EE1 Two-element AC circuits

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We analyse the following ac circuits

- series RC
- series RL
- parallel RC
- parallel RL

Series RC

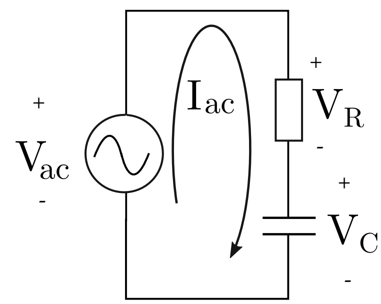

Figure 1: Series RC circuit

Let

$$\omega = 200\pi \tag{1}$$
$$V_{ac} = 200e^{jwt} \tag{2}$$
$$R = 100\Omega \tag{3}$$
$$C = 20\mu F \tag{4}$$

What else do we know?

- $$Z_C = \frac{1}{j\omega C}$$
- $$V = IZ$$
- voltages around a loop must sum to zero, at all times
- the same current flows in all components of a simple series circuit

What do we not *know?*

- the current flowing in the resistor
- the current flowing in the capacitor
- the current flowing in the supply

One approach is to calculate the total impedance of the circuit at a single frequency, and do all the book work numerically at a single frequency.

$$\begin{aligned} Z_T &= Z_R + Z_C && (5) \\ &= R + \frac{1}{j\omega C} && (6) \\ &= 100 - j80 && (7) \end{aligned}$$

Then we can calculate the current

$$I_{ac} = \frac{V}{Z_T}\frac{200}{100 - j80} \tag{8}$$
$$= \frac{200(100 + j80)}{(100 - j80)(100 + j80)} \tag{9}$$
$$= \frac{20 \times 10^3 + j16 \times 10^3}{10 \times 10^3 + 6.4 \times 10^3} \tag{10}$$
$$= 1.22 + j0.98A \tag{11}$$

we multiply by the complex conjugate $(100 + j80)$ to make the deonominator real

Note that we drop the exponential part of the voltage for convenience. We know it is there, and we'll add it back in later. What does out result mean? We still need to work out the magnitude $|I_{ac}|$ and phase $\angle I_{ac}$. The magnitude is

$$|I_{ac}| = |1.22 + j0.98| \tag{12}$$
$$= \sqrt{1.22^2 + 0.98^2} \tag{13}$$
$$= 1.56A. \tag{14}$$

We can double check our value for I_{ac} by calculating the magnitude of the impedance first

$$|I_{ac}| = \frac{V}{|Z_T|} \tag{15}$$
$$= \frac{200}{\sqrt{100^2 + 80^2}} \tag{16}$$
$$= \frac{200}{128.1} \tag{17}$$
$$= 1.56A \tag{18}$$

which is the same as before.

The phase is

$$\angle I_{ac} = \arctan\frac{\text{Im}\{I_{ac}\}}{\text{Re}\{I_{ac}\}} \tag{19}$$
$$= \arctan\frac{0.98}{1.22} \tag{20}$$
$$= 38.8° \tag{21}$$

We can see the results on a phasor diagram of Figure 2. This is only a qualitative representation - you do not need to draw the phasors to scale in the exam because you will be drawing the phasor diagram *before* you've worked out any values!

That's a basic solution done and dusted. Although it is often far far better to be able to work out the result for the general case, and substitute in numbers as you need them - especially while you are building your knowledge of how these circuits work.

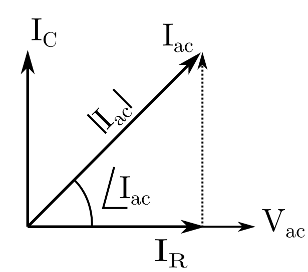

Figure 2: Series RC circuit phasor diagram

So let's calculate the current for any frequency and voltage

$$I_{ac} = \frac{V}{Z_T} \tag{22}$$
$$= \frac{V}{R + \frac{1}{j\omega C}} \tag{23}$$
$$= \frac{jV\omega C}{1 + j\omega RC} \tag{24}$$

The magnitude of the current is

$$|I| = \left|\frac{jV\omega C}{1 + j\omega RC}\right| \tag{25}$$
$$= \frac{|jV\omega C|}{|1 + j\omega RC|} \tag{26}$$
$$= \frac{V\omega C}{1 + (\omega RC)^2} \tag{27}$$

To find the phase of the current, we need to first make the denominator real so we can separate out the real and imaginary parts. We multiply by the complex conjugate

$$I_{ac} = \frac{jV\omega C}{1 + j\omega RC}\frac{1 - j\omega RC}{1 - j\omega RC} \tag{28}$$
$$= \frac{V\omega C(1j - \omega RC)}{1 + (\omega RC)^2} \tag{29}$$
$$\tag{30}$$

Now the phase can be found as

$$\angle I_{ac} = \arctan\frac{\mathrm{Im}\{I_{ac}\}}{\mathrm{Re}\{I_{ac}\}} \tag{31}$$
$$= \arctan\frac{1}{-\omega RC} \tag{32}$$

We can check these equations by substituting in the parameters and checking we get the same answers as before.