

1     **A GENERALISED RANDOM ENCOUNTER MODEL FOR ESTIMATING**  
2                   **ANIMAL DENSITY WITH REMOTE SENSOR DATA**

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## 1. ABSTRACT

**1:** Wildlife monitoring technology has advanced rapidly and the use of remote sensors such as camera traps, and acoustic detectors is becoming common in both the terrestrial and marine environments. Current capture-recapture or distance methods to estimate abundance or density require individual recognition of animals or knowing the distance of the animal from the sensor, which is often difficult. A method without these requirements, the random encounter model (REM), has been successfully applied to estimate animal densities from count data generated from camera traps. However, count data from acoustic detectors do not fit the assumptions of the REM due to the directionality of animal signals.

**2:** We developed a generalised REM (gREM), to estimate absolute animal density from count data from both camera traps and acoustic detectors. We derived the gREM for different combinations of sensor detection widths and animal signal widths (a measure of directionality). We tested the accuracy and precision of this model using simulations of different combinations of sensor detection widths and animal signal widths, number of captures, and models of animal movement.

**3:** We find that the gREM produces accurate estimates of absolute animal density for all combinations of sensor detection widths and animal signal widths. However, larger sensor detection and animal signal widths were found to be more precise. While the model is accurate for all capture efforts tested, the precision of the estimate increases with the number of captures. We found no effect of different animal movement models tested on the accuracy and precision of the gREM.

**4:** We conclude that the gREM provides an effective method to estimate absolute animal densities from remote sensor count data over a range of sensor and animal signal widths. The gREM is applicable for use for count data obtained in both marine and terrestrial environments, visually or acoustically (e.g., big cats, sharks, birds, bats and cetaceans). As sensors such as camera traps and acoustic detectors become more ubiquitous, the gREM will be increasingly useful for monitoring animal populations across broad spatial, temporal and taxonomic scales.

68 1.1. **Keywords.** Acoustic detection, Camera traps, Marine, Population monitor-  
69 ing, Simulations, Terrestrial

## 70 2. INTRODUCTION

71 Animal population density is one of the fundamental measures needed in ecol-  
72 ogy and conservation. The density of a population has important implications for  
73 a range of issues such as sensitivity to stochastic fluctuations (Richter-Dyn & Goel,  
74 1972; Wright & Hubbell, 1983) and risk of extinction (Purvis *et al.*, 2000). Monitor-  
75 ing animal population changes in response to anthropogenic pressure is becoming  
76 increasingly important as humans modify habitats and change climates as never  
77 before (Everatt *et al.*, 2014). Sensor technology, such as camera traps (Rowcliffe &  
78 Carbone, 2008; Karanth, 1995) and acoustic detectors (O’Farrell & Gannon, 1999;  
79 Clark, 1995; Acevedo & Villanueva-Rivera, 2006) are becoming increasingly used  
80 to monitor changes in animal populations (Rowcliffe & Carbone, 2008; Kessel *et al.*,  
81 2014), as they are efficient, relatively cheap and non-invasive (Cutler & Swann,  
82 1999), allowing for surveys over large areas and long periods. However, the prob-  
83 lem of converting sampled count data to estimates of density remains as efforts  
84 must be made to account for detectability of the animals (Anderson, 2001).

85 Methods do already exist for estimating animal density if the distance between  
86 the animal and the sensor can be estimated (e.g., capture-mark recapture meth-  
87 ods (Karanth, 1995) and distance sampling (Harris *et al.*, 2013)). However, these  
88 methods often require additional information that may not be available. For exam-  
89 ple, capture-mark-recapture methods (Karanth, 1995; Trolle & Kéry, 2003; Soisalo  
90 & Cavalcanti, 2006; Trolle *et al.*, 2007) require recognition of individuals; distance  
91 methods require a distance estimation of how far away individuals are from the  
92 sensor barlow2005estimates, marques2011estimating. The development of the ran-  
93 dom encounter model (REM) (a modification of a gas model) enabled animal den-  
94 sities to be estimated from unmarked individuals of a known speed, and sensor  
95 detection parameters (Rowcliffe *et al.*, 2008). The REM method has been success-  
96 fully applied to estimate animal densities from camera trap surveys (Manzo *et al.*,  
97 2012; Zero *et al.*, 2013). However, extending the REM method to other types of  
98 sensors (for example acoustic detectors) is more problematic, because the original

99 derivation assumes a relatively narrow sensor width (up to  $\pi/2$  radians) and that  
100 the animal is equally detectable irrespective of its heading (Rowcliffe *et al.*, 2008).

101 Whilst these restrictions are not problematic for most camera trap makes (e.g.  
102 Reconyx, Cuddeback), the REM could not be used to estimate densities from cam-  
103 era traps with a wider sensor width (e.g. canopy monitoring with fish eye lens  
104 (Brusa & Bunker, 2014)). Additionally, the REM method would not be useful in  
105 estimating densities from acoustic survey data as the acoustic detector angles are  
106 often wider than  $\pi/2$  radians. Acoustic detectors are designed for a range of di-  
107 verse tasks and environments (Kessel *et al.*, 2014), which will naturally lead to  
108 a wide range of sensor detection widths and detection distances. In addition to  
109 this, calls emitted by many animals are directional (breaking the assumption of  
110 the REM method).

111 There has been a sharp rise in interest around passive acoustic detectors in re-  
112 cent years, with a 10 fold increase in publications in the decade between 2000 and  
113 2010 (Kessel *et al.*, 2014). Acoustic monitoring is being developed to study many  
114 aspects of ecology, including the interactions of animals and their environments  
115 (Blumstein *et al.*, 2011; Rogers *et al.*, 2013), the presence and relative abundances of  
116 species (Marcoux *et al.*, 2011), and biodiversity of an area (Depraetere *et al.*, 2012).

117 Acoustic data suffers from many of the problems associated with data from  
118 camera trap surveys in that individuals are often unmarked so capture-make-  
119 recapture methods cannot be used to estimate densities. In some cases the dis-  
120 tance between the animal and the sensor is known, for example when an array of  
121 sensors and the position of the animal is estimated by triangulation (Lewis *et al.*,  
122 2007). In these situations distance-sampling methods can be applied, a method  
123 typically used for marine mammals (Rogers *et al.*, 2013). However, in many cases  
124 distance estimation is not possible, for example when single sensors are deployed,  
125 a situation typical in the majority of terrestrial acoustic surveys (Elphick, 2008;  
126 Buckland *et al.*, 2008). In these cases, only relative measures of local abundance  
127 can be calculated, and not absolute densities. This means that comparison of  
128 populations between species and sites is problematic without assuming equal de-  
129 tectability (Schmidt, 2003). Equality detectability is unlikely because of differences  
130 in environmental conditions, sensor type, habitats, species biology.

131 In this study we create a generalised REM (gREM), as an extension to the cam-  
 132 era trap model of (Rowcliffe *et al.*, 2008), to estimate absolute density from count  
 133 data from acoustic detectors, or camera traps, where the sensor width can vary  
 134 from 0 to  $2\pi$  radians, and the signal given off from the animal can be directional.  
 135 We assessed the accuracy and precision of the gREM within a simulated environ-  
 136 ment, by varying the sensor detection widths, animal signal widths, number of  
 137 captures and models of animal movement. We use the simulation results to rec-  
 138 ommend best survey practice for estimating animal densities from remote sensors.

### 139 3. METHODS

140 **3.1. Analytical Model.** The REM presented by (Rowcliffe *et al.*, 2008) adapts the  
 141 gas model to model count data from camera trap surveys. The REM is derived as-  
 142 suming a stationary sensor with a detection width less than  $\pi/2$  radians. However,  
 143 in order to apply this approach more generally, and in particular to acoustic de-  
 144 tectors, we need both to relax the constraint on sensor detection width, and allow  
 145 for animals with directional signals. Consequently, we derive the gREM for any  
 146 detection width,  $\theta$ , between 0 and  $2\pi$  with a detection distance  $r$  giving a circular  
 147 sector within which animals can be captured (the detection zone)(Figure 1). Ad-  
 148 ditionally, we model the animal as having an associated signal width  $\alpha$  between  
 149 0 and  $2\pi$ (Figure 1, see Appendix S1 for a list of symbols). We start deriving the  
 150 gREM with the simplest situation, the gas model where  $\theta = 2\pi$  and  $\alpha = 2\pi$ .

151 **3.1.1. Gas Model.** Following Yapp (1956), we derive the gas model where sensors  
 152 can capture animals in any direction and animal's signal is detectable from any  
 153 direction( $\theta = 2\pi$  and  $\alpha = 2\pi$ ). We assume that animals are in a homogeneous envi-  
 154 ronment, and move in straight lines of random direction with velocity  $v$ . We allow  
 155 that our stationary sensor can capture animals at a detection distance  $r$  and that if  
 156 an animal moves within this detection zone they are captured with a probability  
 157 of one, while animals outside the zone are never captured.

158 In order to derive animal density, we need to consider relative velocity from  
 159 the reference frame of the animals. Conceptually, this requires us to imagine that  
 160 all animals are stationary and randomly distributed in space, while the sensor  
 161 moves with velocity  $v$ . If we calculate the area covered by the sensor during the

162 survey period we can estimate the number of animals the sensor should capture.  
 163 As a circle moving across a plane, the area covered by the sensor per unit time is  
 164  $2rv$ . The number of expected captures,  $z$ , for a survey period of  $t$ , with an animal  
 165 density of  $D$  is  $z = 2rvtD$ . To estimate the density, we rearrange to get  $D = z/2rvt$ .

166 3.1.2. *gREM derivations for different detection and signal widths.* Different combina-  
 167 tions of  $\theta$  and  $\alpha$  would be expected to occur (e.g., sensors have different detection  
 168 widths and animals have different signal widths). For different combinations  $\theta$   
 169 and  $\alpha$ , the area covered per unit time is no longer given by  $2rv$ . Instead of the size  
 170 of the sensor detection zone having a diameter of  $2r$ , the size changes with the  
 171 approach angle between the sensor and the animal. For any given signal width  
 172 and detector width and depending on the angle that the animal approaches the  
 173 sensor, the width of the area within which an animal can be detected is called the  
 174 profile,  $p$ . The size of the profile (averaged across all approach angles) is defined  
 175 as the average profile  $\bar{p}$ . However, different combinations of  $\theta$  and  $\alpha$  need different  
 176 equations to calculate  $\bar{p}$ .

177 We have identified the parameter space for the combinations of  $\theta$  and  $\alpha$  for  
 178 which the derivation of the equations are the same (defined as sub-models in the  
 179 gREM) (Figure 2). For example, the gas model becomes the simplest gREM sub-  
 180 model (upper right in (Figure 2) and the REM from (Rowcliffe *et al.*, 2008) is an-  
 181 other gREM sub-model where  $\theta < \pi/2$  and  $\alpha = 2\pi$ . We derive one gREM sub-model  
 182 SE2 as an example below (where  $4\pi - 2\alpha < \theta < 2\pi$ ,  $0 < \alpha < \pi$ ) (see Appendix S2 for  
 183 other gREM sub-models).

184 3.1.3. *Example derivation of SE2.* In order to calculate  $\bar{p}$ , we have to integrate over  
 185 the focal angle,  $x_1$  (Figure 3a). This is the angle taken from the centre line of the  
 186 sensor. Other focal angles are possible ( $x_2, x_3, x_4$ ) and are used in other gREM  
 187 sub-models (see Appendix S2). As the size of the profile depends on the approach  
 188 angle, we present the derivation across all approach angles. When the sensor is  
 189 directly approaching the animal  $x_1 = \pi/2$ .

190 Starting from  $x_1 = \pi/2$  until  $\theta/2 + \pi/2 - \alpha/2$ , the size of the profile is  $2r \sin \alpha/2$   
 191 (Figure 3b). During this first interval, the size of  $\alpha$  limits the width of the profile.  
 192 When the animal reaches  $x_1 = \theta/2 + \pi/2 - \alpha/2$  (Figure 3c), the size of the profile is

193  $r \sin(\alpha/2) + r \cos(x_1 - \theta/2)$  and the size of  $\theta/$  and  $\alpha$  both limit the width of the profile  
 194 (Figure 3c). Finally, at  $x_1 = 5\pi/2 - \theta/2 - \alpha/2$  until  $x_1 = 3\pi/2$ , the width of the profile  
 195 is again  $2r \sin \alpha/2$  (Figure 3d) and the size of  $\alpha$  again limits the width of the profile.

196 The profile width  $p$  for  $\pi$  radians of rotation (from directly towards the sensor  
 197 to directly behind the sensor) is completely characterised by the three intervals  
 198 (Figure 3b–d). Average profile width  $\bar{p}$  is calculated by integrating these profiles  
 199 over their appropriate intervals of  $x_1$  and dividing by  $\pi$  which gives

$$\bar{p} = \frac{1}{\pi} \left( \int_{\frac{\pi}{2}}^{\frac{\pi}{2} + \frac{\theta}{2} - \frac{\alpha}{2}} 2r \sin \frac{\alpha}{2} dx_1 + \int_{\frac{\pi}{2} + \frac{\theta}{2} - \frac{\alpha}{2}}^{\frac{5\pi}{2} - \frac{\theta}{2} - \frac{\alpha}{2}} r \sin \frac{\alpha}{2} + r \cos \left( x_1 - \frac{\theta}{2} \right) dx_1 + \int_{\frac{5\pi}{2} - \frac{\theta}{2} - \frac{\alpha}{2}}^{\frac{3\pi}{2}} 2r \sin \frac{\alpha}{2} dx_1 \right) \quad \text{eqn 1}$$

$$= \frac{r}{\pi} \left( \theta \sin \frac{\alpha}{2} - \cos \frac{\alpha}{2} + \cos \left( \frac{\alpha}{2} + \theta \right) \right) \quad \text{eqn 2}$$

200 We then, as with the gas model, use this expression to calculate density

$$201 \quad D = z/vt\bar{p}. \quad \text{eqn 3}$$

202 Rather than having one equation that describes  $\bar{p}$  globally, the gREM must be  
 203 split into submodels due to discontinuous changes in  $p$  as  $\alpha$  and  $\beta$  change. These  
 204 discontinuities can occur for a number of reasons such as a profile switching be-  
 205 tween being limited by  $\alpha$  and  $\theta$ , the difference between very small profiles and  
 206 profiles of size zero and the fact that the width of a sector stops increasing once  
 207 the central angle reaches  $\pi$  radians (i.e., a semi circle is just as wide as a full circle.)

208 As a visual example, if  $\alpha$  is small, there is an interval between Fig. 3c and 3d  
 209 where the ‘blind spot’ would prevent animals being detected at all giving  $p = 0$ .  
 210 This would require an extra integral in our equation as simply putting our small  
 211 value of  $\alpha$  into eqn 1 would not give us this integral of  $p = 0$ .

212 gREM submodel specifications were done by hand, and the integration was  
 213 done using SymPy (SymPy Development Team, 2014) in Python (Appendix S3).  
 214 The gREM submodels were checked by confirming that: 1) submodels adjacent  
 215 in parameter space were equal at the boundary between them; 2) submodels that  
 216 border  $\alpha = 0$  had  $p = 0$  when  $\alpha = 0$ ; 3) average profile widths  $\bar{p}$  were between 0 and



217  $2r$  and; 4) each integral, divided by the range of angles that it was integrated over,  
 218 was between 0 and  $2r$ . The scripts for these tests are included in Appendix S3 and  
 219 the R (R Development Core Team, 2010) implementation of the gREM is given in  
 220 Appendix S4.

221 **3.2. Simulation Model.** We tested the accuracy and precision of the gREM by de-  
 222 veloping a spatially explicit simulation of the interaction of sensors and animals  
 223 using different combinations of sensor detection widths, animal signal widths,  
 224 number of captures, and models of animal movement. 100 simulations were run  
 225 where each consisted of a 7.5 km by 7.5 km square (with periodic boundaries). A  
 226 stationary sensor of radius  $r$  was set up in the exact centre of each simulation, cov-  
 227 ering 7 sensor detection widths  $\theta$  between 0 and  $2\pi$  ( $2/9\pi$ ,  $4/9\pi$ ,  $6/9\pi$ ,  $8/9\pi$ ,  $10/9\pi$ ,  
 228  $14/9\pi$ ,  $2\pi$ ). Each simulation was populated with a density of 70 animals  $\text{km}^{-2}$ , cal-  
 229 culated from the equation in Damuth (1981) as the expected density of mammals  
 230 of weighing 1 g. This density therefore represents the highest likely density of in-  
 231 dividualls, given that the smallest mammal is around 2 g Jones *et al.* (2009). A total  
 232 of 3937 individuals per simulation were created which were placed randomly at  
 233 the start of the simulation. Individuals were assigned 11 signal detection widths  
 234  $\alpha$  between 0 and  $\pi$  ( $1/11\pi$ ,  $2/11\pi$ ,  $3/11\pi$ ,  $4/11\pi$ ,  $5/11\pi$ ,  $6/11\pi$ ,  $7/11\pi$ ,  $8/11\pi$ ,  $9/11\pi$ ,  
 235  $10/11\pi$ ,  $\pi$ ).

236 Each simulation lasted for  $N$  steps (14400) of duration  $T$  (15 minutes) giving a  
 237 total duration of 150 days. The individuals moved within each step with a distance  
 238  $d$ , with an average speed,  $v$ .  $d$ , was sampled from a normal distribution with  
 239 mean distance,  $\mu_d = vT$ , and standard deviation  $\sigma_d = vT/10$ . An average speed,  
 240  $v = 40 \text{ km days}^{-1}$ , was chosen as this represents the largest day range of terrestrial  
 241 animals (Carbone *et al.*, 2005), and represents the upper limit of realistic speeds.  
 242 At the end step, individuals were allowed to either remain stationary for a time  
 243 step (with a given probability,  $S$ ), change direction (with a maximum angle,  $A$ )  
 244 between 0 and  $\pi$ . This resulted in 7 different movement models where: (1) simple  
 245 movement, where  $S$  and  $A = 0$ ; (2) stop-start movement, where (i)  $S = 0.25$ ,  $A = 0$ ,  
 246 (ii)  $S = 0.5$ ,  $A = 0$ , (iii)  $S = 0.75$ ,  $A = 0$ ; (3) random walk movement, where (i)  $S =$

247 0,  $A = \pi/3$ , (ii)  $S = 0$ ,  $A = 2\pi/3$ , (iii)  $S = 0$ ,  $A = \pi$ . Individuals were counted as they  
 248 moved in and out of the detection zone of the sensor per simulation.

249 We calculated the estimated animal density from the gREM by summing the  
 250 number of captures per simulation and inputting these values into the correct  
 251 gREM submodel. gREM accuracy was determined by comparing the density in  
 252 the simulation with the estimated density. High accuracy is indicated by the mean  
 253 difference between the estimated and actual values not being significantly differ-  
 254 ent from zero (Wilcoxon signed-rank test). gREM precision was determined by  
 255 the standard deviation of estimated densities. We used this method to compare  
 256 the accuracy and precision of all the gREM submodels. As these submodels are  
 257 derived for different combinations of  $\alpha$  and  $\theta$ , the accuracy and precision of the  
 258 submodels was used to determine the impact of different values of  $\alpha$  and  $\theta$ .

259 The influence of the number of captures and animal movement models on accu-  
 260 racy and precision was investigated using 4 different gREM submodels represen-  
 261 tative of the range  $\alpha$  and  $\theta$  values (submodels NW1, SW1, NE1, and SE3, Figure 2).  
 262 Using these four submodels, we calculated how long the simulation needed to  
 263 run to generate a range of different capture numbers (from 10 to 100 captures in  
 264 10 unit intervals), and estimated animal density. These estimated densities were  
 265 compared to the real density to assess the impact on the accuracy and precision  
 266 on the gREM of different simulation lengths. We also used these four submodels  
 267 to compare the accuracy and precision of a simple movement model, to stop-start  
 268 movement models and random walk movement models. The gREM assumes that  
 269 individuals move continuously with straight-line movement (simple movement  
 270 model) and we therefore assessed the impact of breaking the gREM assumptions.

## 271 4. RESULTS

272 **4.1. Analytical model.** The equation for  $\bar{p}$  has been newly derived for each sub-  
 273 model in the gREM, except for the gas model and REM which have been calculated  
 274 previously. However, many models, although derived separately, have the same  
 275 expression for  $\bar{p}$ . Figure 4 shows the expression for  $\bar{p}$  in each case. The general  
 276 equation for density, using the correct expression for  $\bar{p}$  is then substituted into  
 277 eqn 3. Although more thorough checks are performed in Appendix S3, it can be

278 seen that all adjacent expressions in Figure 4 are equal when expressions for the  
279 boundaries between them are substituted in.

## 280 4.2. Simulation model.

281 4.2.1. *gREM submodels*. All gREM submodels showed a high accuracy, i.e., the  
282 mean difference between the estimated and actual values was not significantly  
283 different from zero across all models, corrected for multiple tests (all gREM sub  
284 models Wilcoxon signed-rank test,  $p > 0.002$ )(Figure 5). However, the precision of  
285 the submodels do vary, where the gas model is the most precise and the SW7 sub  
286 model the least precise, having the smallest and the largest interquartile range, re-  
287 spectively (Figure 5). The standard deviation of the error between the estimated  
288 and true densities is strongly related to both the sensor and signal widths (Fig-  
289 ure 6), such that larger widths have lower standard deviations (greater precision).  
290 However, even smaller sensor and signal widths have a relatively high level of  
291 precision.

292 4.2.2. *Number of captures*. Within the four gREM submodels tested (NW1, SW1,  
293 SE3, NE1), the accuracy was not affected by the number of captures, where the  
294 mean difference between the estimated and actual values was not significantly dif-  
295 ferent from zero across all capture rates, corrected for multiple tests (all gREM sub  
296 models Wilcoxon signed-rank test,  $p > 0.008$ )(Figure 7). However, the precision  
297 was dependent on the number of captures across all four of the gREM submodels,  
298 where precision increases as number of captures increases(Figure 7). For all gREM  
299 submodels, the the coefficient of variation falls to 10% at 100 captures.

300 4.2.3. *Movement models*. Within the four gREM submodels tested (NW1, SW1, SE3,  
301 NE1), neither the accuracy or precision was affected by the amount of time spent  
302 stationary. The mean difference between the estimated and actual values was not  
303 significantly different from zero for each category of stationary time (0, 0.25, 0.5  
304 and 0.75), corrected for multiple tests (all gREM sub models Wilcoxon signed-rank  
305 test,  $p > 0.12$ )(Figure 8a). Altering the maximum change in direction in each step  
306 (0,  $\pi/3$ ,  $2\pi/3$ , and  $\pi$ ) did not affect the accuracy or precision of the four gREM

submodels tested (all gREM sub models Wilcoxon signed-rank test,  $p > 0.05$ )(Figure 8b).

## 5. DISCUSSION

We have developed the gREM such that it can be used to estimate density from acoustic sensors and camera traps. This has entailed a generalisation of the gas model and the REM in (Rowcliffe *et al.*, 2008) to be applicable to any combination of sensor width and signal directionality. We have used simulations to show, as a proof of principle, that these models are accurate and precise. The precision of the gREM was found to be dependent on the width of the sensor and the call, and the number of captures.

**5.1. Analytical model.** The gREM was derived for different combinations of  $\alpha$  and  $\theta$  resulting in 25 different submodels, the expression for  $\bar{p}$  are equal for many of these submodels resulting in 8 different equations including the previously derived gas model and REM. These submodels were tested for consistency with adjacent expressions being equal at their boundaries. These new submodels will allow researchers to evaluate the absolute density of animals that have previously been difficult to study with noninvasive methods such as remove sensors. The gREM allows the data from acoustic detectors to be used where an animal has a directional calls, this could be used for a range of animals including bats, songbirds, Cetaceans and forest primates.

There are a number of positive extensions to the gREM which could be developed in the future. The original gas model was formulated for the case where both subjects, either animal and detector, or animal and animal, are moving (Hutchinson & Waser, 2007). Indeed any of the models with animals that are equally detectable in all directions ( $\alpha = 2\pi$ ) can be trivially expanded for moving by substituting the sum of the average animal velocity and the sensor velocity for  $v$  as used here. However, when the animal has a directional call, the extension becomes less simple. The approach would be to calculate again the mean profile width. However, for each angle of approach, one would have to average the profile width for an animal facing in any direction (i.e. not necessarily moving towards the sensor) weighted by the relative velocity of that direction. There are a number of

situations where a moving detector and animal could occur and as such may be advantage to have a method of estimating densities from the data collected, e.g. an acoustic detector based off a boat when studying Cetacea or sea birds (Yack *et al.*, 2013). Another interesting, and so far unstudied problem, is edge effects caused by trigger delays (the delay between sensing an animal and attempting to record the encounter) and time expansion acoustic detectors which repeatedly turn on an off during sampling. Both of these have potential biases as animals can move through the detection zone without being detected. The models herein are formulated assuming constant surveillance and so the error quickly becomes negligible. For example, if it takes longer for the recording device to be switched on than the length of some animal calls there could be a systematic underestimation of density.

**5.2. Accuracy and Precision.** We tested each of the gREM submodels for accuracy and precision through a simulation. All the submodels produced estimated densities that were not significantly different from the true density of the simulation. Therefore based on these simulations we believe that the gREM has the potential to produce accurate estimates for many different species, using either camera traps or acoustic detectors. However the precision of the gREM differed between submodels. For example, when the sensor and signal width were smaller then the precision of the model was reduced, so when choosing a sensor for use in a gREM study the detection width should be maximised, and if the study species has a narrow signal directionality other aspects of the study protocol should be used to compensate.

The precision of the gREM is greatly affected by the number of captures that are collected, the coefficient of variation falls dramatically between 10 and 60 captures and then after this continues to slowly reduce. At 100 captures the submodels reach 10% coefficient of variation, and therefore we believe at this point the models are precise. The length of surveys in the field will need to be adjusted so that enough data is collected to reach this level of precision, populations of fast moving animals or populations with large densities will require less survey effort than those with slow moving or low densities.

368 The gREM was both accurate and precise for all the movement models we  
 369 tested against, stop-start movement and correlated random walks. However these  
 370 movement models are still simple representations of true animal movement which  
 371 often consist of multiple be dependent on multiple factors such as behavioural  
 372 state and and existence of home ranges (?). The accuracy of the gREM may be  
 373 affected by the interaction between the movement model and the size of the detec-  
 374 tion radius. In figure 8b we studied a relatively long step length compared to the  
 375 size of the detection radius, and therefore the chance of catching the same animal  
 376 multiple times within a short space of time was reduced. However if the ratio of  
 377 step length to detection radius was smaller then this may decrease the precision of  
 378 the model, however this should not decrease its accuracy.

379 Although we have used simulations to validate the gREM submodels, much  
 380 more robust testing is needed. Although difficult, proper field test validation  
 381 would be required before the models could be fully trusted. The REM (Rowcliffe  
 382 *et al.*, 2008) has already been field tested, and both Rowcliffe *et al.* (2008) and Zero  
 383 *et al.* (2013) both found that the REM was an effective manner of estimating animal  
 384 densities (Rowcliffe *et al.*, 2008; Zero *et al.*, 2013). In some taxa gold standard meth-  
 385 ods of estimating animal density exist, such as capture mark recapture. Where  
 386 these gold standard exist, and have been proved to work, a simultaneous gREM  
 387 study could be completed to test the accuracy under field conditions. An eas-  
 388 ier way to continue to evaluate the models is to run more extensive simulations  
 389 which break the assumptions of the analytical models. The main element that  
 390 cannot be analytically treated is the complex movement of real animals. There-  
 391 fore testing these methods against true animal traces, or more complex movement  
 392 models would be required.

393 Within the simulation we have assumed an equal density across the entire world,  
 394 however in a field environment the situation would be much more complex, with  
 395 additional variation coming from local changes in density between camera sites.  
 396 In the simulation we ran the speed of the animal as  $40 \text{ km days}^{-1}$ , the largest day  
 397 range of terrestrial animals (Carbone *et al.*, 2005), other speed values should not  
 398 alter the accuracy or the precision of the gREM. We also assume perfect knowl-  
 399 edge of the average speed of an animal and size of the detection zone, and instant

400 triggering of the camera. All of which may lead to possible bias or a decrease in  
401 precision.

402 **5.3. Implications for conservation.** The gREM is therefore available for the esti-  
403 mation of density of a number of taxa of importance to conservation, zoonotic dis-  
404 eases and ecosystem services. The models provided are suitable for certain groups  
405 for which there are currently no, or few, effective methods for density estimation.  
406 Any species that would be consistently recorded at least once when within range  
407 of a detector would be a suitable subject for the gREM, such as bats (Kunz *et al.*,  
408 2009), songbirds (Buckland & Handel, 2006), Cetaceans (Marques *et al.*, 2009) or  
409 forest primates (Hassel-Finnegan *et al.*, 2008). Within increasing technological ca-  
410 pabilities, this list of species is likely to increase dramatically.

411 Importantly the methods are noninvasive and do not require human marking or  
412 naturally identifying marks (as required for mark-recapture models). This makes  
413 them suitable for large, continuous monitoring projects with limited human re-  
414 sources. It also makes them suitable for species that are under pressure, species  
415 that cannot naturally be individually recognised or species that are difficult or  
416 dangerous to catch.

## 417 6. ACKNOWLEDGMENTS

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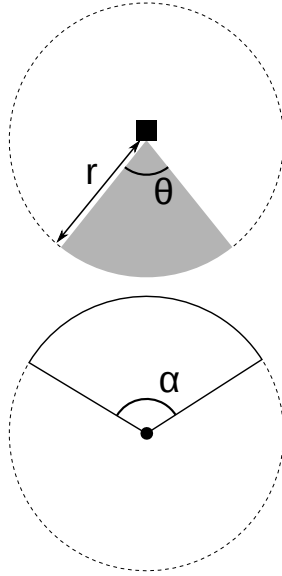


FIGURE 1. Representation of sensor detection width and animal signal width. The filled square and circle represent a sensor and an animal, respectively;  $\theta$ , sensor detection width (radians);  $r$ , sensor detection distance; dark grey shaded area, sensor detection zone;  $\alpha$ , animal signal width (radians). Dashed lines around the filled square and circle represents the maximum extent of  $\theta$  and  $\alpha$ , respectively.

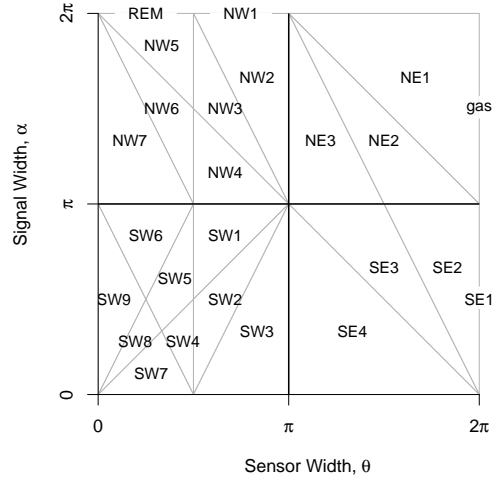


FIGURE 2. Locations where derivation of the average profile  $\bar{p}$  is the same for different combinations of sensor detection width and animal signal width. Symbols within each polygon refer to each gREM submodel named after their compass point, except for Gas and REM which highlight the position of these previously derived models within the gREM. Symbols on the edge of the plot are for submodels with  $\alpha, \theta = 2\pi$

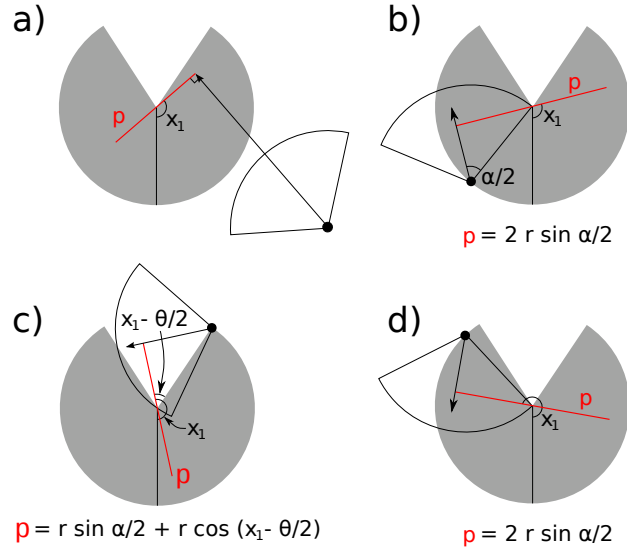


FIGURE 3. An overview of the derivation of SE2. The filled circles represent animals, with the animal signal shown as a unfilled sector and the direction of movement shown as an arrow. The detection zone of the sensors are shown as filled grey sectors with a detection distance of  $r$ . The SYMBOL shows the direction the sensor is facing;  $\theta$ , sensor detection width;  $\alpha$ , animal signal width. The profile  $p$  (the line an animal must pass through in order to be captured) is shown in red and  $x_1$  is the focal angle, where (a) shows the location of  $x_1$ . The derivation of  $p$  changes as the animal approaches the sensor from different directions where (b) is the derivation of  $p$  when  $x_1$  is in the interval  $[\frac{\pi}{2}, \frac{\pi}{2} + \frac{\theta}{2} - \frac{\alpha}{2}]$ , (c)  $p$  when  $x_1$  is in the interval  $[\frac{\pi}{2} + \frac{\theta}{2} - \frac{\alpha}{2}, \frac{5\pi}{2} - \frac{\theta}{2} - \frac{\alpha}{2}]$  and (d)  $p$  when  $x_1$  is in the interval  $[\frac{5\pi}{2} - \frac{\theta}{2} - \frac{\alpha}{2}, \frac{3\pi}{2}]$ . The resultant equation for  $p$  is shown beneath each figure.

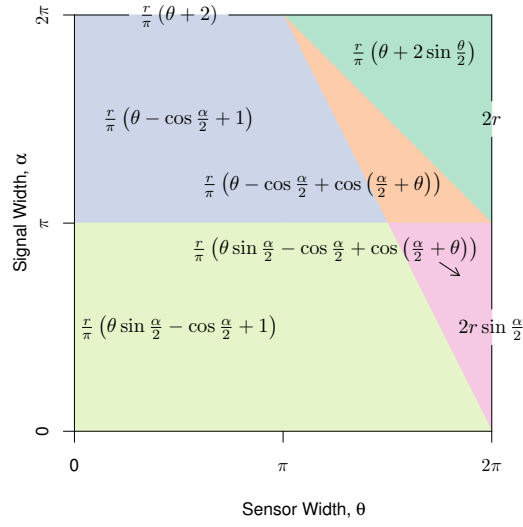


FIGURE 4. Expressions for the average profile width,  $\bar{p}$ , given sensor and signal widths. Despite independent derivation within each block, many models result in the same expression. These are collected together and presented as one block of colour. Expressions on the edge of the plot are for submodels with  $\alpha, \theta = 2\pi$ .

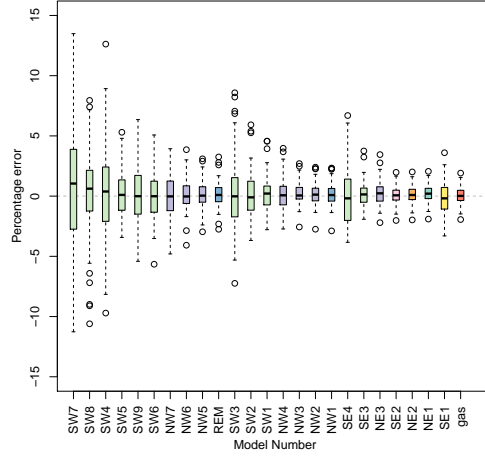


FIGURE 5. Simulation model results of the accuracy and precision for gREM submodels. The percentage error between estimated and true density for each gREM submodel is shown within each box plot, where the black line represents the median percentage error across all simulations, boxes represent the the middle 50% of the data. Box colours correspond to the expressions for average profile width  $\bar{p}$  given in `f:equalModelResults`.



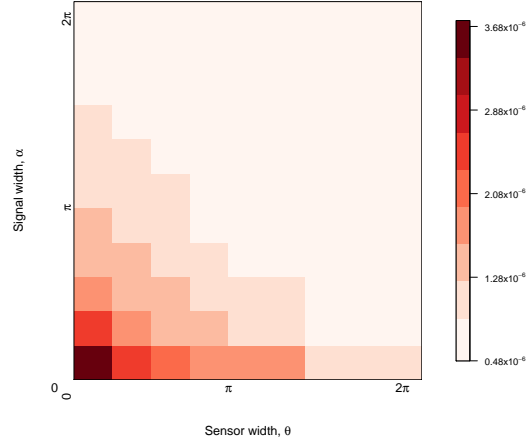


FIGURE 6. Simulation model results of the gREM precision given a range of sensor and signal widths, shown by the standard deviation of the error between the estimated and true densities. Standard deviations are shown from deep red to pink, representing high to low values between  $0.483 \times 10^{-6}$  to  $3.74 \times 10^{-6}$ .

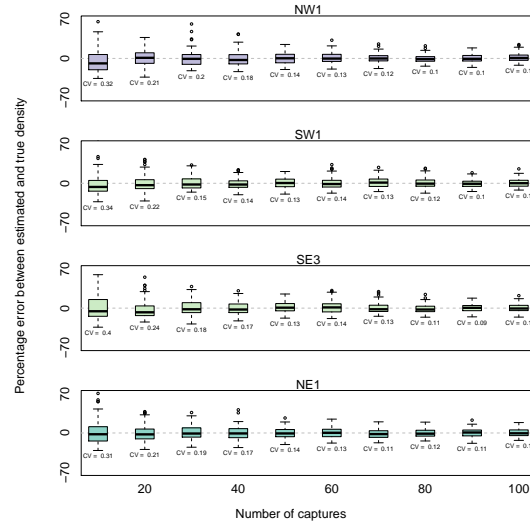


FIGURE 7. Simulation model results of the accuracy and precision of four gREM submodels (NW1, SW1, SE3 and NE1) given different numbers of captures. The percentage error between estimated and true density within each gREM sub model for capture rate is shown within each box plot. Sensor and signal widths vary between submodels. The colour of each box plot corresponds to the expressions for average profile width  $\bar{p}$  given in Figure 4.

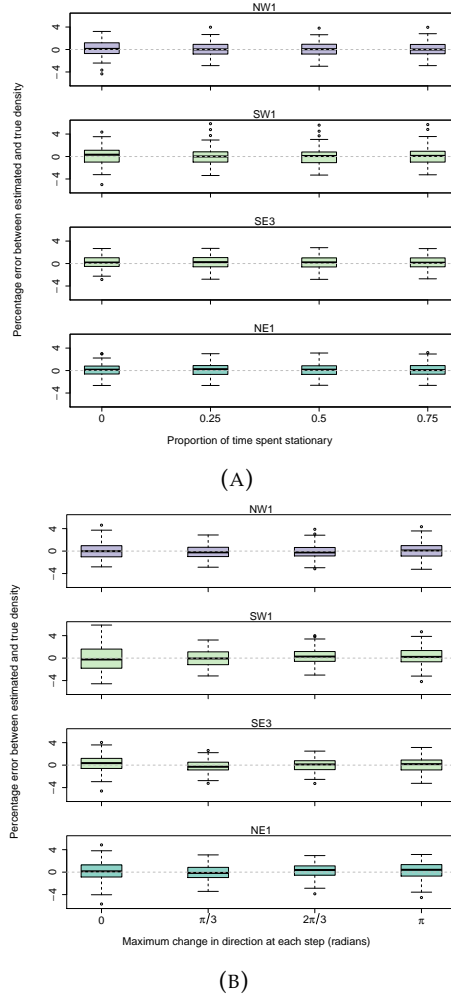


FIGURE 8. Simulation model results of the accuracy and precision of four gREM submodels (NW1, SW1, SE3 and NE1) given different movement models where (A) amount of time spent stationary (stop-start movement) and (B) maximum change in direction at each step (correlated random walk model). The percentage error between estimated and true density within each gREM sub model for the different movement models is shown within each box plot. The simple model is represented where time and maximum change in direction equals 0. The colour of each box plot corresponds to the expressions for average profile width  $\bar{p}$  given in 4.