## 1 Chernoff via Bernstein:

If you have a bunch of variables  $A_i$  that are  $\leq L$  and with zero sum (0-1 coin flips are just a shifted and scaled version of this), then

$$\Pr_{A_i \forall i} \left[ \sum_{i=1}^s A_i \ge t \right] \tag{1}$$

with probability

$$\exp\left(\frac{t^2/2}{\mathbf{Var}_{A_i \forall i} \left[\left(\sum_{i=1}^s A_i\right)\right] + Lt/3}\right) \tag{2}$$

or alternately

$$\max\left(\exp\left(\frac{t^2/2}{2Var(x)}\right), \exp\left(\frac{t^2/2}{Lt/3}\right)\right) \tag{3}$$

## 2 Implications

2.1 If you have a  $\pm 1$  coin, and you want to figure out the number of flips you need such that you get less than s/10 from 0 (the mean) with 1/2 probability.

Here, of course, s/10 can be replaced with  $s\epsilon$ . Substitute:

- 1. L = 1
- 2. t = s/10
- 3. var = s (variance of independent variables add).

TIMOTHY: The following is wrong: we already know what t is.... here s must equal something like 100 or 50. Then the condition of obtaining a bounded sum with at least 1/2 probability is equivalent to showing that  $t^2/2 = O(s)$ , (from the variance term in th denominator of Bernstein, the otehr term is negligible [unproven!]). Therefore,  $s = O(\sqrt{t})$ , up to a constant of size 2-ish.

2.2 If you have some not-yet-specified parameter n and a bunch of  $\pm$  coins, and  $t = \epsilon s$ , (an epsilon fraction of the number of flips), and you want the sum of your coins to be large with small probability in n, then...

Here, substitute:

- 1. L = 1
- 2.  $t = \epsilon s$
- 3. variance thing = s.

Then if we want the probability to be  $\frac{1}{n^{O(1)}}$ , noting that n is a parameter completely separate from anything else we've done so far, then set:

$$t^2/s = \log n \tag{4}$$

or

$$t^2/s = \epsilon^2 s^2/s = \epsilon^2 s = \log n \tag{5}$$

or  $s = \frac{\log n}{\varepsilon^2}$ . As usual, the Lt/3 term becomes non-dominant, as it is  $\epsilon s$  and is much smaller than s.