1 Chernoff via Bernstein:

If you have a bunch of variables A_i that are $\leq L$ and with zero sum (0-1 coin flips are just a shifted and scaled version of this), then

$$\Pr_{A_i \forall i}\left[\sum_{i=1}^{s} A_i \geq t\right] \tag{1}$$

with probability

$$\exp\left(\frac{t^2/2}{\mathbf{Var}_{A_i \forall i}\left[(\sum_{i=1}^{s} A_i)\right] + Lt/3}\right) \tag{2}$$

or alternately

$$\max\left(\exp\left(\frac{t^2/2}{2Var(x)}\right), \exp\left(\frac{t^2/2}{Lt/3}\right)\right) \tag{3}$$

2 Implications

2.1 If you have a ± 1 coin, and you want to figure out the number of flips you need such that you get less than $s/10$ from 0 (the mean) with $1/2$ probability.

Here, of course, $s/10$ can be replaced with $s\epsilon$. Substitute:

1. $L = 1$
2. $t = s/10$
3. $var = s$ (variance of independent variables add).

Then the condition of obtaining a bounded sum with at least $1/2$ probability is equivalent to showing that $t^2/2 = O(s)$, (from the variance term in th denominator of Bernstein, the otehr term is negligible [unproven!]). Therefore, $s = O(\sqrt{t})$, up to a constant of size 2-ish.

2.2 If you have some not-yet-specified parameter n and a bunch of $\pm$ coins, and $t = \epsilon s$, (an epsilon fraction of the number of flips), and you want the sum of your coins to be large with small probability in n, then...

Here, substitute:

1. $L = 1$

2. $t = \epsilon s$

3. $variance - thing = s$.

Then if we want the probability to be $\frac{1}{n^{O(1)}}$, noting that n is a parameter completely separate from anything else we've done so far, then set:

$$t^2/s = \log n \tag{4}$$

or

$$t^2/s = \epsilon^2 s^2/s = \epsilon^2 s = \log n \tag{5}$$

or $s = \frac{\log n}{\varepsilon^2}$. As usual, the $Lt/3$ term becomes non-dominant, as it is ϵs and is much smaller than s.