Random Walks with Quits

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Abstract

Proposal: Random walks with dropoffs.
Input: Graph G, starting point s, ending point t, value $c(G)$.
Schema: Walk from s. At each timestep and vertex (for some x):

1. Quit-or-walk with probability 1.
2. Walk with probability $d \cdot c$.
3. Quit with probability $1 - d \cdot c$.

The number of times that an agent following this random walk is expected to be at point t is:

$$\sum_{t=0}^{\infty} \chi_t^T (c \cdot A)^t \chi_s$$

or:

$$\chi_t^T (I - cA)^{-1} \chi_s.$$

This is since the transition matrix is: $v \to c \cdot Av$. Assuming constant c, (so c large compared to say, $d_m ax$), this has Rayleigh property. However, if c is dependent on $d_m ax$ then it's not so clear. Allegedly, this number increases with the number of edges (so similarity increases).

1 Further Directions

1.1 Probability the Random Walk Ends

The probability a walk from t hits s at all is: (unproven):

$$\frac{\chi_t^T (I - cA)^{-1} \chi_s}{\chi_s^T (I - cA)^{-1} \chi_s}.$$

1.2 Similarity to Distance

Now define the metric: $(\chi_i - \chi_j)^T (I - cA)^{-1} (\chi_i - \chi_j)^T$. I would love this to be decreasing in A, because distances should follow Rayleigh. However, this is not the case. If I have a long thin line starting from s and ending at t, adding two thick edges to the ends will drastically increases $\chi_s^T (I - cA)^{-1} \chi_s$ but not $\chi_s^T (I - cA)^{-1} \chi_t$.

We may also consider effective resistance with the matrix:

$$(I - cA)^{-1}$$

which bears similarity to the Laplacian. I suspect that the effective resistance using this matrix will be connected. I suspect this will also have the Rayleigh property, as adding an edge to A (without increasing c) will INCREASE (in Loewner ordering) the matrix $(I - cA)^{-1}$. (what.) Since this is a rank 2 update.

2 Problem with this Approach

The problem with this approach: I think random walks with quits has behavior governed more or less by geodesic distance. This is since probability of quitting is exponential in path length.

Alsoo this doesn't work well with a finer and finer grid. Somehow, the finer and finer grid seems to have odd behavior that a regular grid does not have.