Assignment #1 (not mandatory)

feedback session for this assignment: 30-11-2020

General info: In non-Corona times, each practical class begins with a brief recap of the assignment with the lecturer pointing out the main pitfalls that students might have struggled with during their preparations. As you see below, the assignment contains many hints that are supposed to guide you. Additionally, I have taped a short video that contains the type of recap that you would usually have gotten in class. I recommend that you watch the video before implementing the assignment in Matlab. Our practical class (via Zoom) will then focus exclusively on individual interactions. There, you can share your screen and get feedback on your code or interpretation of your results. We will be using breakout sessions for that purpose and I will move from breakout room to breakout room dealing with the feedback. I will only provide feedback/help during these classes and strongly recommend that you come to class prepared to get the most out of it.

Also, I always recommend that you use a seed in the assignments.

1. Simulate an MA(2) process

1. Write a function that delivers one realization of length T of an MA(2) process, $\mathbf{y} = (y_1, y_2, \cdots, y_T)'$ that is defined as:

   $$y_t = \mu + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \varepsilon_t,$$

   where $\varepsilon_t \overset{iid}{\sim} \mathcal{N}(0,1)$. Draw the vector of innovations, ε_t, from a standard normal distribution and be aware of the fact that the vector of ε needs to be larger than $T \times 1$ in order to generate $\{y_t\}_{t=1}^{T}$.

   Hint: The function should take as an input parameters μ, θ_1, θ_2, and the number T. The output of the function should be the resulting MA(2) realization.

2. Call the function from task 1.1 to simulate an MA(2) process with parameter values:

   a) $\mu = 0$, $\theta_1 = 0.7$, $\theta_2 = 0.3$ and $T = 100$

   b) $\mu = 4$, $\theta_1 = 5$, $\theta_2 = -17$ and $T = 100$

3. Plot the realizations of the MA(2) process for both parameter specifications (in separate graphs). In each graph, include a line that represents the expected value of the respective process and make your graphs look professional.
   Hint: Take a look at the description of the command `plot(X1,Y1,...,Xn,Yn)` for further hints on plotting lines into MATLAB graphs. `set` can be used to color the graph appropriately.

4. Use the simulated data to compute the empirical autocorrelations of first order for both parameter alternatives and compare this to the theoretical value (print them to the command window). Interpret the result.
   Hint: You can use the command `autocorr`.

5. Compute ensemble means and variances for 30 realizations of your process (for both parameter alternatives). For each process, visualize the ensemble means and variances in one plot.
   Hint: Do not write a new function! Instead, use a loop to call repeatedly the function in task 1.1, and generate a matrix which contains the different realizations of the process in its columns. Make sure to compute the moments across ensembles – you are not interested in time series means/ variances here (it can be done by transposing the matrix or using extra inputs in the `mean` function).

6. Compute autocorrelations (first order) by using the 30 realizations (ensembles), i.e. compute $\text{Corr}(y_1, y_2)$ and $\text{Corr}(y_2, y_3)$,..., $\text{Corr}(y_{99}, y_{100})$. Plot the resulting sequences of autocorrelations and interpret them.
   Hint: You could use the MATLAB command `corr`. It computes the correlation between columns of a matrix and returns the corresponding correlation matrix, i.e. if `X` is an $N \times T$ matrix then `corr(X)` returns a $T \times T$ (symmetric) matrix of correlations. The i^{th} row, j^{th} column element of the correlation matrix gives the correlation between column i and column j. You need to read out the correlations you want to have.

7. Given your results of the previous questions: Evaluate whether the conditions are met, such that the MA(2) processes in task 1.2 are (weakly) stationary. In particular, consider your results from tasks 1.3, 1.5, and 1.6 and argue what they tell you about the stationarity of the processes. Does the stationarity depend on the choice of the parameters θ_1 and θ_2?

2. Simulate an AR(1) process

1. Write a function that delivers one realization of length T of an AR(1) process, $\mathbf{y} = (y_1, y_2, \cdots, y_T)'$ that is defined as:

$$y_t = c + \phi y_{t-1} + \varepsilon_t,$$

   where $\varepsilon_t \overset{iid}{\sim} \mathcal{N}(0, 1)$. Draw the vector of innovations ε_t from a standard normal distribution and initialize the vector $\mathbf{y}$. y_0 is the starting value of the series and not part of the final vector $\mathbf{y}$. The elements of $\mathbf{y}$ are generated recursively as:

$$\begin{aligned}
y_1 &= c + \phi y_0 + \varepsilon_1 \\
y_2 &= c + \phi y_1 + \varepsilon_2 \\
y_3 &= c + \phi y_2 + \varepsilon_3 \\
&\vdots \\
y_T &= c + \phi y_{T-1} + \varepsilon_T
\end{aligned}$$

2. Call your function for these parameter specifications:

   a) $c = 0$, $\phi = 1$, $y_0 = 0$ and $T = 100$ (Random Walk)
   b) $c = 0.7$, $\phi = 1$, $y_0 = 0$ and $T = 100$ (Random Walk with Drift)
   c) $c = 4.5$, $\phi = 0.1$, $y_0 = 5$ and $T = 100$
   d) $c = 4.4$, $\phi = -0.1$, $y_0 = 4$ and $T = 100$
   e) $c = 4.5$, $\phi = 0.9$, $y_0 = 45$ and $T = 100$

   Hint: Write the body of the function by using a loop statement. Use as input c, ϕ, y_0 and T, and as output the resulting AR(1) realization.

3. Plot the realizations of the process for all parameter specifications defined above. For the series defined in c) – e), include into your graph a line that represents the expected value of the respective process. Make your graph look professional.

4. Take a closer look at the graphs for the series defined in c) – e). How does the choice of ϕ affect the behavior of the AR(1) process?

5. Use the simulated series to compute the autocorrelation of first order for the parameter alternatives c) – e) and compare the result to the theoretical value (print them to the command window). Interpret the result.
   Hint: You can use the command `autocorr`.

6. Analyze the stationarity properties of this AR(1) process by using simulations: For that purpose, compute ensemble means and variances for 30 realizations of the processes. For each process, visualize the means and variances in one plot. Interpret the graphs.

7. Compute autocorrelations (first order) by using the 30 realizations (ensembles). Plot the resulting sequences of autocorrelations and interpret the graphs.

8. Given your results of the previous questions: Evaluate if the AR(1) processes generated in task 2.2 are (weakly) stationary. Refer explicitly to your results from tasks 2.3, 2.6 and 2.7 and argue what they tell you about the stationarity of the processes. For which parameter values is a Gaussian AR(1) process stationary?

3. Check the stationarity of an AR(3) process

1. Consider the following AR(3) processes:

   a) $(1 - 0.85L - 0.3L^2 + 0.2L^3)y_t = \varepsilon_t$
   b) $(1 - 1.0L + 0.2L^2 - 0.2L^3)y_t = \varepsilon_t$
   c) $(1 - 0.4L + 0.5L^2 - 0.5L^3)y_t = \varepsilon_t$

   Write a function that constructs the **F**-matrix for an AR(3). Use ϕ_1, ϕ_2, and ϕ_3 as input arguments. The function should return the **F**-matrix as well as its eigenvalues. Call the function for specifications a) – c).
   Hint: Use the command `eig`.

2. Create a vector that holds $\mathbf{F}^j(1,1)$ for $j = 1, \ldots, 100$. For this purpose, write a loop in which you compute $\mathbf{F}^j$ and read out the first row/first column element which is stored in a vector. Plot the resulting series with j on the x-axis. Do this for the three specifications in task 3.1. Make your graphs look professional and interpret your plots.
   Hint: The first element of the resulting vector is $\mathbf{F}(1,1)$, the second one is the first row/first column element of $\mathbf{F}^2$, and so on.

3. Similar to the previous task, create a vector that holds $\mathbf{F}^j(1,1)$ for $j = 1, \ldots, 100$. However, now, use the fact that $\mathbf{F}^j = \mathbf{T}\boldsymbol{\Lambda}^j\mathbf{T}^{-1}$. Compare your results for tasks 3.2 and 3.3. Do you get similar results? Explain.

4. Write a function that returns a vector with evaluations of the respective $1 - \phi_1 z - \phi_2 z^2 - \phi_3 z^3$ polynomial.
   Hint: Input arguments of the function should be: ϕ_1, ϕ_2, ϕ_3 (i.e. the AR(3) parameters) and a vector of values at which you want to evaluate the polynomial.

5. For each of the three specifications in task 3.1, call the function from task 3.4. As we are interested in the roots of the polynomial, make sure to evaluate the polynomial in a plausible range (choose your input vector accordingly). Plot the resulting polynomials and make your graphs look professional.

6. Refer to your results from tasks 3.1, 3.2, and 3.5 and explain what they tell you about the stationarity of processes a) – c). How are the eigenvalues of the $\mathbf{F}$-matrix related to the respective plot of the lag-polynomial?