project1_pt3_corr

March 10, 2023

[ ]:
```
import numpy as np
from scipy.integrate import solve_ivp, odeint
from scipy.fftpack import fft
from scipy.signal import find_peaks, hilbert
from tqdm import tqdm
import matplotlib.pyplot as plt
import pywt
import nolds
import pickle
# Define the FitzHugh-Nagumo model equations
def fitzhugh_nagumo_coupled(t, xy,alpha , w2, a, b, c, k1, k2):
    x, y , x1, y1= xy
    dxdt = alpha*(y + x - (x**3)/3 + (k1 + c*x1))
    dydt = -(1/alpha) * (w2*x - a + b*y)
    dx1dt = alpha*(y1 + x1 - (x1**3)/3 + (k2 + c*x))
    dy1dt = -(1/alpha) * (w2*x1 - a + b*y1)
    return [dxdt, dydt, dx1dt, dy1dt]

alpha = 3
a = 0.7
b = 0.8
w2 = 1
x0 = 0
y0 = 0
x10 = 0
y10 = 0
xy0=[x0, x10, y0, y10]

n =30
c_vals = np.linspace(-0.3,0,num=n)
k1_vals = np.array([-0.3 for i in range(n)])
k2_vals = np.linspace(-0.65,-0.9, num=n)
t_span = [0, 200]
t_eval = np.linspace(0,100,5000)
```

[ ]:
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lyap_map_x1 = np.zeros((c_vals.shape[0], k1_vals.shape[0]))
lyap_map_x2 = np.zeros((c_vals.shape[0], k1_vals.shape[0]))
```

```
for i, c in enumerate(c_vals):
    for j, k2 in tqdm(enumerate(k2_vals)):
        k1 = k1_vals[len(k1_vals)-1-j]
        sol = solve_ivp(fitzhugh_nagumo_coupled, t_span, xy0,␣
↪args=(alpha,w2,a,b,c,k1,k2),t_eval=t_eval,dense_output=True,rtol=1e-8)

        s1 = sol.y[0]
        s2 = sol.y[2]

        #Apply hamming filter
        window = np.hamming(s1.shape[0])
        s1 = s1 * window
        s2 = s2 * window

        #Calculate R index
        lyap_x1 = nolds.corr_dim(s1, emb_dim=10)
        lyap_x2 = nolds.corr_dim(s2, emb_dim=10)

        #Save to response map
        lyap_map_x1[i, j] = lyap_x1
        lyap_map_x2[i, j] = lyap_x2
```

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[ ]:
```python
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    pickle.dump(lyap_map_x1, f)

with open("lyapmapx2.pkl", 'wb') as f:
    pickle.dump(lyap_map_x2, f)
```

[ ]:
```python
lyap_map_x1
```

[ ]:
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```

[ ]:
```
K = []
C = []
for i, c in tqdm(enumerate(c_vals)):
    C.append(c)
for j, k2 in enumerate(k2_vals):
    K.append((k2+0.3)/(-0.3))



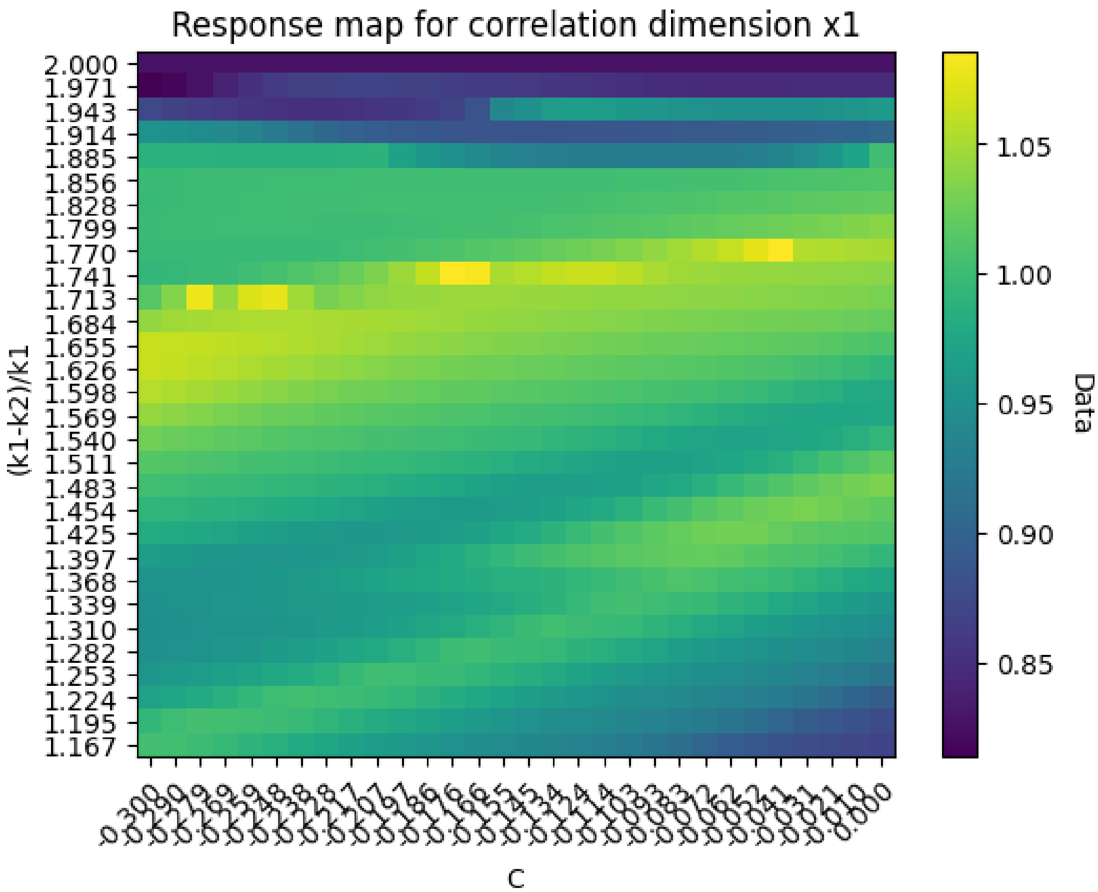
# plt.imshow(Rmap, cmap='viridis', origin='lower', aspect='auto',)
fig, ax = plt.subplots()
im = ax.imshow(lyap_map_x1, cmap='viridis', origin='lower', aspect='auto',)

# Set tick labels
ax.set_xticks(np.arange(len(C)))
ax.set_yticks(np.arange(len(K)))
ax.set_xticklabels([f"{c:.3f}" for c in C])
ax.set_yticklabels([f"{k:.3f}" for k in K])

plt.setp(ax.get_xticklabels(), rotation=45, ha="right", rotation_mode="anchor")
cbar = ax.figure.colorbar(im, ax=ax)
cbar.ax.set_ylabel('Data', rotation=-90, va="bottom")

# Add axis labels
ax.set_xlabel('C')
ax.set_ylabel('(k1-k2)/k1')
ax.set_title("Response map for correlation dimension x1")
# Show plot
plt.show()
```

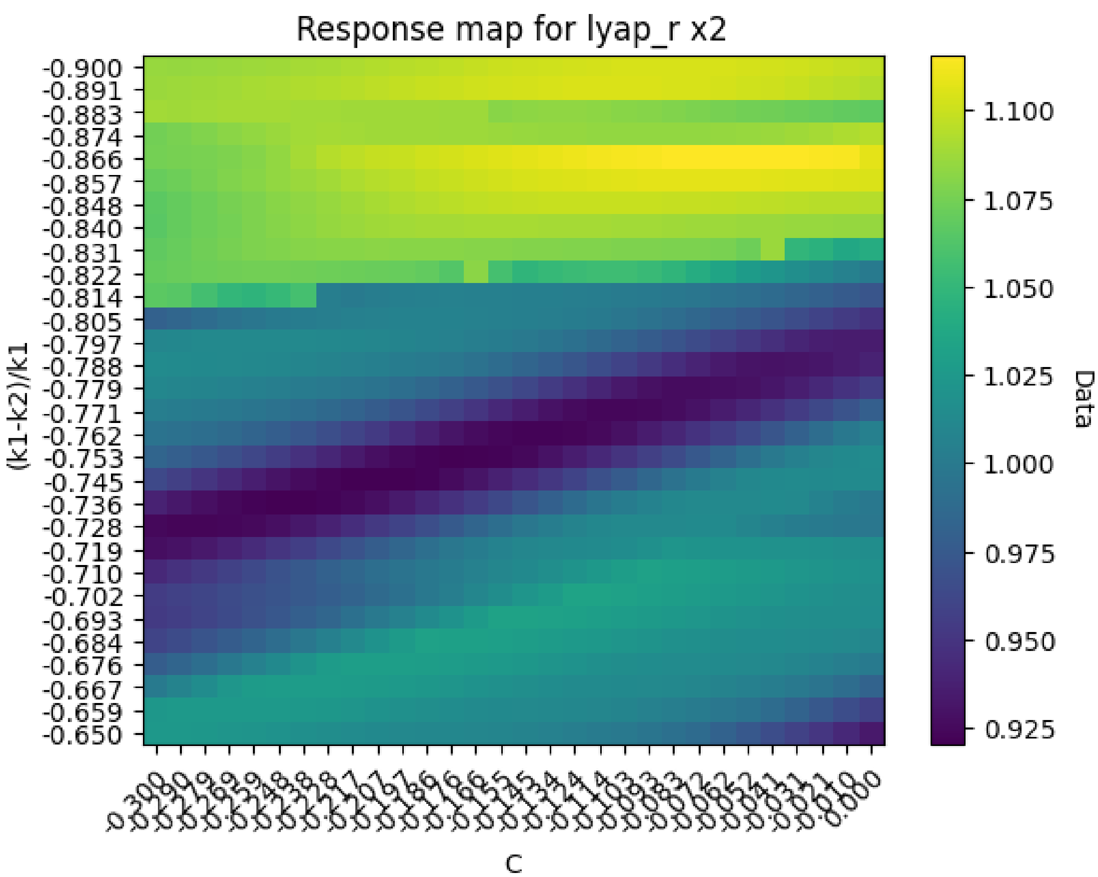
```
30it [00:00, 154581.23it/s]
```



[ ]:
```python
K = []
C = []
for i, c in tqdm(enumerate(c_vals)):
    C.append(c)
# for j, k2 in enumerate(k2_vals):
#     K.append((k2+0.3)/(-0.3))
for k2 in k2_vals:
    K.append(k2)

# plt.imshow(Rmap, cmap='viridis', origin='lower', aspect='auto',)
fig, ax = plt.subplots()
im = ax.imshow(lyap_map_x2, cmap='viridis', origin='lower', aspect='auto',)

# Set tick labels
ax.set_xticks(np.arange(len(C)))
ax.set_yticks(np.arange(len(K)))
ax.set_xticklabels([f"{c:.3f}" for c in C])
ax.set_yticklabels([f"{k:.3f}" for k in K])
```

```
plt.setp(ax.get_xticklabels(), rotation=45, ha="right", rotation_mode="anchor")
cbar = ax.figure.colorbar(im, ax=ax)
cbar.ax.set_ylabel('Data', rotation=-90, va="bottom")

# Add axis labels
ax.set_xlabel('C')
ax.set_ylabel('(k1-k2)/k1')
ax.set_title("Response map for correlation dimension x2")
# Show plot
plt.show()
```

30it [00:00, 197533.94it/s]



[ ]:

```
```

[ ]:

```
k1 = -0.3
k2 = -0.872
c = -0.200
```

```
# k1 = -0.3
# k2 = -0.68
# c = -0.3
sol = solve_ivp(fitzhugh_nagumo_coupled, t_span, xy0,␣
 ↪args=(alpha,w2,a,b,c,k1,k2), t_eval=t_eval, dense_output=True,rtol=1e-8)
t = sol.t
s1 = sol.y[0]
s2 = sol.y[2]
```

[ ]:
```
nolds.lyap_r(s1)
```

```
/Users/timgu/opt/anaconda3/envs/ece1786/lib/python3.8/site-
packages/nolds/measures.py:240: RuntimeWarning: signal has very low mean
frequency, setting min_tsep = 1250
  warnings.warn(msg.format(min_tsep), RuntimeWarning)
```

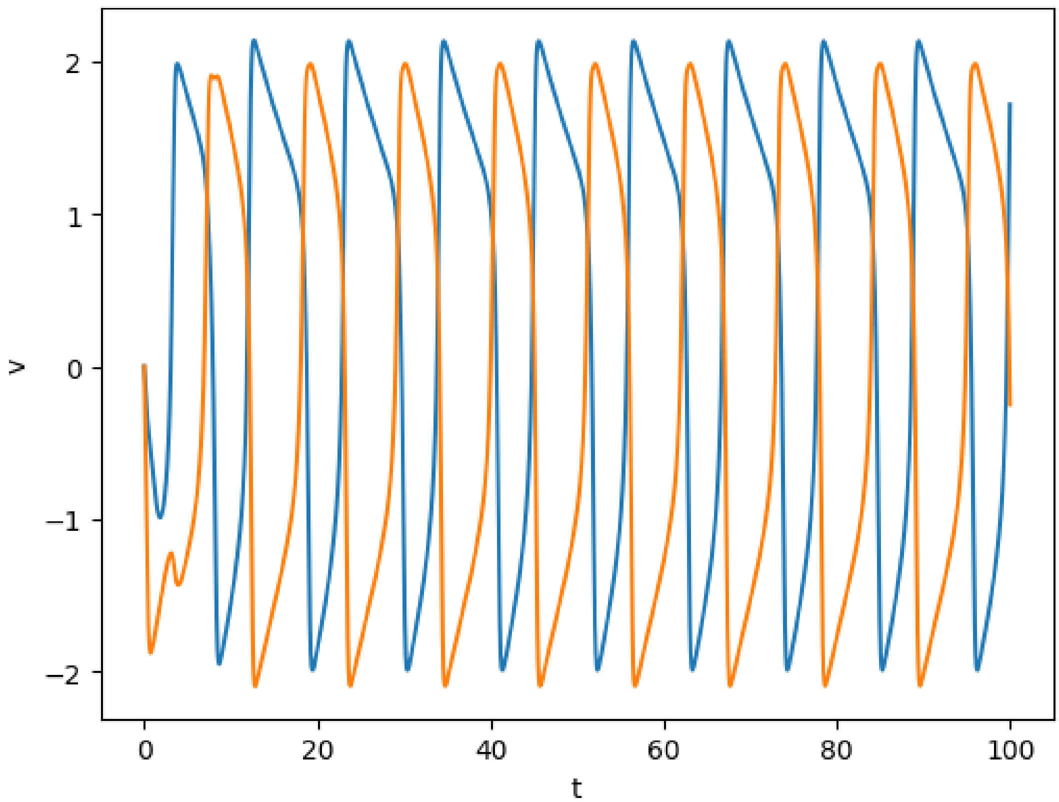
[ ]: 0.0018533649301170285

[ ]:
```
nolds.lyap_r(s2)
```

```
/Users/timgu/opt/anaconda3/envs/ece1786/lib/python3.8/site-
packages/nolds/measures.py:240: RuntimeWarning: signal has very low mean
frequency, setting min_tsep = 1250
  warnings.warn(msg.format(min_tsep), RuntimeWarning)
```

[ ]: 0.0022231453343441072

[ ]:
```
# plt.plot(t[0:100], (np.abs(np.fft.fft(v))**2)[0:100])
plt.plot(t, s1)
plt.plot(t, s2)
plt.xlabel('t')
plt.ylabel('v')
plt.show()
```



2
1
0
−1
−2
v
0
20
40
60
80
100
t