Assignment #9

feedback session for this assignment: 22-02-2021

General remarks: This assignment is a simulation study that considers cointegration and IRFs. Additionally, we make use of various notations that you got to know in class.

1. Vector Error Correction Model (VECM):
In this assignment, we continue to analyze the dependencies between stock prices of the same company traded on different markets. Throughout the assignment, p^h refers to the ln-price for a stock of the company in the home market (denoted in the home currency), p^f gives the ln-price for a stock of the company in a foreign market (denoted in the foreign currency), and r^x denotes the ln-exchange rate (foreign to home market). In contrast to assignment 8, we will not combine p^f and r^x, however, we will use a VECM that explicitly includes all three series, such that $\mathbf{y}_t = (p_t^h,\ r_t^x,\ p_t^f)'$.

1. We consider a VECM of the general form

$$\Delta \mathbf{y}_t = \boldsymbol{\zeta}_1 \Delta \mathbf{y}_{t-1} - \mathbf{B}\mathbf{A}'\mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{1}$$

   where $\varepsilon_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Omega})$ and assume that we know its true parameters:

$$\begin{bmatrix} \Delta p_t^h \\ \Delta r_t^x \\ \Delta p_t^f \end{bmatrix} = \begin{bmatrix} 0.2 & 0.1 & 0.1 \\ 0.1 & 0.5 & 0.0 \\ 0.1 & -0.2 & 0.2 \end{bmatrix} \begin{bmatrix} \Delta p_{t-1}^h \\ \Delta r_{t-1}^x \\ \Delta p_{t-1}^f \end{bmatrix} + \begin{bmatrix} -0.2 \\ 0 \\ 0.8 \end{bmatrix} (1\ -1\ -1) \begin{bmatrix} p_{t-1}^h \\ r_{t-1}^x \\ p_{t-1}^f \end{bmatrix} + \begin{bmatrix} \varepsilon_t^h \\ \varepsilon_t^x \\ \varepsilon_t^f \end{bmatrix}, \tag{2}$$

   where

$$\boldsymbol{\Omega} = \begin{bmatrix} 0.0010 & 0.0000 & 0.0001 \\ 0.0000 & 0.0020 & -0.0001 \\ 0.0001 & -0.0001 & 0.0020 \end{bmatrix}. \tag{3}$$

   Write $\boldsymbol{\zeta}_1$, $\mathbf{B}$, $\mathbf{A}$, and $\boldsymbol{\Omega}$ into your program.

2. Compute the $\boldsymbol{\Phi}_1$ and $\boldsymbol{\Phi}_2$ matrices that are implied by the VECM in Equation (2).
   Hint: Remember the relation of the $\boldsymbol{\Phi}$ and $\boldsymbol{\zeta}$ matrices that is outlined in the videos. This link is also summarized on slide 376 in the script.

3. Compute and report $\boldsymbol{\Phi}(1)$.

4. Compute the rank of $\boldsymbol{\Phi}(1)$ and interpret it.
   Hint: Use the MATLAB function `rank`.

5. Write a function to simulate the VAR(2) in ln-levels:

$$\mathbf{y}_t = \boldsymbol{\Phi}_1 \mathbf{y}_{t-1} + \boldsymbol{\Phi}_2 \mathbf{y}_{t-2} + \boldsymbol{\varepsilon}_t. \tag{4}$$

   Set $\mathbf{y}_{-1} = \mathbf{y}_0 = \mathbf{0}$, such that $\mathbf{y}_1 = \boldsymbol{\varepsilon}_1$ and make sure that you draw the innovations from $\mathcal{N}(\mathbf{0}, \boldsymbol{\Omega})$ (drawing from a standard normal is not enough). Input arguments of your function should be $\boldsymbol{\Phi}_1$, $\boldsymbol{\Phi}_2$, $\boldsymbol{\Omega}$ and T and your function should return the simulated $\{\mathbf{y}_t\}_1^T$ series as a $(T \times 3)$ matrix.
   Hint: If you want to draw a vector of observations $\mathbf{x}$ from a multivariate normal distribution $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Omega})$, you can do so by: $\mathbf{x} = \boldsymbol{\mu} + \texttt{chol}(\boldsymbol{\Omega})'\mathbf{z}$, where $\mathbf{z}$ is a vector of draws from a standard normal distribution and `chol` refers to the Cholesky decomposition function that is provided in MATLAB.

6. Call your function from Task 1.5 four times for $T = 3{,}000$. For each call, plot the three series into one graph (four graphs in total). Make sure your graphs look professional and elaborate by looking at the graphs and from theoretical considerations: How many stochastic trends exist?

7. Write a function that has one simulated $\mathbf{y}_t = (p_t^h,\ r_t^x,\ p_t^f)'$ series as input argument and returns an estimated cointegrating (1×3) vector. The estimation is based on Phillips's triangular representation by performing a regression of p_t^h on a constant, r_t^x, and p_t^f.

8. Use a loop to call your data simulation and estimation function (Task 1.7) 500 times. Store the estimated cointegrating vectors in a (500×3) matrix.

9. Plot kernel densities for the second and third column of your (500×3) matrix from Task 1.8 in one graph. Interpret your results and make your graphs look professional.

10. Estimate the $\mathbf{\Psi}(1)$ matrix in the Beveridge-Nelson decomposition by simulation. Do so by forward iterating the VAR(2) in Equation (4) using $\mathbf{y}_{-1} = \mathbf{y}_0 = \mathbf{0}$. For your simulation, use $s = 2{,}000$. In order to obtain the first column of $\mathbf{\Psi}(1)$, set $\varepsilon_1^h = 1$ and $\varepsilon_1^x = \varepsilon_1^f = 0$. Set all future $\boldsymbol{\varepsilon}_t = \mathbf{0}$. The $\mathbf{y}_{2000}$ that is obtained in this fashion gives you the first column of $\mathbf{\Psi}(1)$. In order to obtain the second column, set $\varepsilon_1^x = 1$ and $\varepsilon_1^h = \varepsilon_1^f = 0$. Set all future $\boldsymbol{\varepsilon}_t = \mathbf{0}$. The $\mathbf{y}_{2000}$ that is obtained in this fashion gives you the second column of $\mathbf{\Psi}(1)$. The same procedure is applied for the third column of $\mathbf{\Psi}(1)$. Print the resulting $\mathbf{\Psi}(1)$ to the output window and interpret it economically.

11. Compute $\mathbf{\Psi}(1)$ using the formula on slide 354 in the script and print it to the output window. Compare your result to that of Task 1.10. Compute the rank of $\mathbf{\Psi}(1)$, print it to the command window, and interpret it.
    Hint: The MATLAB function `null` computes orthogonal complements of the **rows** of a matrix.

12. Compute $\mathbf{\Phi}(1) \cdot \mathbf{\Psi}(1)$ using either $\mathbf{\Psi}(1)$ from 1.10 or 1.11. Print the result to the command window, and interpret it.

2. Identification of W:
We are interested in the effect of the idiosyncratic shocks $\mathbf{u}_t = (u_t^h,\ u_t^x,\ u_t^f)$ that are related to the composite shocks by:

$$\begin{bmatrix} \varepsilon_t^h \\ \varepsilon_t^x \\ \varepsilon_t^f \end{bmatrix} = \mathbf{W} \begin{bmatrix} u_t^h \\ u_t^x \\ u_t^f \end{bmatrix}, \qquad \text{where} \quad \mathbf{u}_t = \begin{bmatrix} u_t^h \\ u_t^x \\ u_t^f \end{bmatrix} \sim (\mathbf{0}, \mathbf{I}_3). \tag{5}$$

For this purpose, we want to obtain $\mathbf{W}$ from $\mathbf{\Omega} = \mathbf{W}\mathbf{W}'$, which does not identify $\mathbf{W}$ uniquely. Hence, we want to impose the following restrictions on $\mathbf{W}$:

$$\mathbf{W} = \begin{bmatrix} * & * & 0 \\ 0 & * & 0 \\ * & * & * \end{bmatrix}. \tag{6}$$

1. Can we identify $\mathbf{W}$ from $\mathbf{\Omega}$ using these three zero restrictions? What is the economic meaning of each restriction?

2. It may not be apparent at first glance, but you can obtain the $*$-elements in Equation (6) using a Cholesky decomposition of a variance-covariance matrix of the composite shocks. In order to do so, you need to reorder the entries of $\mathbf{\Omega}$ to account for a new variable ordering $y_t^r = (r_t^x,\ p_t^h,\ p_t^f)'$. We call this new variance-covariance matrix $\mathbf{\Omega}^r$. Note that the same values occur in both matrices, they are just ordered differently (for example, $\mathrm{Var}(\varepsilon_t^h)$ is the $[1,1]$-element in $\mathbf{\Omega}$ and the $[2,2]$-element in $\mathbf{\Omega}^r$). Construct $\mathbf{\Omega}^r$ and print it to the command window.

3. The identification scheme in Equation (6) entails for the reordered variables:

$$\begin{bmatrix} \varepsilon_t^x \\ \varepsilon_t^h \\ \varepsilon_t^f \end{bmatrix} = \mathbf{W}^r \begin{bmatrix} u_t^x \\ u_t^h \\ u_t^f \end{bmatrix}, \qquad \text{where} \quad \mathbf{W}^r = \begin{bmatrix} * & 0 & 0 \\ * & * & 0 \\ * & * & * \end{bmatrix}. \tag{7}$$

   Obtain $\mathbf{W}^r$ from a Cholesky decomposition of $\mathbf{\Omega}^r$. Note that $\mathbf{W}^r$ holds the same elements as $\mathbf{W}$, but they are in a different ordering.
   Hint: Use the MATLAB function `chol` to perform the decomposition and keep in mind that `chol` returns an upper triagonal matrix.

4. Reorder the elements of $\mathbf{W}^r$ to obtain $\mathbf{W}$. Print $\mathbf{W}$ to the output window.

5. Study the effects of one unit shocks in the idiosyncratic innovations on $\mathbf{y}_t$ by plotting impulse response functions. Proceed in the following way: To study the effects of a one unit shock in u^h, set $\mathbf{u}_1 = (1,\ 0,\ 0)'$, and obtain the respective $\boldsymbol{\varepsilon}_1$ by means of Equation (5). Now, similar to what you already did in Task 1.10, iterate the system forwards (consider $s = 0, \ldots, 10$) by setting all future $\mathbf{u}_t = \mathbf{0}$. Do the same for one unit shocks in u_1^x and u_1^f and plot the resulting impulse response functions comprehensively. Interpret your results.