Advanced Control Lab 2 - Preparatory Work

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Question 1

Assume that the 500 rotary encoder measures rotation angle of a DC motor rotor. The rotor is connected to the arm through a gearbox with 4 : 1 gear ratio (4 rotor turns into one arm rotation). Obtain analytically the constant that converts the encoder pulses to the arm rotation angle in degree units.

Solution:

For such a given rotary encoder, we have 1000 pulses per revolution, thus

$$v = \frac{360}{500} = 0.72 \left[\frac{\text{degree}}{\text{pulses}}\right] \tag{1}$$

Considering the influence of the gearbox, we have the constant that converts the encoder pulses to the arm rotation angle in degree units being calculated as:

$$K = \frac{0.72}{4} = 0.18 \left[\frac{\text{degree}}{\text{pulses}}\right] \tag{2}$$

Meaning, for 1 pulse in the encoder, the arm rotates $0.18°$.

Question 2

Neglecting the dynamics of an electric circuit, the behavior of a DC motor with an inertial load can be described by the following transfer function (assumes that the input to the system is the supplied to the motor voltage and the output is the load turn angle):

$$P(s) = \frac{k_p}{s(\tau_p s + 1)} \tag{3}$$

Time constant τ_p and static gain k_p are positive and defined by the motor and load parameters (see Lab 2 for more details). The motor and the load are controlled by a proportional controller k_c as in Figure 1.

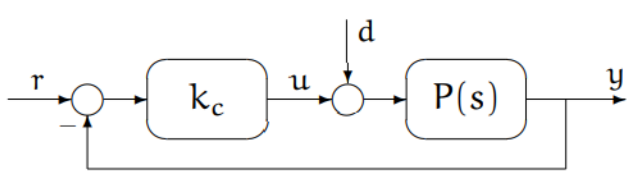

Figure 1: The closed loop with a proportional controller

Answer the following questions:

1. Is $P(s)$ stable? What is the range of k_c that stabilizes the closed loop?

Solution:

$P(s)$ is not stable since it has a pole $p = 0$. Consider the characteristic polynomial of the closed-loop system:

$$\chi_{cl} = N_c N_p + D_c D_p = \tau_p s^2 + s + k_p k_c \tag{4}$$

Based on the Routh Criterion, all of the coefficients of a second order χ_{cl} should be positive:

$$\tau_p > 0, \quad k_p k_c > 0 \tag{5}$$

Given positive k_p, we have $k_c > 0$.

2. What is a steady-state error to the unit steps in reference r and disturbance d as functions of τ_p, k_p, and k_c?

Solution:

Method 1 Transfer function for e_{ss} analysis

$$T(s) = \frac{y}{r}(s) = \frac{k_c P}{1 + k_c P} = \frac{k_c k_p}{\tau_p s^2 + s + k_p k_c} \tag{6}$$

$$T_d(s) = \frac{y}{d}(s) = \frac{P}{1 + k_c P} = \frac{k_p}{\tau_p s^2 + s + k_p k_c} \tag{7}$$

And the steady state error:

$$e_{ss,r} = 1 - T(0) = 0, \quad e_{ss,d} = -T_d(0) = -\frac{1}{k_c} \tag{8}$$

Method 2 The unified approach for e_{ss} analysis

Input	Type 0	Type 1	Type 2	Type 3	Remark
Step $\mathbf{1}(t)$	$\frac{R(0)}{1/F(0)+k_p}$	0	0	0	$k_p = B(0)$
Ramp $t \cdot \mathbf{1}(t)$	∞	$\frac{R(0)}{k_v}$	0	0	$k_v = \lim_{s\to 0}[sB(s)]$
Acceleration $\frac{t^2}{2} \cdot \mathbf{1}(t)$	∞	∞	$\frac{2R(0)}{k_a}$	0	$\mathrm{k}_a = \lim_{s\to 0}[s^2 B(s)]$

Table 1: steady state error

and the schemes for error analysis are:

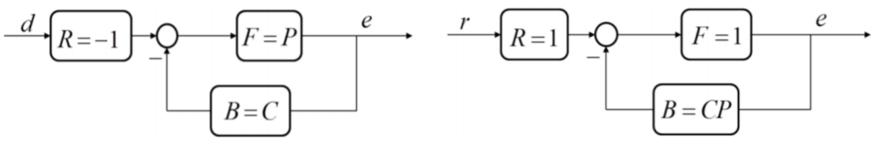

Figure 2: The error-equivalent diagrams for disturbance rejection and tracking

In our case, for step reference tracking, $B = CP$ contains 1 integrator, thus it's of type 1, leading to $e_{ss,r} = 0$; while for step disturbance rejection, $B = C$ contains 0 integrator, thus it's of type 0, leading to $e_{ss,d} = -\frac{1}{k_c}$.

3. Simulate in Simulink the closed loop in Figure 1 under $\tau_p = 0.04$, $k_p = 1.2$, reference $r = 1(t)$, disturbance $d = 0$, and the following controller gains $k_c = 5, 10, 40$. After the simulation, plot in MATLAB the system response (y) under the controller gains above. Utilize the block `Scope` to save signals with respect to simulation time. How does the value of the proportional controller influence on the closed loop system response (overshoot, rise time, and settling time)? Use Nyquist plot and root locus to explain the answer.

Solution:

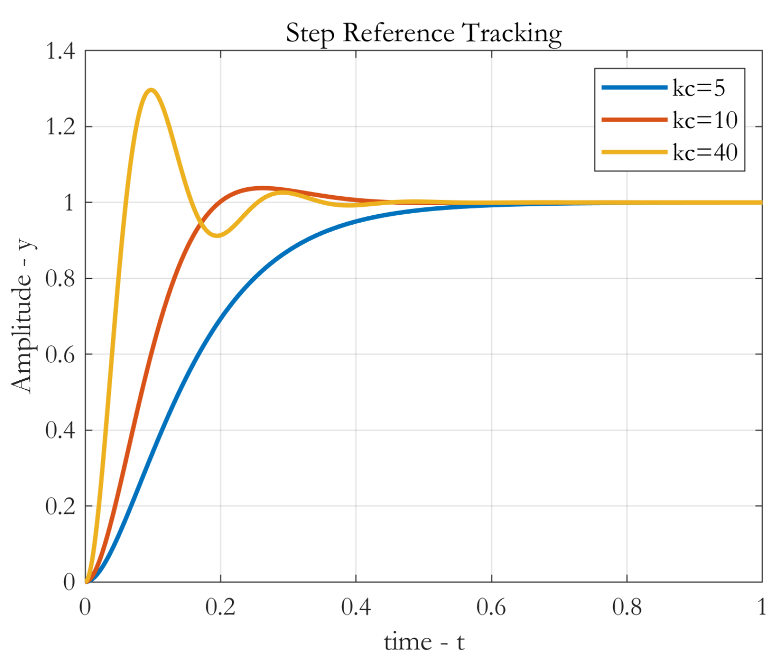

Figure 3: Step Reference Tracking

The results are shown in Fig.3, and the simulink scheme and the codes are shown below:

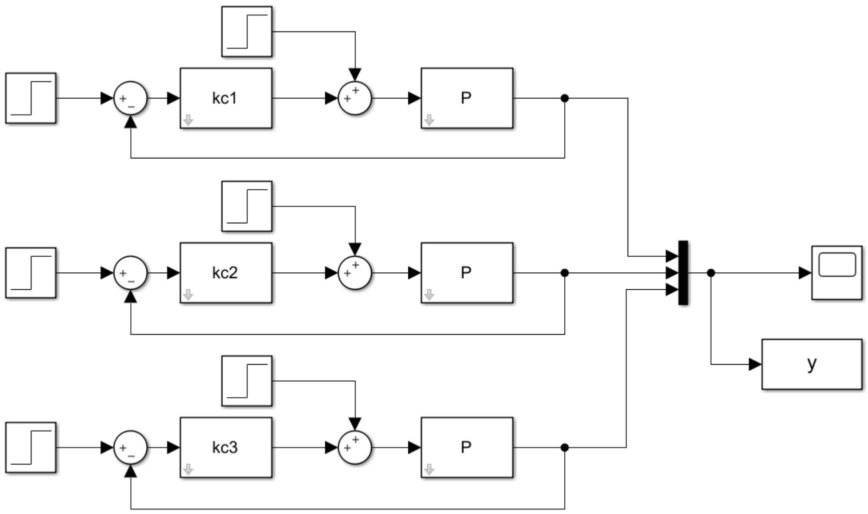

Figure 4: Simulink scheme

```
clear; clc;
s = tf('s');
tau=0.04; kp=1.2; kc1=5; kc2=10; kc3=40;

P = kp / (s*(tau*s+1));

sim('Lab2_PW');

figure(1)
plot(y, 'linewidth', 2);
grid on;
title('Step Reference Tracking');
xlabel('time - t'); ylabel('Amplitude - y');
legend('kc=5','kc=10','kc=40');
```

After implementing `stepinfo`, we get the information of each system:

```
%For closed loop with controller C1 = 5 :
RiseTime: 2.8259e-01;   SettlingTime: 4.9532e-01;   Overshoot: 0;

%For closed loop with controller C2 = 10 :
RiseTime: 1.2670e-01;   SettlingTime: 3.4153e-01;   Overshoot: 3.7803e+00;

%For closed loop with controller C3 = 40 :
RiseTime: 4.0646e-02;   SettlingTime: 3.1373e-01;   Overshoot: 2.9618e+01;
```

where we can find that when the gain of controller increases, rise time decreases, settling time decreases, overshoot increases.

To justify this through Nyquist plot:

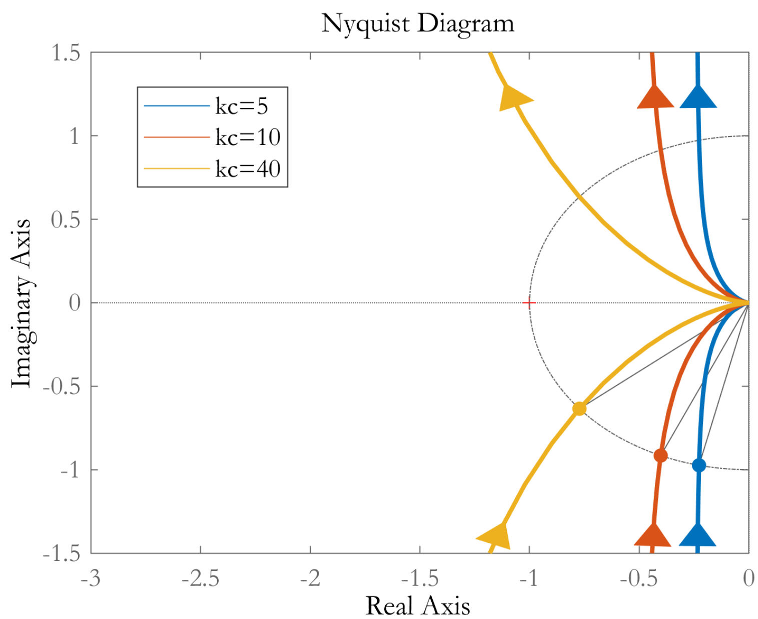

Figure 5: Nyquist plot of $L_i = k_{c_i}P(s)$

We can notice from the Nyquist plot that when k_c increases, phase margin μ_{ph} decreases, thus the system has a tendency to become unstable, which is indicated by the appearance of oscillations in Fig.3.

To justify this through Root Locus:

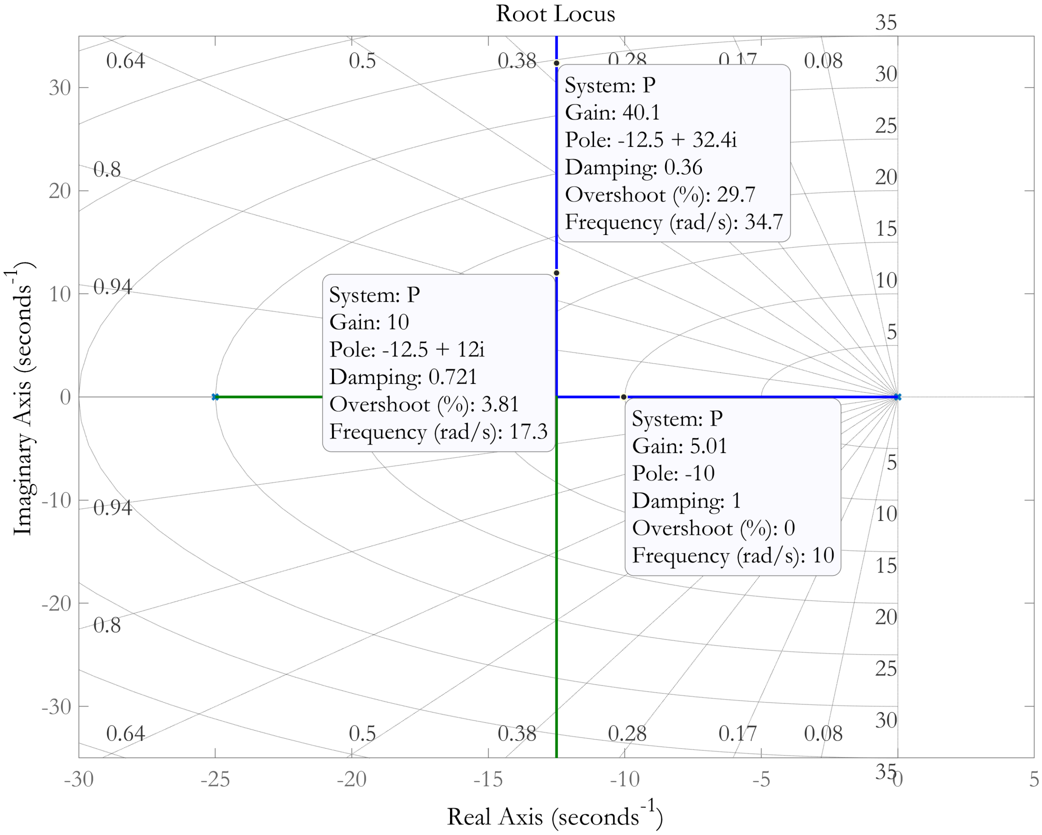

Figure 6: Root Locus of $P(s)$

According to some definitions,

$$p_{1,2} = -\omega_n(\zeta \pm i\sqrt{1-\zeta^2}), \quad OS = \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right), \quad t_{s,5\%} \approx \frac{3}{\zeta\omega_n} \tag{9}$$

We conclude that for poles located on real axis, i.e. when $k_c = 5$, we have no overshoot since $\zeta = 1$, while for poles on the vertical part of the RL-curve, ζ decreases and $\zeta\omega_n \equiv 12.5$, resulting in an increasing overshoot and a theoretically constant settling time. However, in fact the setting time sightly changes, mainly because the formula for settling time is an experimental result which has some errors.

4. Assume k_p = 1.2 and τ_p = 0.04, and find the value of proportional controller which satisfies the overshoot of 5%.

Solution:

The general formula for a second order closed-loop system and the system in our case:

$$G(s) = \frac{K_{ss}\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \quad T(s) = \frac{k_c k_p}{\tau_p s^2 + s + k_p k_c} \tag{10}$$

Set equal accordingly, and we have

$$K_{ss} = 1, \quad \omega_n = \sqrt{\frac{k_c k_p}{\tau_p}} = \sqrt{30k_c}, \quad 2\zeta\omega_n = \frac{1}{\tau_p} = 25 \tag{11}$$

For $OS = 5\%$, we have the value of ζ:

$$\zeta = \frac{|\ln(OS)|}{\sqrt{\pi^2 + |\ln(OS)|^2}} \approx 0.6901 \tag{12}$$

Therefore, we can get the value of proportional controller:

$$\omega_n = \frac{25}{2\zeta} = 18.1131 \quad \Rightarrow \quad \boldsymbol{k_c = \frac{\omega_n^2}{30} \approx 10.9362} \tag{13}$$

The result:

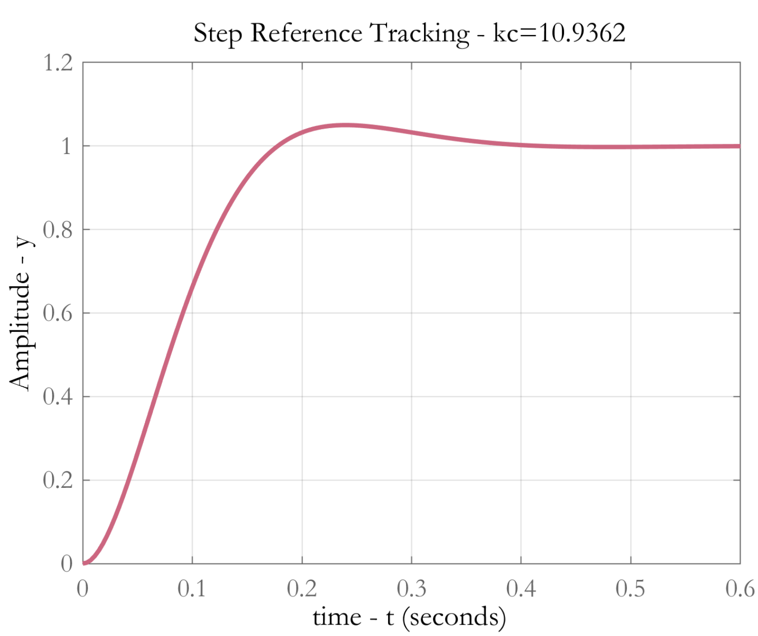

Figure 7: Step response

With the `stepinfo`:

`RiseTime: 1.1578e-01;    SettlingTime: 3.3100e-01;    Overshoot: 5.0000e+00.`

5. The file named `Data.mat` contains the noised response in a finite time range t for $r = 1 - e^{-80t}$ and $d = 0$ of the closed loop system when $k_c = 0.2$. Write the program in MATLAB to obtain the values of k_p and τ_p in accordance with the given response. The solution should include the code in MATLAB, and the response plot which proves the correctness of the result.

Solution:

The general linear 2nd-order system takes the form, in our case:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{14}$$

whose step response has the parameters evaluated as:

$$t_r \approx \frac{1.6\zeta^3 - 0.17\zeta^2 + 0.92\zeta + 1.02}{\omega_n}, \qquad t_{s,1\%} \approx -\frac{\ln 0.01}{\zeta\omega_n} = \frac{4.6}{\zeta\omega_n} \tag{15}$$

The file `Data.mat` contains the reference tracking of the experimental system where we can manually derive the rise time and settling time so as to evaluate ζ and ω_n:

$$\zeta = 0.9145, \quad \omega_n = 5.3509 \tag{16}$$

Therefore, a theoretical simulated response can be derived, as shown:

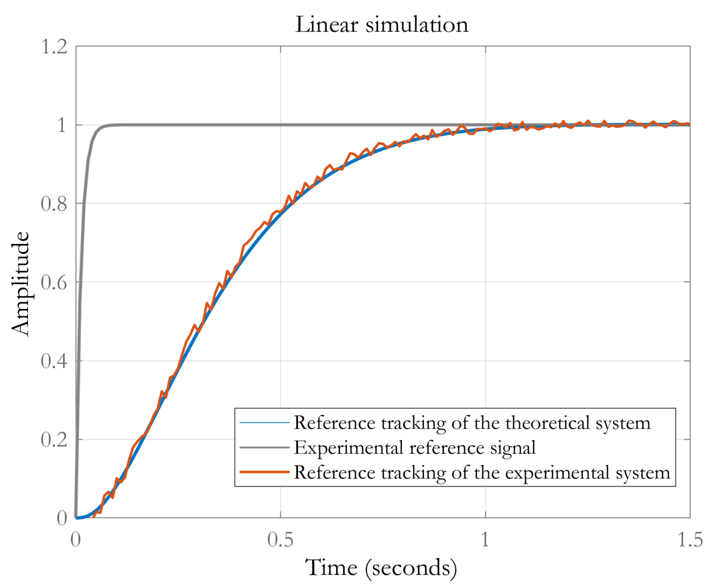

Figure 8: Reference Tracking

Judging from [Eq.(10)], we can deduce that

$$\tau_p = \frac{1}{2\zeta\omega_n} = 0.1022, \qquad k_p = \frac{\omega_n^2 \tau_p}{k_c} = 14.6275 \tag{17}$$

And the code for Q5 is shown below:

```
clear; clc;
load('Data.mat');
syms z w;
s = tf('s');
r = 1-exp(-80*t);
kc = 0.2;

% find rise time:
t90 = min(find(y≥0.9));
t10 = max(find(y≤0.1));
tr = t(t90)-t(t10);

% find setting time (1%)
t99 = min(find(abs(1-y)≤0.01));
ts = t(t99);

% solve for zeta and omega_n
eqn = [tr==(1.6*z^3-0.17*z^2+0.92*z+1.02)/w, ts==4.6/(z*w)];
var = [z,w];
[zeta, wn] = solve(eqn, var);
z = double(vpa(zeta));  w = double(vpa(wn));
z = z(find(sqrt(z.^2)==real(z))); w = w(find(sqrt(w.^2)==real(w)));

G = w^2/(s^2+2*z*w*s+w^2);

% find tau_p and kp
tau_p = 1/(2*z*w);
kp = w^2*tau_p/kc;

figure()
hold on;
lsim(G,r,t);
set(findobj(gcf,'type','line'),'linewidth',2);
plot(t,r, 'linewidth',1, 'color', [.5,.5,.5]);
plot(t,y, 'linewidth',1.5);
title('Linear simulation'); grid;
legend('Reference tracking of the theoretical system', ...
       'Experimental reference signal', ...
       'Reference tracking of the experimental system');
```

6. Repeat the previous section, but utilize MATLAB function `tfest` for an estimation of the closed loop transfer function. The description of this function is `sys = tfest(data,np,nz)`. The time-domain data is specified by `data = iddata(y,r,`τ_s`)` where τ_s is the sample time of the experimental data.

Solution:

The estimated transfer function is:

$$T(s) = \frac{30.15}{s^2 + 10.09s + 30.14} \tag{18}$$

where we can derive that

$$\omega_n = \sqrt{30.15} = 5.4909, \qquad \zeta = 0.9188 \tag{19}$$

Again, judging from [Eq.(10)], we can deduce that

$$\tau_p = \frac{1}{2\zeta\omega_n} = 0.0991, \qquad k_p = \frac{\omega_n^2 \tau_p}{k_c} = 14.9404 \tag{20}$$

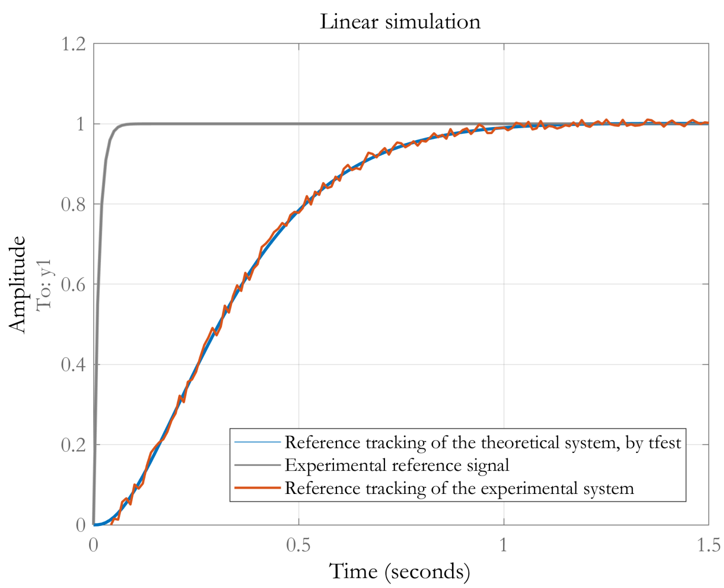

Figure 9: Reference Tracking

The code for Q6:

```
load('Data.mat');
r = 1-exp(-80*t);
ts = t(2)-t(1);
data = iddata(y,r,ts);
G = tfest(data,2,0);
```

7. Compare the values of k_p and τ_p obtained from the two previous sections.

Solution:

From section 5 we have:

$$\tau_p = \frac{1}{2\zeta\omega_n} = 0.1022, \qquad k_p = \frac{\omega_n^2 \tau_p}{k_c} = 14.6275$$

From section 6 we have:

$$\tau_p = \frac{1}{2\zeta\omega_n} = 0.0991, \qquad k_p = \frac{\omega_n^2 \tau_p}{k_c} = 14.9404$$

The results from two different sections are very close to each other. The method we use in section 6 have a better tracking performance. So, we think method in section 6 is more accurate. In section 5 we use two approximation to get t_r and $t_{s,1\%}$ and it seems the approximation we used is quite reasonable.

All of above conclude the Advanced Control Lab 2 - Preparatory Work.