# A Randomized Algorithm for Coded Cooperative Data Exchange [1]

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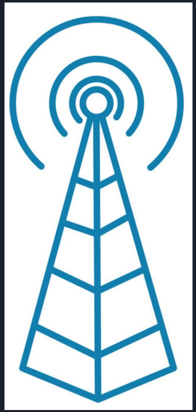
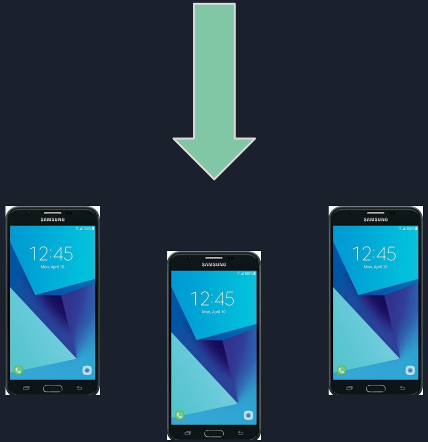
# Problem Description

- Information exchange between wireless clients
- Each client
  - holds a subset of packets
  - knows which packets are available to other clients
  - can broadcast packets it processes
- Broadcasting
  - To every client via noiseless channel.
  - One linear combination of packets each time
- Goal
  - Every client obtains every packet
  - Minimize the total number of transmissions



[3]




[2]

# Naïve Solutions

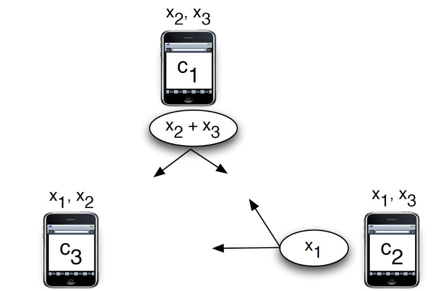
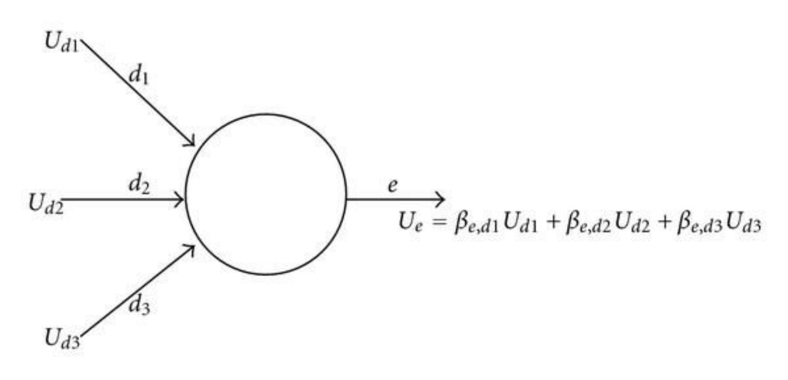


Fig. 1. Coded data exchange among three clients.

[1]

- Each client broadcast its packets to others
- One naive solution is to have each client sends uncoded transmissions. In figure 1: $c_1 \rightarrow x_2 \quad c_2 \rightarrow x_3 \quad c_3 \rightarrow x_1$
- Not always optimal
- With linear coding, $c_2$ broadcasts $x_1$, and $c_1$ broadcasts a linear combination of $(x_2 + x_3)$

# Linear Coding




[4]

- Packets = codewords in a finite field
  - $\mathbb{F}_q$ has q elements, q is prime
  - Ex: q=5, packet=[0 1 4 3 2]
- Linear combination of packets = codeword
  - $p_i = \sum_{i=0}^{n} \gamma_i x_i$ Ex: q=5, $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} + 3 * \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$
- Typically for error correcting
- Reducing bits sent
  - Encoding vector - coefficients
  - Linear combination and vector sent
  - Recover packets

# Randomized Data Exchange (RDE) Algorithm

Iterate until all clients can recover all packets

- Select client $c_{t_i}$, the client who has the most information at iteration $i$
- Create encoding vector $\gamma_i^j = \begin{cases} \text{random element of } \mathbb{F}_q & \text{if } c_{t_i} \text{ has } x_j \\ 0 & \text{otherwise} \end{cases}$
- Compute $p_i = \sum_{x_j \in X} \gamma_i^j x_j$
- Broadcast $p_i$ and $\gamma_i$

# Theorem

The algorithm computes, with probability at least $1 - \frac{kn}{q}$ an optimal solution for the data exchange problem, provided that the size q is larger than n.

(Sprintson et al. 2010)

(Given k clients, n packets, and finite field of size q)

# Motivation of Proof of Optimality

- Iteration i: Select client $c_{t_i}$ with most information
- This client has information that *some* clients need -- $C'$
- $\mathbb{P}(\gamma_i \textit{ doesn't} \text{ increase rank of some client } c_j \in C')$
- $\mathbb{P}\left(< \gamma_i, \zeta_j >= 0\right)$
- $\zeta_j$ -- contains information that client $c_j$ *needs*

$$< \gamma_i , \zeta_j >= 0$$

$$\sum_{k=0}^{m} \gamma_k \zeta_k = 0$$

$$\gamma_0 \zeta_0 + \sum_{k=1}^{m} \gamma_k \zeta_k = 0$$

$$\gamma_0 = -\frac{\sum_{k=1}^{m} \gamma_k \zeta_k}{\zeta_0}$$

Chosen i.i.d uniformly from a finite field of size q

# Motivation of Proof of Optimality

- Iteration i: Select client $c_{t_i}$ with most information
- This client has information that *some* clients need -- $C'$
- $\mathbb{P}(\gamma_i \textit{ doesn't} \text{ increase rank of some client } c_j \in C') = \frac{1}{q}$
- $\mathbb{P}(\gamma_i \textit{ doesn't} \text{ increase rank of } \textit{all} \text{ clients } c \in C') \leq \frac{k}{q}$
- $\mathbb{P}(\gamma_i \textit{ does} \text{ increase rank of all clients } c \in C') \geq 1 - \frac{k}{q}$

$$\left(1 - \frac{k}{q}\right)^{OPT} \geq \left(1 - \frac{k}{q}\right)^{n} \geq 1 - \frac{kn}{q}$$

# Description of Experiments

- Bit counting
- Naïve algorithm
  - Optimizations
  - Counts size of packets that would be sent
- RDE algorithm
  - Counts size of linear combinations and encoding vectors
- Run types
  - has_one, 25%, 50%, 75%, missing_one
  - Random 10%, ..., random 90%
- Input sizes
- Field Size

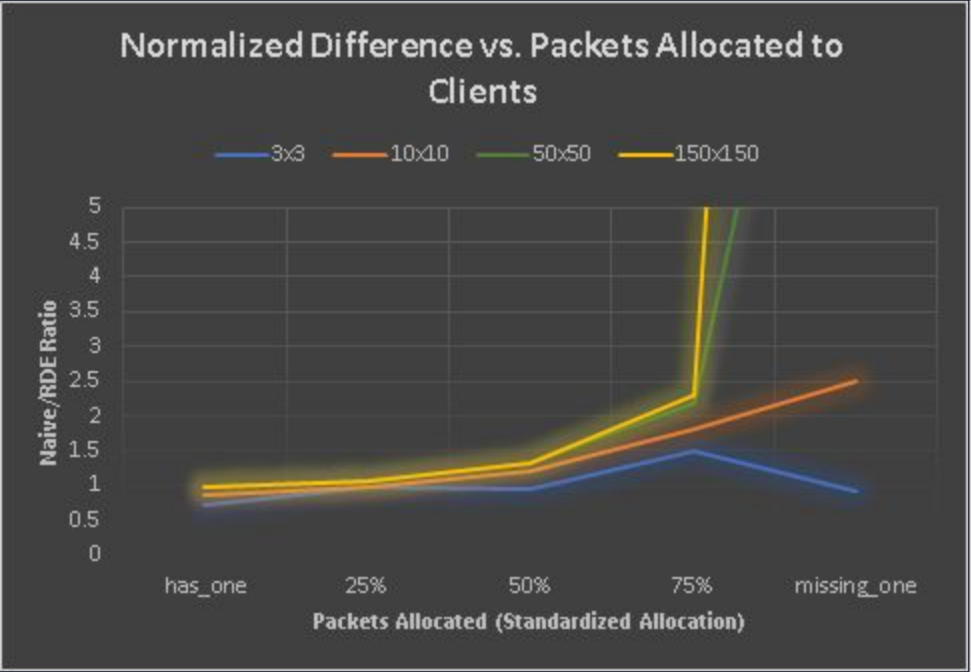
# Sample of results

- Calculated average over 10 runs of RDE
- Ratio
  - (Naïve bits) / (RDE bits)

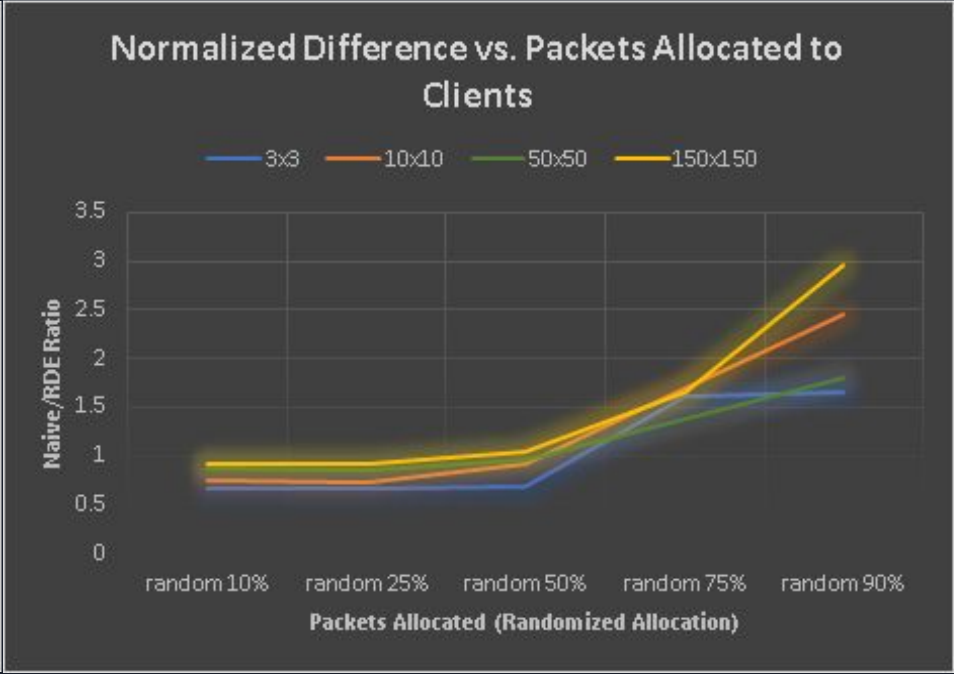
| input_smallmed_gen.txt | has_one |
|---|---|
| 562 | 636 |
| 562 | 651 |
| 562 | 654 |
| 562 | 634 |
| 562 | 655 |
| 562 | 635 |
| 562 | 630 |
| 562 | 635 |
| 562 | 635 |
| 562 | 625 |
| | 639 |

| input_smallmed_gen.txt | missing_one |
|---|---|
| 562 | 223 |
| 562 | 237 |
| 562 | 215 |
| 562 | 232 |
| 562 | 220 |
| 562 | 222 |
| 562 | 225 |
| 562 | 218 |
| 562 | 229 |
| 562 | 236 |
| | 225.7 |

# Results

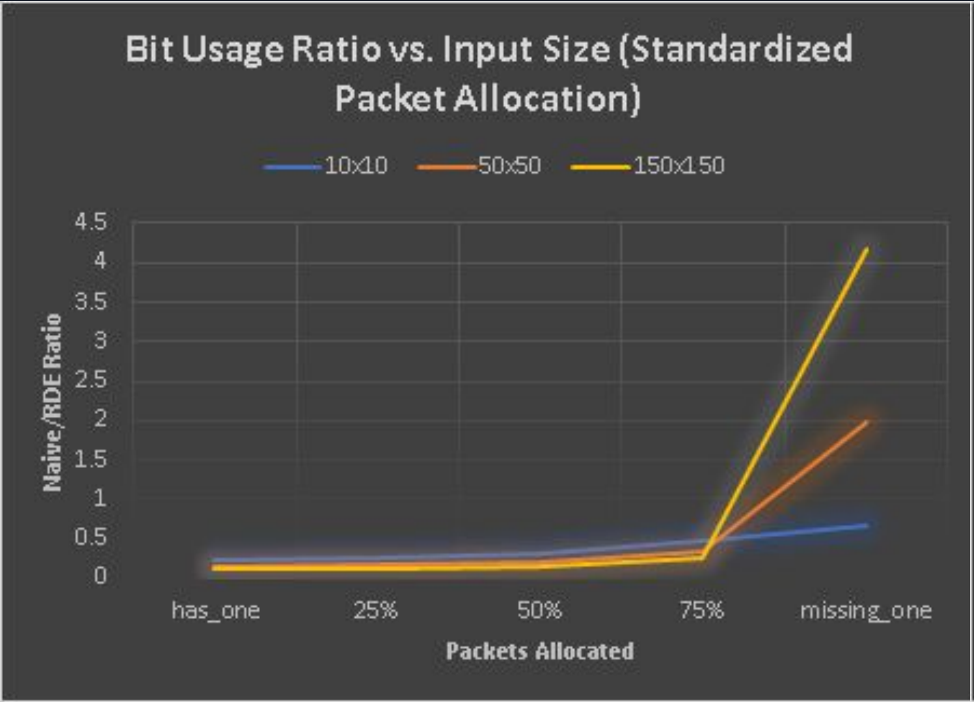


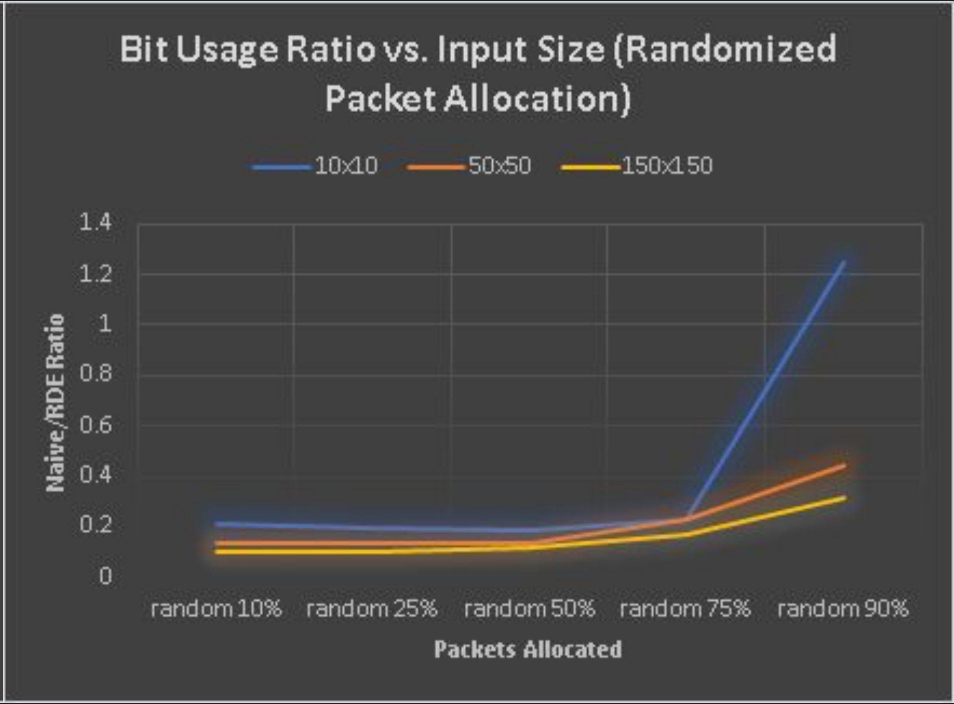
Normalized Difference vs. Packets Allocated to Clients
3x3
10x10
50x50
150x150
Naive/RDE Ratio
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
has_one
25%
50%
75%
missing_one
Packets Allocated (Standardized Allocation)

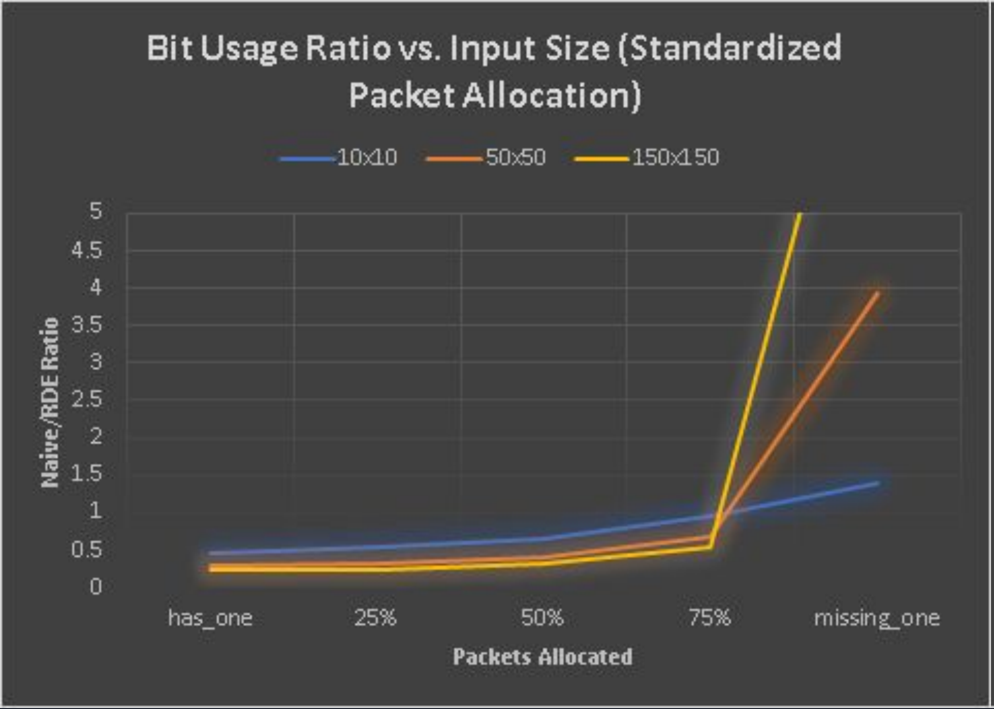
Normalized Difference vs. Packets Allocated to Clients
3x3
10x10
50x50
150x150
Naive/RDE Ratio
0
0.5
1
1.5
2
2.5
3
3.5
random 10%
random 25%
random 50%
random 75%
random 90%
Packets Allocated (Randomized Allocation)

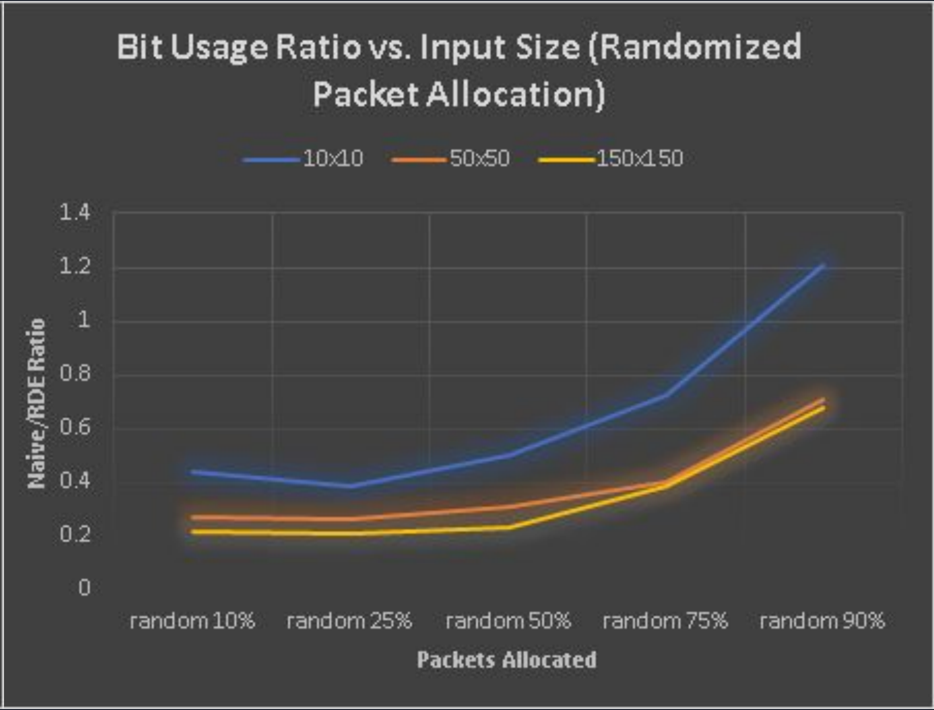
# Results (Binary Codewords)



# Results (Reduced Field Size)



# Summary

- Algorithm finds an optimal solution with probability $1 - \frac{kn}{q}$
  - Guarantee certain probability of success
  - E.g. $q = 4kn \Rightarrow \frac{3}{4}$ prob. of success
- Multiple (simulated) iterations
- Linear coding is a useful technique in general

# References


- [1] Sprintson, Alexander, et al. "A randomized algorithm and performance bounds for coded cooperative data exchange." *ISIT*. 2010.
- [2] https://www.walmart.com/ip/Straight-Talk-Samsung-Galaxy-J7-Sky-Pro-16GB-LTE-No-Contract-Prepaid-SmartPhone-Black/840262752
- [3] https://www.vectorstock.com/royalty-free-vector/radio-base-station-linear-icon-concept-radio-base-vector-20917854
- [4] https://www.hindawi.com/journals/ijdmb/2011/857847/fig3/