

# Adapting Frequent Itemsets Mining for Social Media Text by Strengthening the Closed Property

## ABSTRACT

Abstract

## 1. INTRODUCTION

The nature of text in social media poses a challenge when applying traditional information retrieval and text mining algorithms, as detailed in section 2. Text in social media is usually short, lacks context and structure, and is created at a very high rate. The sheer volume of social media streams mandates the use of efficient algorithms, and the short length of individual documents makes it possible to apply Frequent Itemsets Mining. This family of algorithms is very fast and efficient, however it is not readily suited for application on text. For example, {obama, won}, {sham, and, travesty, @realDonaldTrump} and {flag, in, her, hair} are itemsets mined from Tweets posted on November 6th, 2012 using terms as items. They appear frequently in the last couple of hours of the US elections day, but they are not as frequent as {of, the} and other English language constructs. Even if a maximum frequency threshold is set, risking to filter out important itemsets, a lot of non-English language constructs will be mined because the proportion of posts in English is much higher than other languages. In this paper we propose methods for adapting Frequent Itemsets Mining to be effective on social media text, without degrading its efficiency. As a result, the mined Frequent Itemsets provide the vocabulary associated with events and can be used in various ways for search and summarization.

Frequent itemsets include “trending topics”, as well as topics that have sustained interest for longer periods of time without sudden peaks. We use the term “trending topics” as defined by Twitter in a blog post explaining why a topic about WikiLeaks wasn’t trending as was expected by internet activists<sup>1</sup>. The blog post explains that a topic is trending if a sudden peak is detected, and links this to the

<sup>1</sup><http://blog.twitter.com/2010/12/to-trend-or-not-to-trend.html>

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, to republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee.

CIKM '13

Copyright 20XX ACM X-XXXXX-XX-X/XX/XX ...\$10.00.

change in the velocity of the conversation. Since the number of posts associated with such a trending topic must be high, Frequent Itemsets Mining will detect it. Furthermore, it detects topics that have a steady high volume of posts.

The rest of the paper is organized as follows:

## 2. FREQUENT ITEMSET MINING FOR SOCIAL MEDIA

Short non-canonical text

Works regardless of language... not only English

### 2.1 Preliminaries

A *frequent itemset* is a set of items that occur together a number of times higher than a certain threshold, called the *support* threshold. Traditionally, Frequent Itemset Mining is applied to a *database* of *transactions* made at a retail store. This terminology is suitable for market basket data and we will stick to it out of convention, even though we are mining text where the terms corpus and document are normally used. Because of the dynamic nature of social media, the input we give to mining algorithms is an *epoch* of data rather than a database. An epoch of data is all documents posted in a certain period of time, the length of this period is the epoch’s *span*. We also adapt the support threshold to the dynamicity of the volume of documents posted at different times of the day. We define the *minimum support threshold* as the threshold at the hour of the least volume during the day, an absolute number that is then converted to a ratio  $\alpha$ . The actual support threshold for a certain *span* of time is thus  $\alpha$  multiplied by the number of documents in the *epoch*.

Following is the notation used in this paper:

- $I = \{i_1, i_2, \dots, i_n\}$ : The set of possible items (vocabulary).
- $d_a = \{i_{a1}, i_{a2}, \dots\}$ : A document made up of a set of items, not necessarily terms. Each document has a sequential id denoted by the subscript letter.
- $E^{span} = \langle d_a, d_b, \dots \rangle$ : An epoch of data of a certain span, such as an hour, made up of a sequence of documents.
- $S \subset I$ : An itemset, its support is given by  $\|D_S\|$ .
- $D_S$ : All documents containing itemset  $S$ .
- $\|\cdot\|$ : The norm operator; gives the size of the operand.

## 2.2 Background

The two basic operations of Frequent Itemset Mining algorithms are *Candidate Generation* and *Solution Pruning*. The original Apriori algorithm by Agrawal et al. [?] generates candidates of length  $K$  ( $K$ -itemsets) by merging frequent itemsets of length  $(K-1)$  ( $(K-1)$ -itemsets) that differ in only 1 item, usually the last item given a certain total ordering of items. By using only frequent  $(K-1)$ -itemsets for generating candidate  $K$ -itemsets a lot of possible  $K$ -itemsets are implicitly pruned, based on that all subsets of a frequent itemset has to be frequent (the Apriori property). This still generates a very large number of candidates, specially in early iterations of the algorithm. Consider, for example, the generation of candidate 2-itemsets from a corpus of documents. This requires producing all unordered pairs of 1-itemsets (terms), after pruning out rare ones that appear less than the minimum support. In many domains, including text mining, the number of frequent 1-itemsets is large enough to prohibit generating a number of candidates in the order of this number squared.

## 3. BASIC ALGORITHM

To overcome the bottleneck of *Candidate Generation*, many algorithms are proposed to take hints from the transaction space rather than operating blindly in the item space, each based on a certain property that helps pruning out more candidates. In this paper we expand on LCM [?], an algorithm based on a property of a certain class of itemsets called *Closed Itemsets*. A formal definition of closed itemsets is given in equation 1:

$$\mathcal{C} = \{S_c : \nexists S_d \text{ where } S_c \subset S_d \text{ and } \|D_{S_c}\| = \|D_{S_d}\|\} \quad (1)$$

The properties of Closed Itemsets are the following:

1. An itemset is closed if adding any item to it will reduce its support.
2. A subset of a closed itemset is not necessarily closed, but one or more closed subset must exist for any itemset (formally this could be the empty set, given that any item that appears in all transactions is removed in a preprocessing step).
3. If a closed  $K$ -itemset can be extended any further then one of its supersets will be closed, however not necessarily a  $(K+1)$  superset. Itemsets that cannot be extended any further are called *Maximal Itemsets*, and they are a subclass of closed itemsets.

Besides being much smaller than the solution space of frequent itemsets, the solution space of closed itemsets can be navigated efficiently by using an arbitrary total ordering of items such that any closed itemset is considered an extension of exactly one of its subsets. Thus during candidate generation all its other subsets do not need to be extended by items that would lead to the longer closed itemset. This is called *Prefix Preserving Closure Extension (PPC-Extension)* and it is proposed and formally proved in [?]. This is achieved by following three rules, which we state after a few definitions to facilitate their statement. First, an item is *larger/smaller* than another item if it comes later/earlier in the total ordering. This terminology comes from that LCM is most efficient if the items are ordered in ascending order of their frequency.

Second, the *suffix* of an itemset is one or more items whose removal does not result in an itemset with higher support. Notice that they will necessarily be at the end of the itemset, regardless of the total ordering. Finally, we call the first item added to the suffix of the itemset its *suffix head*. With this terminology, the rules for *PPC-Extension* are:

1. An itemset can be extend only by items *larger* than its *suffix head*. Extending by *smaller* items will lead to closed itemsets already generated.
2. After forming an itemset  $S$ , add to its *suffix* all items whose frequency within  $D_S$  is equal to  $\|D_S\|$ .
3. If any item in the *suffix* is *smaller* than the suffix head, prune this solution branch. All closed itemsets within this branch have already been generated.

Table 1 is an example of how *PPC-Extension* is used to generate closed itemsets starting from the 1-itemset 'barack'. The upper table enumerates  $D_{barack}$ . The lower table shows steps of itemsets generation. The current solution along with its frequency is in column 2, solutions marked by an \* are the closed itemsets emitted. All possible extension items and their frequencies are in column 3 with the one being considered bolded. Column 4 is a comment explaining the step. At each step, a pass is done on  $D_{itemset}$  to enumerate and count possible extension items. To enforce a support threshold infrequent extension items are removed, but in this example there isn't such a threshold. Notice that the number of steps is linear in the number of closed itemsets, and the only additional storage required besides the storage of the documents is that of the possible extension items. Of course this is a simplified example, but it shows in essence how LCM achieves its low run time and memory requirements. We refer the interested reader to [?] for a theoretical proof that the algorithm runs in linear time in the number of closed itemsets, and that this number is quadratic in the number of documents. We proceed by describing how to implement this algorithm using an inverted index.

### 3.1 An inverted index based implementation

We show in 1 how to implement LCM and PPO-Extension using an inverted index. The algorithm also lends itself to distributed implementations easily. For example, a Map/Reduce implementation is easy since the only operations are counting (line 14) and projection (line 22). However, the fast execution time and the low memory requirements of the algorithm makes it possible that a distributed implementation will cause an overhead. In the implementation shown, it is not necessary that the index tokens list follow the total ordering; all itemsets of length 1 will be considered anyway.

The algorithm described works well on the sparse data typical to text mining. We experimented with other algorithms, and this one is superior to all of them. As the focus of this paper is not Frequent Itemsets Mining, but rather its adaptation to social media text, we give only a brief account of the experience. As mentioned before, Apriori based algorithms suffer from the candidate generation bottleneck when the number of distinct items is high. Another famous class of mining algorithms is the FP-Growth [?] based algorithms. FP-Growth skips the candidate generation step, and instead creates a succinct representation of the data as a special prefix tree called the FP-tree. An FP-tree imposes the invariant that within a branch the frequency is non-increasing from

Doc. Id	Document	Doc. Id	Document
a	barack & mitt	b	brack obama & mitt romney
c	brack obama & romney	d	brack obama

Documents (two per row)

Step	Current Solution	Possible Extension Items	Comments
1	{barack} (4)*	<b>mitt</b> (2), obama (3), romney (2)	Items ordered lexicographically
2	{barack, mitt} (2)*	<b>obama</b> (1), romney (1)	Extension items reenumerated & counted
3	{barack, mitt, obama} (1)	romney (1)	Rule 2: 'romney' appears in all $D_{itemset}$
4	{barack, mitt, obama, romney} (1)*		Rule 2: 'obama' is the <i>suffix head</i>
5	{barack} (4)	mitt (2), <b>obama</b> (3), romney (2)	Nothing more to add, back to 'barack'
6	{barack, obama} (3)*	<del>mitt</del> (1), <b>romney</b> (2)	Rule 1: skipping 'mitt', adding 'romney'
7	{barack, obama, romney} (2)*	<del>mitt</del> (1)	Rule 1: Nothing more to add.
8	{barack} (4)	mitt (2), obama (3), <b>romney</b> (2)	Back to 'barack', adding 'romney'
9	{barack, romney} (2)	<del>mitt</del> (1), obama (2)	Rule 2: add obama to suffix after 'romney'
10	{barack, romney, obama} (2)	<del>mitt</del> (1)	Rule 3: suffix isn't ordered, prune solution

Closed itemsets containing 'barack'

Table 1: Generation of closed itemsets by Prefix Preserving Closure Extension

**Input:**  $\alpha$ : Minimum support ratio  
**Data:**  $E$ : Epoch of documents  
**Result:**  $C$ : Closed itemsets having support  $\alpha$  within  $E$

```

1  $C \leftarrow \{\emptyset, \|E\|\}$ ; //  $\emptyset$  is a closed itemset
2  $X \leftarrow$  Inverted index of  $E$ ;
3 foreach  $i \in X.tokens$  do
4    $D_{\{i\}} \leftarrow X.postingsList[i]$ ;
5   if  $\|D_{\{i\}}\| \geq \alpha\|E\|$  then  $LCM(\{i\}, i, D_{\{i\}})$ ;
6 end
7 return  $C$ ;
8 Function  $LCM(S: Current\ itemset, i_{sh}: Suffix\ head,$ 
9  $D_S: Documents\ containing\ S)$  is
10   frequency[1... $i_n$ ]  $\leftarrow 0$ ;
11   suffix  $\leftarrow \{i_{sh}\}$ ;
12   foreach  $d \in D_S$  do
13     foreach  $i \in d$  do
14       frequency[ $i$ ]++;
15       if frequency[ $i$ ] =  $\|D_S\|$  then suffix.add( $i$ );
16     end
17   end
18   if  $\exists j : i_{sh} \succ suffix[j]$  then return;
19    $C.add(\langle S \cup suffix, \|D_S\| \rangle)$ ;
20   foreach  $i \succ i_{sh}$  and  $i \notin suffix$  do
21     if frequency[ $i$ ]  $\geq \alpha\|E\|$  then
22        $D \leftarrow D_S \cap i$ ; // Results of query  $S$  AND  $i$ 
23        $LCM(S \cup suffix \cup \{i\}, i, D)$ 
24     end
25   end
26 end

```

Algorithm 1: LCM Frequent Itemsets Mining

the root down to the leaves. While this class of algorithms performs better than the Apriori based ones, it suffers from the sparsity of the data since the data structure is succinct only if it can find common prefixes within the constraints of its invariant. Thus, LCM is the most efficient algorithm for Frequent Itemsets Mining on social media text.

One type of uninteresting itemsets that are produced just because the definition of closed itemsets cover them is itemsets of length less than 2 - the empty itemset and individual tokens. In the rest of this paper we exclude them altogether simply by not producing an itemset whose length is less than 2.

## 4. MINING TERM N-GRAMS

While extremely fast, Frequent Itemsets Mining produces numerous itemsets that are not interesting. Actually it was originally proposed as a preliminary stage to Association Rules Mining, which sifts through the relatively high number of itemsets and produces a smaller number of association rules. The first type of uninteresting itemsets is the language constructs that bear no information, such as "such as", and itemsets that are made up of all the different fragments of the language construct along with other items. The latter type is an artifact of itemset mining when the final itemset has such a language construct within it; for example, {we, did, it, #teamobama}. If we can treat sequential language constructs, and really any other multiword expression, as one item we reduce the number of itemsets produced in these two ways.

There are many measures of association that can be used to detect multiword expressions [?], but each measure is good under certain conditions and has special properties. We experimented with various measures, and in fact we found out that a very good measure for identifying multiword Named Entities is Yule's Q; a measure of association and disassociation derived from the odds ratio. However, we have finally found that for the purpose of preprocessing before Frequent Itemsets Mining what works best is tokenizing the documents into term N-Grams with varying N.

We start by tokenizing into unigrams, then we create two term bigrams for each unigram appearing with a probability

above a certain threshold,  $\eta$ , by attaching to it the unigrams before and after it. We do this recursively by increasing  $N$  and creating two  $(N+1)$ -Grams for each  $N$ -Gram above the threshold until there are no more  $N$ -Grams above the threshold. We explain this by walking through the example of tokenizing “Barack Obama president of the United States”:

1. First the tokens will be: ‘barack’, ‘obama’, ‘president’, ‘of’, ‘the’, ‘united’, ‘states’.
2. Assume that after tokenizing the whole epoch we find that the probability of ‘obama’, ‘of’ and ‘the’ within the current epoch is more than  $\eta$ . Therefore the tokens are changed to: ‘barack’, ‘barack + obama’, ‘obama + president’, ‘president’, ‘president + of’, ‘of + the’, ‘the + united’, ‘united’, ‘states’.
3. Assume that the bigrams ‘barack + obama’ and ‘of + the’ still appear with a probability higher than  $\eta$ . Therefore trigrams are generated by attaching the unigrams before and after them, and the tokens become: ‘barack’, ‘barack + obama + president’, ‘obama + president’, ‘president’, ‘president + of’, ‘president + of + the’, ‘of + the + united’, ‘the + united’, ‘united’, ‘states’.
4. At  $N=3$ , we find that no trigrams still appear with probability greater than  $\eta$ , and thus terminate.

Note that in the above procedure we didn’t prevent overlap, because there is no guarantee that the  $N$ -Gram created makes any sense. The goal is to create a single unit out of frequent sequences and this is achieved because the  $N$ -Gram keeps growing as long as it is frequent, which will stop as soon as a unigram that is not part of the expression is attached to it. We can look at this as flattening the Zipfian distribution by creating more tokens with less frequency each, but the distribution of the input doesn’t affect the results of itemsets mining - only its runtime can be affected.

Figure 1 shows the effect of increasing the maximum length of  $N$ -Grams from 1 to 5 on the number of tokens, the number of itemsets mined, and the runtime. The value of  $\eta$  used is 0.00012, which is the probability of the term ‘obama’ within the whole collection of Tweets. The values are averaged across

## 5. STRONGLY CLOSED ITEMSETS ALLIANCES

The closed property of an itemset is very easily violated by modifying one document that contains the itemset and removing one of its items. While an update operation is not supported in the model of frequent itemsets mining, a similar effect happens when people are writing about a certain fine grained topic. For example, on November 9th, 2012 many people where tweeting that “Justin Bieber and Selena Gomez broke up”. If all Tweets which use the verb “break up” to report the topic also contain the two names in full, then there will be one closed itemset with all 6 items. However, a Tweet can contain any of 8 other combinations of the four names that fully convey the meaning. We can consider that Tweets with any of these combinations are modifications of the *maximal* closed itemset. Therefore instead of 1 maximal itemset about the topic there will be 9 closed ones. Now consider that it is possible to say that “Justin and Selena broke up” as well as “Justin broke up with Selena”, resulting in 2

maximal itemsets. This increase in the number of maximal itemsets because of slight variations in the language is intensified by the length limit on Tweets, specially when people try to make space for their comment about a Tweet they are retweeting. We therefore need a property between the easily violated closed property and the very strict maximal property which also results in a large number of redundant itemsets.

We define a *strongly closed* itemset as a closed itemset whose frequency comprises more than a certain proportion of the frequency of its least frequent subset, or which has at least one strongly closed superset. This property chooses closed itemsets which violate the closed property of their subsets by a significant amount. It also filters out itemsets that are short language constructs, because they appear very frequently and none of their numerous supersets comprise a significant proportion of their high frequency. Notice that within the documents containing a closed itemset,  $D_{itemset}$ , the probability of a closed superset ( $\|D_{superset}\|/\|D_{itemset}\|$ ) is called the confidence of the rule “*itemset*  $\rightarrow$  *superset*.” This property is the basic property used for association rules mining, and some algorithms for mining itemsets based on a variation called *all confidence* have been proposed [?]. The strongly closed property builds on the notion of confidence by choosing itemsets which would result in rules of high confidence. The minimum acceptable confidence of the resulting rule is a parameter  $\kappa$  that can vary between 0 and 1 to increase the strength of the strongly closed property.

Even though choosing strongly closed itemsets filters out many redundant itemsets formed because of slight variations of the same topic, it is still not a strong enough property. The intersection of the sets of documents containing two strongly closed itemsets can be different by only 1 document from either of the sets. For example, the closed itemset {justin, selena} can have three closed supersets: {justin, selena, broke, up, #jelena}, {justin, selena, broke, up} and {justin, selena, #jelena}. Figure 5 illustrates how all supersets can be strongly closed, while they are still redundant itemsets mined from almost the same documents. Each circle in the figure represents the set of documents containing the superset formed by concatenating the words in the circle with the words in its container. The figure also shows, as dashed circles, examples of itemsets that would be filtered out by the strongly closed property.

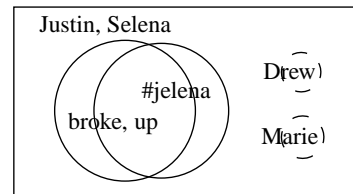
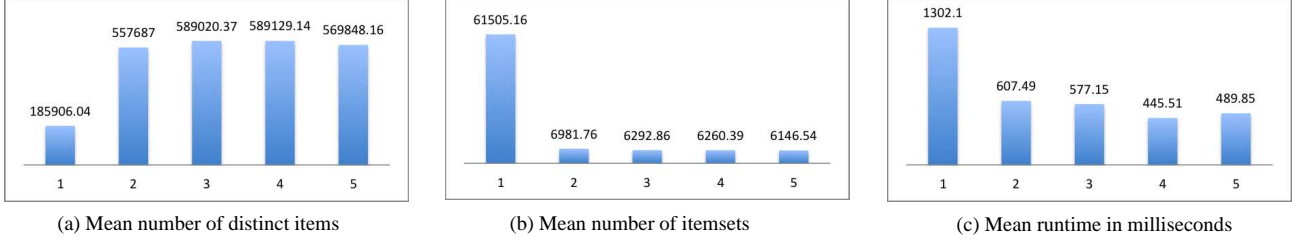


Figure 2: Strongly and not strongly closed itemsets

To remove such redundancy, we merge strongly closed itemsets with high cosine similarity into an itemset *alliance*. The itemset alliance is a bag of items formed by taking the union of all member itemsets. This concentrates the numerous itemsets about a certain topic into one unit. The alliance also include itemsets that are not supersets of one another; for example, itemsets that differ because of items that come earlier in the total ordering and are not pertaining



**Figure 1: Effect of the maximum N-Gram length on the mining of a 1hr epoch of data**

to the topic, such as “with” and “and” in the Justin-Selena example. The use of cosine similarity limits the candidates for merging to itemsets that are very likely about the same topic, since the Inverse Document Frequency (IDF) value of terms pertaining to a certain topics is typically much higher than the IDF value of other terms. The IDF and cosine similarity formulae used are given in equations 2 and 3 respectively.

$$IDF(i) = \log \frac{\|E\|}{\|D_i\|} \quad (2)$$

$$\cos(S_1, S_2) = \frac{\sqrt{\sum_{i \in S_1 \cap S_2} IDF(i)^2}}{\sqrt{\sum_{i \in S_1} IDF(i)^2} * \sqrt{\sum_{i \in S_2} IDF(i)^2}} \quad (3)$$

To avoid forming alliances that are not coherent about a certain topic, we calculate the difference between the posting lists of every two itemsets being merged. The merge is permitted only if the difference is below the maximum number of different document allowed for the required closed strength, given by equation 4. In case of merging an itemset with its superset, this difference can be calculated directly from the sizes of the posting lists. Otherwise, the difference can be efficiently calculated from the postings lists since they are sorted, and in fact it is enough to check if the difference exceeds the maximum number allowed.

$$\Delta(S_1, S_2, \kappa) = (1 - \kappa) * \max(\|D_{S_1}\|, \|D_{S_2}\|), \kappa \in [0, 1[ \quad (4)$$

After joining an alliance an itemset seizes to exist outside of the alliance, so that an itemset can be part of only one alliance. However, when checking which itemsets to consider for merging with a still unallied itemset, its similarity is calculated with the individual itemsets rather than the alliance. Otherwise, an itemset could fail to join an existing alliance because the similarity between the larger bag of items and the itemset is likely to be lower than the similarity between individual member itemsets and the itemset. This can cause cascading many alliances that should have been separate into one large alliance. Such an oversized alliance could also be about different topics. Empirically, we observe that one and only one oversized alliance about different topics is formed. This alliance catches many itemsets made up of low IDF terms, thus not interesting. Topics with high IDF terms in them cannot have high cosine similarity with topics of only low IDF terms. The size of the bag of items in this alliance is significantly larger than other alliances, making it easy to distinguish and discard it.

Expanding on the previous observation that adding a high IDF term to an itemset with low IDF terms only prevents

achieving a high enough cosine similarity, we introduce an optimization that improves both runtime, memory requirements and filtering power. Unlike the original LCM algorithm, our extension requires keeping previous itemsets in memory so that newly generated ones are compared to them to find candidates for alliance. Instead of keeping all previous itemsets in memory we keep only a limited number,  $b$ . This obviously improves runtime and memory requirements, and it can also improve the quality of itemsets chosen for alliance. If the total ordering follows the descending order of items’ frequencies, then, because of the way PPC-Extension produces itemsets, for an itemset  $S_x$  and any candidate subset  $S_{(x-b)}$  that was produced  $b$  itemsets earlier  $\|D_{S_{(x-b)}}\| - \|D_{S_x}\| \geq b$ . This lower bound is achieved when the intersection of the documents containing the current itemset  $S_x$  and all the  $b-1$  supersets before it is exactly the subset  $S_{(x-b)}$ , and there are no more itemsets with the same intersection. In this case each of the  $b$  supersets must have support at most  $\|D_{S_{(x-b)}}\| - 1$  to be considered closed, but because there are  $b$  of them then actually their support is  $\|D_{S_{(x-b)}}\| - b$ . Since the minimum support difference is  $b$ , then the maximum confidence of  $S_x$  is  $\frac{\|D_{S_{(x-b)}}\| - b}{\|D_{S_{(x-b)}}\|}$ . Therefore for a given  $\kappa$  we can determine the buffer size  $b$  as the frequency of itemsets keeps decreasing. In our implementation we use a fixed  $b$  of 1000.

Algorithm 2 shows the described algorithm for merging itemsets alliances. Table ?? shows the number of closed itemsets of length at least 2 without filtering any of them out, as well as after applying the KLD filter and the strongly closed filter, and the number of itemsets alliances. The table also shows the average quality of the itemsets after each stage of reduction. The quality is calculated as Basic Elements? PERPLEXITY? CASCADE MEASURE?

## 6. TEMPORAL MINING

KL-Divergence

## 7. EXPERIMENTAL EVALUATION

Examples from Nov. 6 and 9. Nov. 9 has two events for Bieber.. MTVEMA and break up.

## 8. CONCLUSION

## 9. ACKNOWLEDGEMENT

**Input:**  $\theta$ : Minimum cosine similarity,  
 $\kappa$ : Minimum closed strength  
**Data:** S: Frequent Itemsets produced by LCM  
**Result:** A: Frequent Itemsets alliances

```

1 for  $i \leftarrow 2$  to  $\|S\|$  do
2    $C \leftarrow \emptyset$ ; // Candidates for merging with  $S_i$ 
3    $T \leftarrow \emptyset$ ; // Subsets of current itemset
4   for  $j \leftarrow 1$  to  $i - 1$  do
5     if  $\|S_i \cap S_j\| = \min(\|S_i\|, \|S_j\|)$  then
6        $C.add(S_j)$ ;
7       if  $\|S_i\| > \|S_j\|$  then  $T.add(S_j)$ ;
8     else if  $\cos(S_i, S_j) \geq \theta$  then
9        $C.add(S_j)$ ;
10    end
11  end
12   $M.score \leftarrow \infty$ ; // Best merge candidate's score
13  foreach  $S_c \in C$  do
14    if  $S_c \in T$  then
15       $\delta \leftarrow \|D_{S_c}\| - \|D_{S_i}\|$ ;
16    else
17       $\delta \leftarrow \text{difference}(D_{S_i}, D_{S_c})$ ; // Stops early
18    end
19    if  $\delta \leq \Delta(S_i, S_c, \kappa)$  and  $\delta < M.score$  then
20       $M \leftarrow S_c$ ;
21       $M.score \leftarrow \delta$ ;
22    end
23  end
24  if  $M.score < \infty$  then
25     $A[M].itemset \leftarrow A[M].itemset \cup S_i \cup M$ ;
26  end
27 end
28 return A;

```

**Algorithm 2:** Merging itemsets into alliances