A PROOF OF THEOREM 3

We first show that the problem is NP. Given an aligned instance R, all the three conditions can be verified in polynomial time. Condition (1) is verified by comparing the tuples with each other in $O(|R|^2)$ time. For condition (2), the time constraint θ can be checked by traversing the tuples in $O(|R|)$ time and the map constraint δ is examined by searching the nearest neighbor for each tuple $r \in R$ in $O(|R| \log |M|)$ time. Condition (3) can simply be checked by comparing $|R|$ and κ in $O(1)$.

To show NP-hardness of the problem, we build a reduction from the *maximum 3-dimensional matching* [10, 12], one of Karp's 21 NP-complete problems [13].

Let A, B, C be finite disjoint sets, and $P_c \subseteq A \times B \times C$, i.e., $P_c = \{(a, b, c) | a \in A, b \in B, c \in C\}$. For $p_1(a_1, b_1, c_1), p_2(a_2, b_2, c_2) \in P$, we say that p_1 and p_2 intersect on some coordinate, denoted by $p_1 \leftrightarrow p_2$, if either $a_1 = a_2$, $b_1 = b_2$ or $c_1 = c_2$. We use $P \subseteq P_c$ to denote a 3-dimensional matching if no element in P intersects with others, i.e., $\forall p_1, p_2 \in P, p_1 \nleftrightarrow p_2$. The *maximum 3-dimensional matching* is to find the matching P^* among all the possible values of P with the largest amount of triples. The decision problem is, given an integer κ, to decide whether there exists a 3-dimensional matching P such that $|P| \geq \kappa$.

For an instance of the *maximum 3-dimensional matching* problem with P_c, to construct a similarity alignment problem under map constraint, we first set $\theta = +\infty$, i.e., the time constraint is satisfied by any two aligned tuples. Then, we create S, T, F in our problem by assigning unique timestamps to them. In the meantime, let each $s_i \in S$ correspond to $a_i \in A$, each $t_j \in T$ correspond to $b_j \in B$ and each $f_k \in F$ correspond to $c_k \in C$. Next, if $(a_i, b_j, c_k) \in P_c$, we assign $s_i[V_{speed}]$, $t_j[V_{torque}]$ and $f_k[V_{fuel}]$ unique combination of values, and then insert $(s_i[V_{speed}], t_j[V_{torque}], f_k[V_{fuel}])$ into M. Let map constraint $\delta = 0$, since each $(s_i[V_{speed}], t_j[V_{torque}], f_k[V_{fuel}])$ is unique, we have $R_c = \{(s_i, t_j, f_k) | (s_i[V_{speed}], t_j[V_{torque}], f_k[V_{fuel}]) \in M\}$. Therefore, we have $(s_i, t_j, f_k) \in R_c$ if and only if $(a_i, b_j, c_k) \in P_c$.

Next, we will show that, for each satisfied 3-dimensional matching P, we have $|P| \geq \kappa$, if and only if the aligned instance $R = \{(s_i, t_j, f_k) | (a_i, b_j, c_k) \in P\}$ corresponding to P in our problem is also the set that satisfies (1) three candidate keys U_{speed}, U_{torque}, U_{fuel}, (2) time constraint $\Theta(r) \leq 2\theta$ and map constraint $\Delta(r, M) \leq \delta$ satisfied for each $r \in R$, and (3) $|R| \geq \kappa$.

First, according to the definition, we suppose $|P| \geq \kappa$. Since $\forall p_1, p_2 \in P$, $p_1 \nleftrightarrow p_2$, for the corresponding R, we will have $\forall r_1, r_2 \in R$, $r_1 \nsim r_2$, thus condition (1) is satisfied. For condition (2), first recall that we construct our alignment problem by supposing the time constraint θ large enough. Moreover, we generate R_c by satisfying map constraint, and thus condition (2) is satisfied automatically. Finally, since we have $(s_i, t_j, f_k) \in R$ if and only if $(a_i, b_j, c_k) \in P$, $|R| = |P| \geq \kappa$ holds, condition (3) is satisfied.

Conversely, suppose that $|R|$ satisfies condition (1), (2), (3). Following similar steps, according to the condition (1), $\forall r_1, r_2 \in R$, $r_1 \nsim r_2$, the corresponding P has $p_1 \nleftrightarrow p_2, \forall p_1, p_2 \in P$. Next, according to condition (3), $|R| \geq \kappa$. Since $(s_i, t_j, f_k) \in R$ if and only if $(a_i, b_j, c_k) \in P$, we will get $|P| = |R| \geq \kappa$. The conditions of the 3-dimensional matching problem are satisfied.

B PROOF OF PROPOSITION 4

We first show that our Result Search Algorithm 2 is a special t-optimal local search algorithm [16]. Given an integer $t > 0$, for the current set of candidates R we have found, the t-optimal local search algorithm looks for a set B of $p \leq t$ tuples outside R ($B \subseteq R_c - R$) that overlap at most $p - 1$ candidates $A \subseteq R$ while the candidates in B do not overlap with each other. The algorithm will replace $A \subseteq R$ with B, i.e, $R \longleftarrow (R - A) \cup B$. Let $t = 2$, $A = \{r\}$ and $B = \{r_1, r_2\}$, it is equivalent to our Result Search Algorithm 2.

For k-dimensional time series and parameter t for Result Search, the approximation ratio ξ satisfies

$$\xi = \frac{k(k-1)^r - k}{2(k-1)^r - k}, \qquad if\ t = 2r - 1; \tag{28}$$

$$\xi = \frac{k(k-1)^r - 2}{2(k-1)^r - 2}, \qquad if\ t = 2r. \tag{29}$$

The proof is presented in [11]. In our scenario, given $t = 2$ and $k = 3$, it is a factor-2 approximation..