Effective Resistances is a Hypermetric

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June 7, 2018

Abstract

The effective resistance is a widely used metric. It is known to be of negative type.

Negative type metrics are known to contain a wide class of metrics. In particular, it is known that L2 is contained inside L1, which is contained inside the space of Hypermetrics. The space of Hypermetrics is in turn contained inside the space of negative type metrics.

We aim to precisely determine how effective resistance metrics fit into this heirarchy of metrics: which metric spaces contain all effective resistance metrics, and which metrics are contained inside the space of effective resistance metrics?

In this paper, we show that L2 is not contained in the effective resistance metrics, and that the effective resistance metrics are contained in the hypermetrics. To do this, we introduce both an easily computable criterion for determining whether a given distance can be induced by an effective resistance distance, and a generalization of the Rayleigh Monotonicity law. These two may be of independent interest when studying properties of Effective Resistances.

1 Introduction

1.1 Effective Resistances

1. Effective Resistances are a metric.

2. They're metrics of negative type, a fact useful for computing them quickly. (Spielman Srivastava).

3. They measure expected commute time, the probability that any given edge is contained in a random spanning tree, and more.

1.2 Negative Type Metrics

1. Negative type metrics are known to contain: L2, L1, Hypermetrics, ...
2. Theory of Isometric Embeddings is of independent interest.
3. Known that $l_2 \subset l_1 \subset HyperMetrics \subset NegType$.
4. Question: Where does effective resistance fit in?
5. Conjecture: Effective Resistance is contained inside l_1.
6. Theorem: Effective Resistances are contained in Hypermetrics, are not contained in l_2, and l_2 does not contain the effective resistance distance.

1.3 Hypermetrics

1. Polygonal inequalities satisfied.
2. Mostly of mathematical interest.

2 Overview

3 A Quadratic Time Criterion for Effective Resistance.

4 A Generalization of Rayleigh's Monotonicity

5 Effective Resistance Obeys the Hypermetric Inequalities.

6 Open Problems

Hello.