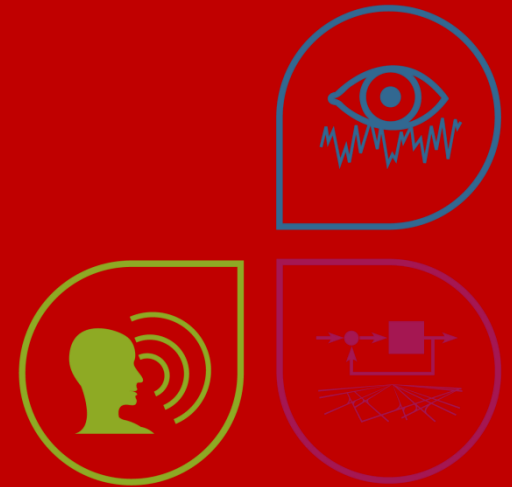


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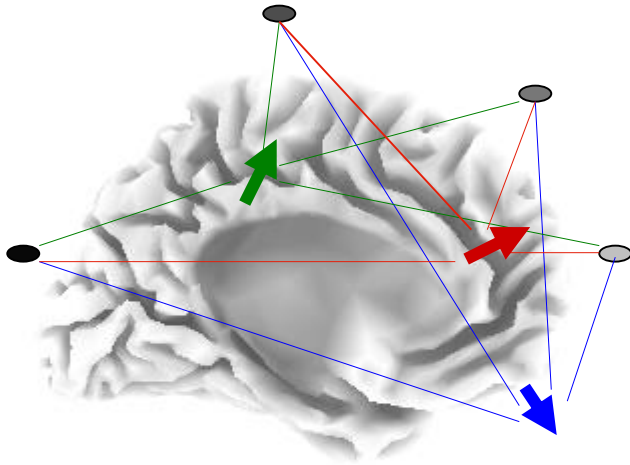
Grenoble INP



Riemannian Geometry for BCIs: Intuition and Theory



Generative Model of EEG



X N sensor measurements ● ● ● ●

S $P \leq N$ source dipolar fields ↗ ↘ ↙

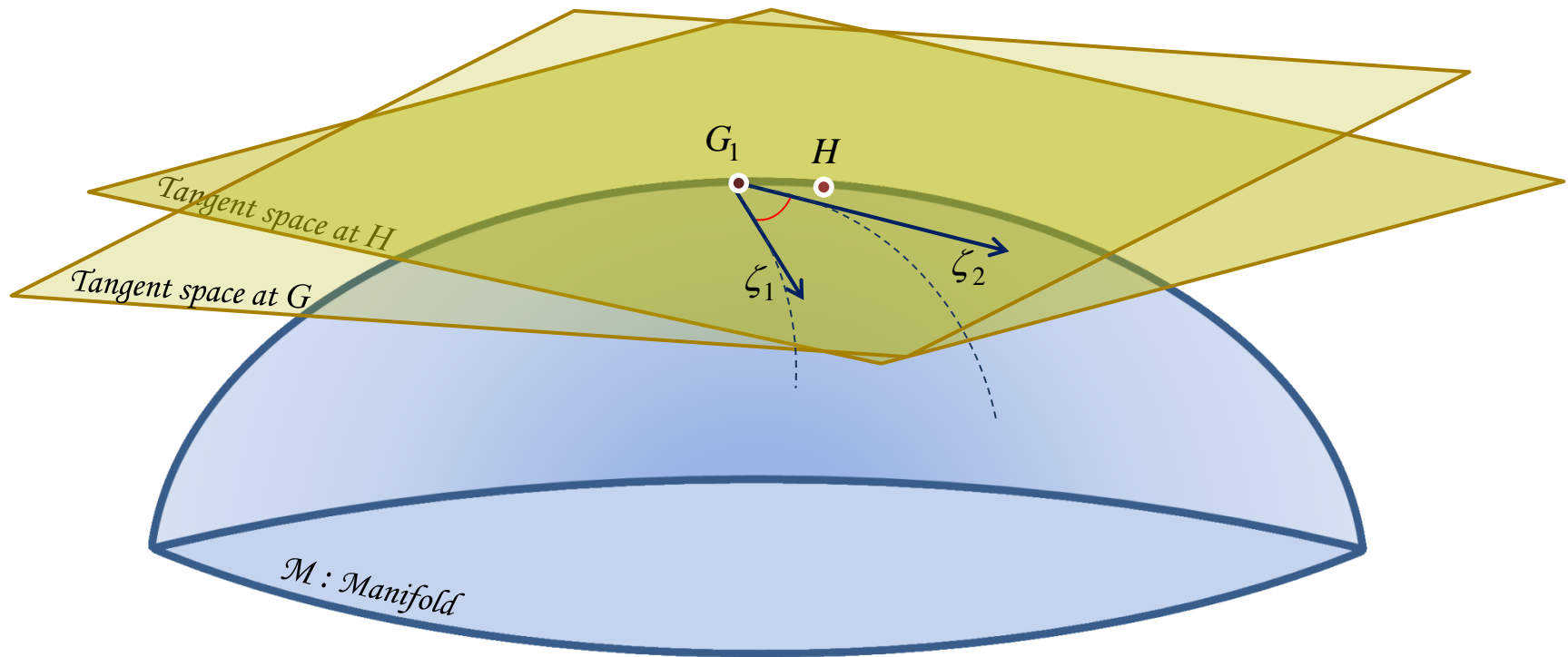
$$X = AS$$



$$\text{Cov}(X) = A \text{Cov}(S) A^T$$

Constant for a given task

Riemannian Geometry of Covariance Matrices



The tangent space at point G is the Euclidean vector space containing the tangent vectors to all curves on \mathcal{M} passing through G .

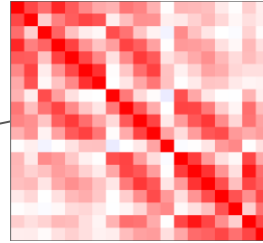
Inner product on tangent space (metric) \rightarrow Riemannian Manifold

Fisher Metric of the SPD Manifold: $\text{tr}(G^{-1}\zeta_1 G^{-1}\zeta_2)$

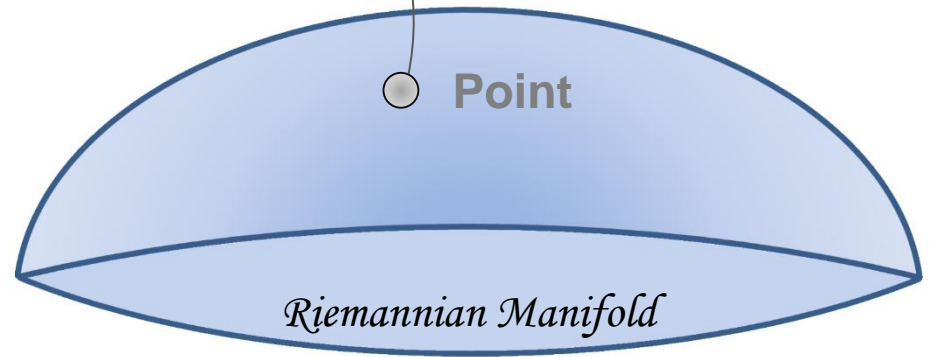
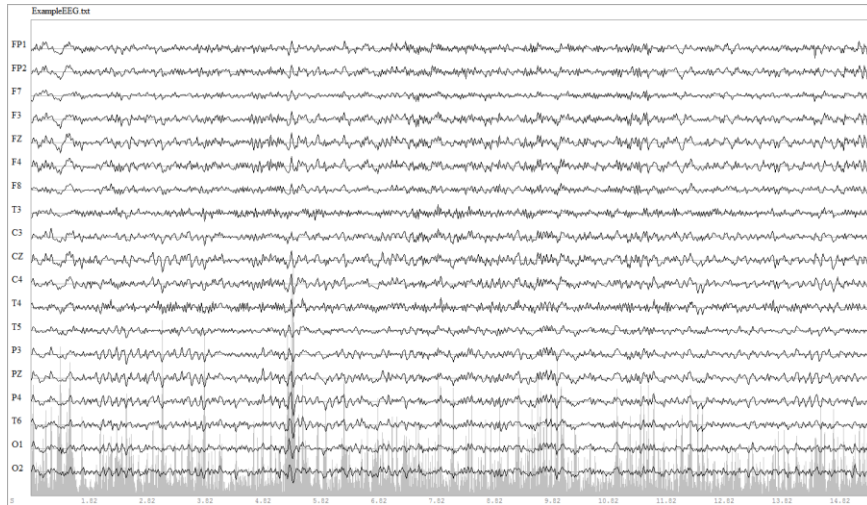
Norm: $\|G^{-1}\zeta\|_F$

Mapping EEG data on the Manifold of SPD matrices

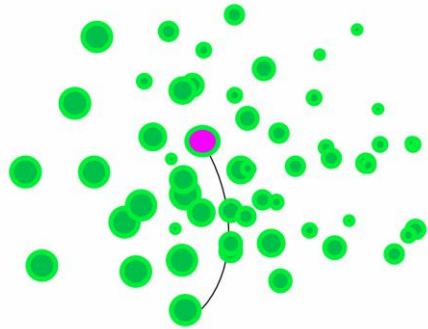
Covariance matrix



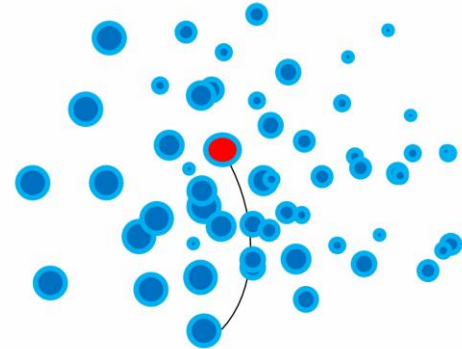
EEG Data



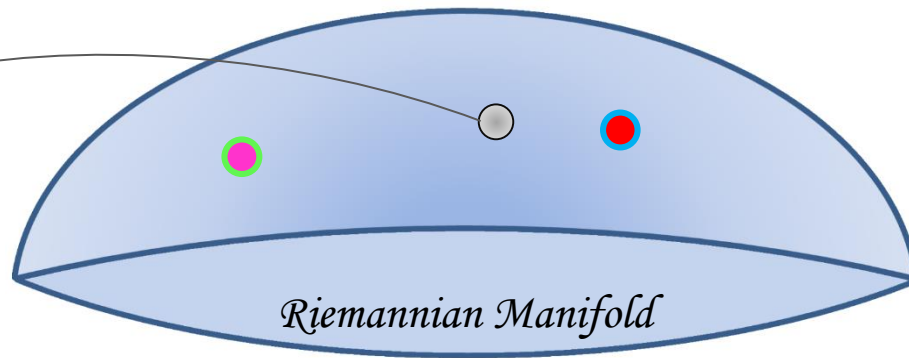
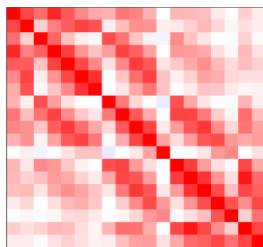
Minimum Distance to Mean (MDM) Classifier



Class 1

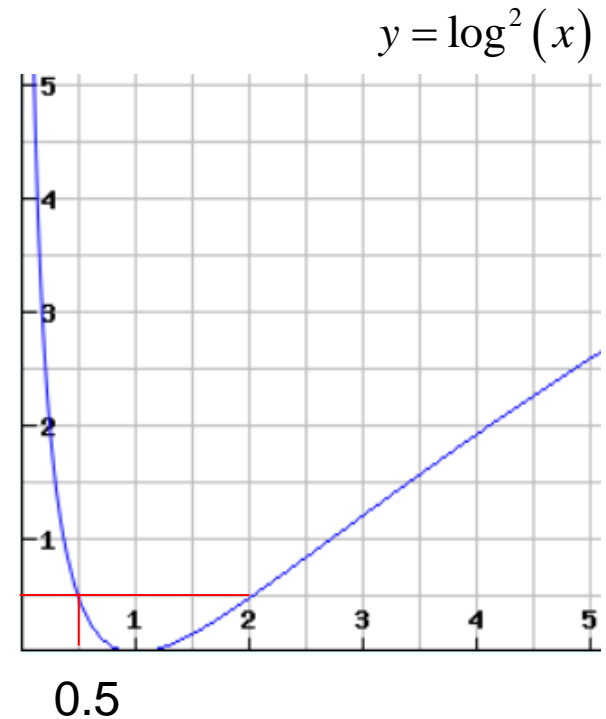


Class 2



The Fisher-Rao distance

$$\delta^2(C_1, C_2) = \sum_n \log^2 \lambda_n(C_1^{-1} C_2)$$



Invariances

Congruence (Affine) $\delta(C_1, C_2) = \delta(BC_1B^T, BC_2B^T)$

Inversion (Self-Duality) $\delta(C_1, C_2) = \delta(C_1^{-1}, C_2^{-1})$

Congruence Invariance

(equivalence of sensor and source space)

First Order

$$X = AS$$

Second Order

$$\text{cov}(X) = A \text{cov}(S) A^T$$

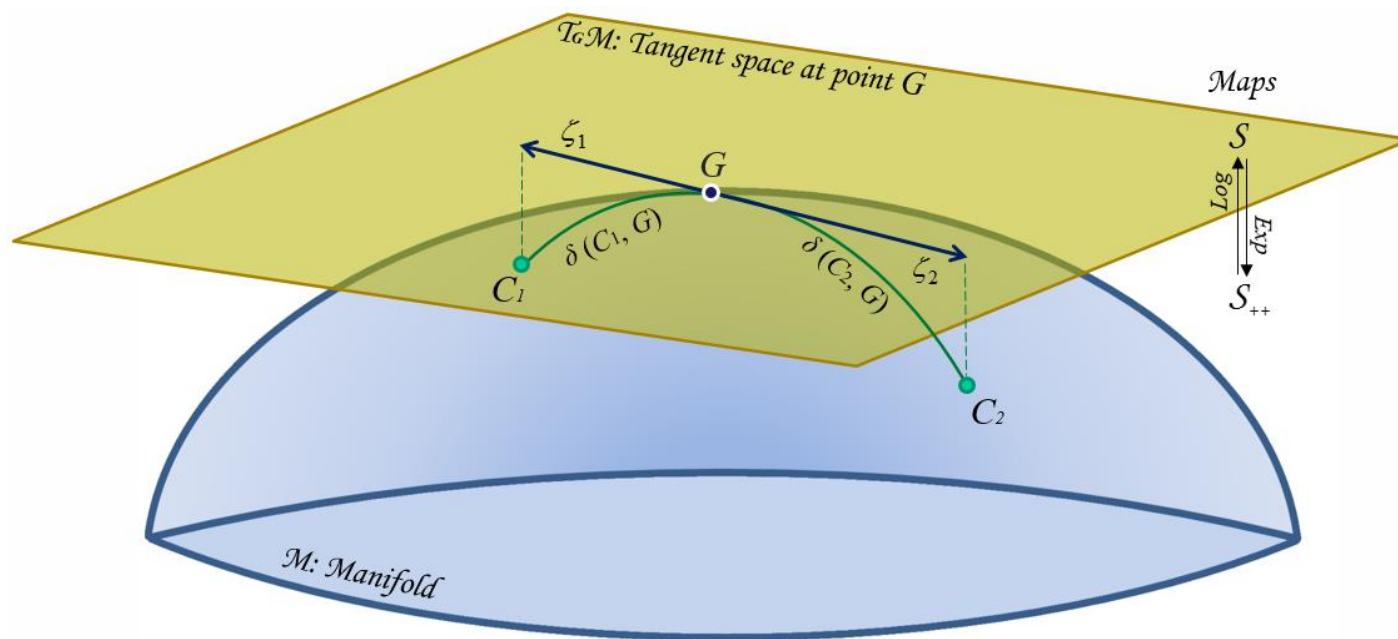
Inversion Invariance

(adjust to heterogeneity of source power)

$$\lambda(C) = (\overbrace{10, 1}^{\text{noise}}, \underbrace{0.1}_{\text{signal}})$$

$$\lambda(C^{-1}) = (0.1, 1, 10)$$

The PD Geodesic (t-weighted mean)



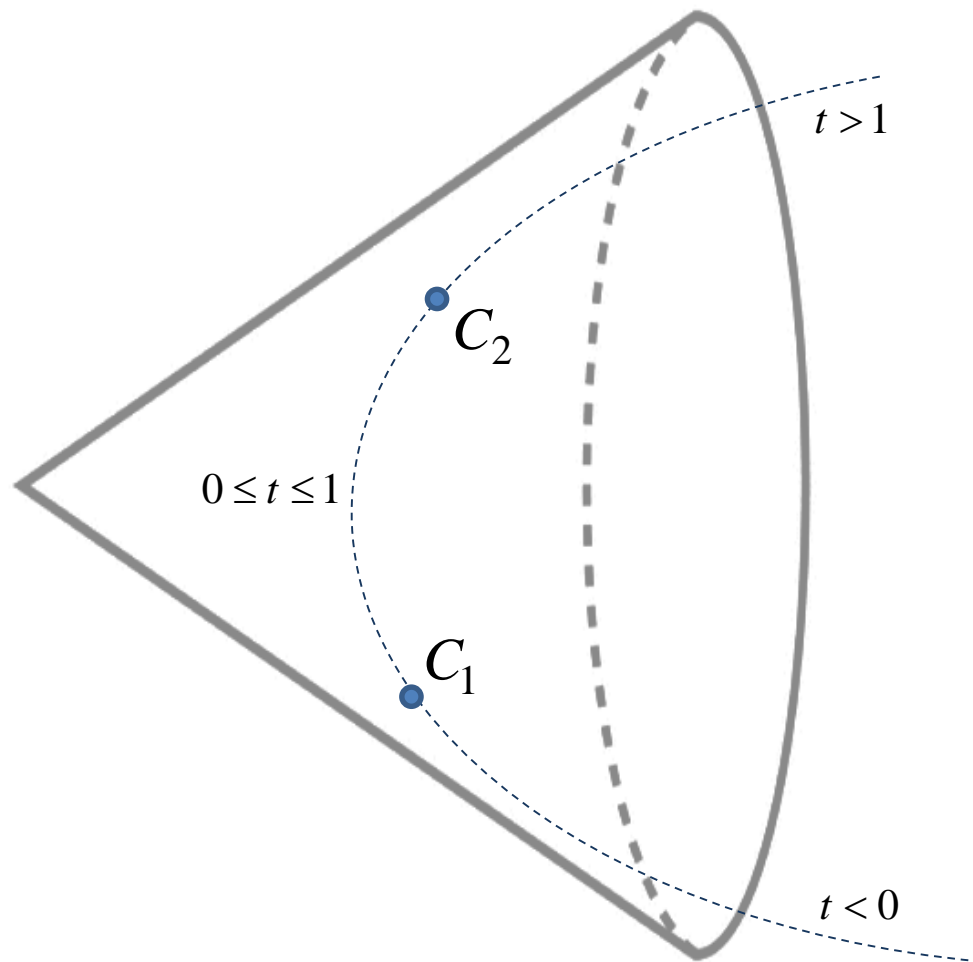
$$C_1 \#_t C_2 = C_1^{1/2} \left(C_1^{-1/2} C_2 C_1^{-1/2} \right)^t C_1^{1/2}, \quad t \in [0, 1]$$

$$C_1 \#_0 C_2 = C_1$$

$$C_1 \#_1 C_2 = C_2$$

$$C_1 \#_{1/2} C_2 = G \quad (\text{Midpoint} = \text{Geometric Mean})$$

Interpolation & Extrapolation

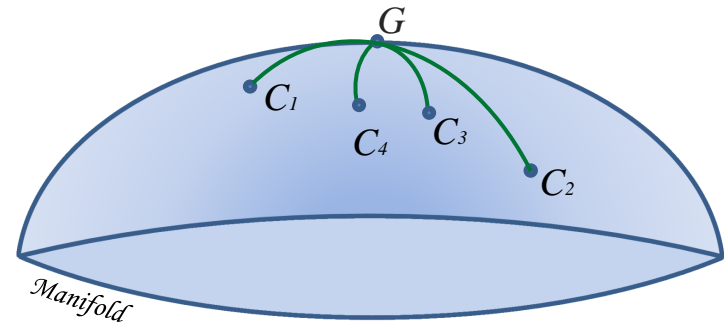


$$C_1 \#_t C_2 = C_1^{\frac{1}{2}} \left(C_1^{-\frac{1}{2}} C_2 C_1^{-\frac{1}{2}} \right)^t C_1^{\frac{1}{2}}$$

Geometric Mean

(Karcher mean, Barycenter, Centre of Mass...)

$$\arg \min_G \sum_k \delta^2(G, C_k)$$

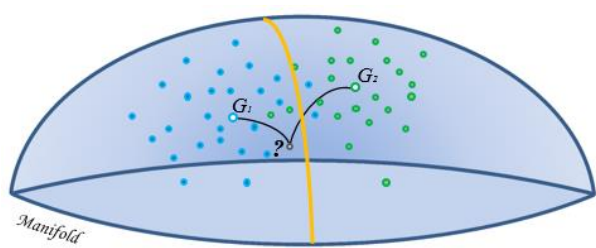
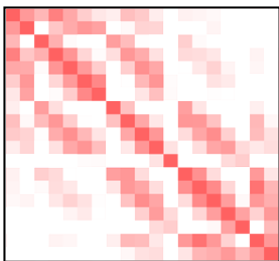
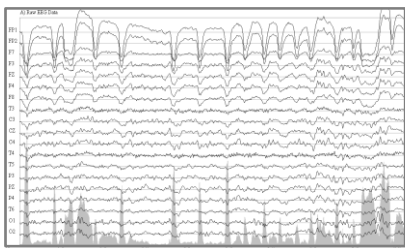


Always exists, is unique and satisfies
$$\sum_k \left[\text{Log} \left(G^{-1/2} C_k G^{-1/2} \right) \right] = 0$$

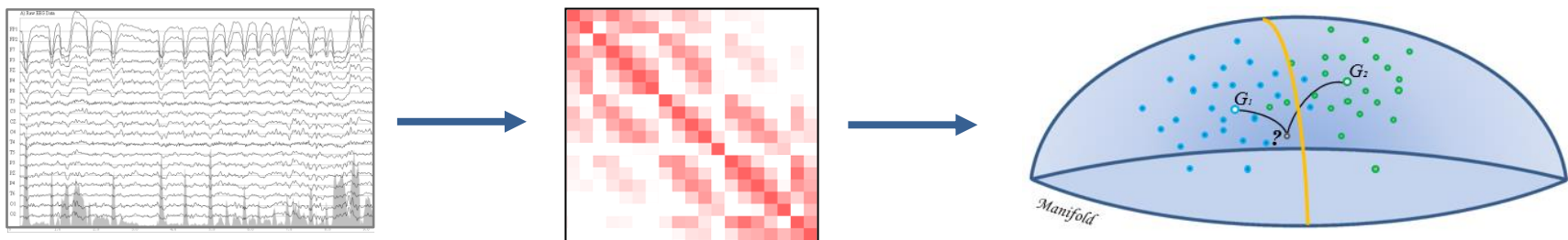
Existence and uniqueness first proved by Elie Cartan on Lie groups

Moakher M (2005) SIAM. J. Matrix Anal. Appl, 26 (3), 735-747.

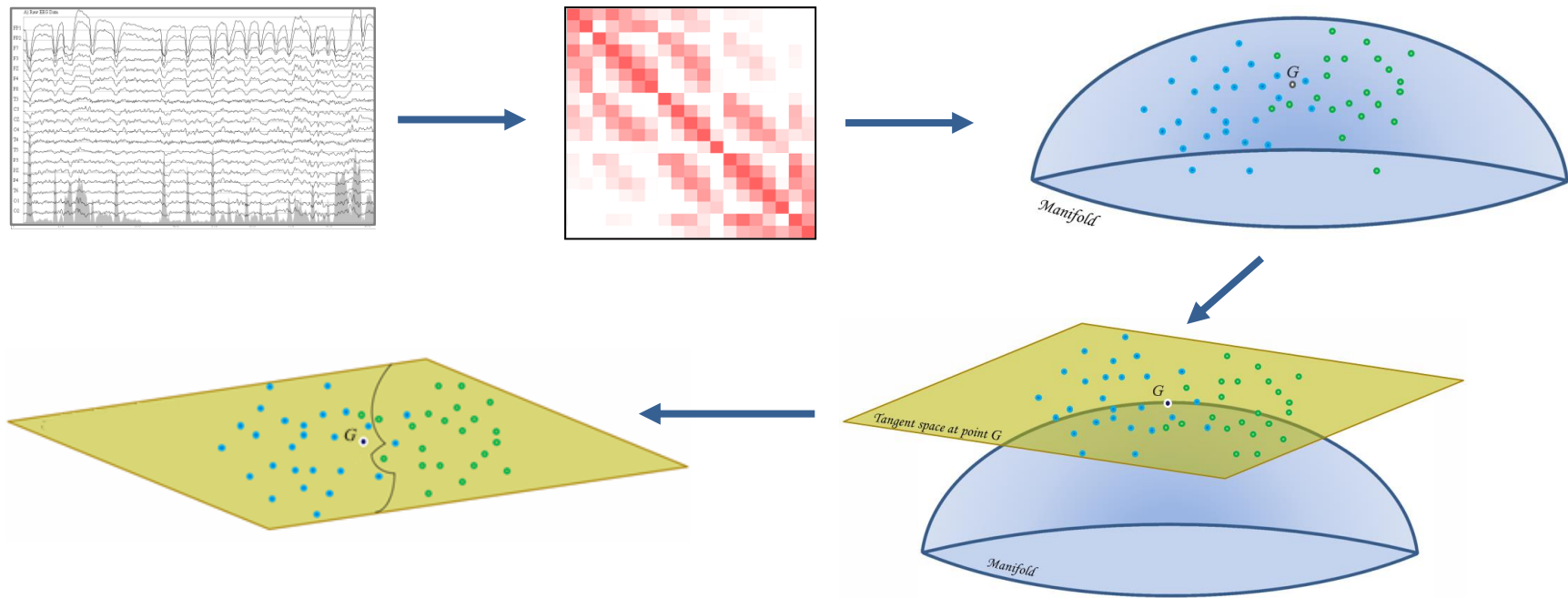
ML on Manifold



ML on Manifold



ML on Tangent Space



International BCI Decoding Competitions

Nom de la Compétition	Événement ou Organisateur	Clôture	Participants	Score (%)
DecMeg 2014	BIOMAG 2014 Conference	27/07/2014	301	75.5
BCI Challenge	IEEE NER 2015 Conference	24/02/2015	311	87.2
Grasp&Lift EEG Challenge	WAY European Project	31/08/2015	452	98.1
Decoding Brain Signals	Microsoft	01/07/2016	688	93.7
Biomag2016 competition	BIOMAG 2016 Conference	25/09/2016	7	95.6

Congedo, Bhatia & Barachant (2017)

Riemannian Geometry for EEG-based Brain-Computer Interfaces; a Primer and a Review.

Brain-Computer Interfaces, 4(3), 155-174.

Code for Riemannian geometry

(**Julia** – Python – R – Matlab – C++ – Delphi)

<https://sites.google.com/site/marcocongedo/science/code-resources>

P300-BCI Data

<https://sites.google.com/site/marcocongedo/science/EEG-data>

7 *Databases*

273 *Subjects*

1446 *Sessions*

BCI in Virtual Reality and on a PC

Description: Visual P300 BCI experiment on PC and virtual reality (passive head-mounted display) (2018)

Subjects: 21
Sessions: 2
Electrodes: 16 wet Silver/Silver Chloride electrodes
EEGMachine: g.tec g.USBamp
Sampling Rate: 512 sps
Database ID: VR.EEG.2018-GIPSA

Brain Invaders Multi-User Cooperation vs. Competition

Description: Multi-User Brain Invaders: Cooperation versus Competition condition (2015)

Subjects: 44
Sessions: 2
Electrodes: 32 wet Silver/Silver Chloride electrodes
EEGMachine: g.tec g.USBamp
Sampling Rate: 512 sps
Database ID: bi2015b

Brain Invaders Multi-User flash duration

Description: Brain Invaders calibration-less P300-based BCI with modulation of flash duration (2015)

Subjects: 50
Sessions: 3
Electrodes: 32 wet Silver/Silver Chloride electrodes
EEGMachine: g.tec g.USBamp
Sampling Rate: 512 sps
Database ID: bi2015a

Brain Invaders Solo vs. Multi-User Collaboration

Description: Multi-User Brain Invaders in Solo versus Collaboration condition (2014)

Subjects: 38
Sessions: 3
Electrodes: 32 wet Silver/Silver Chloride electrodes
EEGMachine: g.tec g.USBamp
Sampling Rate: 512 sps
Database ID: bi2014b

Brain Invaders dry electrodes

Description: Brain Invaders calibration-less P300-based BCI with dry electrodes (2014)

Subjects: 71
Sessions: up to three per participant
Electrodes: 16 metal dry electrodes
EEGMachine: g.tec g.USBamp
Sampling Rate: 512 sps
Database ID: bi2014a

Brain Invaders Adaptive vs. Non-Adaptive

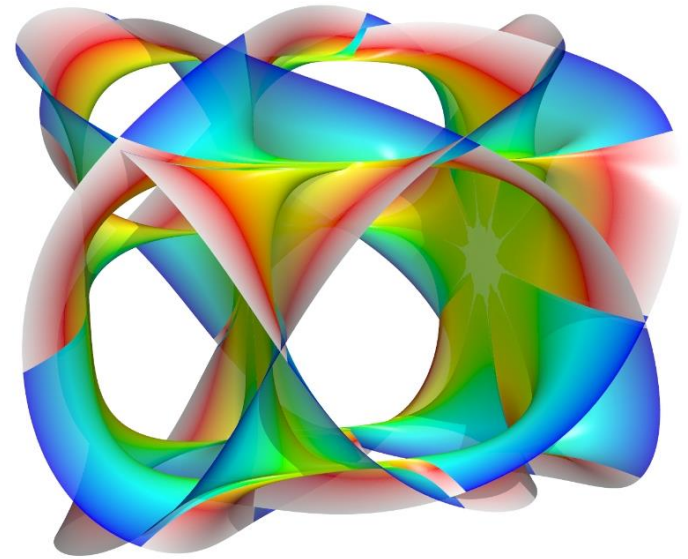
Description: First Brain Invaders dataset of a BCI working without calibration (2013)

Subjects: 24
Sessions: 8 for subjects 1-7, one for the remaining.
Electrodes: 16 wet Silver/Silver Chloride electrodes
EEGMachine: g.tec g.USBamp
Sampling Rate: 512 sps
Database ID: BLEEG.2013-GIPSA

Building Brain Invaders

Description: First Brain Invaders BCI dataset (2012)

Subjects: 25
Sessions: 2
Electrodes: 16 wet Silver/Silver Chloride electrodes
EEG Machine: TMSI Porti
Sampling Rate: 128 sps
Database ID: BLEEG.2012-GIPSA



Collaborators @GIPSA-lab

Prof. C. Jutten

Ingeneer: A. Andreev

Post-Doc: A. Barachant, P. Zanini

PhD Students: L. Korczowski, F. Barachant, P.L.C. Rodrigues, A. Bleuzé

