Title

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Abstract

Buser shows that the isoperimetric cut of a surface with Ricci curvature bounded below, can be bounded as

$\lambda_1 \leq C(H + H^2)$

where H is the isoperimetric ratio of a cut.

The first proof technique uses an "area minimizing current", which shows that the curvature of the cut must be constant.

The second proof technique doens't use this: instead, they use an ϵ net to build a new division from the initial cut X (which can be 'wavy'). This new 'cut' (doesn't have to be a hypersurface, isn't null, but can be full dimensional) is $\hat{X}$, and they do this by defining $\hat{X}$ to be the locus of points where a ball of radius r has exactly half the mass in A and half the mass in B. This 'smooths' wavy cuts.

Then they build an epsilon net around $\hat{X}$, and extend that to an epsilon net of the whole manifold. Finally, they build a little collar around $\hat{X}$, and use some properties of the hyperbolic spheres (only useful since hyperbolic sphere is some manifold lower bound on some volume terms relevant in the calculation, on objects of lower-bounded Ricci curvature) to establish the final bound.

In short, they build a Rayleigh quotient using $\hat{X}$, a smoothed version of X (that isn't exactly a 'cut', but still divides manifold M into two pieces), via a collar of radius r around $\hat{X}$. The Rayleigh quotient is 1 on the parts outside of $\hat{X}$, while it is $d(p, \hat{X})/r$ everywhere else.

ϵ-nets are used for the analysis, but I am not sure they are used for the construction.