

Alternative Symmetrizations of Hitting Times in Graphs

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Abstract

Suppose you have a point in a grid, that gets finer and finer. This is the fundamental operation: bisecting a quad (rather than bisecting a line, as in effective resistance).

The idea is: bisecting every quad should behave nicely with this measure.

As an example, consider a uniform circular material. Consider p resistance in two dimensions, for any $p < 2$. At radius r , each ‘edge’ has flow $\frac{1}{r^{d-1}}$, so each flow has cost $\frac{1}{r^{p(d-1)}}$. However, there are r^{d-1} edges in a radial shell. So the total energy loss on a shell is $r^{(d-1)(1-p)}$.

I would like this value to satisfy:

$$\frac{\int_0^a r^{(d-1)(1-p)} dr}{\int_a^{2a} r^{(d-1)(1-p)} dr}$$

to be constant. As long as $(d-1)(1-p) + 1 \neq 0$, then the fraction is:

$$\frac{a^{(d-1)(1-p)+1} \Big|_0^r}{a^{(d-1)(1-p)+1} \Big|_r^{2r}}$$

So any $(d-1)(p-1) < 1$ satisfies, or $p < 1 + \frac{1}{d-1}$.