Title

Timothy Chu
Carnegie Mellon University
tzchu@andrew.cmu.edu

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Abstract

In this note, we sketch a general idea to show that edge q-power metrics and NN $q-1$-power metrics are approximations of each other.

In CMS17, it is proven that the nearest neighbor metric equals the edge squared metric. Formally:

Definition 0.1. ***Edge q-power metrics*** *are defined as taking a point set P, taking the Euclidean distance between points l, and applying the function:*

$$f(l) := (l/2)^q/q$$

Then the shortest path on the resulting graph is computed. This shortest path is the edge q-power metric.

Definition 0.2. ***NN q-power metrics*** *wtih respect to point set P are defined as taking the cost function:*

$$d(x, P)^q$$

and integrating this over x along a path.

CMS'17 showed that the edge q-power metric and the NN $q-1$ power metric were equal for $q = 2$, and not true for $q < 2$ (even for just two points, with the exception of $q = 1$. For $q \leq 1$, the shortest path is always a straight line).

1 Open Questions

1. What are the approximation factors you get from using CFMSV 15?
2. What are the approximation factors you get from naively using CMS 17?

3. What are the approximation factors you can get using flows on screw simplices? (Using a naive potential function).

4. Can you prove or disprove that you can use: certain restrictive classes of potential functions that generate flows? (List three or so classes of potential functions to restrict our search. Really, there's an entire suite of such functions.)