Suppose I have the following Lemma, which is very similar to Lemma 7.6 in Chu et. al. (CGPSSW 18).

Lemma 0.1. Let K be a collection of bicliques. Then K can be ϵ sketched with $O(n\epsilon^{-1} + n(K)\epsilon^{-0.5})$ edges.

Here, n(K) is the total number of vertices in a collection of bi-cliques.

Now: suppose we wanted to take a vertex v with d multi-edges adjacent to v. Here, d can be larger than n.

If each of the multi-edges have leverage score less than ε , then the resulting (weighted) clique obtained via the Schur complement has a total leverage score less than $d\varepsilon$.

Let this clique be K. Then since

$$L_G^{\dagger/2} L_K L_G^{\dagger/2} \le d\varepsilon,$$

then

$$x^T L_K x \le d\varepsilon x^T L_G x$$

Er, that's a real bad bound. I'd hoped it to be $d\varepsilon/n$.