equation 4) needs to be solved resulting in the standard form of the VAR.

$$\begin{aligned} \mathbf{y}_t &= \mathbf{B}_0^{-1}\mathbf{k} + \mathbf{B}_0^{-1}\mathbf{B}_1\mathbf{y}_{t-1} + \mathbf{B}_0^{-1}\mathbf{u}_t \\ &= \mathbf{c} + \mathbf{\Phi}_1\mathbf{y}_{t-1} + \varepsilon_t \end{aligned} \tag{5}$$

In the standard form, the new composite shocks are not correlated with the regressors anymore since only the past of the dependent variables ($\mathbf{y}_{t-1}$) is used as a regressor. Hence, the VAR in its standard form (Equation 5) is considered for the following estimations.
The composite innovations in the standard form are constructed by $\varepsilon_t = \mathbf{B}_0^{-1}\mathbf{u}_t$. Here, one can see that the composite shocks are functions of all idiosyncratic shocks, making the composite shocks contemporaneously correlated. Equation 6 shows exemplarily the construction of ε_{1t}, letting β_{1i} be the first row i^{th} column element of $\mathbf{B_0^{-1}}$, for the time being.

$$\varepsilon_{1t} = \beta_{11}u_{1t} + \beta_{12}u_{2t} + \beta_{13}u_{3t} + \beta_{14}u_{4t} \tag{6}$$

Simultaneously, all ε_{it} emerge similarly which shows the dependence of the composite shocks on all idiosyncratic shocks. Therefore, they are contemporaneously correlated.

4.2 Practical implementation

Estimating the VAR in its standard form (Equation 5) yields the following results for $\hat{\mathbf{c}}$ and $\hat{\mathbf{\Phi}}_1$:

$$\hat{\mathbf{c}} = \begin{bmatrix} 0.0251 \\ -0.1515 \\ 0.0012 \\ 0.0017 \end{bmatrix}, \quad \hat{\mathbf{\Phi}}_1 = \begin{bmatrix} 0.0085 & -0.0040 & -1.3046 & -0.7060 \\ 0.9394 & 0.3214 & 17.4903 & 11.6129 \\ 0.0077 & -0.0010 & 0.4090 & 0.3163 \\ -0.0043 & 0.0016 & 0.2995 & 0.3067 \end{bmatrix}$$

To conduct orthogonalized impulse response function (IRF) analysis, the idiosyncratic shocks ($\mathbf{u}_t$) need to be backed out from the composite shocks (ε_t). Using the residuals of the estimated model in Equation 5, an estimate for the variance-/covariance-matrix $\hat{\mathbf{\Omega}}$ of the composite shocks is obtained.

$$\hat{\mathbf{\Omega}} = \begin{bmatrix} 0.0019 & -0.0098 & -0.0000 & -0.0000 \\ -0.0098 & 0.4215 & 0.0001 & 0.0011 \\ -0.0000 & 0.0001 & 0.0000 & -0.0000 \\ -0.0000 & 0.0011 & -0.0000 & 0.0001 \end{bmatrix}$$

Further, the following relationship holds:

$$\begin{aligned} E[\varepsilon_t \varepsilon_t'] &= \mathbf{\Omega} \\ &= \mathbf{B}_0^{-1}\mathbf{D}(\mathbf{B}_0^{-1})' \end{aligned} \tag{7}$$

However, when trying to derive $\mathbf{B}_0$ and $\mathbf{D}$ from $\mathbf{\Omega}$, one faces an under-identification problem: The system consists of 16 equations, of which 6 equations are identical. That leads to only 10 unique equations while facing 16 unknown parameters. Here, Cholesky decomposition can help to tackle this under-identification of degree $\frac{n^2-n}{2} = 6$ (as $n = 4$). Assuming that some variables do not influence others contemporaneously, a hierarchical structure on the variables of interest can be imposed. Ordering the system accordingly makes $\mathbf{B}_0$ a lower triangular matrix. This assumption that 6 parameters of the systems are equal to zero imposes the number of restrictions on unknown parameters for the equation to be identified. Then, the Cholesky decomposition can be used to decompose any real symmetric positive definite matrix (here, $\mathbf{\Omega}$) into matrices $\mathbf{P}$, $\mathbf{A}$ and $\mathbf{C}$ such that

$$\begin{aligned} \mathbf{\Omega} = \mathbf{P}\mathbf{P}' &= \mathbf{A}\mathbf{C}\mathbf{C}'\mathbf{A}' \\ &= \mathbf{B}_0^{-1}\mathbf{C}\mathbf{C}'(\mathbf{B}_0^{-1})' \end{aligned} \tag{8}$$

where $\mathbf{P}$ and $\mathbf{A}$ are lower triangular matrices and $\mathbf{C}$ is a diagonal matrix with positive elements. Using a pre-specified MATLAB function, the matrix $\mathbf{P}$ is computed.

$$\mathbf{P} = \begin{bmatrix} 0.0431 & 0 & 0 & 0 \\ -0.2280 & 0.6079 & 0 & 0 \\ -0.0006 & 0.0000 & 0.0063 & 0 \\ -0.0009 & 0.0016 & -0.0050 & 0.0049 \end{bmatrix}$$

Now, the other matrices of interest can be obtained from the following relationships:

$$\mathbf{C} = \begin{bmatrix} p_{11} & 0 & 0 & 0 \\ 0 & p_{22} & 0 & 0 \\ 0 & 0 & p_{33} & 0 \\ 0 & 0 & 0 & p_{44} \end{bmatrix} = \begin{bmatrix} 0.0431 & 0 & 0 & 0 \\ 0 & 0.6079 & 0 & 0 \\ 0 & 0 & 0.0063 & 0 \\ 0 & 0 & 0 & 0.0049 \end{bmatrix}$$

$$\mathbf{A} = \mathbf{P}\mathbf{C}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -5.2867 & 1 & 0 & 0 \\ -0.0134 & 0.0000 & 1 & 0 \\ -0.0199 & 0.0026 & -0.7873 & 1 \end{bmatrix}$$

$$\hat{\mathbf{D}} = \mathbf{C}\mathbf{C}' = \begin{bmatrix} 0.0019 & 0 & 0 & 0 \\ 0 & 0.3695 & 0 & 0 \\ 0 & 0 & 0.0000 & 0 \\ 0 & 0 & 0 & 0.0000 \end{bmatrix}$$

$$\hat{\mathbf{B}}_0 = \mathbf{A}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 5.2867 & 1 & 0 & 0 \\ 0.0132 & 0.0000 & 1 & 0 \\ 0.0167 & -0.0026 & 0.7873 & 1 \end{bmatrix}$$

5 Conclusion

After obtaining the parameter matrices of the SVAR, the idiosyncratic shocks can be recursively backed out using $\mathbf{A}\mathbf{u}_t = \varepsilon_t$. Finally, this can be inserted into the VMA representation of the VAR to determine the impact of a shock in one system variable (e.g. u_{1t}) on all other variables (orthogonalized IRF analysis).

References

James D. Hamilton. Time series analysis. *Princeton University Press*, 1994.

Appendices

A Analysis of the interest rate series (r)

Lag	$\hat{\rho}$	s.e.($\hat{\rho}$)	t-statistic	SBC	AIC
0	0.9676	0.0214	-1.5098	-0.6441	-0.6834
1	0.9521	0.0201	-2.3767	$\mathbf{-0.7592}$	-0.8184
2	0.9502	0.0207	-2.4035	-0.7142	-0.7935
3	0.9539	0.0213	-2.1668	-0.6756	-0.7751
4	0.9573	0.0218	-1.9576	-0.6353	-0.7552
5	0.9582	0.0220	-1.9022	-0.6227	-0.7632
6	0.9467	0.0216	-2.4654	-0.6575	-0.8188
7	0.9545	0.0222	-2.0505	-0.6253	-0.8076
8	0.9515	0.0224	-2.1689	-0.6153	-0.8187
9	0.9667	0.0222	-1.4996	-0.6393	-0.8641
10	0.9591	0.0224	-1.8262	-0.6166	-0.8629
11	0.9611	0.0226	-1.7213	-0.6008	$\mathbf{-0.8689}$
12	0.9632	0.0232	-1.5888	-0.5539	-0.8440
13	0.9655	0.0236	-1.4641	-0.5183	-0.8306
14	0.9678	0.0241	-1.3361	-0.4700	-0.8046
15	0.9719	0.0239	-1.1734	-0.4740	-0.8312
16	0.9687	0.0242	-1.2934	-0.4407	-0.8207

Table 5: Application of ADF case 2 to series r for $k = 0, 1, ..., 16$

B Analysis of the Consumer Price Index series (p)

Lag	$\hat{\rho}$	s.e.($\hat{\rho}$)	t-statistic	SBC	AIC
0	1.0079	0.0094	0.8397	-1.2520	-1.3109
1	1.0095	0.0096	0.9981	-1.2306	-1.3095
2	1.0004	0.0089	0.0400	-1.3677	-1.4668
3	1.0043	0.0090	0.4795	-1.3642	-1.4836
4	0.9928	0.0076	-0.9365	-1.6858	-1.8257
5	0.9972	0.0076	-0.3731	$\mathbf{-1.7008}$	$\mathbf{-1.8614}$
6	0.9947	0.0077	-0.6887	-1.6720	-1.8534
7	0.9942	0.0079	-0.7270	-1.6263	-1.8289
8	0.9911	0.0080	-1.1039	-1.6042	-1.8280
9	0.9913	0.0083	-1.0562	-1.5540	-1.7992
10	0.9920	0.0084	-0.9448	-1.5201	-1.7870
11	0.9914	0.0086	-0.9957	-1.4825	-1.7712
12	0.9916	0.0088	-0.9577	-1.4369	-1.7476
13	0.9938	0.0090	-0.6849	-1.3977	-1.7308
14	0.9954	0.0091	-0.5017	-1.3548	-1.7104
15	0.9960	0.0093	-0.4278	-1.3084	-1.6866
16	0.9973	0.0094	-0.2869	-1.2745	-1.6756

Table 6: Application of ADF case 4 to series p for $k = 0, 1, ..., 16$

C Analysis of the Gross Domestic Product series (g)

Lag	$\hat{\rho}$	s.e.($\hat{\rho}$)	t-statistic	SBC	AIC
0	0.9892	0.0163	-0.6615	13.8106	13.7517
1	0.9839	0.0164	-0.9861	13.8250	13.7461
2	0.9715	0.0152	-1.8760	13.6875	13.5885
3	0.9798	0.0154	-1.3159	13.6927	13.5734
4	0.9685	0.0148	-2.1336	13.6123	13.4724
5	0.9799	0.0146	-1.3729	**13.5787**	**13.4181**
6	0.9756	0.0148	-1.6454	13.6052	13.4237
7	0.9768	0.0152	-1.5227	13.6535	13.4510
8	0.9693	0.0153	-1.9976	13.6624	13.4386
9	0.9737	0.0158	-1.6674	13.7008	13.4556
10	0.9795	0.0160	-1.2806	13.7264	13.4595
11	0.9766	0.0164	-1.4273	13.7724	13.4836
12	0.9753	0.0169	-1.4638	13.8234	13.5126
13	0.9747	0.0173	-1.4597	13.8732	13.5401
14	0.9735	0.0178	-1.4825	13.9264	13.5709
15	0.9718	0.0181	-1.5553	13.9460	13.5677
16	0.9672	0.0186	-1.7611	13.9872	13.5861

Table 7: Application of ADF case 4 to series g for $k = 0, 1, ..., 16$

D Analysis of the Money Stock M1 series (m)

Lag	$\hat{\rho}$	s.e.($\hat{\rho}$)	t-statistic	SBC	AIC
0	1.0035	0.0118	0.2941	18.0882	18.0293
1	0.9934	0.0115	-0.5703	**18.0174**	**17.9385**
2	0.9934	0.0120	-0.5488	18.0643	17.9653
3	0.9925	0.0125	-0.5968	18.1112	17.9919
4	0.9913	0.0130	-0.6722	18.1581	18.0182
5	0.9864	0.0135	-1.0051	18.1940	18.0334
6	0.9858	0.0142	-1.0020	18.2429	18.0615
7	0.9943	0.0148	-0.3830	18.2609	18.0584
8	0.9954	0.0155	-0.2973	18.3034	18.0796
9	0.9914	0.0165	-0.5244	18.3488	18.1036
10	0.9851	0.0173	-0.8606	18.3898	18.1229
11	0.9870	0.0181	-0.7182	18.4369	18.1482
12	0.9954	0.0189	-0.2430	18.4657	18.1549
13	0.9895	0.0201	-0.5224	18.5073	18.1742
14	0.9841	0.0209	-0.7604	18.5538	18.1983
15	0.9827	0.0221	-0.7795	18.6082	18.2300
16	0.9985	0.0229	-0.0673	18.6171	18.2160

Table 8: Application of ADF case 4 to series m for $k = 0, 1, ..., 16$

E Analysis of the transformed interest rate series ($\tilde{r}$)

Lag	$\hat{\rho}$	s.e.($\hat{\rho}$)	t-statistic	SBC	AIC
0	0.3592	0.0757	-8.4707	**−0.7618**	-0.8012
1	0.3596	0.0924	-6.9321	-0.7158	-0.7752
2	0.3050	0.1061	-6.5539	-0.6840	-0.7636
3	0.2330	0.1205	-6.3649	-0.6493	-0.7492
4	0.2271	0.1346	-5.7405	-0.6381	-0.7585
5	0.3901	0.1454	-4.1951	-0.6559	-0.7971
6	0.2857	0.1529	-4.6701	-0.6364	-0.7984
7	0.2740	0.1626	-4.4637	-0.6227	-0.8058
8	0.1098	0.1674	-5.3182	-0.6643	**−0.8687**
9	0.2279	0.1835	-4.2085	-0.6336	-0.8595
10	0.1268	0.1929	-4.5260	-0.6205	-0.8680
11	0.1131	0.2080	-4.2631	-0.5770	-0.8463
12	0.1142	0.2216	-3.9976	-0.5443	-0.8357
13	0.0703	0.2359	-3.9410	-0.4988	-0.8125
14	0.0156	0.2444	-4.0281	-0.5062	-0.8424
15	0.0426	0.2595	-3.6902	-0.4707	-0.8296
16	-0.0226	0.2748	-3.7210	-0.4223	-0.8041

Table 9: Application of ADF case 2 to series $\tilde{r}$ for $k = 0, 1, ..., 16$

F Analysis of the transformed Consumer Price Index series ($\tilde{p}$)

Lag	$\hat{\rho}$	s.e.($\hat{\rho}$)	t-statistic	SBC	AIC
0	0.2167	0.0791	-9.9003	-9.9579	-9.9973
1	0.5590	0.0926	-4.7639	-10.1184	-10.1778
2	0.5312	0.1000	-4.6877	-10.0769	-10.1565
3	0.8035	0.0892	-2.2024	$\mathbf{-10.4178}$	-10.5177
4	0.7604	0.0898	-2.6684	-10.4126	$\mathbf{-10.5330}$
5	0.7932	0.0923	-2.2414	-10.3800	-10.5211
6	0.7927	0.0950	-2.1822	-10.3311	-10.4931
7	0.8222	0.0968	-1.8366	-10.3001	-10.4832
8	0.8118	0.0989	-1.9025	-10.2562	-10.4605
9	0.7996	0.1013	-1.9777	-10.2117	-10.4375
10	0.7765	0.1036	-2.1575	-10.1715	-10.4190
11	0.8072	0.1040	-1.8540	-10.1747	-10.4441
12	0.7933	0.1051	-1.9676	-10.1583	-10.4497
13	0.7695	0.1079	-2.1366	-10.1202	-10.4339
14	0.7493	0.1114	-2.2504	-10.0712	-10.4074
15	0.7310	0.1150	-2.3401	-10.0239	-10.3828
16	0.7370	0.1188	-2.2141	-9.9740	-10.3558

Table 10: Application of ADF case 2 to series $\tilde{p}$ for $k = 0, 1, ..., 16$

G Analysis of the transformed Gross Domestic Product series ($\tilde{g}$)

Lag	$\hat{\rho}$	s.e.($\hat{\rho}$)	t-statistic	SBC	AIC
0	0.2362	0.0788	-9.6936	-9.6879	-9.7273
1	0.4496	0.0969	-5.6833	-9.7262	-9.7856
2	0.3398	0.1052	-6.2752	-9.7222	-9.8018
3	0.5465	0.1127	-4.0243	-9.7869	-9.8868
4	0.4188	0.1147	-5.0691	$\mathbf{-9.8243}$	$\mathbf{-9.9447}$
5	0.4902	0.1237	-4.1217	-9.8026	-9.9437
6	0.4764	0.1319	-3.9703	-9.7542	-9.9162
7	0.5648	0.1379	-3.1556	-9.7362	-9.9192
8	0.4995	0.1423	-3.5183	-9.7084	-9.9128
9	0.4213	0.1471	-3.9349	-9.6947	-9.9205
10	0.4291	0.1565	-3.6479	-9.6433	-9.8908
11	0.4280	0.1653	-3.4597	-9.5929	-9.8623
12	0.4301	0.1735	-3.2839	-9.5473	-9.8388
13	0.4493	0.1820	-3.0255	-9.4950	-9.8087
14	0.4580	0.1852	-2.9272	-9.4930	-9.8292
15	0.4721	0.1923	-2.7457	-9.4460	-9.8049
16	0.4621	0.1979	-2.7185	-9.4092	-9.7910

Table 11: Application of ADF case 2 to series $\tilde{g}$ for $k = 0, 1, ..., 16$

H Analysis of the transformed Money Stock M1 series ($\tilde{m}$)

Lag	$\hat{\rho}$	s.e.($\hat{\rho}$)	t-statistic	SBC	AIC
0	0.0863	0.0808	-11.3069	$\mathbf{-6.1702}$	$\mathbf{-6.2097}$
1	0.1444	0.1103	-7.7550	-6.1283	-6.1877
2	0.1707	0.1317	-6.2978	-6.0822	-6.1618
3	0.1603	0.1493	-5.6248	-6.0357	-6.1357
4	0.1274	0.1659	-5.2611	-5.9893	-6.1098
5	0.1011	0.1825	-4.9254	-5.9416	-6.0828
6	0.0996	0.1992	-4.5212	-5.8924	-6.0544
7	0.0425	0.2109	-4.5396	-5.8809	-6.0640
8	0.0836	0.2276	-4.0263	-5.8341	-6.0384
9	0.1316	0.2423	-3.5844	-5.7864	-6.0122
10	0.1034	0.2547	-3.5209	-5.7419	-5.9893
11	-0.0227	0.2657	-3.8490	-5.7100	-5.9793
12	0.0162	0.2795	-3.5201	-5.6802	-5.9716
13	0.0076	0.2949	-3.3655	-5.6273	-5.9410
14	-0.0545	0.3075	-3.4297	-5.5916	-5.9278
15	-0.1521	0.3225	-3.5722	-5.5479	-5.9068
16	-0.2257	0.3411	-3.5937	-5.4965	-5.8783

Table 12: Application of ADF case 2 to series $\tilde{m}$ for $k = 0, 1, ..., 16$