Assignment #4 (not mandatory)

feedback session for this assignment: 11-01-2021

1. Simulate a random walk

1. Use your function from the first assignment to simulate a random walk that is defined as follows:

$$y_t = y_{t-1} + u_t, \quad u_t \sim i.i.d.\, \mathcal{N}(0,1) \tag{1}$$

for $T = 100$ and $y_0 = 0$.

2. Case 1

1. Estimate ρ in
$$y_t = \rho y_{t-1} + u_t$$
using OLS. To do so, implement an OLS estimator: $\hat{\rho} = (\mathbf{X}'\mathbf{X})^{-1}(\mathbf{X}'\mathbf{y})$, where $\mathbf{y} = [y_1, \ldots, y_T]'$ and $\mathbf{X} = [y_0, \ldots, y_{T-1}]'$.

2. Estimate the standard error of $\hat{\rho}$ and calculate the $t-$statistic for the null hypothesis that the true value of ρ equals 1.
Hint: The variance of $\hat{\rho}$ can be estimated as $\widehat{\mathrm{Var}}(\hat{\rho}) = s^2(\mathbf{X}'\mathbf{X})^{-1}$, where (in Case 1) $s^2 = \frac{1}{T-1}\sum_{t=1}^{T}(y_t - \hat{\rho}y_{t-1})^2$.

3. Compute the test statistic: $T(\hat{\rho} - 1)$.

4. Write a function around the OLS estimation. The input argument should be the time series simulated in task 1. The output is a row vector that contains the estimate for $\hat{\rho}$, the corresponding standard error, the value of $T(\hat{\rho} - 1)$, and the $t-$statistic associated with $H_0 : \rho = 1$.

5. Repeat the data simulation and estimation of ρ $n = 10{,}000$ times and store your results in a matrix. In order to do so, write a loop to call the two respective functions multiple times and collect the parameter estimates for ρ and the corresponding standard errors and $t-$values in a matrix.

6. Compute quantiles of the simulated $t-$statistics (0.01, 0.025, 0.05, 0.1, 0.9, 0.95, 0.975, and 0.99). Do the same for $T(\hat{\rho} - 1)$.
Hint: Use the MATLAB function `quantile`.

7. Use your results from the previous task and print the quantiles nicely to the command window. Perform this task for sample sizes $T = 100$ and $T = 1000$ and for the 0.01, 0.025, 0.05, 0.1, 0.9, 0.95, 0.975, and 0.99 quantiles. Do so for the values of $t-$statistic and for $T(\hat{\rho} - 1)$.
Hint: You can check your results for plausibility by comparing them to Tables B.5 ($T(\hat{\rho} - 1)$) and B.6 ($t-$statistic) on pages 762 and 763 in Hamilton (1994).

8. Illustrate the distribution of the n values of each of $\hat{\rho} - 1$, $\sqrt{T}(\hat{\rho} - 1)$, $T(\hat{\rho} - 1)$ and the $t-$statistic. Make your graphs look professional and interpret them.

3. Case 2

1. Use the function from task 1 to simulate a random walk with $T = 100$.
2. Estimate α and ρ in
$$y_t = \alpha + \rho y_{t-1} + u_t$$
using OLS. To do so, implement an OLS estimator: $\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}(\mathbf{X}'\mathbf{y})$, where
$$\mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_T \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} \hat{\alpha} \\ \hat{\rho} \end{bmatrix}, \qquad \text{and} \qquad \mathbf{X} = \begin{bmatrix} 1 & y_0 \\ \vdots & \vdots \\ 1 & y_{T-1} \end{bmatrix}.$$
3. Estimate the standard error of $\hat{\rho}$ and calculate the $t-$statistic for the null hypothesis that the true value of ρ equals 1.
Hint: The variance-covariance matrix of $\mathbf{b}$ can be estimated as $\widehat{\mathrm{Var}}(\mathbf{b}) = s^2(\mathbf{X}'\mathbf{X})^{-1}$, where (in case 2) $s^2 = \frac{1}{T-2}\sum_{t=1}^{T}(y_t - \hat{\alpha} - \hat{\rho} y_{t-1})^2$.
4. Repeat tasks 2.3 to 2.8 for case 2.

4. Case 4

1. Use the function from task 1 to simulate a random walk with $T = 100$.
2. Estimate α, δ and ρ in
$$y_t = \alpha + \delta t + \rho y_{t-1} + u_t$$
using OLS. To do so, implement an OLS estimator: $\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}(\mathbf{X}'\mathbf{y})$, where
$$\mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_T \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} \hat{\alpha} \\ \hat{\delta} \\ \hat{\rho} \end{bmatrix}, \qquad \text{and} \qquad \mathbf{X} = \begin{bmatrix} 1 & 1 & y_0 \\ \vdots & \vdots & \vdots \\ 1 & T & y_{T-1} \end{bmatrix}.$$
3. Estimate the standard error of $\hat{\rho}$ and calculate the $t-$statistic for the null hypothesis that the true value of ρ equals 1.
Hint: The variance-covariance matrix of $\mathbf{b}$ can be estimated as $\widehat{\mathrm{Var}}(\mathbf{b}) = s^2(\mathbf{X}'\mathbf{X})^{-1}$, where (in case 4) $s^2 = \frac{1}{T-3}\sum_{t=1}^{T}(y_t - \hat{\alpha} - \hat{\delta} t - \hat{\rho} y_{t-1})^2$.
4. Repeat tasks 2.3 to 2.8 for case 4.