In[286]:=

```
Clear[t, m, n, o, p]
```
lösche

```
f[a_, b_, c_, d_, e_, x_] = e + d x + c x^2 + b x^3 + a x^4
df[a_, b_, c_, d_, e_, x_] = D[f[a, b, c, d, e, x], {x, 1}]
```
leite ab

```
ddf[a_, b_, c_, d_, e_, x_] = D[f[a, b, c, d, e, x], {x, 2}]
```
leite ab

```
energyAtAPoint[a_, b_, c_, d_, e_, x_] = (ddf[a, b, c, d, e, x])^2
energy[a_, b_, c_, d_, e_, t_] =
 Integrate[energyAtAPoint[a, b, c, d, e, x], {x, 0, t}]
```
integriere

```
constraints = {
   f[a, b, c, d, e, 0] == m,
   df[a, b, c, d, e, 0] == n,
   f[a, b, c, d, e, t] == o,
   df[a, b, c, d, e, t] == p
  };

Print["constraintsSolutions:"]
```
gebe aus

```
constraintsSolutions = Solve[constraints, {b, c, d, e}]
```
löse

```
Print["Constraints with solutions:"]
```
gebe aus

```
constraintsSimplified = constraints /. constraintsSolutions[[1]]
energyWithSolutions[a_, t_] =
 Evaluate[energy[a, b, c, d, e, t] /. constraintsSolutions[[1]]]
```
werte aus

```
Print["Energy function:"]
```
gebe aus

```
energySimplified[a_, t_] = Simplify[energyWithSolutions[a, t]]
```
vereinfache

```
Print["solution: (a, b, c, d, e)"]
```
gebe aus

```
(*without loss of generality: t is always 1*)

amin = ArgMin[{energySimplified[a, 1]}, a]
```
Argument des Minimums

```
bmin = First[Evaluate[b /. constraintsSolutions /. {t → 1, a → amin}] ]
```
erstes··· werte aus

```
cmin =
 First[Evaluate[c /. constraintsSolutions /. {t → 1, a → amin, b → bmin} ] ]
```
erstes··· werte aus

```
dmin =  First[Evaluate[
```
erstes··· werte aus

```
   d /. constraintsSolutions /. {t → 1, a → amin, b → bmin, c → cmin} ] ]
emin =  First[Evaluate[e /. constraintsSolutions /.
```
erstes··· werte aus

```
    {t → 1, a → amin, b → bmin, c → cmin, d → dmin} ] ]
Print["Check:"]
```
prüfe

```
Simplify[constraints /. {t → 1, a → amin, b → bmin, c → cmin, d → dmin, e → emin}]
```
vereinfache

```
Print["Target function:"]
```
gebe aus

```
f [amin, bmin, cmin, dmin, emin, x]


averageSpeedInTimespan [from_, to_] :=
```

```
(1 / (to - from)) * Integrate[df[amin, bmin, cmin, dmin, emin, x], {x, from, to}]
                    integriere
Print["AverageSpeedInTimespan:"]
gebe aus
Evaluate[averageSpeedInTimespan[begin, end]]
werte aus
```

Out[287]= $e + d\,x + c\,x^2 + b\,x^3 + a\,x^4$

Out[288]= $d + 2\,c\,x + 3\,b\,x^2 + 4\,a\,x^3$

Out[289]= $2\,c + 6\,b\,x + 12\,a\,x^2$

Out[290]= $\left(2\,c + 6\,b\,x + 12\,a\,x^2\right)^2$

Out[291]= $4\,c^2\,t + 12\,b\,c\,t^2 + 12\,b^2\,t^3 + 16\,a\,c\,t^3 + 36\,a\,b\,t^4 + \frac{144\,a^2\,t^5}{5}$

constraintsSolutions:

Out[294]= $\left\{\left\{b \to -\frac{-2\,m + 2\,o - n\,t - p\,t + 2\,a\,t^4}{t^3},\ c \to -\frac{3\,m - 3\,o + 2\,n\,t + p\,t - a\,t^4}{t^2},\ d \to n,\ e \to m\right\}\right\}$

Constraints with solutions:

Out[296]= $\left\{\text{True},\ \text{True},\ \text{True},\ n + 4\,a\,t^3 - \frac{2\left(3\,m - 3\,o + 2\,n\,t + p\,t - a\,t^4\right)}{t} - \frac{3\left(-2\,m + 2\,o - n\,t - p\,t + 2\,a\,t^4\right)}{t} == p\right\}$

Out[297]= $\frac{144\,a^2\,t^5}{5} - 16\,a\,t\left(3\,m - 3\,o + 2\,n\,t + p\,t - a\,t^4\right) + \frac{4\left(3\,m - 3\,o + 2\,n\,t + p\,t - a\,t^4\right)^2}{t^3} - 36\,a\,t\left(-2\,m + 2\,o - n\,t - p\,t + 2\,a\,t^4\right) + \frac{12\left(3\,m - 3\,o + 2\,n\,t + p\,t - a\,t^4\right)\left(-2\,m + 2\,o - n\,t - p\,t + 2\,a\,t^4\right)}{t^3} + \frac{12\left(-2\,m + 2\,o - n\,t - p\,t + 2\,a\,t^4\right)^2}{t^3}$

Energy function:

Out[299]= $\frac{4\left(15\,m^2 + 15\,o^2 - 15\,o\,(n + p)\,t + 15\,m\,(-2\,o + (n + p)\,t) + t^2\left(5\,n^2 + 5\,n\,p + 5\,p^2 + a^2\,t^6\right)\right)}{5\,t^3}$

solution: (a, b, c, d, e)

Out[301]= 0

Out[302]= $2\,m + n - 2\,o + p$

Out[303]= $-3\,m - 2\,n + 3\,o - p$

Out[304]= n

Out[305]= m

Check:

Out[307]= {True, True, True, True}

Target function:

Out[309]= $m + n\,x + (-3\,m - 2\,n + 3\,o - p)\,x^2 + (2\,m + n - 2\,o + p)\,x^3$

AverageSpeedInTimespan:

Out[312]= $\frac{1}{-\text{begin} + \text{end}}\left(3\,\text{begin}^2\,m - 2\,\text{begin}^3\,m - 3\,\text{end}^2\,m + 2\,\text{end}^3\,m - \text{begin}\,n + 2\,\text{begin}^2\,n - \text{begin}^3\,n + \text{end}\,n - 2\,\text{end}^2\,n + \text{end}^3\,n - 3\,\text{begin}^2\,o + 2\,\text{begin}^3\,o + 3\,\text{end}^2\,o - 2\,\text{end}^3\,o + \text{begin}^2\,p - \text{begin}^3\,p - \text{end}^2\,p + \text{end}^3\,p\right)$

In[32]:=
```
Quit[]
```

In[25]:=
```
clear[m, n, o, p]
m := -1.4
n := -0.5
o := 1.3
p := 0.6
targetF[x_] = f[amin, bmin, cmin, dmin, emin, x]
Plot[targetF[x], {x, 0, 2}]
```

graphische Funktionsdarstellung

Out[30]= $\{-1.4 - 0.5\,x + 8.5\,x^2 - 5.3\,x^3\}$

Out[31]=