Assignment #5

feedback session for this assignment: 18-01-2020

General remark: This assignment deals with an assessment of the power of a Dickey-Fuller test. It relies heavily on Assignment 4. Thus, before starting this assignment, make sure that your estimation function in Assignment 4 is correct (to do so, compare the quantiles).

1. Power of the Dickey-Fuller test:

1. Use your function from the last assignment to obtain 10,000 estimates of ρ for case 1 of the Dickey Fuller test for $T = 1000$. Compute their mean. Illustrate $\hat{\rho}$ using a kernel density and include the mean parameter estimate into your plot. What do you notice? Repeat this for cases 2 and 4. Is there a difference?
   Hint: Plot all three kernel densities in one graph.
2. Compute $T(\hat{\rho} - 1)$ and $\frac{\hat{\rho}-1}{s.e.(\hat{\rho})}$ for the three cases. For both test statistics plot the kernel densities for the three cases into one graph (so, one plot for $T(\hat{\rho} - 1)$ and one plot for $\frac{\hat{\rho}-1}{s.e.(\hat{\rho})}$). Describe.
3. Simulate a stationary AR(1) process:
$$y_t = 1 + 0.85y_{t-1} + \varepsilon_t, \tag{1}$$
   with $T = 100$, $\varepsilon_t \overset{iid}{\sim} \mathcal{N}(0, 1)$, and starting value $y_0 = 1/(1 - \rho)$. Hand this series to your function from the last assignment and compute the Dickey-Fuller test statistics $T(\hat{\rho} - 1)$ and $\frac{\hat{\rho}-1}{s.e.(\hat{\rho})}$ for case 1. The difference is that the true process is NOT a unit root process, but the stationary AR(1) in equation (1). Make sure you are using the correct y_0 in your AR simulation function and in your OLS estimation function. Repeat the simulation and computation of the test statistic $n = 10{,}000$ times, which provides a distribution of the test statistics under the stationary alternative. For both test statistics, plot the kernel densities to illustrate the distributions of the test statistics under the null hypothesis of a unit root and the stationary alternative in one graph. Repeat for cases 2 and 4.
4. Write a function that calculates the relative frequency with which a type II error is made. The null is the unit root, the alternative is the stationary AR(1) in equation (1). The type II error probability is therefore the probability of failing to reject the incorrect null hypothesis of a unit root. The type II error depends on the significance level, which you should fix at 5 percent. The tests are one-sided (you reject the null hypothesis for small values of the test statistic). Input arguments of your procedure should be the simulated test statistic series ($T(\hat{\rho} - 1)$ and $\frac{\hat{\rho}-1}{s.e.(\hat{\rho})}$) for the random walk and for the AR(1) with $\rho = 0.85$ (four input arguments). Output arguments are the two Type II errors (one for each test statistic).
5. Interpret the results from tasks 1.3 and 1.4. Which test specification (case and test statistic) has the largest power for this specific process?
   Hint: Power: probability of rejecting the false H_0.
6. Repeat tasks 1.3 - 1.5 assuming that for the stationary alternative, $\rho = 0.98$ and $\rho = 0.3$ (again use $y_0 = 1/(1 - \rho)$), as well as $T = 100$ and $T = 1000$. Also, consider $\rho = 0.85$ and $T = 1000$. Interpret your results.

2. Power of the Dickey-Fuller test: Case 3:

1. Use your function from the last assignment to simulate $T = 1000$ observations of an AR(1) process of the form:
$$y_t = 1 + y_{t-1} + \varepsilon_t, \tag{2}$$
where $y_0 = 0$ and $\varepsilon_t \overset{iid}{\sim} \mathcal{N}(0,1)$. Estimate ρ for case 3 of the Dickey-Fuller test. Your function should return $\hat{\rho}$, the corresponding standard error, and $\frac{\hat{\rho}-1}{s.e.(\hat{\rho})}$.
Hint 1: The estimated regression for case 3 includes a constant but no time trend.
Hint 2: Standard errors can be computed in the same way as with case 2.
2. Repeat the steps in task 2.1 $n = 10{,}000$ times and store the output of your estimation procedure in vectors.
3. Estimate the kernel density of $\frac{\hat{\rho}-1}{s.e.(\hat{\rho})}$. Furthermore, construct a sequence that starts at -5 and ends at 4.9 with increments of size 0.1. Use this sequence to evaluate the density of a standard normal distribution. Plot both (kernel) densities nicely in one graph and interpret your findings
Hint: The MATLAB function `normpdf(X,`μ`,`σ`)` computes the pdf using the normal distribution at each of the values in `X` using mean, μ and standard deviation, σ.
4. Simulate a trend-stationary AR(1) process:
$$y_t = 1 + 0.85 y_{t-1} + 0.1t + \varepsilon_t, \tag{3}$$
with $T = 100$, $\varepsilon_t \overset{iid}{\sim} \mathcal{N}(0,1)$, and starting value $y_0 = 1/(1-\rho)$. Again, estimate ρ using case 3 of the Dickey-Fuller test and compute $\frac{\hat{\rho}-1}{s.e.(\hat{\rho})}$. Repeat the simulation and computation of the test statistic $n = 10{,}000$ times, which provides a distribution of the test statistic under the trend-stationary alternative in equation (3). Plot the kernel densities to illustrate the distribution of $\frac{\hat{\rho}-1}{s.e.(\hat{\rho})}$ under the null hypothesis of a unit root and the trend-stationary alternative in one graph.
5. Call your function from task 1.4 to calculate the relative frequency with which a Type II error is made (only consider $\frac{\hat{\rho}-1}{s.e.(\hat{\rho})}$). Report the Type II error along with the respective power of the test in the output window.
6. Repeat tasks 2.4 - 2.5 using $\rho = 0.3$ (instead of $\rho = 0.85$) for the trend-stationary alternative. For both choices of ρ (0.85 and 0.3), study the effect of increasing the length of the simulated series to $T = 1000$. Interpret your results.