What is the gradient of Effective Resistance between two vertices with respect to a single edge's resistance? This is part of the "Is Effective Resistance a Hyper-metric" problem.

Using the "min-energy-flow" formulation of Effective Resistance, I show that the gradient is equal to the square of the flow on that edge.

1 Why?

Let the flow on an edge be f in any min energy flow with boudnary constraints. Suppose the resistance of that edge is lowered by ε.

The min-energy flow must decrease by at least $\epsilon \cdot f^2$. If you raised the resistance of that edge by ε, the min-energy flow must increase by at most $\epsilon \cdot f^2$. This is because those two values are the change in flow-energy assuming the underlying flow-with-boundary-conditions doesn't change.

Since the partial of Effective Resistance with respect to a single edge is continuous (no proof!), we conclude it must be exactly f^2 (questionable claim!)

2 Extensions?

I Believe the above claim holds for q-flow-energy-minizers, where this is defined to be th minimum energy flow from point A to point B where energy is calculated by taking f^q.