Today: $\boxed{\text{Ken}}$

$\boxed{2.1}$ Logical Form & Equivalence

$\boxed{2.2}$ Conditional Statements

$\boxed{2.3}$ Valid & Invalid Arguments

Last time:

Syllabus review

$\boxed{2.1}$ Logical Form & Equivalence

$\boxed{2.2}$ Conditional Statements

If it's not raining, then Jon rides a bicycle to work.

$p \to q \equiv \neg q \to \neg p$

Definition

Two statement forms are called logically equivalent if, and only if, they have identical truth values for each possible substitution of statements for their statement variables. We denote statement forms p and q logically equivalent $p \equiv q$

Two statements are called logically equivalent if, and only if, they have logically equivalent forms when identical component statement variables are used to replace identical component statements.

Inequalities

① strict $2 < 3$

② without qualification or "weak" (disjunction)

$2 \leq 3$

$2 < 3 \lor 2 = 3$

③ chained (conjunction)

$2 < 3 < \pi$

$2 < 3 \land 3 < \pi \longrightarrow 2 < \pi$ transitivity

④ trichotomy (exclusive or) XOR

$x < y \lor y < x \lor x = y$

Tautologies & Contradictions

① A tautology is a statement form that is always true regardless of the truth values of the individual statements substituted for its statement variables. A statement whose form is a tautology is a tautological statement.

$p \vee \neg p \equiv T \equiv t$

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

law of the excluded middle

② A contradiction is a statement form that is always false regardless of the truth values of the individual statements substituted for its statement variables. A statement whose form is a contradiction is a contradictory statement.

$p \wedge \neg p \equiv \perp \equiv c$

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Theorem 2.1.1 Logical Equivalences

1. commutativity
   $p \vee q \equiv q \vee p$
   $p \wedge q \equiv q \wedge p$

2. associativity
   $(p \vee q) \vee r \equiv p \vee (q \vee r)$
   $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

3. distributivity
   $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
   $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

4. identity
   $p \wedge T \equiv p$
   $p \vee \perp \equiv p$

5. negation laws
   $p \vee \neg p \equiv T \equiv t$ (tautology)
   $p \wedge \neg p \equiv \perp \equiv c$ (contradiction)

6. negation elimination (double negative law)
   $\neg(\neg p) \equiv p$

7. idempotence
   $p \vee p \equiv p$
   $p \wedge p \equiv p$

8. universal bound
   $p \vee T \equiv T$
   $p \wedge \perp \equiv \perp$

9. DeMorgan's laws
   $\neg(p \vee q) \equiv \neg p \wedge \neg q$
   $\neg(p \wedge q) \equiv \neg p \vee \neg q$

10. absorption
    $p \vee (p \wedge q) \equiv p$
    $p \wedge (p \vee q) \equiv p$

11. tautology & contradiction negation
    $\neg T \equiv \perp$
    $\neg \perp \equiv T$

#52 Show

$\neg(p \vee \neg q) \vee (\neg p \wedge \neg q) \equiv \neg p$

using Theorem 2.1.1

$\cancel{\neg(p \vee \neg q) \vee (\neg p \wedge \neg q) \equiv \neg p}$

$\cancel{(\neg p \wedge \neg\neg q) \vee (\neg p \wedge \neg q) \equiv \neg p}$

$\cancel{(\neg p \wedge q) \vee (\neg p \wedge \neg q) \equiv \neg p}$

$\neg(p \vee \neg q) \vee (\neg p \wedge \neg q) \equiv (\neg p \wedge \neg\neg q) \vee (\neg p \wedge \neg q)$	DeMorgan's laws
$\equiv (\neg p \wedge q) \vee (\neg p \wedge \neg q)$	double negation law
$\equiv \neg p \wedge (q \vee \neg q)$	distributivity
$\equiv \neg p \wedge t$	negation laws
$\equiv \neg p$	identity law

$p \rightarrow q \equiv \neg p \vee q$

(via truth tables in exercise 13(a))

p	$\neg p$	q	$p \rightarrow q$	$\neg p \vee q$
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T

$\neg(p \to q) \equiv \neg(\neg p \vee q)$ "Awesome law"

$\equiv \neg\neg p \wedge \neg q$ DeMorgan's law

$\equiv p \wedge \neg q$ double negative law

via Theorem 2.1.1

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

e.g. There exists $a, b \in \mathbb{Z}$ such that a divides b implies there exists $k \in \mathbb{Z}$ such that $b = ka$.

$p \to q$

$\neg(p \to q) \equiv p \wedge \neg q$

$\neg(p \to q) \not\equiv p \to \neg q$

Given $p \to q$

contrapositive $p \to q \equiv \neg q \to \neg p$

converse $q \to p \not\equiv p \to q$

inverse $\neg p \to \neg q \not\equiv p \to q$

necessary OR sufficient conditions

$p \to q$

p is a sufficient condition for q

q is a necessary condition for p

also necessary AND sufficient condition

$p \leftrightarrow q$

p is a necessary AND sufficient condition for q

q is a necessary AND sufficient condition for p

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vacuous truth or true by default

"F implies T"

#43 Show

$(\neg p \vee q) \vee (p \wedge \neg q)$

is tautologous using truth tables

$\boxed{2.3}$ Definition

An argument is a sequence of statements, and an argument form is a sequence of statement forms. All statements in an argument and all statement forms in an argument form, except for the final one, are called premises (or assumptions or hypotheses). The final statement or statement form is called the conclusion. The symbol $\therefore$ meaning "therefore" is normally placed just before the conclusion.

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e.g.

$$\begin{array}{l} \sqrt{2} < e \\ e < \pi \\ \therefore \sqrt{2} < e < \pi \end{array}$$

also abbreviated

$$\begin{array}{l} p \\ q \\ \therefore p \wedge q \end{array}$$

To say that an argument form is valid means that no matter what particular statements are substituted for the statement variables in its premises, if the resulting premises are all true, then the conclusion is also true. To say an argument is valid means that its form is valid.

Testing an Argument Form for Validity

① Identify the premises and conclusion of the argument form.

② Construct a truth table showing the truth values of all the premises and the conclusion.

③ A row of the truth table in which all the premises are true is called a critical row. If there is a critical row in which the conclusion is false, then it is possible for an argument of the given form to have true premises and a false conclusion, and so the argument form is invalid.

If the conclusion in every critical row is true, then the argument form is valid.

Caution: If at least one premise of an argument is false, then we have no information about the conclusion: It might be true or it might be false.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

critical row

p
q
$\therefore p \wedge q$

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