# Forecasting Canadian Inflation Using Dynamic Model Averaging


**Abstract**

This paper forecasts CPI inflation and core-CPI inflation measures for Canada using Dynamic Model Averaging (DMA) over the period 1990Q1 to 2019Q2. DMA combines a state space model with a markov chain model which allows the significance of the predictors and the model itself to change over time. We compare the forecasting performance of DMA against traditional forecasting models. There are two key results. First, we find that DMA improves upon forecasts produced by standard econometric models for the short run for both CPI and core-CPI inflation. Second, we find that DMA's forecasting performance is more ambiguous in the medium and the long run.



**Keywords:** Bayesian, State space model, Phillips curve, Inflation

**JEL Classification:** E31, E37, C11, C53

## 1. Introduction

We discuss the issues surrounding the econometric models used for forecasting, use Canadian data to conduct a related statistical exercise by applying Dynamic Model Averaging (DMA) and Dynamic Model Selection (DMS) to forecast CPI inflation measures, and discuss the implications of the findings. Inflation forecasts play a crucial role in assisting central banks to monitor economic performance and conduct monetary policy. Since the introduction of the inflation targeting framework in 1991, the Bank of Canada has used core consumer price index (CPI) inflation as an operational guide to conduct its monetary policy and has been successful in keeping inflation relatively low and stable, even during periods of economic volatility (Bank of Canada, 2016). Inflation forecasts are also important for corporations (financial and non-financial) that may negotiate long-term contracts with suppliers or need to set the prices of their products in advance. These forecasts are also of interest for non-profit institutions serving households, such as trade unions, that may need to negotiate future compensation packages on behalf of their members.

As a special case of Bayesian Model Averaging, DMA was first introduced by Raftery, Kárny, and Ettler (2010), where it is described as a combination of a state space model with a markov chain model. DMA is a two-step technique that estimates a number of time varying parameter models and then averages them over a set of recursively updating weights. This technique makes it feasible for a large number of unique models to be computed and averaged in real time by allowing for some computations to be replaced by empirically sensible approximations. Since DMA's introduction in 2010, the model has been widely used for forecasting in situations where model uncertainty and structural breaks may lead to the coefficients of the independent variables and the model itself to alter over time. For example, it was used by Wang, Ma, Wei, and Wu (2016) to forecast realized volatility. In addition to DMA forecasts, we also produce forecasts using DMS. DMS is also a two-step technique which estimates a number of time varying parameter models and then selects the model that is assigned the highest weight.

Our research question is the following: Do DMA and DMS forecasts of CPI inflation and core-CPI inflation measures outperform traditional forecast models in Canada? The traditional models we consider are autoregressive integrated moving average models (ARIMA), a vector autoregression model (VAR), a autoregression model (AR) and a simple model that simply takes the mean of the endogenous series as the forecasted values. To our knowledge, DMA and DMS have not been applied to forecast inflation in Canada, making our paper a clear and important contribution to the extant literature. We use quarterly data, over the period 1990Q1 to 2019Q2 for core-CPI inflation measures and 1993Q1 to 2019Q2 for CPI inflation, and consider out of sample performance. There are two main findings. First, DMA and DMS produce superior forecasts compared to all the benchmarks in the short run. Second, DMA and DMS forecasts are better in the medium and long run than those from traditional models for core-CPI inflation measures but not for CPI inflation.

The rest of this paper is as follows. Section 2 provides a review of the literature and outlines our contribution to the literature. Section 3 outlines the data and methods. In Section 4, we discuss the results. Section 5 concludes with a summary and policy implications.

## 2. What we know about inflation forecasting

### *2.1 Related Literature*

The switch in Canadian monetary policy regime in 1991 lead to permanent change in the time series properties of CPI inflation and core-CPI inflation. Since the introduction of the inflation targeting framework in Canada in 1991, the CPI inflation and core-CPI inflation series have become less volatile. The lower volatility has led to lower forecasting errors in forecasts produced by a variety of different models due to lower predictability and increased uncertainty of Canadian inflation rates. Thus, this decrease in volatility has made it more difficult for Bank of Canada to improve inflation forecasts over what can be achieved by standard econometric models. (Champagne, Bellisle & Sekkel, 2018). This is consistent with the phenomenon of the disappearance of the predictable components of fluctuation pertaining to inflation in the US since the mid-1980s (Stock & Watson, 2007; Tulip, 2009). In addition to decreased volatility, studies also find a shift in inflation persistence in Canada due to the change in monetary policy. Inflation persistence refers to the duration of the shocks hitting inflation and can usually be measured using a simple univariate time series model or a more structural model. Pre-inflation targeting Canadian economy was characterized as having high inflation persistence and this held true through the period between the collapse of the Bretton Woods regime and the introduction of inflation targeting. Since the introduction of inflation targeting, inflation persistence has been low for the Canadian economy. In fact, the Canadian inflation series is characterized as being close to white noise under inflation targeting (Benati, 2008; Burdekin & Siklos, 1999; Siklos, 1999).

The recent data release of historical staff projections of economic aggregates by Bank of Canada has made it possible to study the central bank's forecasting accuracy for both the CPI inflation and the core-CPI inflation and to pinpoint areas for improvement. Since the projections are available from 1982Q1, the evaluations also help us assess the effect of the monetary policy regime change on the accuracy of the bank's inflation forecasts. It is interesting to note that while the decline in the volatility of CPI inflation post-1991 lead to a drop in the forecasting errors associated with the bank's inflation forecasts, this reduction was lower than the drop in forecasting errors associated with the forecasts produced by the benchmarks used for evaluation. Due to the higher volatility of non-core inflation, it is not surprising that forecasting errors in non-core inflation are the largest contributors to forecasting errors in the CPI inflation projections by Bank of Canada. Short-term staff projections perform better than forecasts produced by standard econometric models and even professional forecasters. However, at a forecasting horizon of three years, forecasts from a simple benchmark are just as if not more accurate than the predictions made by Bank of Canada's staff. Another interesting observation from the evaluation of the bank's forecasts is that the forecast for core inflation performs worse than forecasts from the private sector and simple benchmarks (Champagne, Bellisle, & Sekkel, 2018; Matheson, 2019).

Due to the important role inflation forecasts play in implementation of monetary and fiscal policy in Canada, there is always search for new techniques that may improve upon the shortcomings of Bank of Canada's staff economic projections. There is extensive literature examining the predictive accuracy of a variety of alternative forecasting methods ranging from simple autoregression models to dynamic factor models.

The shortcomings of the above-mentioned models are threefold. First, the coefficients of the explanatory variables can change over time. This can be due to a structural break in the data, a change in the monetary or fiscal policy or a variety of other reasons. As a result, models which fail to account for these parameter changes may produce unreliable forecasts. For example, structural breaks in macroeconomic data may make forecasts from the generalized Phillips curve model, as used by Stock and Watson (1999), unreliable because it fails to account for parameter change. Second, in using a large number of explanatory variables to forecast inflation, many models suffer from the problem of over-parameterization. Due to this limitation, standard time series models, such as the VAR and ARIMA, are only able to incorporate a dozen times series at most into their evaluation. This is the case even when hundreds of different macroeconomic aggregates are available. Many researchers have turned to diffusion indexes to address this issue. For example, Stock and Watson (1998) used 224 different time series as explanatory variables to produce US inflation forecasts that outperform standard econometric benchmarks.

Third, the relevant model for forecasting inflation can change over time. Many researchers find that the performance of the Phillips curve is episodic. (Gabrielyan, 2019; Stock & Watson, 2007, 2008) That is, it sometimes forecasts better than a univariate model, such as in an inflation targeting economy, and in some situations a univariate analysis performs better, such as in a fixed exchange or a pre-inflation targeting regime. Researchers have turned to Bayesian model averaging to address this issue of model uncertainty (Avramov, 2002).

### *2.2 Contribution to the literature*

This paper examines the predictive power of DMA and DMS for inflation in Canada and thereby makes its contribution to extant literature on inflation forecasting. To our knowledge no other study has applied this technique to forecast of inflation in Canada. DMA has produced consistently reliable forecasts for US inflation that have outperformed standard econometric benchmarks and even improved upon the Greenbook forecasts produced by the Federal Reserve Board of Governors. (Koop and Korobilis, 2012). However, the Canadian economy is characterized as a small open economy. This characteristic combined with the inflation-targeting monetary policy regime presents a new environment for DMA application. Canadian inflation time series also has some unique attributes. For instance, US data has been observed to be useful in forecasting Canadian inflation. (Gosselin & Tkacz, 2001, 2010). As Canada was one of the first countries to introduce an inflation-targeting policy, the time series to assess the usefulness of DMA and DMS in a small open inflation-targeting economy is one of the best.

## 3. Data and Methodology

### *3.1 Data*

Bank of Canada uses core inflation as an operational guide to conduct its monetary policy and has been successful in keeping inflation low and stable despite economic volatility. To sort through the temporary effects of the most volatile components of the total Consumer Price Index (CPI) the bank had historically used Core Consumer Price Index (CPIX), which is defined as CPI excluding Food and Energy (CPIXFET) from 1980Q1 to 2001Q1, and as CPI excluding the 8 most volatile components (CPIX) from 2001Q2 to 2013Q4. Both these measures exclude effects of changes in indirect taxes. (Champagne, Bellisle, & Sekkel, 2018). However, in recent years the reliability of CPIX has declined due to the volatility in energy prices. Thus, the bank has made the decision to replace it with three new measures of core CPI, namely CPI-common, CPI-trim and CPI-median. Total inflation in the Canadian economy is captured by CPI inflation and core inflation is approximately captured by CPI-common inflation. For this reason, we choose two measures of inflation: CPI inflation and CPI-common inflation as our dependent variables (Bank of Canada, 2016).These measures of inflation are defined as follows:

1. CPI Inflation: Year-over-year percentage change in Consumer Price Index including all items and excluding the effect of indirect taxes.
2. CPI-common Inflation: Year-over-year percentage change in CPI-common. Where CPI common is a measure of core inflation that tracks the common price changes in the CPI basket using a factor model.

In conducting unit root tests on the dependent variables using the Augmented Dickey Fuller and Philips-Perron test, we find both CPI inflation and CPI-common inflation series to be stationary at levels. This is consistent with Ng and Perron (2001) which shows that unit root tests find inflation series in Canada to be stationary at 5% significance level when using Bayesian information criterion (BIC),

We work with quarterly estimates of 8 independent variables: Unemployment Rate, Real Personal Consumption Expenditure, Real Gross Domestic Product, Housing Starts, M1 Money Stock, Dow Jones Industrial Average, 3 Month Treasury Bill Rate, Government of Canada marketable bonds 5-10-year average yield. All dependent variables are seasonally adjusted. The values of CPI-common inflation are collected for the period of 1990Q1 to 2019Q2 and the values of CPI inflation are collected for the period of 1993Q1 to 2019Q2. All variables are transformed to be stationary. When obtaining results from the Bayesian forecasting methods we include all exogenous variables as well as two lags of the dependent variables.

*3.2 Methodology*

Our approach to forecasting inflation involves using two approximations which are introduced in Raftery et al. (2010). First, let there be *m* explanatory variables and let Z be the set of all the explanatory variables. We have $\mathrm{K} = 2^{\mathrm{m}}$ possible models, where each model has a different subset of Z as its explanatory variables, denoted by $x^{(k)}$ for *k* = 1, … *K*. A single model from this set of K models can be described by the equations:

$$y_t = (x_t^{(k)})^T \theta_t^{(k)} + \epsilon_t^{(k)} \qquad (1)$$

$$\theta_t^{(k)} = \theta_{t-1}^{(k)} + \delta_t^{(k)} \qquad (2)$$

where $\epsilon_t^{(k)} \sim N(0, H_t^{(k)})$ and $\delta_t^{(k)} \sim N(0, Q_t^{(k)})$

Hence, each model from the set of K potential models is specified as a time-varying parameter model where $x_t^{(k)}$is the column vector of dependent variables in the $k^{th}$ regression model and $\theta_t^{(k)}$ is the column vector of regression coefficients. Normally the estimation of $\theta_t^{(k)}$, given the actual values of the dependent variable up to the preceding time period, would require for us to run the MCMC algorithm. However, we instead estimate $\theta_t^{(k)}$using an approximation introduced by Raftery et al. (2010). The primary benefit of using this approximation is that we only need to use Kalman filtering for each time-varying parameter model. The original equation used for the estimation of $\theta_t^{(k)}$ is as follows:

$$\theta_t |\, y^{t-1} \sim N(\hat{\theta}_{t-1}, \Sigma_{t|t-1}) \qquad (3)$$

$$\Sigma_{t|t-1} = \Sigma_{t-1|t-1} + Q_t \qquad (4)$$

We then replace the estimation of the variance term with the following approximation:

$$\Sigma_{t|t-1} = \frac{1}{\lambda}\, \Sigma_{t-1|t-1} \qquad (5)$$

In this approximation $\lambda$ is a forgetting factor between 0 and 1. It tells us that an observation j periods in the past will have a weight of $\lambda^j$. For example, $\lambda$ = 0.99 implies that observations 5 periods in the past have 80% as much weight as observations from the last period. A lower value of $\lambda$ allows the parameters of the time-varying parameter models to transition faster in response to structural breaks. $\lambda$ = 1 implies that there is no time variation in the parameters (Koop & Korobilis, 2012).

Let $L_t = k$ if the process is covered by the $k^{th}$ model at time t and let $Y^t = (y_1, y_2, \dots, y_t)$ i.e. information up to time t. DMA involves calculating posterior probability for each time-varying parameter model. Posterior probability is the probability that the process we are trying to estimate is covered by the $k^{th}$ model at time t, given information up to time period t-1. Posterior probability at time t for model k is represented by $\pi_{t|t-1,k}$.To obtain a single point forecast in dynamic model averaging we first compute $\pi_{t|t-1,k} = \Pr\,(L_t = k|Y^{t-1})$ for each k = 1, 2, .... K and average the forecast given by each for the K models over their posterior probabilities. The calculation for posterior probabilities could be performed by specifying a transition matrix and using the MCMC algorithm. However, this would be computationally burdensome because if the number of explanatory variables used is large, then we would have a large number of models to estimate. Thus, we use the following approximation specified by Raftery et al. (2010). for calculating posterior probabilities:

$$\pi_{t|t-1,k} \approx \frac{(\pi_{t-1|t-1,k})^{\alpha}}{\sum_{l=1}^{K}(\pi_{t-1|t-1,l})^{\alpha}} \qquad (6)$$

Here $\alpha$ is another forgetting factor that lies between 0 and 1. It tells us how much weight is given to forecasting performance from the previous periods as compared to forecasting performance from the last period. For example, if $\alpha$ = .99 then forecasting performance from 5 years ago would receive 80% as much weight as forecasting performance from the last period. If $\alpha$ = .95 then forecasting performance from 5 years ago would receive 35% as much weight as forecasting performance from the last period (Koop & Korobilis, 2012). Conventionally, $\alpha$ lies between 0.95 and 0.99, depending upon how much weight forecasting performance in the past period holds.

The following approximation again allows for the estimation for posterior probabilities using a method comparable to updating equations in the Kalman filter. The updating equation here is:

$$\pi_{t|t,k} = \frac{\pi_{t|t-1,k}\, p_k(y_t|y^{t-1})}{\sum_{l=1}^{K}\pi_{t|t-1,l}\, p_l(y_t|y^{t-1})} \qquad (7)$$

These estimations are all conditional on $H_t^{(k)}$. As we are forecasting inflation there is a high possibility that the variance of errors i.e. $H_t^{(k)}$, is changing with time. Thus, according to Koop & Korobilis (2012), $H_t^{(k)}$ is best approximated by an exponentially weighted moving average.

$$\hat{H}_{t+1|t}^{(k)} = \kappa \hat{H}_{t|t-1}^{(k)} + (1-\kappa)\left(y_t - z_t^{(k)}\hat{\theta}_t^{(k)}\right)^2 \qquad (8)$$

Finally, at time-period t = 0, we assume that all the models are equally probable and so $\pi_{0,k} = \frac{1}{K}$ for each k = 1,.....,K. The coefficient column vectors are initially set to 0 i.e. $\theta_0^{(k)} = 0$ for each k = 1, ....., K. The final equation for the forecast is given by:

$$\hat{y}^{DMA} = \sum_{k=1}^{K} \pi_{t|t-1,k} \hat{y}_t^{(k)} = \sum_{k=1}^{K} \pi_{t|t-1,k} \left(x_t^{(k)}\right)^T \hat{\theta}_{t-1}^{(k)} \quad (9)$$

Dynamic model averaging involves estimating *K* time-varying parameter models and averaging them through a set of recursively updating weights to obtain a single point forecast. Dynamic model selection on the other hand involves selecting the forecast produced by the model with the highest posterior probability.

## 4. **Results**

### *4.1 Comparing Different Forecasting Methods*

In terms of forecasts we present results for the following:

1. Mean of the Series
2. ARIMA

3. Univariate AR (1) Model
4. Vector Autoregressive model (VAR)
5. DMA & DMS $\lambda = \alpha = .99$ and h = 1
6. DMA & DMS $\lambda = \alpha = .99$ and h = 4
7. DMA & DMS $\lambda = \alpha = .99$ and h = 8
8. DMA & DMS $\lambda = \alpha = .95$ and h = 1
9. DMA & DMS $\lambda = \alpha = .95$ and h = 4
10. DMA & DMS $\lambda = \alpha = .95$ and h = 8
11. DMA $\lambda = 1, \alpha = .99$ and h = 1 (special case of DMA where coefficients do not vary with time)
12. TVP model contain intercept, two lags of inflation and all the predictors, $\lambda = .99, \alpha = 1$ and h = 1 (Special case of DMA or DMS where coefficients are allowed to vary but 100% of the prior weight is attached to the model with all the predictors)
13. BMA $\lambda = \alpha = 1$ and h = 1

We perform dynamic model averaging and dynamic model selection on all eight exogenous and two lags of the dependent variable. We use an uninformative prior over the models so that in the first period all models are equally likely. We use a data based prior on the initial condition of the states. The data based prior builds the prior using historical data. Further, in accordance with the suggestion made by Koop & Korobilis (2012), we set the decay factor for the exponentially weighted moving average approximation as 0.98. We present forecast evaluations for 1-step ahead, 4-step ahead and 8-step ahead forecasts for both CPI inflation and CPI-common inflation in Table 1 and Table 2 respectively.

Forecasting accuracy of the various econometric models is evaluated using Root Mean Squared Error (RMSE) and Theil inequality coefficient $U_1$. The RMSE of a particular forecast depends upon the scale of the dependent variable and a smaller RMSE indicates that a forecast is more accurate. Theil inequality coefficient $U_1$ is a value between 0 and 1, with 0 indicating a perfect forecast and the accuracy of forecasts decreasing as Theil $U_1$ approaches 1.

**Table 1. Comparing Different Forecasting Methods: CPI-Common Inflation**

| | h=1 | | h=4 | | h=8 | |
|---|---|---|---|---|---|---|
| | RMSE | Theil U1 | RMSE | Theil U1 | RMSE | Theil U1 |
| DMA ($\alpha = \lambda = .95$) | 0.1164 | 0.0308* | 0.2506 | 0.0671 | 0.2791* | 0.0751* |
| DMS ($\alpha = \lambda = .95$) | 0.1197 | 0.0317 | 0.2443* | 0.0656* | 0.2909 | 0.0783 |
| DMA ($\alpha = \lambda = .99$) | 0.1164* | 0.0309 | 0.2764 | 0.0744 | 0.3248 | 0.0875 |
| DMS ($\alpha = \lambda = .99$) | 0.1191 | 0.0315 | 0.2873 | 0.0770 | 0.3389 | 0.0912 |
| BMA ($\alpha = \lambda = 1$) | 0.1166 | 0.0309 | 0.2816 | 0.0900 | 0.3327 | 0.0900 |
| DMA ($\alpha = 1, \lambda = .99$) | 0.1167 | 0.0309 | 0.2781 | 0.0879 | 0.3262 | 0.0879 |
| DMS ($\alpha = 1, \lambda = .99$) | 0.1184 | 0.0314 | 0.2858 | 0.0916 | 0.3396 | 0.0916 |
| DMA-NTVP ($\alpha = .99, \lambda = 1$) | 0.1165 | 0.0309 | 0.2809 | 0.0894 | 0.3308 | 0.0894 |
| VAR(4) | 0.1531 | 0.0415 | 0.4731 | 0.1395 | 0.4731 | 0.1395 |
| ARIMA | - | - | 1.0640 | 0.3784 | 1.0640 | 0.3784 |
| Mean of series | 0.3010 | 0.0795 | 0.3010 | 0.0795 | 0.3010 | 0.0795 |

Notes: * indicates lowest value

**Table 2. Comparing Different Forecasting Methods: CPI Inflation**

| | h=1 | | h=4 | | h=8 | |
|---|---|---|---|---|---|---|
| | RMSE | Theil U1 | RMSE | Theil U1 | RMSE | Theil U1 |
| DMA ($\alpha = \lambda = .95$) | 0.2856 | 0.0803 | 0.4093 | 0.1158 | 0.3920 | 0.1117 |
| DMS ($\alpha = \lambda = .95$) | 0.3030 | 0.0854 | 0.4126 | 0.1163 | 0.3856 | 0.1101 |
| DMA ($\alpha = \lambda = .99$) | 0.2822 | 0.0797 | 0.4082 | 0.1157 | 0.4125 | 0.1193 |
| DMS ($\alpha = \lambda = .99$) | 0.2897 | 0.0818 | 0.4103 | 0.1165 | 0.4072 | 0.1180 |
| BMA ($\alpha = \lambda = 1$) | 0.2826 | 0.0799 | 0.4091 | 0.1160 | 0.4155 | 0.1208 |
| DMA ($\alpha = 1, \lambda = .99$) | 0.2834 | 0.0801 | 0.4088 | 0.1159 | 0.4108 | 0.1188 |
| DMS ($\alpha = 1, \lambda = .99$) | 0.2913 | 0.0822 | 0.4148 | 0.1177 | 0.4119 | 0.1195 |
| DMA NTVP ($\alpha = .99, \lambda = 1$) | 0.2815* | 0.0796* | 0.4080 | 0.1157 | 0.4179 | 0.1215 |
| VAR(1) | 0.2818 | 0.0799 | 0.3675 | 0.1044* | 1.3092 | 0.1044* |
| AR(1) | 0.2816 | 0.0800 | 0.3674* | 0.1045 | 0.3674* | 0.1045 |
| Mean of series | 0.3888 | 0.1117 | 0.3888 | 0.1117 | 0.3888 | 0.1117 |

Notes: * indicates lowest value

Table 1 and Table 2 present our comparison results amongst different forecasting methods. We observe that our results vary considerably depending on the forecasting horizon. For h=1, DMA and DMS produce forecasts that are more accurate than the benchmarks. This holds true for both CPI-common and CPI inflation forecasts. For h=1, the DMA variant which has non-time-varying parameters and allows for gradual model change produces the most accurate CPI inflation forecasts. However, when evaluating CPI-common forecasts in the short run, we find that two different Bayesian methods perform better than other techniques. Specifically, the DMA variant with $\alpha = \lambda = .95$, which allows for faster model and parameter change in response to a structural break produces the CPI-common forecast with the lowest Theil $U_1$ and the DMA variant with $\alpha = \lambda = .99$, which assigns more weight to past observations and therefore allows for more gradual parameter and model change produces the CPI-common forecasts with the lowest RMSE. Hence, there is ambiguity in which DMA variant produces the best CPI-common inflation forecasts. Another key result in the short run is that forecasting using DMS holds no advantage over using the DMA approach. Specifically, DMS forecasts have greater RMSE and Theil $U_1$ compared to DMA forecasts.

For medium and long forecasting horizon, the results are more ambiguous. For h=4, we observe that AR, VAR or the mean of the endogenous series produce forecasts that are more accurate that the ones produced by DMA and DMS for CPI inflation. In-fact, just taking the mean of the CPI inflation series as the forecasted values produces a forecast that has lower RMSE and Theil $U_1$ than the ones produced by DMA and DMS. Amongst CPI-common inflation forecasts, we observe that the Bayesian forecasting techniques perform better. Specifically, we can see that DMS with $\alpha = \lambda = .95$ produces the most accurate CPI-common forecasts.

For h=8, DMA with $\alpha = \lambda = .95$ produces CPI-common forecasts that are more accurate than the forecasts produced by any of the benchmarks. Looking at CPI inflation forecasts, we observe that there is ambiguity on whether the VAR(1) or the AR(1) model produces the most accurate forecasts.

To summarize, we make a few key observations when examining the question of which Bayesian forecasting model produces the most accurate forecasts across different forecasting horizons. We see that DMA and DMS with $\alpha = \lambda = .95$ performs better than all other Bayesian forecasting techniques when forecasting CPI-common inflation across all forecasting horizons. This shows a clear preference for techniques that allow for faster model and parameter change when forecasting CPI-common inflation. In forecasting CPI inflation rates for h=1, we notice a preference for DMA variants which allows for more gradual model and parameter changes.

Overall, we observe that forecasts produced by DMA and DMS for CPI-common inflation improve upon the forecasting accuracy of the benchmarks across all forecasting horizons. . For CPI inflation forecasts, DMA and DMS produce forecasts that are more accurate than the benchmarks in the short run. However, for h=4 and h=8, the forecasts produced by the benchmarks are more accurate.

*4.3 Which variables are good predictors of inflation?*

Theoretically, in assigning posterior probabilities to each time-varying parameter model at a given time, dynamic model averaging ensures that the most relevant model is being used for forecasting at that particular time. In our empirical application, we want to ensure that this theoretical change is actually taking place. Thus, we analyze posterior inclusion probabilities (PIP) of predictors to determine if there is a change in which predictors are considered useful by

DMA and DMS for forecasting the CPI inflation and CPI-common inflation during our sample period. PIP is the probability that a particular variable has a non-zero coefficient at a particular point in time. For a given exogenous variable in our analysis, the PIP of that variable is calculated as the sum of the posterior probabilities for all the individual models that include that particular variable (Sala-i-Martin, Doppelhofer & Miller, 2004).

In keeping with Koop & Korobilis (2012), we define a variable to be important in forecasting CPI-common and CPI inflation rates at a particular point in time if it has a PIP of 0.5 or higher at that particular time. We present the results from our analysis in the in Appendix 7 and 8. We analyze results from one-step ahead, four-step ahead and eight-step ahead forecasts.

In our analysis, we can see that DMA and DMS are selecting the best predictors for CPI-common and CPI inflation rates at different time periods. Barring the first lagged value of CPI-common and CPI inflation, there are no other variables which are consistently considered to be important predictors across different forecasting horizons and throughout the entire sample period. The PIP of our numerous explanatory variables is continuously changing based on the past predictive accuracy of the individual time-varying parameter models and this suggests that the models used for forecasting by DMA and DMS are continuously changing throughout the sample period. Furthermore, in our empirical application we observe that both gradual and sudden changes in the importance of an individual predictor are possible. For instance, in forecasting CPI-common inflation, there is an abrupt increase in the PIP of GDP at market prices in 2007 for h=4. On the other hand, when forecasting CPI inflation there is a gradual increase in the importance of household final consumption expenditure starting in 2001 for h=1. Overall, we observe that our numerous explanatory variables continuously gain and lose importance throughout the sample period. Hence, we can conclude that DMA and DMS overcome the shortcoming of model change faced by traditional models.

### *4.4 Sensitivity Analysis*

In our results, we present forecasts using Bayesian forecasting techniques for a set range of forgetting factors. Specifically, for $\alpha, \lambda \in [0.95, 0.99]$. Here a value close to .99 for the forgetting factors implies that the change in the forecasting model is gradual and the parameters are relatively stable. A value of the forgetting factors closer to .95 implies rapid change in the forecasting model and parameter instability. We restrict our analysis to this range of forgetting factors as Koop and Korobilis (2012) find that this range is reasonable for most empirical approximations.

We now present a sensitivity analysis to show that the forecasts produced by DMA and DMS are robust to a reasonable degree of change in the forgetting factors $\alpha$ and $\lambda$. There is a need for this analysis because Raftery et al. (2010) set $\alpha = \lambda = .99$, as they find their results to be relatively insensitive to changes in the forgetting factors between the range of .97 to .995. They argue this robustness will be true for different values of the forgetting factors too. Koop and Korobilis (2012) find that this claim of robustness is true for $\alpha, \lambda \in [0.95, 0.99]$ for US inflation. They present results for forecast evaluations using a variety of different values for the forgetting factors, ranging from .80 to .99. Their results indicate that the Root Mean Squared Error and Mean Absolute Error for forecasts produced by DMA and DMS do not change drastically in response to a slight change in the forgetting factors. Moreover, they discover that there is no added advantage to setting $\alpha = \lambda = .80$, which allows both the model and the time varying coefficients to transition faster in response to new information. We follow the same approach as Koop and Korobilis (2012) and present forecast evaluations using a range of different forgetting factors.

**Table 3. Sensitivity Analysis: CPI-Common Inflation**

| | h=1 | |
|---|---|---|
| | RMSE | Theil U1 |
| DMA ($\alpha = .80, \lambda = .80$) | 0.135515 | 0.037259 |
| DMA ($\alpha = .99, \lambda = .95$) | 0.128299 | 0.035341 |
| DMA ($\alpha = .95, \lambda = .99$) | 0.131321 | 0.036285 |
| DMS ($\alpha = .80, \lambda = .80$) | 0.142682 | 0.039167 |
| DMS ($\alpha = .99, \lambda = .95$) | 0.133539 | 0.036741 |
| DMS ($\alpha = .95, \lambda = .99$) | 0.148208 | 0.040856 |
| | h=4 | |
| | RMSE | Theil U1 |
| DMA ($\alpha = .80, \lambda = .80$) | 0.240343 | 0.065582 |
| DMA ($\alpha = .99, \lambda = .95$) | 0.259988 | 0.071918 |
| DMA ($\alpha = .95, \lambda = .99$) | 0.278452 | 0.077475 |
| DMS ($\alpha = .80, \lambda = .80$) | 0.213872 | 0.058541 |
| DMS ($\alpha = .99, \lambda = .95$) | 0.255435 | 0.071051 |
| DMS ($\alpha = .95, \lambda = .99$) | 0.269935 | 0.075380 |
| | h=8 | |
| | RMSE | Theil U1 |

| | | |
|---|---|---|
| DMA ($\alpha = .80, \lambda = .80$) | 0.291026 | 0.081495 |
| DMA ($\alpha = .99, \lambda = .95$) | 0.332902 | 0.094043 |
| DMA ($\alpha = .95, \lambda = .99$) | 0.357947 | 0.101174 |
| DMS ($\alpha = .80, \lambda = .80$) | 0.295048 | 0.082843 |
| DMS ($\alpha = .99, \lambda = .95$) | 0.337043 | 0.095302 |
| DMS ($\alpha = .95, \lambda = .99$) | 0.370756 | 0.104575 |

**Table 4. Sensitivity Analysis: CPI Inflation**

| | h=1 | |
|---|---|---|
| | RMSE | Theil U1 |
| DMA ($\alpha = .80, \lambda = .80$) | 0.342404 | 0.095526 |
| DMA ($\alpha = .99, \lambda = .95$) | 0.291400 | 0.082013 |
| DMA ($\alpha = .95, \lambda = .99$) | 0.285202 | 0.080698 |
| DMS ($\alpha = .80, \lambda = .80$) | 0.366054 | 0.102701 |
| DMS ($\alpha = .99, \lambda = .95$) | 0.305137 | 0.085890 |
| DMS ($\alpha = .95, \lambda = .99$) | 0.287208 | 0.081318 |
| | h=4 | |
| | RMSE | Theil U1 |
| DMA ($\alpha = .80, \lambda = .80$) | 0.446452 | 0.126306 |
| DMA ($\alpha = .99, \lambda = .95$) | 0.421508 | 0.119203 |
| DMA ($\alpha = .95, \lambda = .99$) | 0.419258 | 0.118905 |
| DMS ($\alpha = .80, \lambda = .80$) | 0.448489 | 0.126270 |
| DMS ($\alpha = .99, \lambda = .95$) | 0.430738 | 0.120919 |
| DMS ($\alpha = .95, \lambda = .99$) | 0.419759 | 0.118954 |
| | h=8 | |
| | RMSE | Theil U1 |

| | | |
|---|---|---|
| DMA ($\alpha = .80, \lambda = .80$) | 0.401326 | 0.113135 |
| DMA ($\alpha = .99, \lambda = .95$) | 0.384758 | 0.109816 |
| DMA ($\alpha = .95, \lambda = .99$) | 0.428306 | 0.124124 |
| DMS ($\alpha = .80, \lambda = .80$) | 0.355387 | 0.100007 |
| DMS ($\alpha = .99, \lambda = .95$) | 0.380062 | 0.108135 |
| DMS ($\alpha = .95, \lambda = .99$) | 0.437162 | 0.126919 |

Tables 3 and 4 shows that for α, λ ∈ $[0.80, 0\ .99]$ there is no drastic change in the RMSE and MAPE of the forecasts produced by DMA and DMS. So, our estimation is relatively insensitive to a small change in the values of the forgetting factors. There is added advantage to setting $\alpha = \lambda = .80$ for CPI-common forecasts in the medium run and for CPI inflation forecasts in the long run. This indicates that techniques that allow for rapid model and parameter change are favored when forecasting CPI-common and CPI inflation rates in the medium run and in the long run, respectively. Moreover, we observe a decline in the accuracy of the forecasts for longer forecasting horizons. The result that our estimations are insensitive to small changes in the values of the forgetting factors is consistent with Koop & Korobilis (2012).

## *5. Conclusion*

This paper investigates the performance of DMA and DMS in forecasting Canadian inflation over the period 1990Q1 to 2019Q2. Given the excellent performance of DMA in forecasting the US inflation rate, we apply this Bayesian method towards forecasting CPI Inflation and CPI-common inflation rates in the Canadian economy. Our results in applying DMA and DMS in a small inflation-targeting economy are not as favorable as those observed by Koop & Korobilis (2012). We observe that the forecasting performance of DMA and DMS depends considerably on the forecasting horizon. In the short run, DMA and DMS produce forecasts that are more accurate than those produced by our benchmarks. However, in the medium and the long run we find that standard econometric models such as the AR and the VAR may sometimes produce more accurate forecasts.

In addition, we also find that in the medium and long run, the forecasts produced by taking the mean of the CPI inflation series as the forecasted values produces the more accurate forecasts than produced by DMA or DMS. The inflation targeting framework in Canada ensures that the mean of the CPI inflation series does not stray too far away from the inflation target.

Since Canada adopted its inflation-targeting policy framework in 1992, Bank of Canada has had an excellent track record of maintaining CPI inflation around its 2 percent target rate. The Bank of Canada staff forecasts, specially their "nowcasts" (forecasts for the current quarter), have been significantly more accurate than standard econometric models in forecasting CPI inflation (Champagne, Bellisle, & Sekkel, 2018; Matheson, 2019). However, there still remain potential areas for improvement where an application of DMA and DMS might be useful. For instance, Matheson (2019) found that Bank of Canada staff core inflation forecasts tend to be less accurate than those produced by the private sector and standard econometric models. In our analysis, we find that in the short run DMA and DMS produce core inflation forecasts that are more accurate than forecasts produce by AR and VAR models and a simple mean forecast. The natural next step is to further this investigation by analyzing how the core inflation forecasts produced by DMA and DMS perform in comparison to Bank of Canada staff forecasts for core inflation.

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## *Appendix 1: Transformations*

All variables are transformed to be approximately stationary. We apply the following transformation:

| *Name* | *Description* | *Transformation* |
|---|---|---|
| bonds___5_10_yrs | Government of Canada marketable bonds 5-10-year average yield | First Difference |
| gdp_mprices | Real Gross Domestic Product | First Difference |
| household_fce | Real Personal Consumption Expenditure | First Difference |
| housing | Housing Starts | First Difference |
| industrial_average | Dow Jones Industrial Average | First Difference |
| m1_seasonally_adj | M1 Money Stock | Log First Difference |
| t_bill_3_mth | 3 Month Treasury Bill Rate | Level |
| u_rate | Unemployment Rate | First Difference |
| Inflation_total | CPI Inflation | First Difference |
| Inflation_cpi_common | CPI-Common Inflation | First Difference |

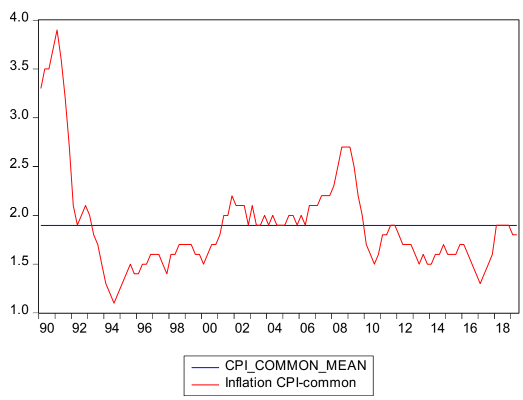
### *Appendix 2: Unit Root Test*

| | Augmented Dickey-Fuller Test Statistic | |
|---|---|---|
| Exogenous | CPI-Common Inflation | CPI Inflation |
| Constant | -4.722654*** | -3.809574*** |
| Constant, Linear Trend | -5.179595*** | -3.791763** |
| None | -2.325943** | -0.459645 |

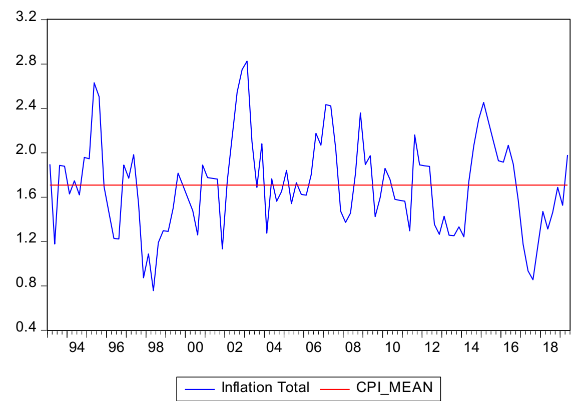
*Note: * denotes 10% significance level; ** denotes 5% significance level; *** denotes 1% significance level*

### *Appendix 3: Mean of the Series*

CPI-Common Inflation




CPI Inflation



*Appendix 4: Vector Autoregressive Model (VAR)*

*Lag Length Criteria*

CPI-Common Inflation

| Lag | LogL | LR | FPE | AIC | SC | HQ |
|---|---|---|---|---|---|---|
| 0 | -56.36865 | NA | 0.131642 | 3.648041 | 3.739649 | 3.678406 |
| 1 | -15.74373 | 73.63266* | 0.013358 | 1.358983 | 1.633809* | 1.450080 |
| 2 | -10.17626 | 9.395103 | 0.012160 | 1.261017 | 1.719059 | 1.412845 |
| 3 | -5.805297 | 6.829636 | 0.011992 | 1.237831 | 1.879091 | 1.450390 |
| 4 | 0.716668 | 9.375325 | 0.010416* | 1.080208 | 1.904685 | 1.353499* |
| 5 | 4.599465 | 5.096171 | 0.010782 | 1.087533 | 2.095227 | 1.421555 |
| 6 | 8.596281 | 4.746219 | 0.011237 | 1.087732 | 2.278643 | 1.482486 |
| 7 | 11.03946 | 2.595882 | 0.013143 | 1.185033 | 2.559161 | 1.640518 |
| 8 | 18.85426 | 7.326372 | 0.011259 | 0.946609* | 2.503953 | 1.462824 |

* indicates lag order selected by the criterion

LR: sequential modified LR test statistic (each test at 5% level)

FPE: Final prediction error

AIC: Akaike information criterion

SC: Schwarz information criterion

HQ: Hannan-Quinn information criterion

CPI Inflation

| Lag | LogL | LR | FPE | AIC | SC | HQ |
|---|---|---|---|---|---|---|
| 0 | -78.17604 | NA | 0.514422 | 5.011002 | 5.102611 | 5.041368 |
| 1 | -40.83257 | 67.68505 | 0.064081 | 2.927035 | 3.201861* | 3.018132 |
| 2 | -35.79005 | 8.509247 | 0.060282 | 2.861878 | 3.319921 | 3.013706 |
| 3 | -34.70076 | 1.702014 | 0.072979 | 3.043797 | 3.685057 | 3.256357 |
| 4 | -29.25751 | 7.824672 | 0.067813 | 2.953594 | 3.778071 | 3.226885 |
| 5 | -18.08409 | 14.66511* | 0.044505* | 2.505256* | 3.512949 | 2.839278* |
| 6 | -15.15308 | 3.480576 | 0.049576 | 2.572068 | 3.762978 | 2.966821 |
| 7 | -12.70689 | 2.599081 | 0.057977 | 2.669180 | 4.043308 | 3.124665 |
| 8 | -6.425948 | 5.888380 | 0.054662 | 2.526622 | 4.083966 | 3.042837 |

* indicates lag order selected by the criterion

LR: sequential modified LR test statistic (each test at 5% level)

FPE: Final prediction error

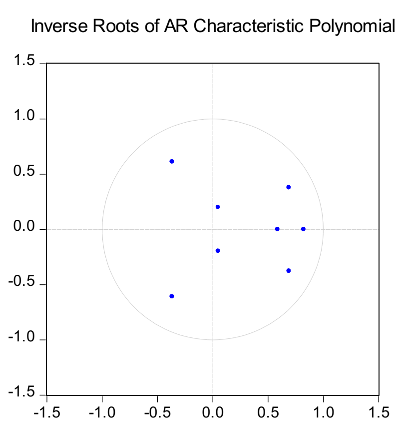
AIC: Akaike information criterion

SC: Schwarz information criterion

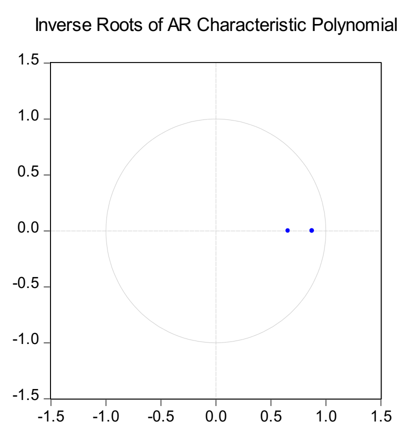
HQ: Hannan-Quinn information criterion

*Eigenvalue Test*

CPI-Common Inflation




CPI Inflation



*Serial Correlation LM Test*

| Lags | LM Statistic | |
|---|---|---|
| | CPI-Common Inflation | CPI Inflation |
| 1 | 10.27539** | 1.707426 |
| 2 | 5.382424 | 1.608988 |
| 3 | 2.778579 | - |
| 4 | 5.485594 | - |
| 5 | 4.585506 | - |

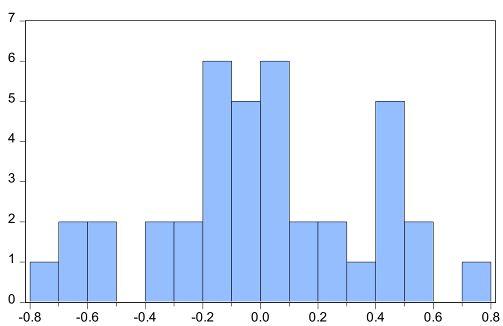
*Note: * denotes 10% significance level; ** denotes 5% significance level; *** denotes 1% significance level*

### *Appendix 5: AutoRegressive Model (AR)*

| | Test statistic |
|---|---|
| Serial Correlation LM Tests | 0.560281 |
| Breusch-Pagan-Godfrey: Heteroskedasticity Test | 0.656665 |

*Note: * denotes 10% significance level; ** denotes 5% significance level; *** denotes 1% significance level*

*Jarque-Bera Normality Test*

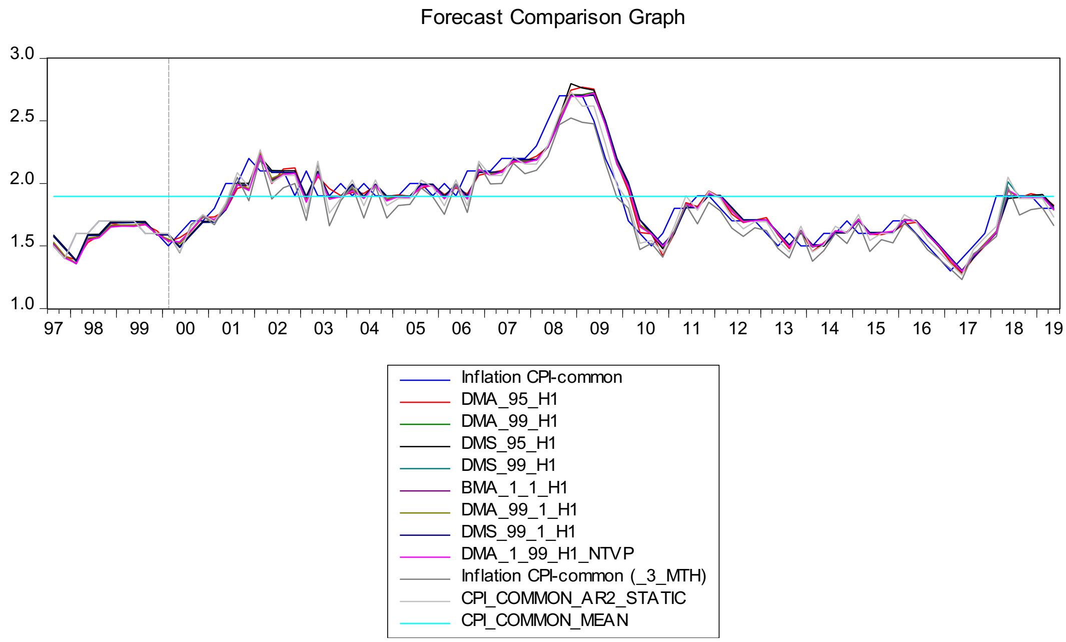



| | |
|---|---|
| Series: Residuals | |
| Sample 1993Q2 2002Q4 | |
| Observations 39 | |
| Mean | -2.72e-16 |
| Median | -0.051970 |
| Maximum | 0.766156 |
| Minimum | -0.724221 |
| Std. Dev. | 0.360749 |
| Skewness | -0.019717 |
| Kurtosis | 2.503353 |
| Jarque-Bera | 0.403346 |
| Probability | 0.817362 |

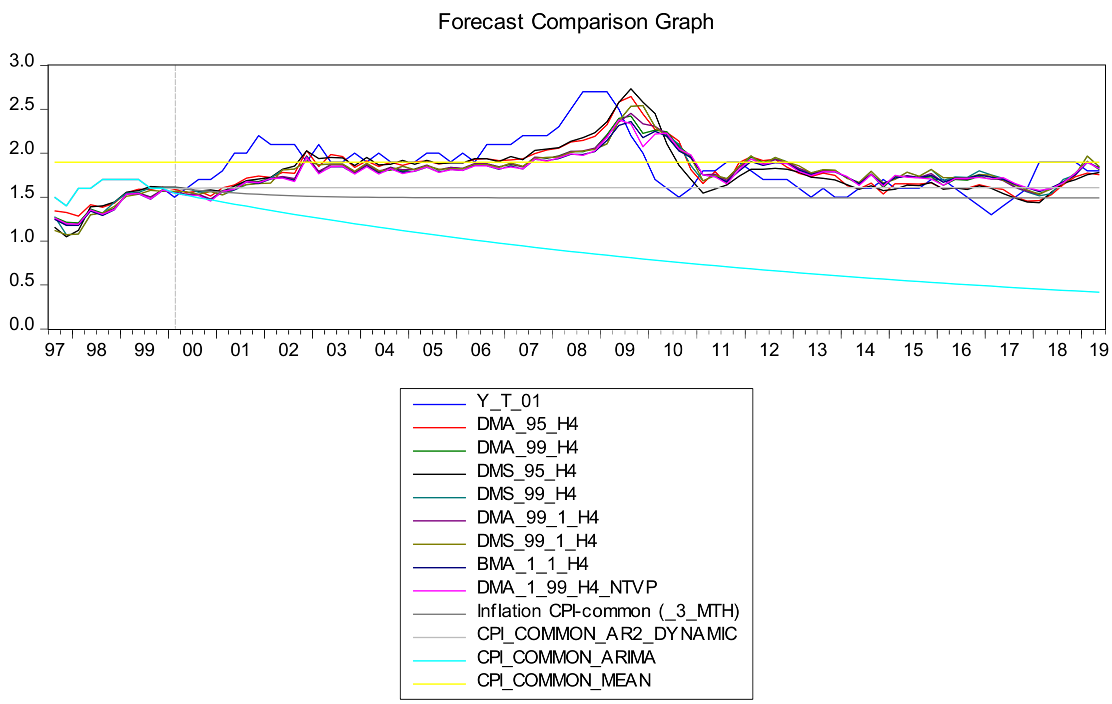
## *Appendix 6: Forecast Comparison Graph*

### *CPI-Common Inflation*

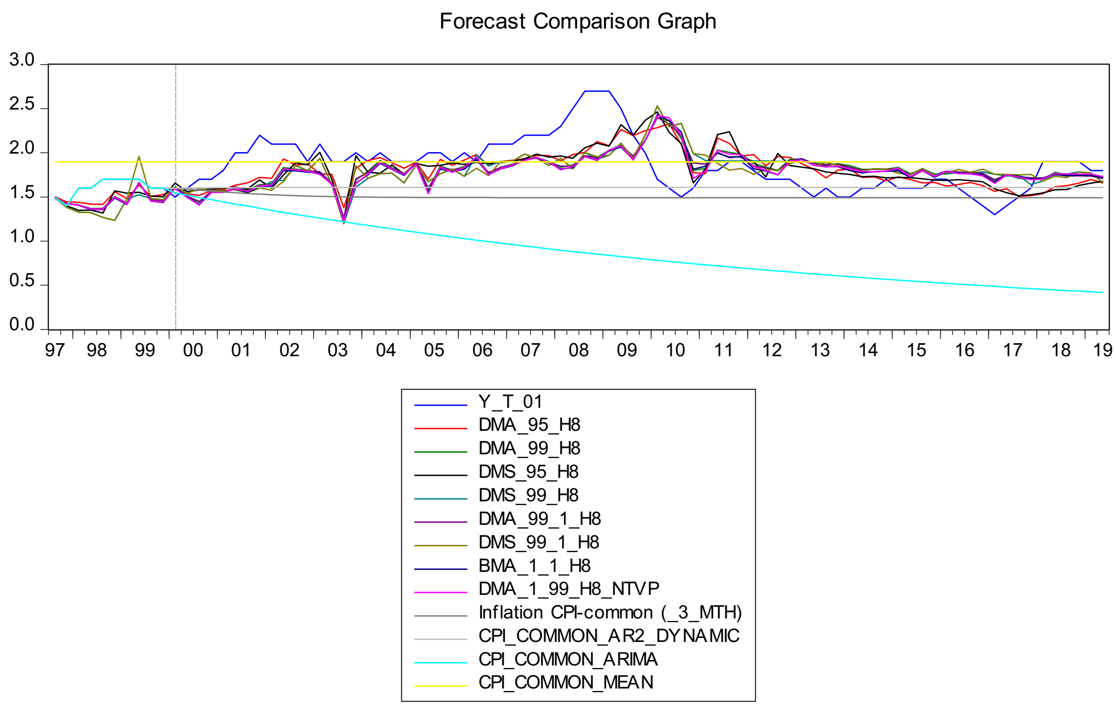
1. h=1




2. h=4

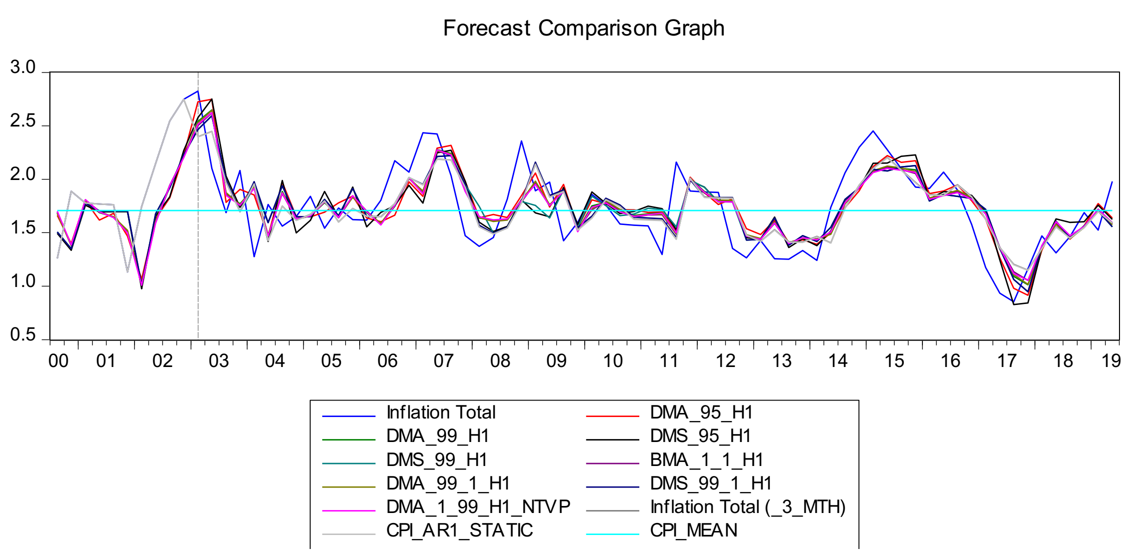


3. h=8

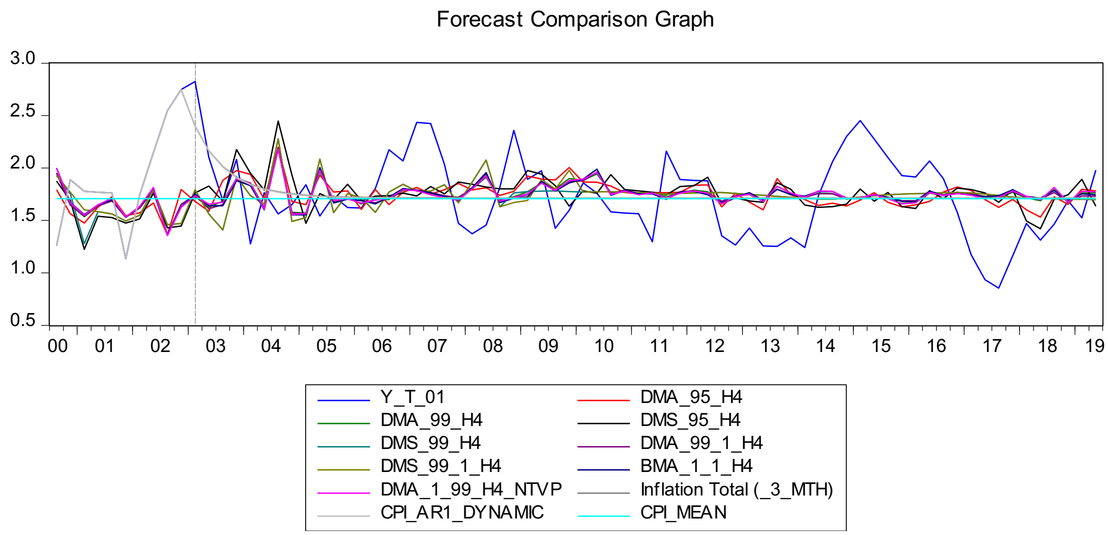



*CPI Inflation*

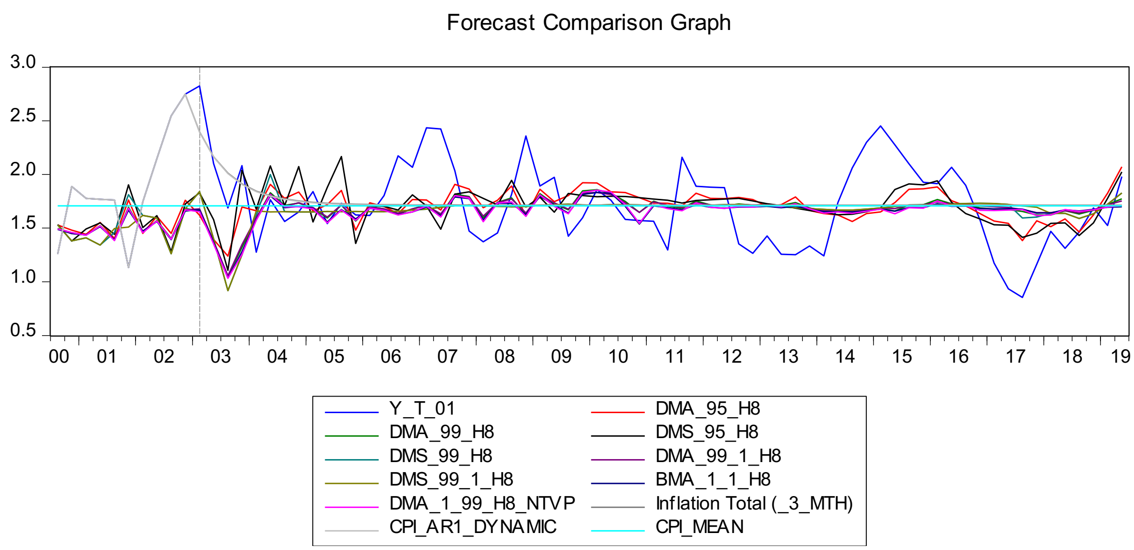
1. h=1



2. h=4

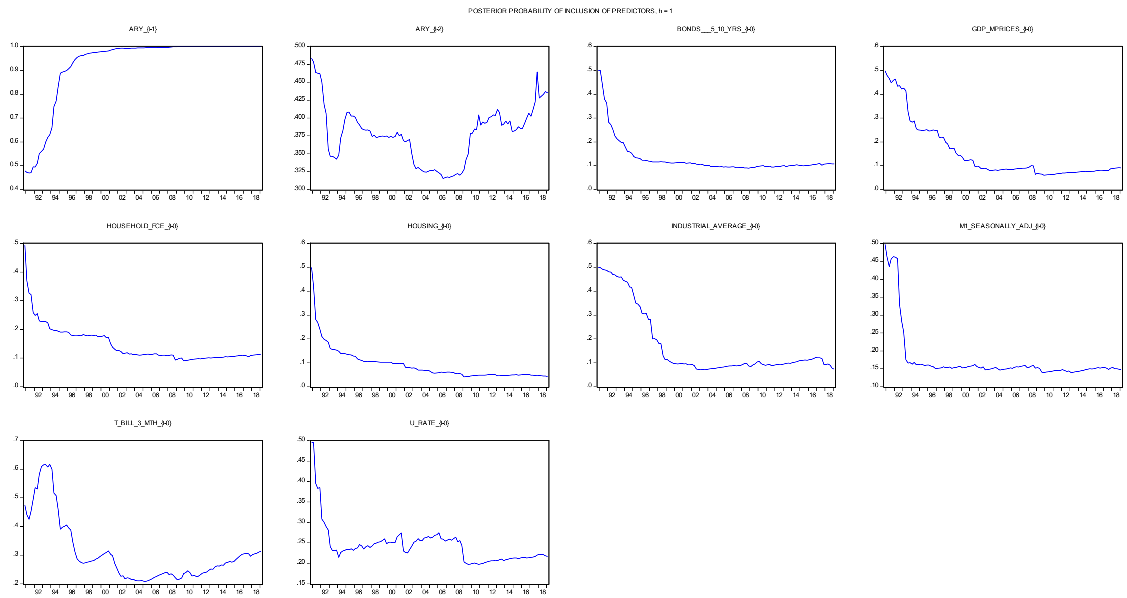

Forecast Comparison Graph
3.0
2.5
2.0
1.5
1.0
0.5
00 01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19
Y_T_01
DMA_95_H4
DMA_99_H4
DMS_95_H4
DMS_99_H4
DMA_99_1_H4
DMS_99_1_H4
BMA_1_1_H4
DMA_1_99_H4_NTVP
Inflation Total (_3_MTH)
CPI_AR1_DYNAMIC
CPI_MEAN

3. h=8

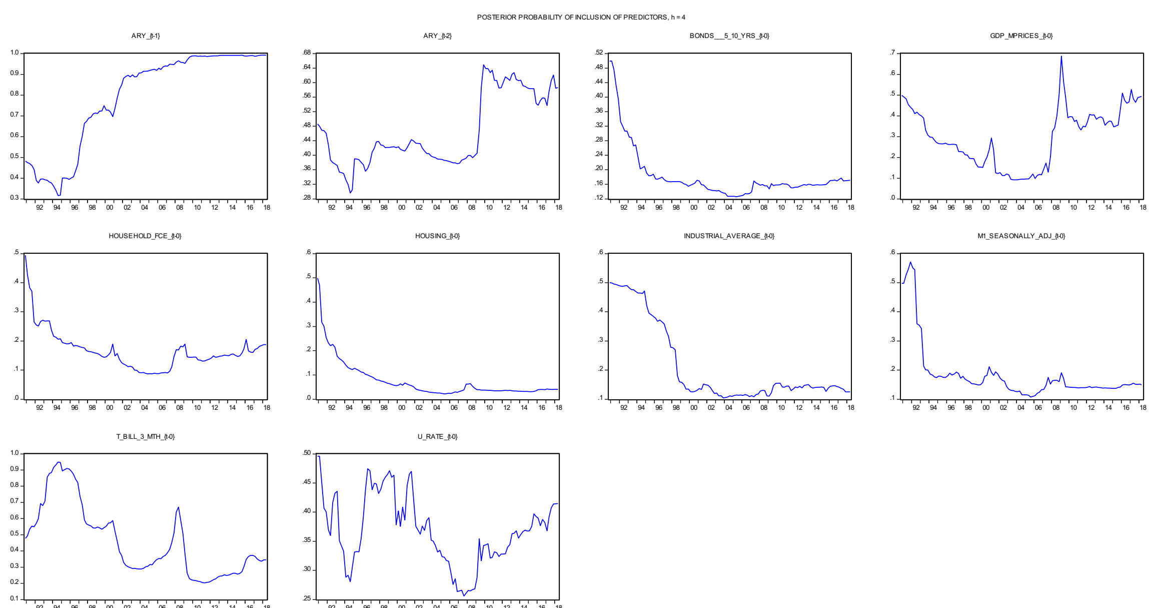

Forecast Comparison Graph
3.0
2.5
2.0
1.5
1.0
0.5
00 01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19
Y_T_01
DMA_95_H8
DMA_99_H8
DMS_95_H8
DMS_99_H8
DMA_99_1_H8
DMS_99_1_H8
BMA_1_1_H8
DMA_1_99_H8_NTVP
Inflation Total (_3_MTH)
CPI_AR1_DYNAMIC
CPI_MEAN

## Appendix 7. Posterior probability of Predictors with Bayesian Methods: CPI-Common Inflation

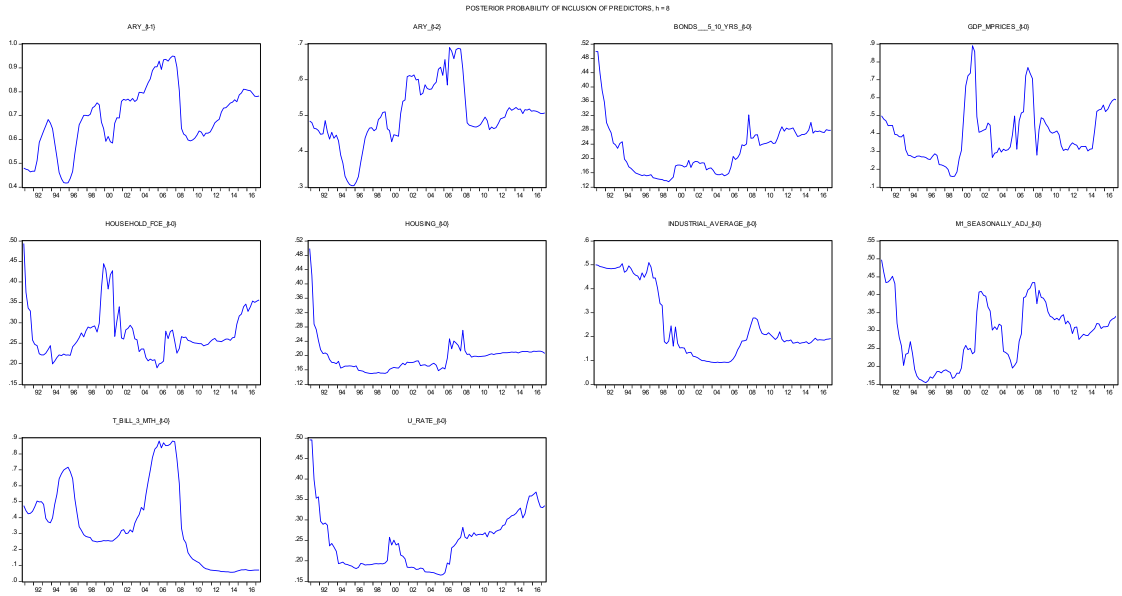
### 1. DMA & DMS $\lambda = \alpha = .99$ and h = 1



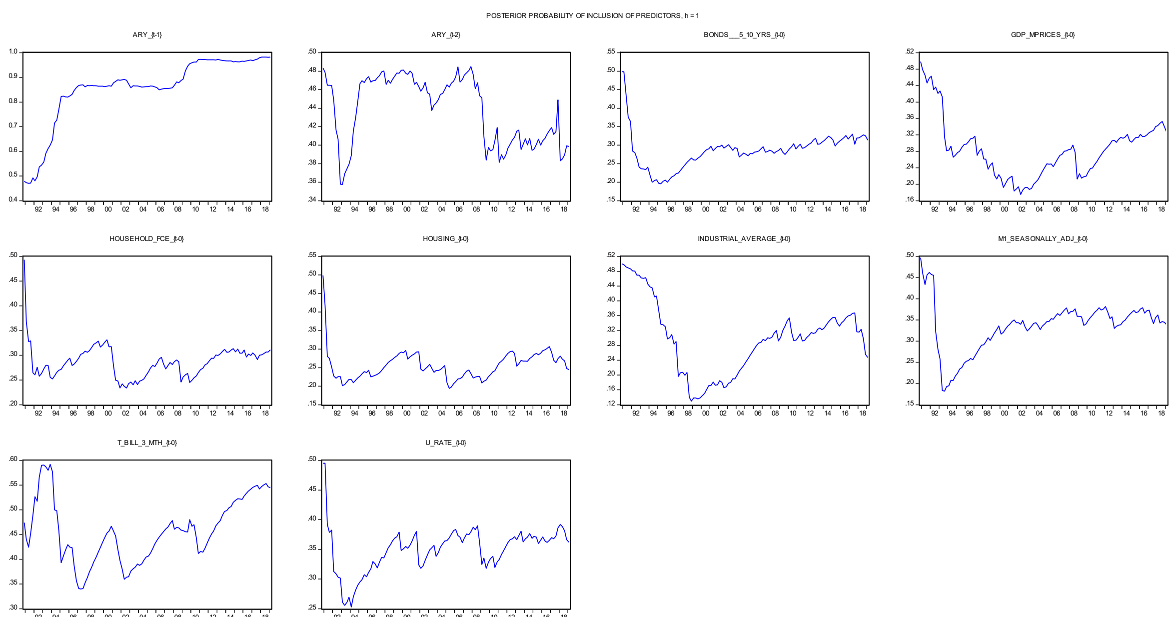

### 2. DMA & DMS $\lambda = \alpha = .99$ and h = 4



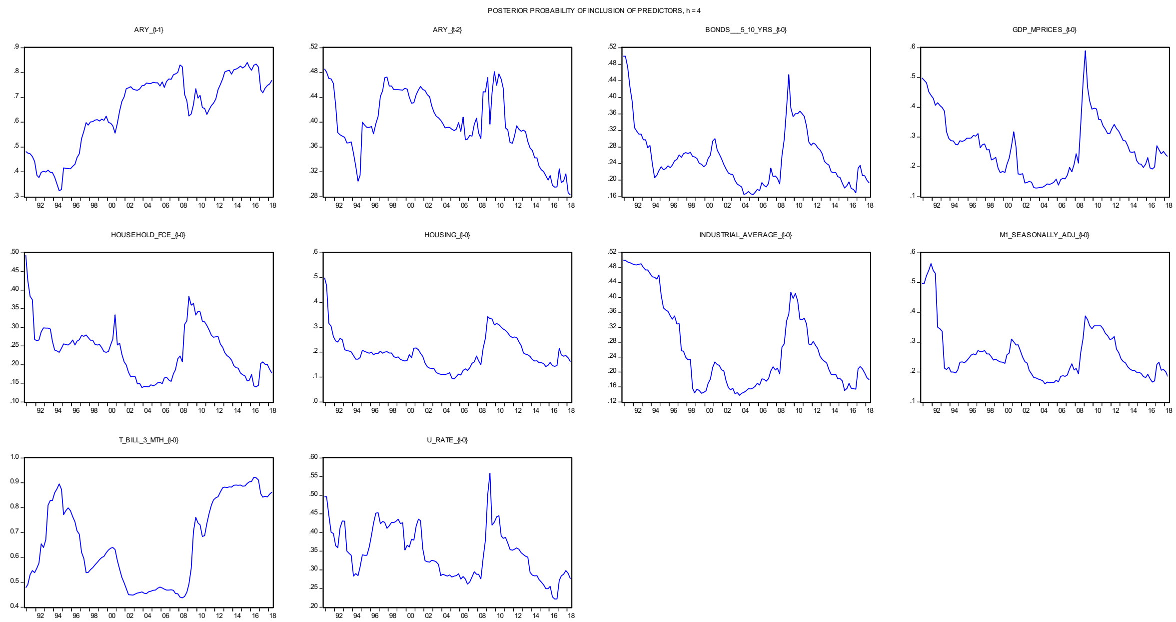
### 3. DMA & DMS $\lambda = \alpha = .99$ and h = 8



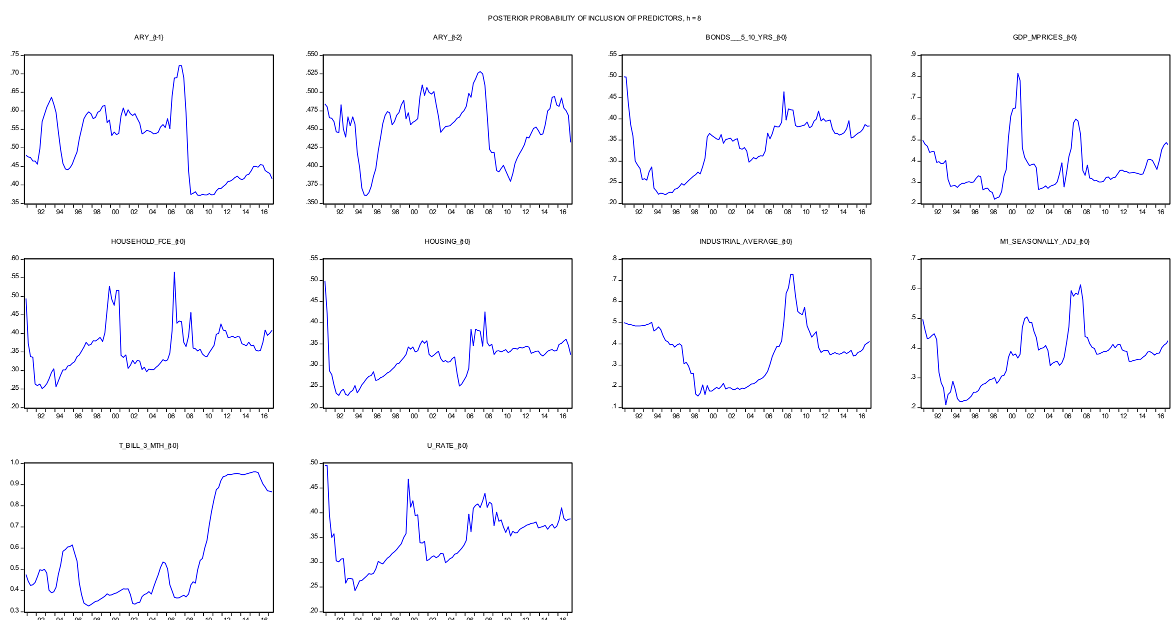

### 4. DMA & DMS $\lambda = \alpha = .95$ and h = 1



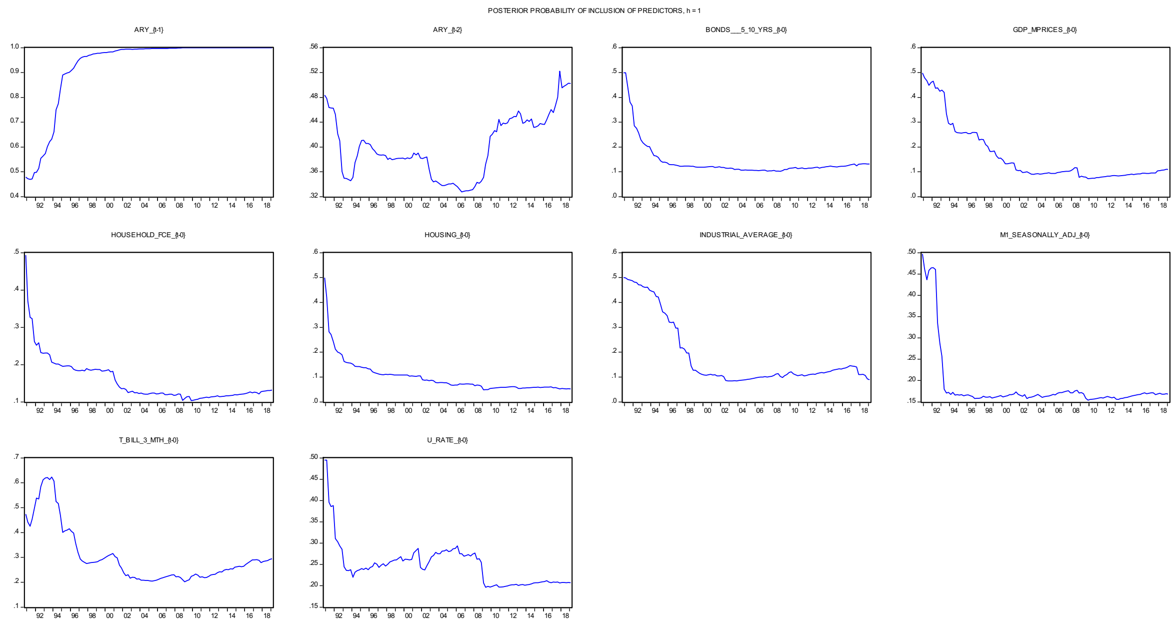
## 5. DMA & DMS $\lambda = \alpha = .95$ and h = 4



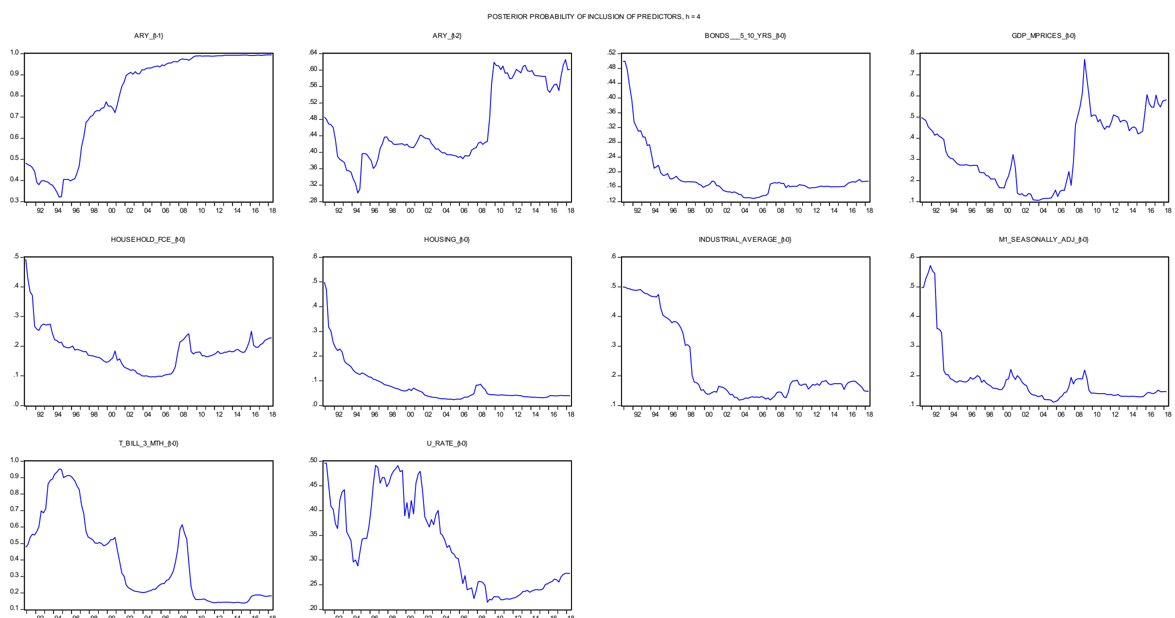

## 6. DMA & DMS $\lambda = \alpha = .95$ and h = 8



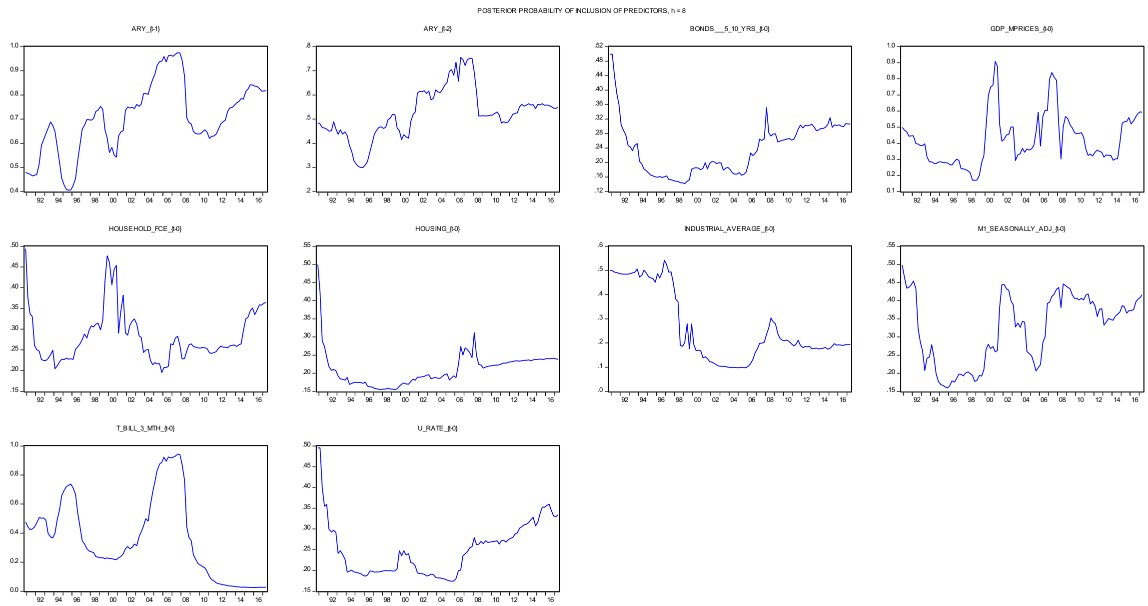
**7. DMA $\lambda = 1$, $\alpha = .99$ and h = 1 (coefficients do not vary with time )**



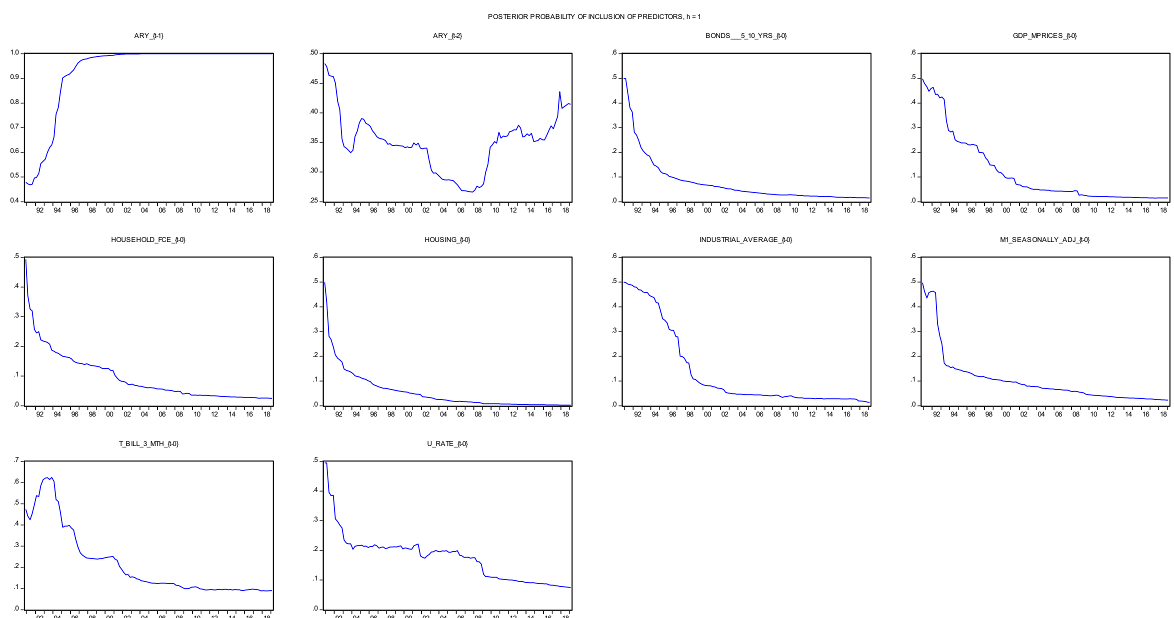

**8. DMA $\lambda = 1$, $\alpha = .99$ and h = 4 (coefficients do not vary with time)**



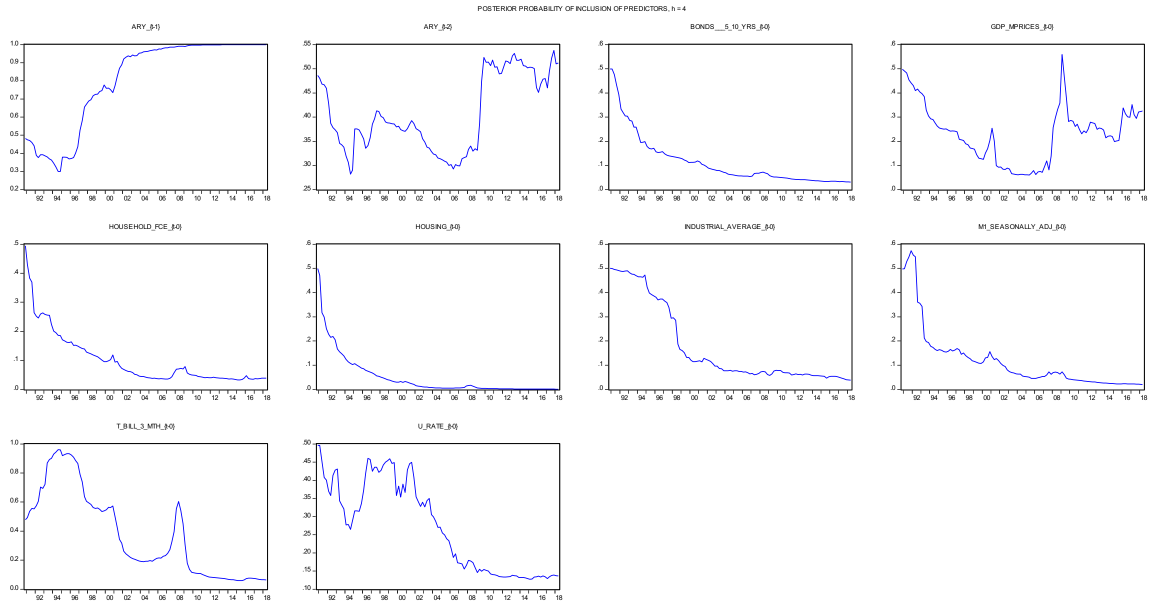
## 9. DMA $\lambda = 1, \alpha = .99$ and h = 8 (coefficients do not vary with time)



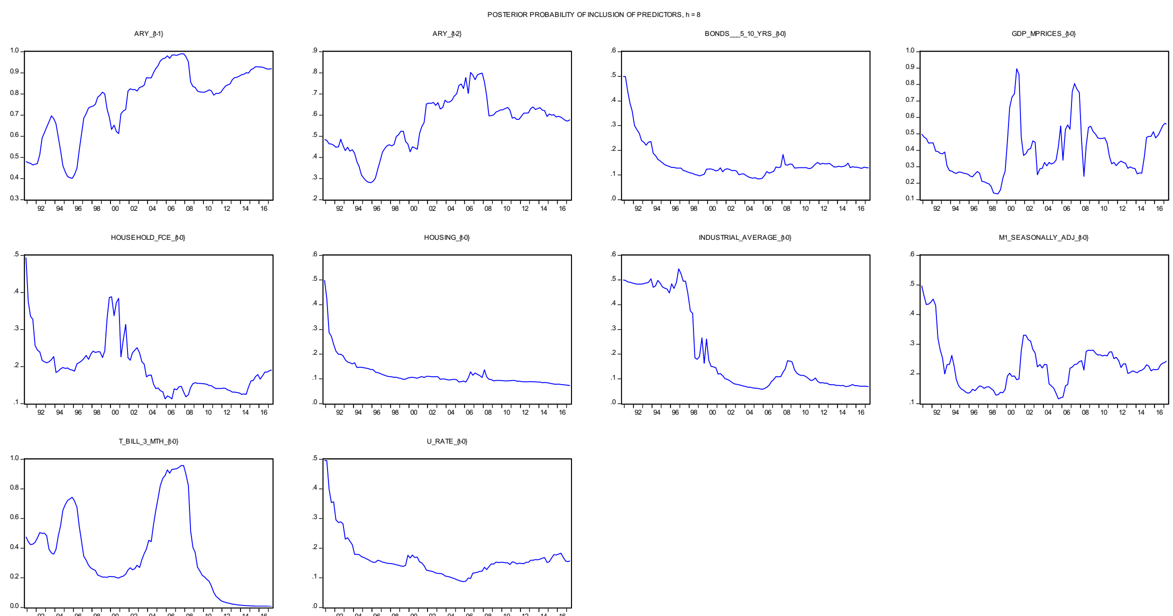

## 10. BMA $\lambda = \alpha = 1$ and h = 1



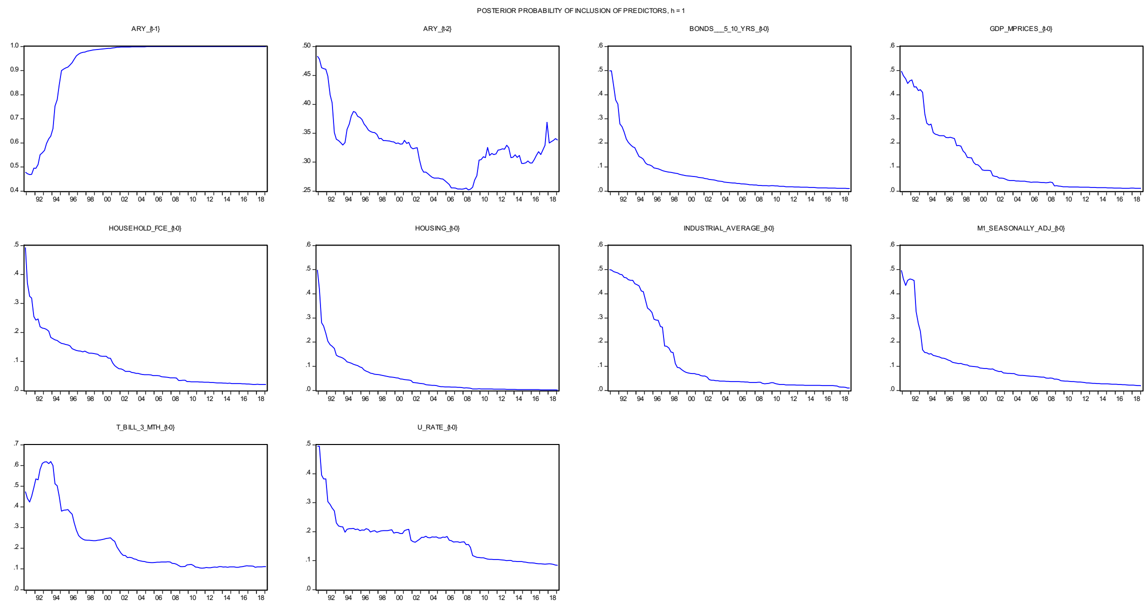
## 11. BMA $\lambda = \alpha = 1$ and h = 4



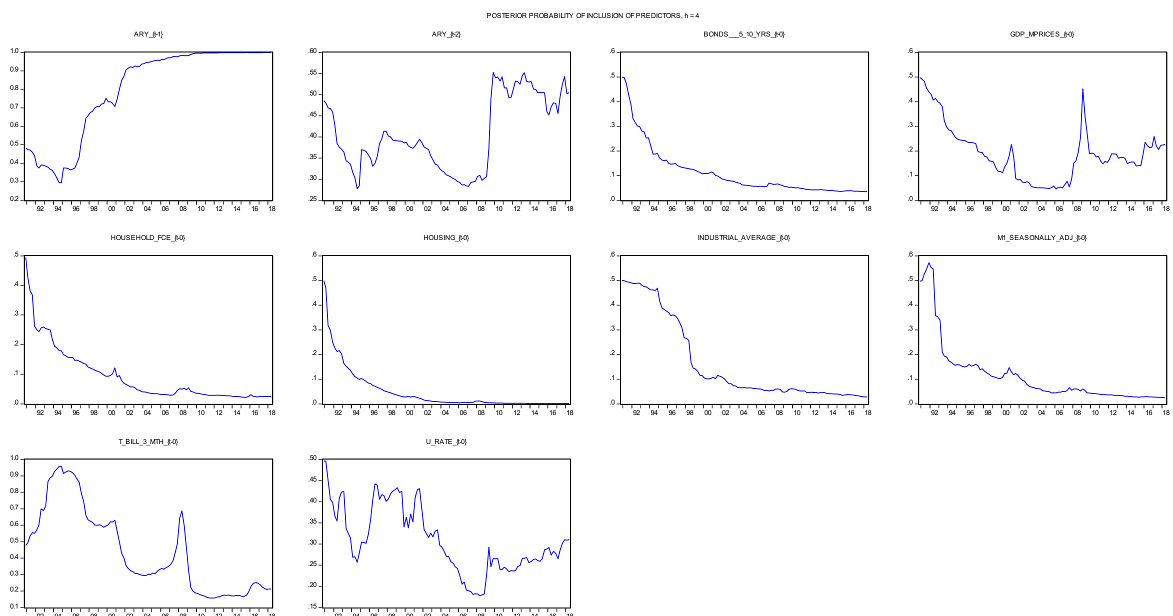

## 12. BMA $\lambda = \alpha = 1$ and h = 8



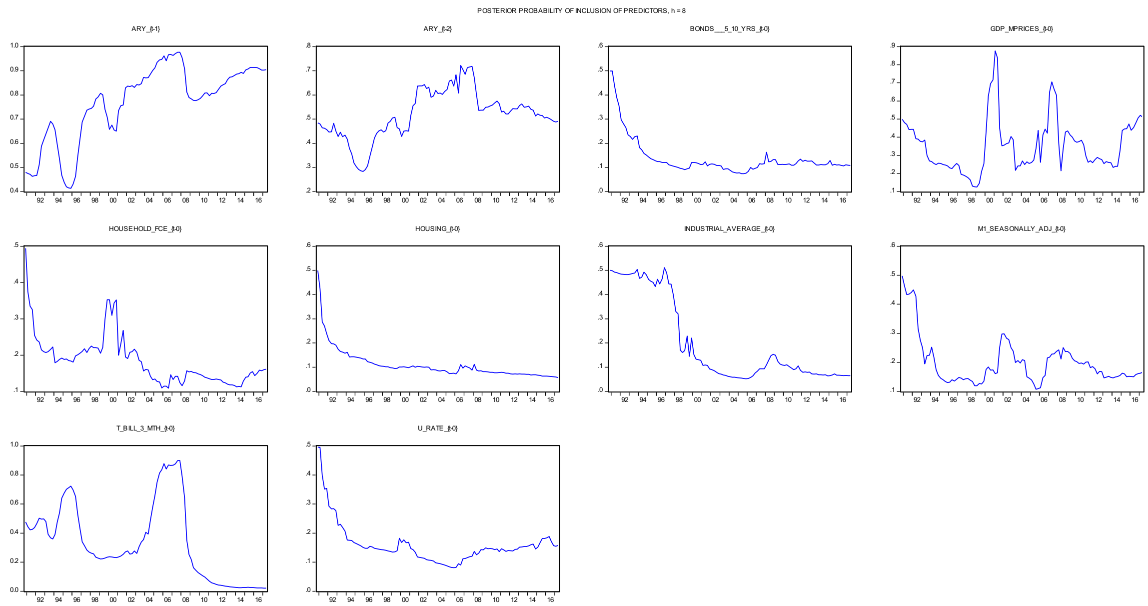
## 13. DMA & DMS $\lambda = .99, \alpha = 1$ and h = 1




## 14. DMA & DMS $\lambda = .99, \alpha = 1$ and h = 4

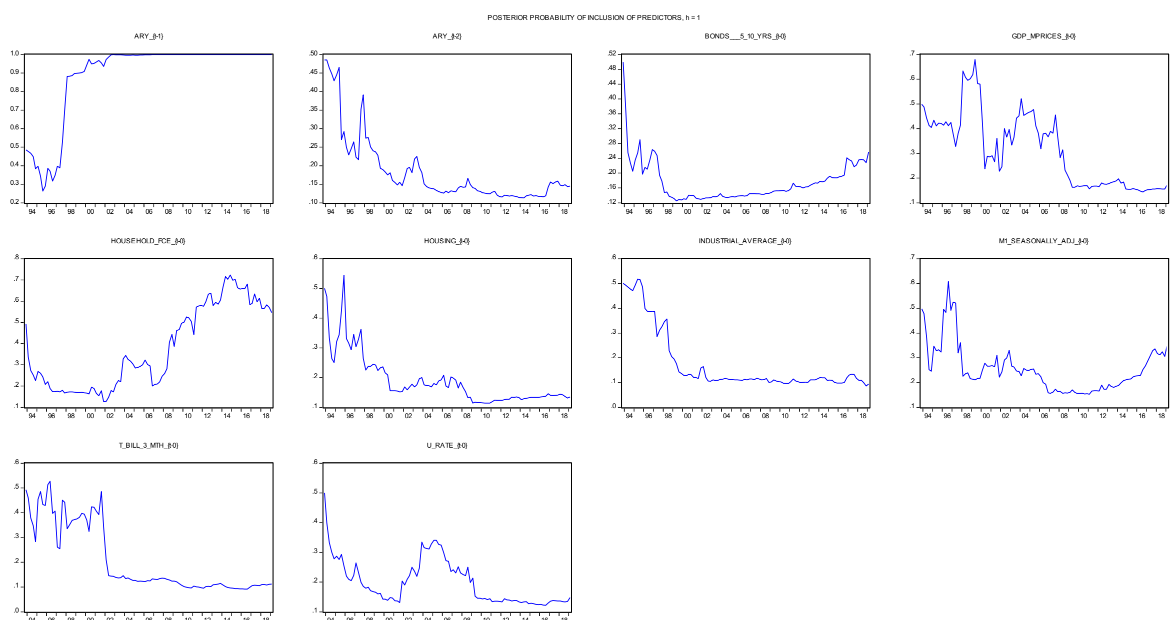


### 15. DMA & DMS $\lambda = .99, \alpha = 1$ and h = 8

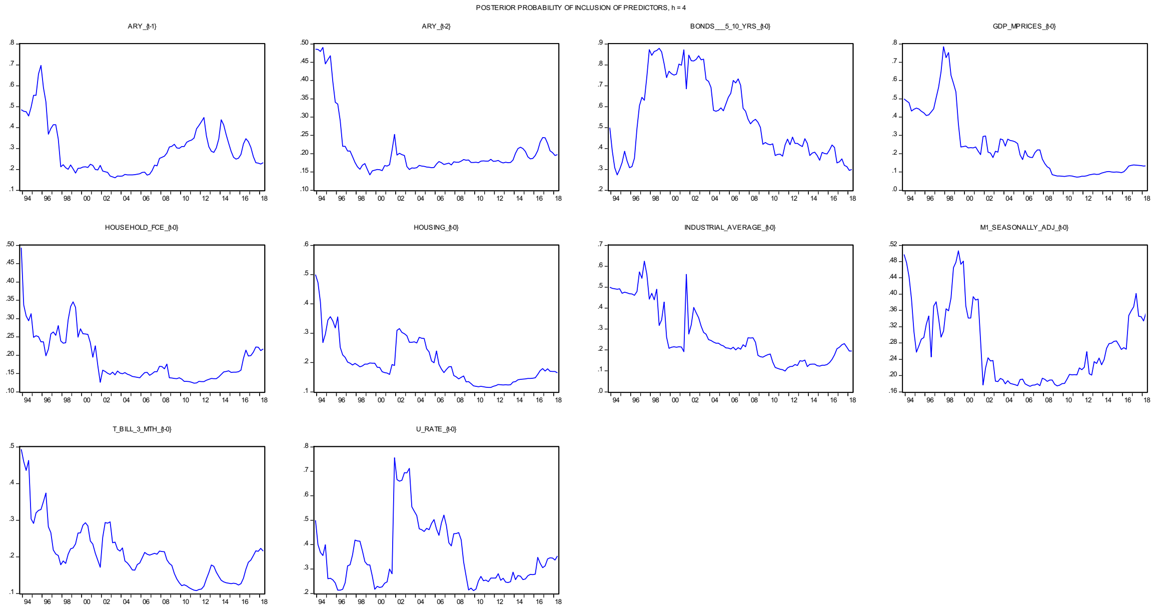



## Appendix 8. Posterior probability of Predictors with Bayesian Methods: CPI Inflation

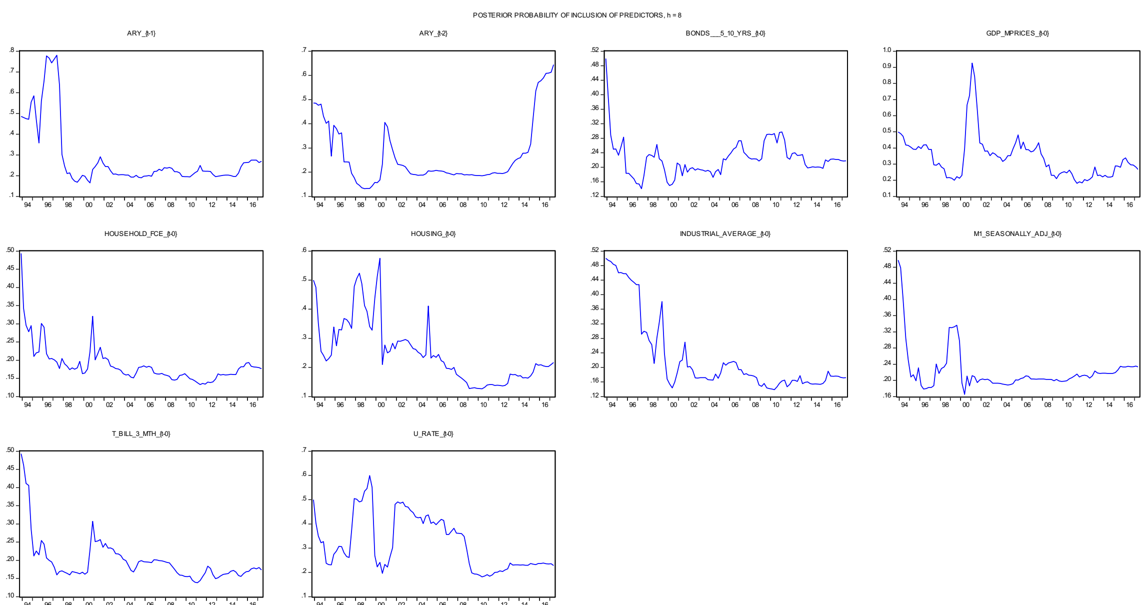
### 1. DMA & DMS $\lambda = \alpha = .99$ and h = 1



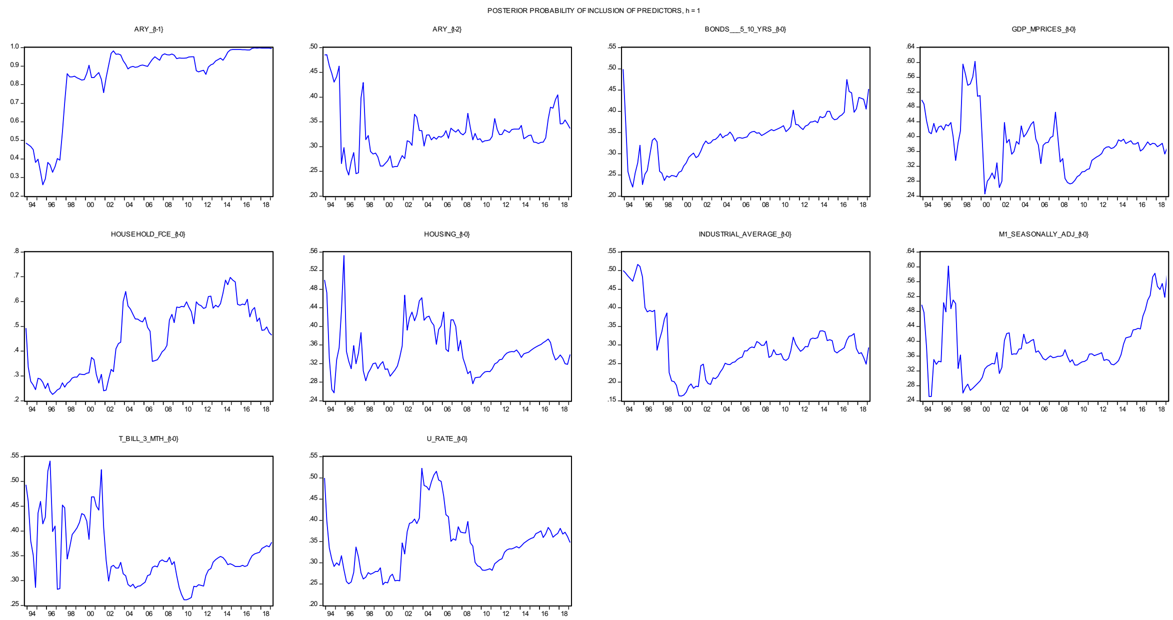
## 2. DMA & DMS $\lambda = \alpha = .99$ and h = 4



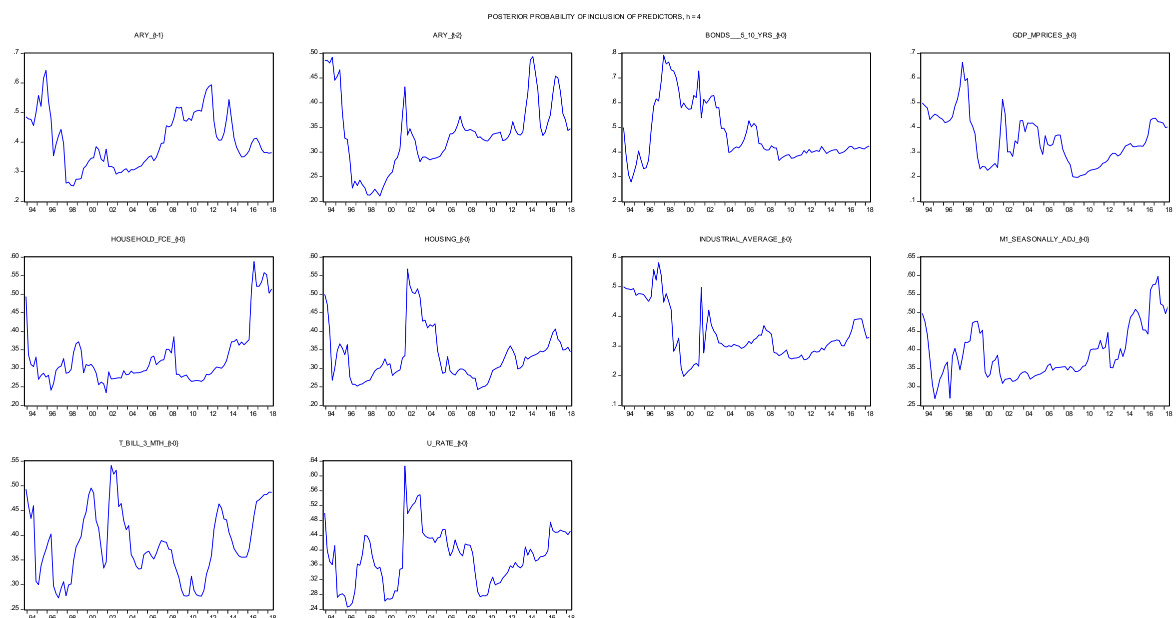

## 3. DMA & DMS $\lambda = \alpha = .99$ and h = 8



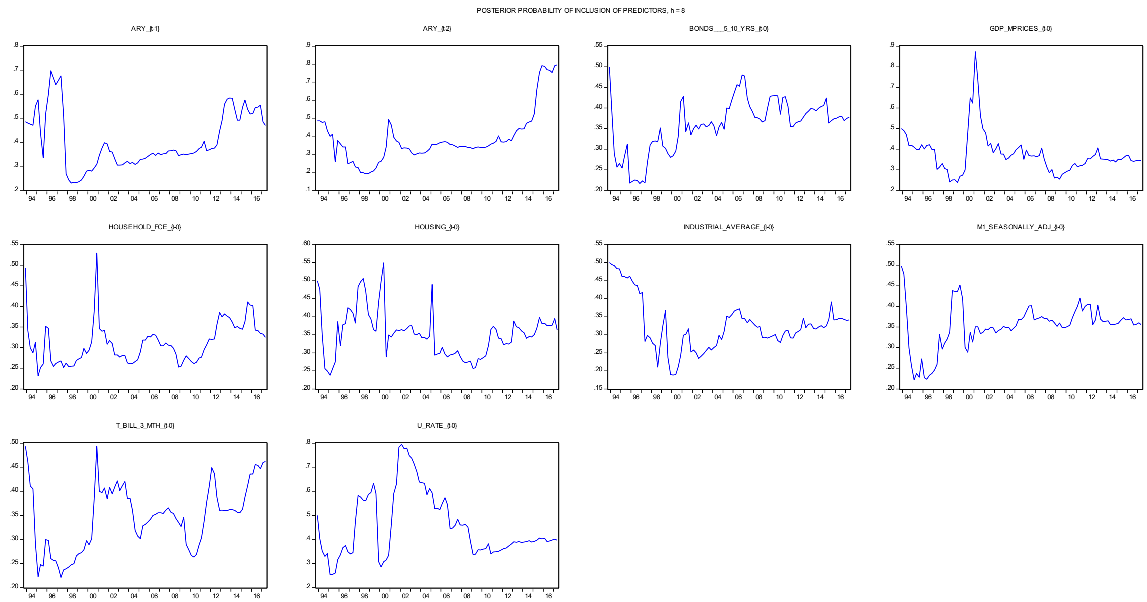
## 4. DMA & DMS $\lambda = \alpha = .95$ and h = 1



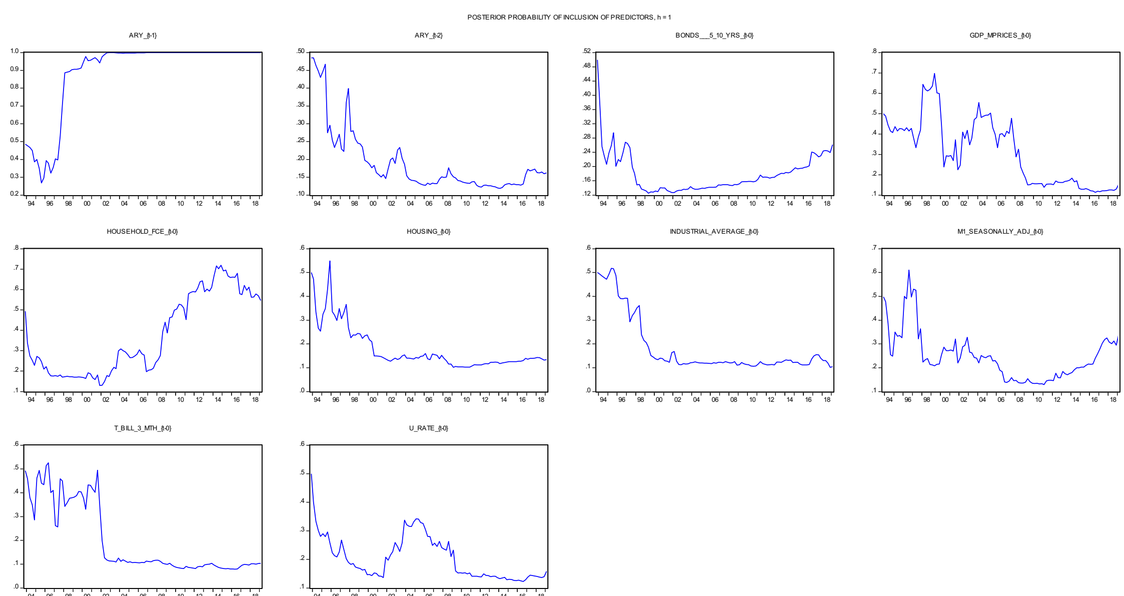

## 5. DMA & DMS $\lambda = \alpha = .95$ and h = 4



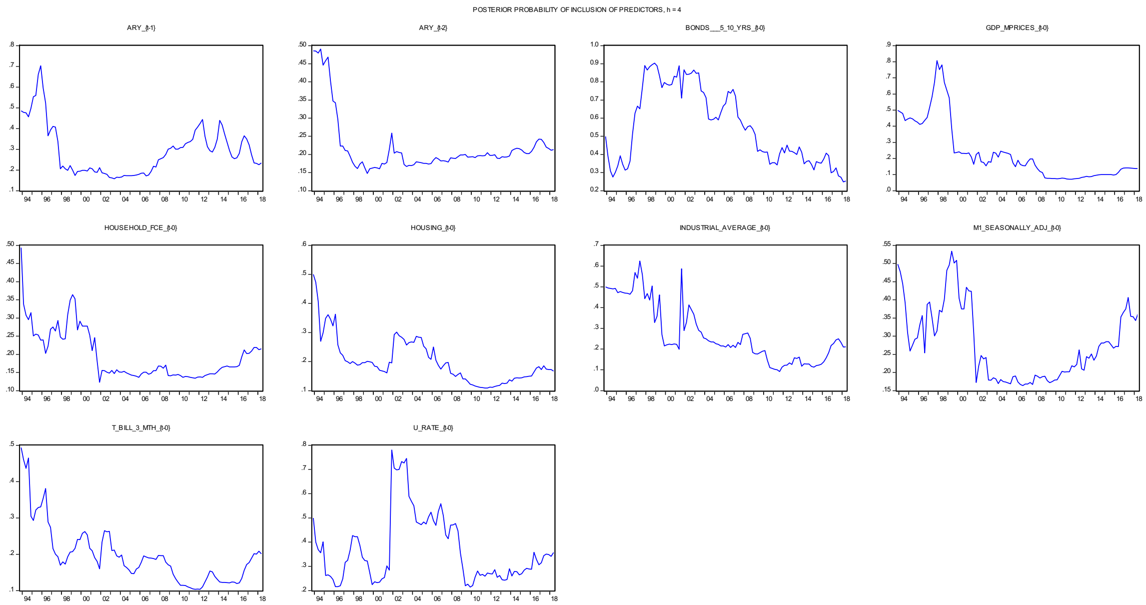
## 6. DMA & DMS $\lambda = \alpha = .95$ and h = 8



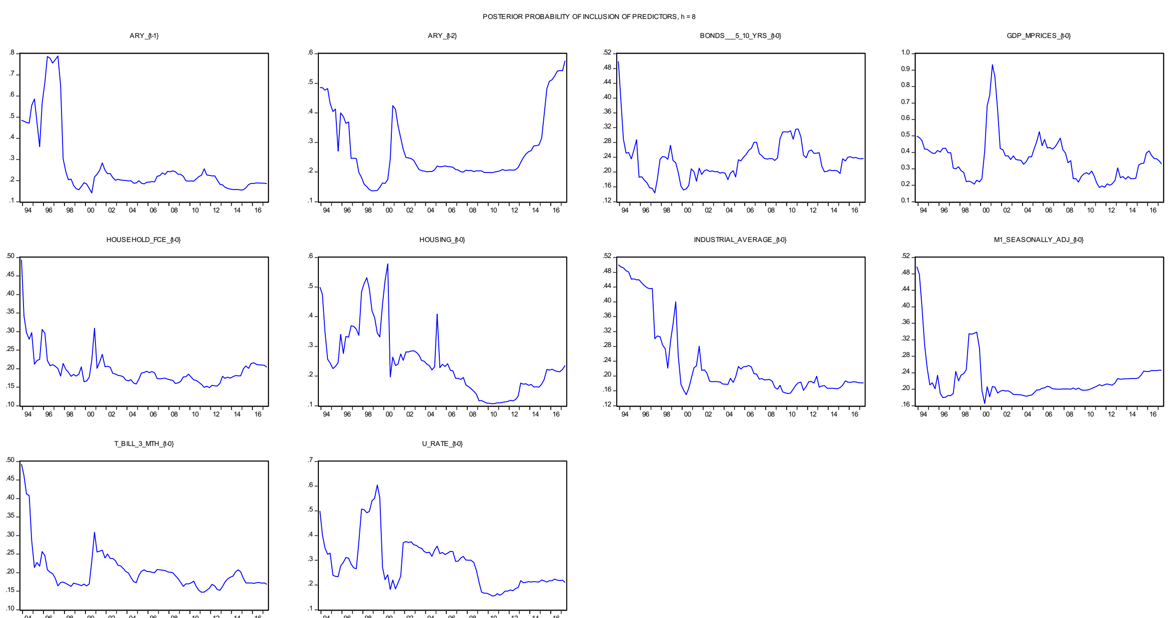

## 7. DMA $\lambda = 1$ , $\alpha = .99$ and h = 1 (coefficients do not vary with time)



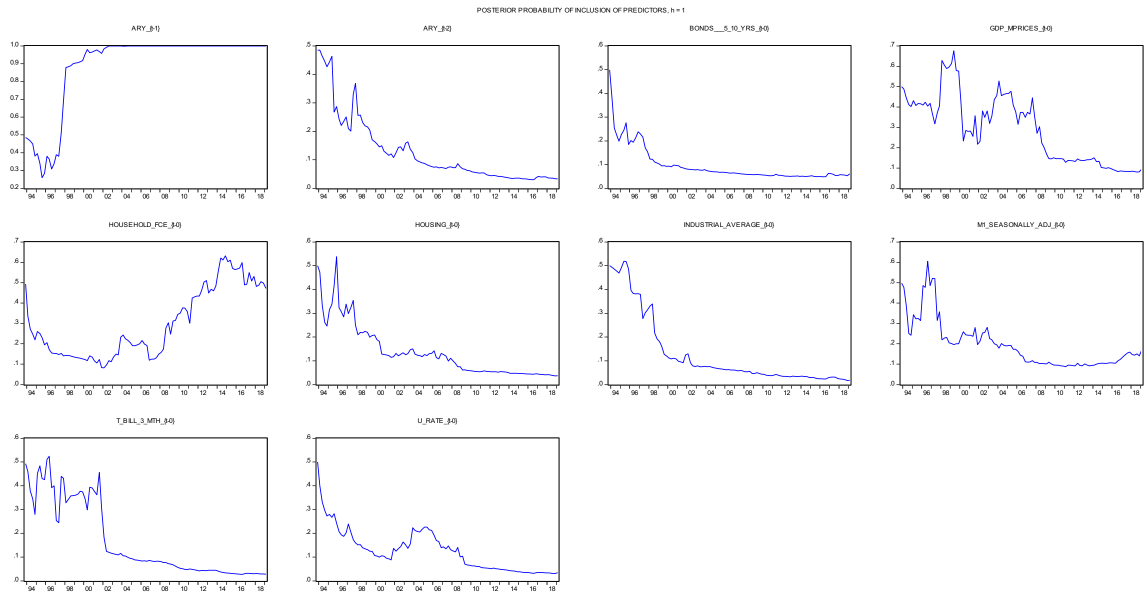
## 8. DMA $\lambda = 1, \alpha = .99$ and h = 4 (coefficients do not vary with time)



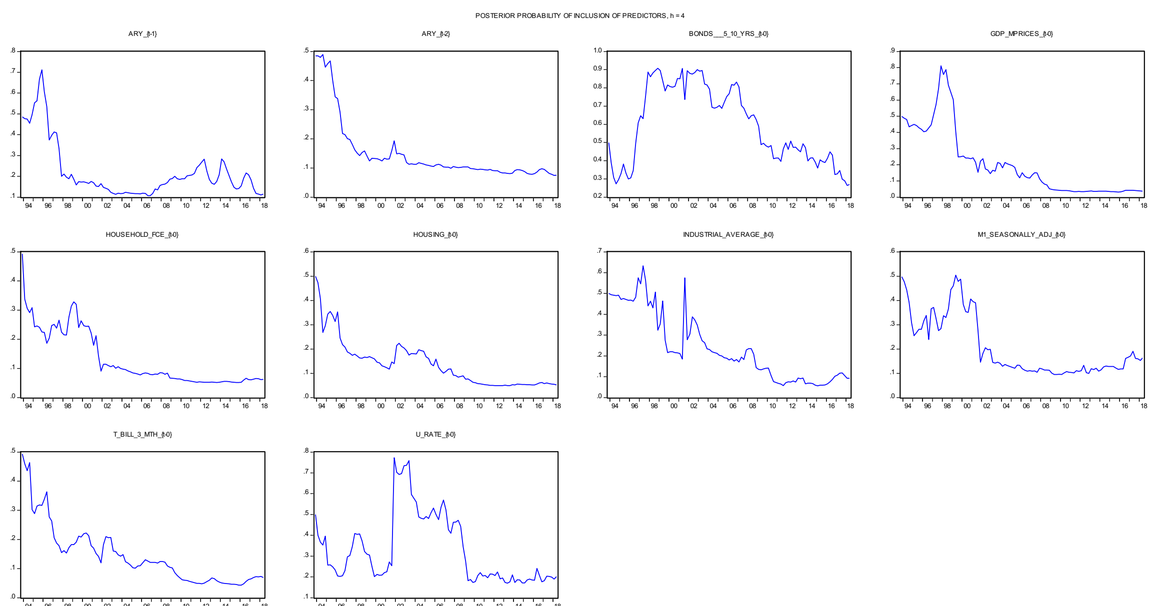

## 9. DMA $\lambda = 1, \alpha = .99$ and h = 8 (coefficients do not vary with time)



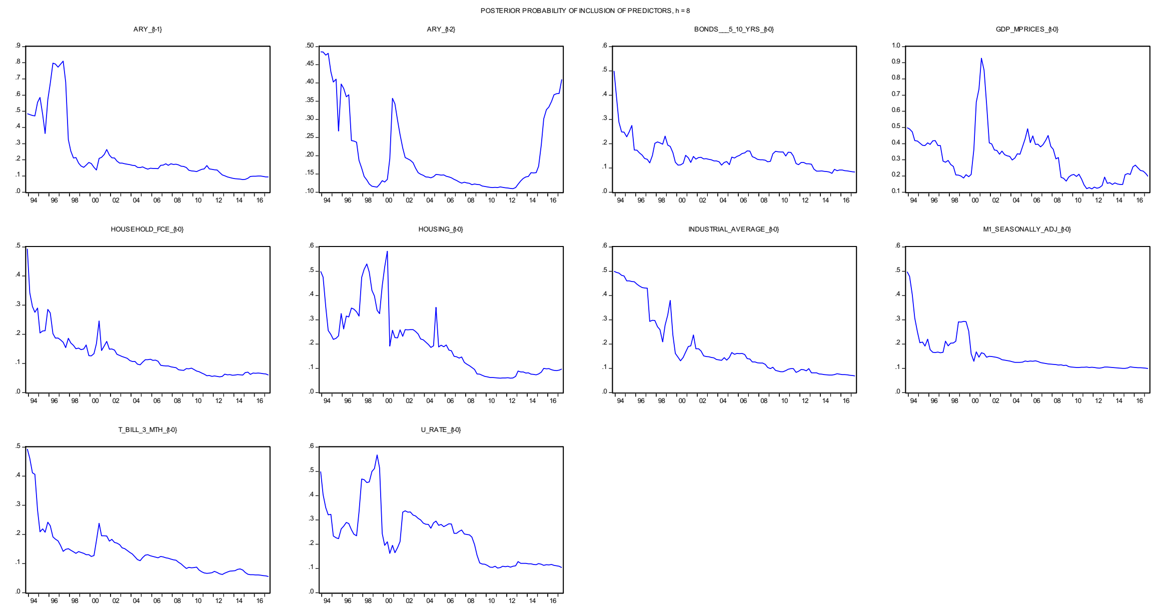
## 10. BMA $\lambda = \alpha = 1$ and h = 1



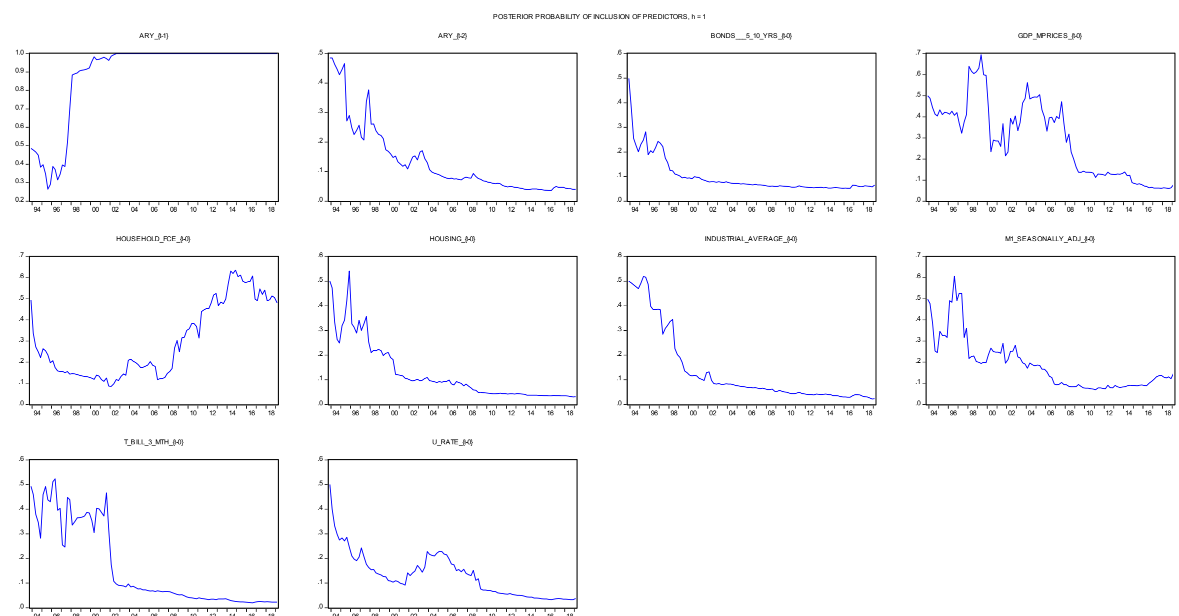

## 11. BMA $\lambda = \alpha = 1$ and h = 4



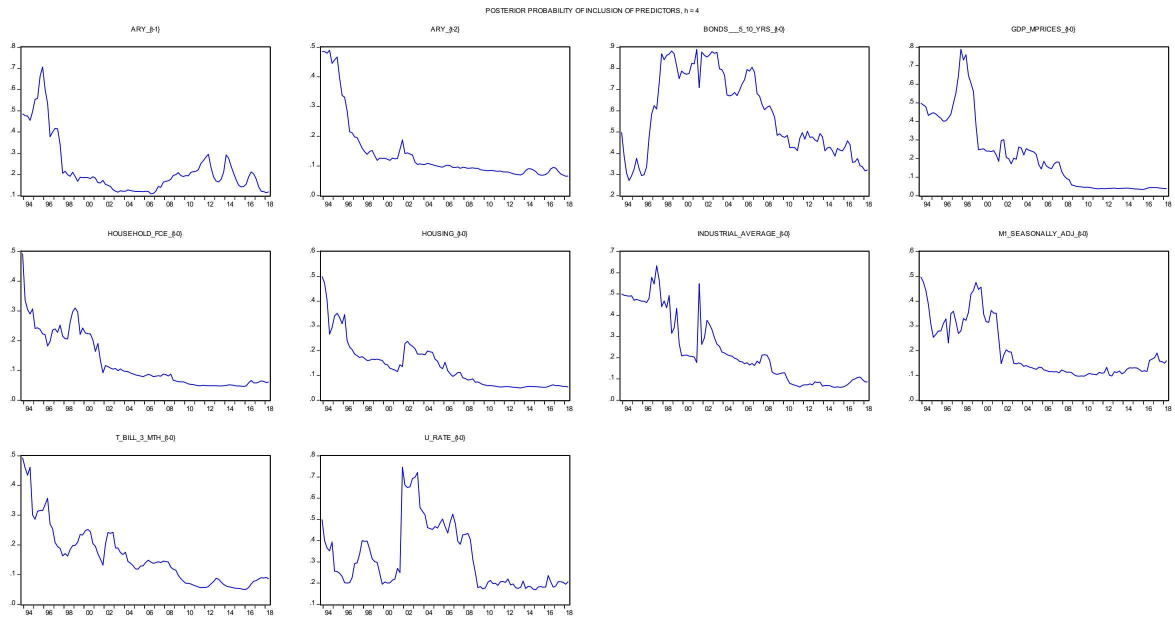
## 12. BMA $\lambda = \alpha = 1$ and h = 8




## 13. DMA & DMS $\lambda = .99, \alpha = 1$ and h = 1



## 14. DMA & DMS $\lambda = .99, \alpha = 1$ and h = 4




## 15. DMA & DMS $\lambda = .99, \alpha = 1$ and h = 8