Alternative Symmetrizations of Hitting Times in Graphs

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Abstract

For various reasons, we propose looking into:

$$\chi_{ij}^T \left(L^\dagger\right)^k \chi_{ij}$$

This came out of an exploration of heat, although I haven't worked out major issues in degree normalization and how to weight this Laplacian in a mesh, when the mesh gets smaller.

1 Shortcomings

I attempted to determine whether the vertices i where: $\chi_i^T \left(L^\dagger\right)^k \chi_{ab}$ is ≥ 0, are connected. The motivation is that when $q = 1$ its connected (voltage threshold cut), and when $q = \infty$ it's connected (Fiedler nodal domain).

Turns out it's not. Take a star with edge weights 1 except one edge with weight 3. Then choose vertex a and b to be the center and the vertex with an edge of weight 3 respectively. This will break connectivity. Maybe figuring out the proof for Fiedler's nodal domains will be helpful.