

Package ‘umfpackr’

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Type Package

Title Sparse Linear Algebra with UMFPACK

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Description Solve linear and non-linear
systems of equations using the sparse linear algebra package UMFPACK.

License GPL-3

Encoding UTF-8

LazyData true

LinkingTo Rcpp

Imports Rcpp

Depends methods, Matrix

RoxygenNote 7.1.1

Roxygen list(markdown = TRUE, old_usage = TRUE)

Suggests testthat, nleqslv, stringr, knitr, R.rsp

NeedsCompilation yes

SystemRequirements

For Linux and other non-Windows platforms the libsuitesparse-dev library is required.

VignetteBuilder knitr,
R.rsp

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umf_solve

*Solves the system of linear equations using UMFPACK***Description**

This function solves the linear equations of the form $Ax = b$ using UMFPACK.

Usage

```
umf_solve(a, b, umf_control = list())
```

Arguments

a	an object of class dgCMatrix (see dgCMatrix-class)
b	the vector b
umf_control	A named list with control parameters passed to UMFPACK. Currently only a single control parameter can be specified: ordering, which specifies the ordering method. Allowed values are "AMD" (the default), "CHOLMOD", "METIS" and "BEST". See the Vignette UMFPACK interface for R for more details. For example, to use METIS ordering specify <code>umf_control = list(ORDERING = "METIS")</code> . The METIS ordering method can handle larger matrices than the standard AMD method.

Value

the solution x

References

- Dennis, J.E. Jr and Schnabel, R.B. (1997), *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*, Siam.
- Davis, T.A. (2004). A column pre-ordering strategy for the unsymmetric-pattern multifrontal method. *ACM Trans. Math. Softw.*, **30(2)**, 165–195.
- Davis, T.A (2004). Algorithm 832: UMFPACK, an unsymmetric-pattern multifrontal method. *ACM Trans. Math. Softw.*, **30(2)**, 196–199.
- Davis, T.A and Duff, I.S. (1997). An unsymmetric-pattern multifrontal method for sparse LU factorization. *SIAM J. Matrix Anal. Applic.*, **18(1)**, 140–158.
- Davis, T.A and Duff, I.S (1999). A combined unifrontal/multifrontal method for unsymmetric sparse matrices. *ACM Trans. Math. Softw.*, **25(1)**, 1–19..

See Also

[umf_solve_nl](#).

umf_solve_nl	<i>Solves a system of non-linear equations using UMFPACK</i>
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Description

This function solves a system of non-linear equations $F(x) = 0$ using Newton's method. UMFPACK is employed to solve the linear equations in each Newton iteration. Optionally a cubic line search is used when a Newton step does not yield a sufficient reduction of the function values.

Usage

```
umf_solve_nl(start, fn, jac, ..., control, global = c("no", "cline"),
             scaling = c("row", "col", "none"), umf_control = list())
```

Arguments

start	initial guess of the solution x
fn	the function F
jac	a function returning the Jacobian of the function as a dgCMatrix object
...	arguments passed to fn and jac
control	a list with control parameters. See Details.
global	The global strategy. Possible values are "no" (no global strategy, the default) and "cline" (cubic line search).
scaling	Scaling method. Possible values are "row", "col" and "none". The default is "row". See Details.
umf_control	A named list with control parameters passed to UMFPACK. Currently only a single control parameter can be specified: ordering, which specifies the ordering method. Allowed values are "AMD" (the default), "CHOLMOD", "METIS" and "BEST". See the Vignette UMFPACK interface for R for more details. For example, to use METIS ordering specify <code>umf_control = list(ORDERING = "METIS")</code> . The METIS ordering method can handle larger matrices than the standard AMD method.

Details

Control options: Argument `control` is a named list containing one or more of the following components:

`ftol` The function value tolerance. Convergence is reached if the largest absolute function value is smaller than `ftol`. The default value is $1e-8$.

`xtol` The relative step size tolerance. If convergence has not been reached yet and if the relative step size of all x values is smaller than `xtol` then the iteration process is terminated with an error. The default value is $1e-8$. The relative step size of $x[i]$ is calculated as $|x[i] - x^*[i]| / \max(|x[i]|, 1)$, where $x^*[i]$ is the value of $x[i]$ at the previous iteration.

`maxiter` The maximum number of iterations. The default is 20 if no global strategy is used (argument `global = "no"`), and 150 if cubic line searching is used (argument `global = "cline"`).

trace A logical. If TRUE then the progress of the iteration is printed. The default is FALSE.

silent A logical. If TRUE then all output is suppressed. The default is FALSE.

cnd_tol The tolerance for the inverse condition of the jacobian. If the inverse condition is smaller than `cnd_tol`, the solution process is terminated with an error, except if control parameter `allow_singular` is set to TRUE. The default is the machine precision (on most platforms about $2e-16$). If the inverse condition is very small but nonzero it may be difficult to find a solution, or the solution may not be meaningful. However, sometimes a good solution can be found even if the condition is quite small. The test for the ill-conditioning of the jacobian can be turned off by setting `cnd_tol` to 0 or a negative number. However, if the matrix is singular (the inverse condition is exactly zero), it is never possible to continue with the solution process (except if control parameter `allow_singular` is set to TRUE). The default value of `cnd_tol` is quite small, in some cases it can be appropriate to use a somewhat larger value (for example $1e-12$)

cnd_method A character vector specifying the method used to estimate the inverse condition number of the jacobian. Possible options are "umfpack" (the default), "condtest" and "kappa". For "umfpack" a rough estimate of the condition as computed by UMFPACK is used, using the expression $\min(\text{abs}(\text{diag}(U)))/\max(\text{abs}(\text{diag}(U)))$, where U is the U matrix of the LU factorisation of the jacobian. Method "condtest" employs function `condtest` of the Matrix package and kappa the function `kappa` of the base package. Method `condtest` is more accurate than the rough estimate of UMFPACK, but takes more time. `kappa` is exact, but is slow for large matrices because this function does not use sparse matrices. Method "condtest" usually gives a reasonable approximation of the inverse condition number. It is recommended to normally use "umfpack", but occasionally use "condtest" for a more accurate check of the condition number.

allow_singular A logical value (default FALSE) indicating if a small correction to the Jacobian is applied when it is singular or too ill-conditioned. The method used is similar to a Levenberg-Marquardt correction and is explained in Dennis and Schnabel (1996) on page 151. If TRUE, then the correction is applied if the inverse condition is exactly zero or if the inverse condition is smaller than control parameter `cnd_tol`.

Scaling of the Jacobian: For each iteration in the Newton method the linear system $Js = F(x)$ is solved, where the Jacobian matrix J are the derivatives of the equations with respect to the variables, and s the Newton step. Scaling can improve the condition of the Jacobian. For *row scaling*, the system is transformed to $D^{-1}Js = D^{-1}F(x)$, where D is a diagonal matrix with row scaling factors. Here the scaling factors are the L1 norms of the rows of J . For *column scaling*, the system is transformed to $JD^{-1}Ds = F(x)$, where D is a diagonal matrix with column scaling factors, calculated from the L1 norms of the columns of J .

The scaling is only used to solve the linear equations and has no effect on the convergence of the Newton algorithm. Thus the iterations are considered to be converged when the maximum value of the unscaled function values $F(x)$ is smaller than `ftol`. It is therefore recommended to define a the function $F(x)$ for which all function values have comparable orders of magnitude and all function arguments x have similar orders of magnitude.

Value

a list with the following components:

solved A logical equal to TRUE if convergence of the function values has been achieved.

iter	the number of iterations
x	the final values of x
fval	the final function values
message	A string equal to "ok" if a solution has been found. Otherwise it describes the reason why the iteration was stopped without success

References

- Dennis, J.E. Jr and Schnabel, R.B. (1997), *Numerical Methods for Unconstrained Optimisation and Nonlinear Equations*, Siam.
- Davis, T.A. (2004). A column pre-ordering strategy for the unsymmetric-pattern multifrontal method. *ACM Trans. Math. Softw.*, **30**(2), 165–195.
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See Also

[umf_solve](#).

Examples

```
library(umfpackr)

dslnex <- function(x, c) {
  y <- numeric(2)
  y[1] <- x[1]^2 + x[2]^2 - c
  y[2] <- exp(x[1]-1) + x[2]^3 - c
  y
}

jacdsln <- function(x, c) {
  n <- length(x)
  Df <- matrix(numeric(n*n),n,n)
  Df[1,1] <- 2*x[1]
  Df[1,2] <- 2*x[2]
  Df[2,1] <- exp(x[1]-1)
  Df[2,2] <- 3*x[2]^2

  return(as(Df, "dgCMatrix"))
}

xstart <- c(2,3)
print(umf_solve_nl(xstart, dslnex, jacdsln, c = 2,
  control = list(trace = TRUE)))
```

```
# now use METIs columns ordering (run this on Linux only)
if (.Platform$OS.type != "windows") {
  print(umf_solve_nl(xstart, dslnex, jacdsln, c = 2,
    control = list(trace = TRUE),
    umf_control = list(ordering = "METIS")))
}
```

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