Assignment #3 (mandatory)

feedback session for this assignment: 14-12-2020

1. Constructing the conditional log-likelihood function of an MA(1)

1. Adapt the MA simulation function from the 1^{st} assignment. Simulate an MA(1) process

$$y_t = \mu + \theta\varepsilon_{t-1} + \varepsilon_t,$$

where $\theta = 0.2$, $\mu = 10$, $\sigma^2 = 1$, $T = 100$, and $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$. Plot the series and make your graph look professional.

2. Assuming that $\varepsilon_0 = 0$, the conditional log likelihood function of an MA(1) process is given by:

$$\ln \mathcal{L} = \sum_{t=1}^{T} \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{(y_t - \mu - \theta\varepsilon_{t-1})^2}{-2\sigma^2}\right]\right) = \sum_{t=1}^{T}\left[\ln\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) - \frac{\varepsilon_t^2}{2\sigma^2}\right],$$

where $\varepsilon_t = y_t - \mu - \theta\varepsilon_{t-1}$.

Write a function that computes the conditional log likelihood contributions for an MA(1). Input arguments should be a column vector of the parameters (μ, θ, σ^2) and the data $\mathbf{y} = (y_1, y_2, \cdots, y_T)'$. It is important that the input arguments are handed to the function in this order: first the vector of parameters, then the series. The function should return a $(T \times 1)$ column vector of log likelihood contributions.

3. Additionally, write a second MATLAB function that computes the value of the conditional log likelihood function. Call your function for

   a) $\mu = 10$, $\theta = 0.2$, and $\sigma^2 = 1$,

   b) $\mu = 5$, $\theta = 0.7$, and $\sigma^2 = 2$.

   Interpret your results.

2. Estimating the parameters of an MA(1)

1. We have uploaded a MATLAB toolbox to Ilias that helps you do statistical inference on maximum likelihood functions. Save the CML toolbox folder in your working directory folder and include the toolbox via:

```
addpath('CML');
```

2. Use the two functions from Tasks 1.2 and 1.3 and the CML toolbox to estimate the parameters μ, θ, and σ^2 of an MA(1) process by conditional maximum likelihood. Note that the function from Task 1.3 must be slightly adapted for this purpose because the

CML function will try to minimize a function (just as you did in the last assignment). Remember that the input arguments for both functions are a column vector of the parameters to be estimated and a data matrix. When using the toolbox, choose the Hessian-based covariance matrix, the fminsearch algorithm for the optimization, and the true values as the starting values.

Hint on using the toolbox: To call the CML function, use the following syntax:

```
[x,f,g,cov,retcode]= CML(@fct1,@fct2,dataset,x0,algorithm,covPar,options);
```

Hint on input arguments: `@fct1` and `@fct2` are function handles for the functions `fct1` and `fct2` that compute the log likelihood function (this is the function that must be adjusted) and the log likelihood contributions, respectively (you have probably called these functions differently – replace `fct1` and `fct2` by the respective names of your functions). `dataset` refers to the name of the data matrix that is used in the analysis. Here, this is the time series $\mathbf{y} = (y_1, y_2, \cdots, y_T)'$. `x0` is a column vector of starting values for the parameters that are estimated (μ, θ and σ^2). `algorithm` refers to the type of algorithm used in the optimization (1: `fminsearch`, 2: `fminunc`, 3: `patternsearch`). `covPar` refers to the type of covariance matrix to be calculated (1: Hessian-based, 2: OPG-based, 3: QML-based). `options` refers to the optimization options. For this problem, set options before you call the CML toolbox as follows:

```
options = optimset('Display','iter','TolX',10 ^-40,'TolFun',10 ^-40,
          'MaxIter',10^10 , 'MaxFunEvals', 100000);
```

Hint on output arguments: `x` is a column vector of the parameter values at the minimum of the function `fct1`. `f` is the value of `fct1` at `x`. `g` is the value of the gradient at `x`. `cov` is the covariance matrix calculated according to `covPar`. `retcode` describes the exit condition of the optimization function used.

3. Compute standard errors for all estimates and display them in the command window.

4. Consider the null hypothesis $H_0 : \theta = 0.4$ and compute the test statistic of a t-test. Assume that you are working with a significance level of 5% and interpret your result.

5. Compute the two-sided p-value of the test-statistic from Task 2.4 and interpret it.
   Hint: `normcdf` computes the cumulative distribution function of the standard normal distribution.

6. Compute the 95% confidence intervals for parameters μ and θ, respectively. Interpret them. Display the confidence intervals in the command window. What do the values within the interval tell you?

7. Compute the standard errors once more, but using the other two options of `covPar`. Report your results in the command window and interpret them.

8. Simulate the MA(1) process in Task 1 again, but with $T = 50{,}000$, and re-do Tasks 2.2–2.7. Interpret your results taking into consideration the change of T and the change in the covariance matrix calculation.

9. Fix $\mu = 10$ and $\sigma^2 = 1$ and compute the value of the conditional log likelihood function for varying θ. Start with $\theta = -0.6$ and increase the value by 0.05 until $\theta = 0.8$. Use $T = 200$. Store the results in a vector and plot the conditional log likelihood function for $\theta \in [-0.6; 0.8]$. Also locate $\hat{\theta}$ and the true value in your graph. Interpret.

10. Simulate the MA(1) for $\theta = 5$, $\mu = 10$ and $\sigma^2 = 1$ and estimate the parameters μ, θ, and σ^2 by conditional ML. Use $T = 200$, select the `fminunc` algorithm and set the starting value for θ to 0.6 and see what happens. Interpret the results.

This assignment can be handed in for grading until **11:55 pm on Friday, January 8th, 2021**. Upload your working MATLAB script file and all your associated function files as well as your report (pdf format) to ILIAS. Please review `Assignments_Checklist2020.pdf` (uploaded on IlIAS) to make sure that your code and report comply with the guidelines.