Technion International
Israel Institute of Technology

Preparatory Work 2

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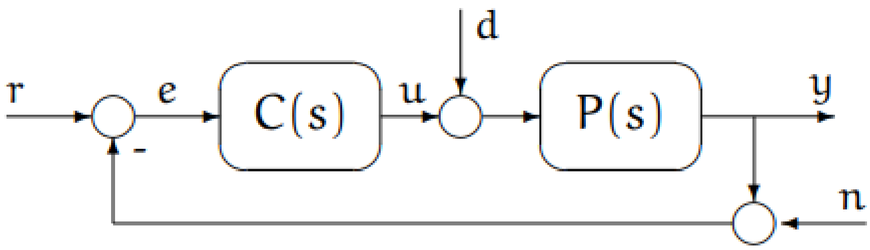
Course: *034406 - Advanced Control Lab* – Lecturer: *Dmitry Shneiderman*
Due date: *April 27th, 2021*



1 Question 1 (70%)

A control system of the DC motor with load, $P(s) = \frac{10}{s(s+10)}$, is shown in Figure 1.

1. Design a minimal order controller $C(s)$ which satisfies the following specifications:
   - zero steady-state error for a step in $r(t)$,
   - the overshoot $OS < 20\%$,
   - the settling time $t_s < 1$ (sec) corresponded to the settling level of $\pm 1\%$.
2. Design a minimal order controller $C(s)$ which satisfies the previously defined specifications and, in addition, results in zero steady-state error for a step in $d(t)$.

The final solution should include (for both controllers):

(a) controller transfer function and design process,

(b) polar plot and Bode plot of $P(s)$ and $C(s)P(s)$,

(c) Bode plot of transfer functions from r to u and from d to u,

(d) system response $y(t)$ and controller response $u(t)$ in closed loop for $r(t) = 1(t)$, $d(t) = 0$, and $n(t) = 0$,

(e) system response $y(t)$ and controller response $u(t)$ in closed loop for $r(t) = 0$, $d(t) = 1(t)$, and $n(t) = 0$.

1.1 Original

1.1.1. Design Process

We take a 0 order controller $C(s) = 4.3266$, which satisfies the specifications. First, we take C=4 and find out that it wasn't fit the requirement. Next, we write a logical loop to increase C by 0.0001 each time, until we meet the requirements. Finally, the loop stop at C=4.3266.

1.1.2. Polar and Bode plot

The results are as follows:

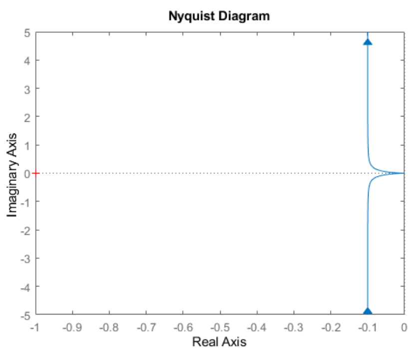


Figure 1: Nyquist for P

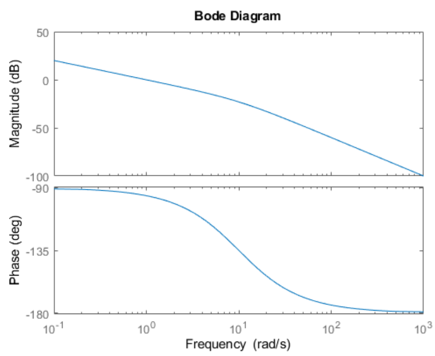


Figure 2: Bode for P

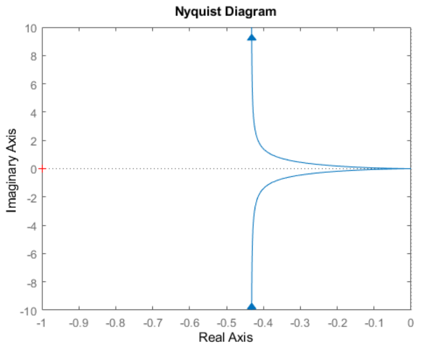


Figure 3: Nyquist for CP

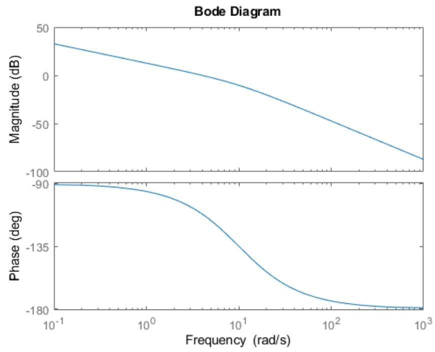


Figure 4: Bode for CP

1.1.3. Bode plot of TF_{ru} and TF_{du}

The results are as follows:

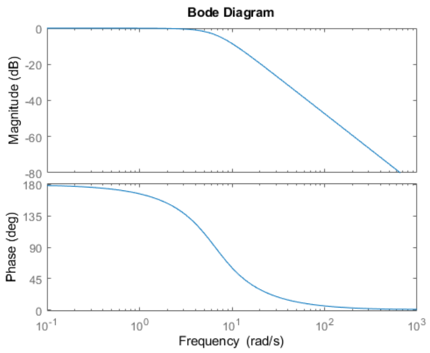


Figure 5: Bode Diagram for TF from r to u

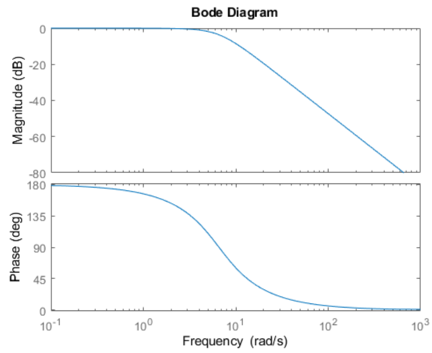


Figure 6: Bode Diagram for TF from r to u

1.1.4. System response for r=1 and for d=1

We can clearly see that for d=1, we have a steady state error.

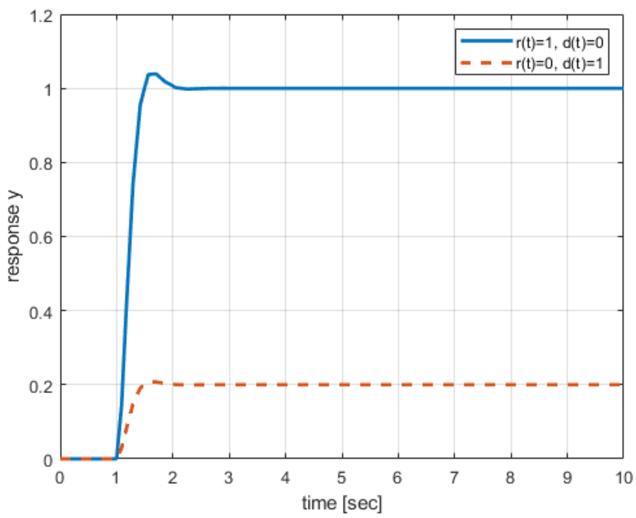


Figure 7: r=1 vs d=1

1.2 Without steady state error of d

1.2.1. Design Process

We introduce Internal Model Principle here to eliminate disturbance in steady state. First, we write down the output of disturbance :

$$Y(s) = T_d(s) \cdot D(s) = \frac{P}{1 + CP} D(s) \tag{1}$$

The effect of disturbance vanish in steady state,

- if D(s) has only left hand side poles
- if the denominator of D(s) is a factor of numerator in $T_d(s)$

In our case, the Disturbance generating polynomial is:

$$d(t) = 1(t) \Longrightarrow D(s) = \frac{1}{s} \tag{2}$$

So our Controller C must contain an integrator $\frac{1}{s}$. In most cases, increasing P and I will both increase the overshoot when performing step response test. So first, we need to check if only P and I can keep the overshoot below 20%, regardless the other requirements.

Choosing $K_i = K_p = 1$, and we get the following response:

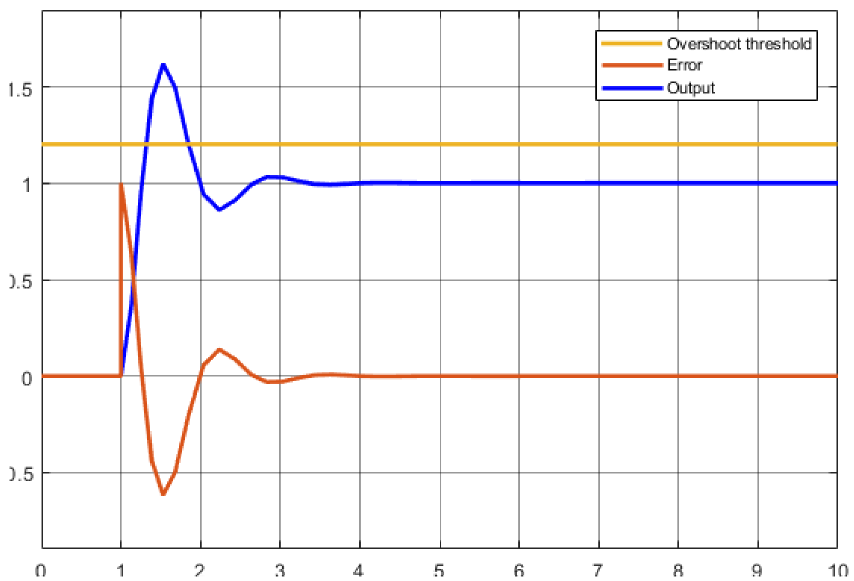


Figure 8: Test PI controller

As long as we use a relatively small K_i and K_p but the overshoot still too high. So we decide to add an differentiate to reduce overshoot. After fine tuning, we choose $K_p = 20,\ K_i = 14,\ K_d = 4$, and the response is shown below:

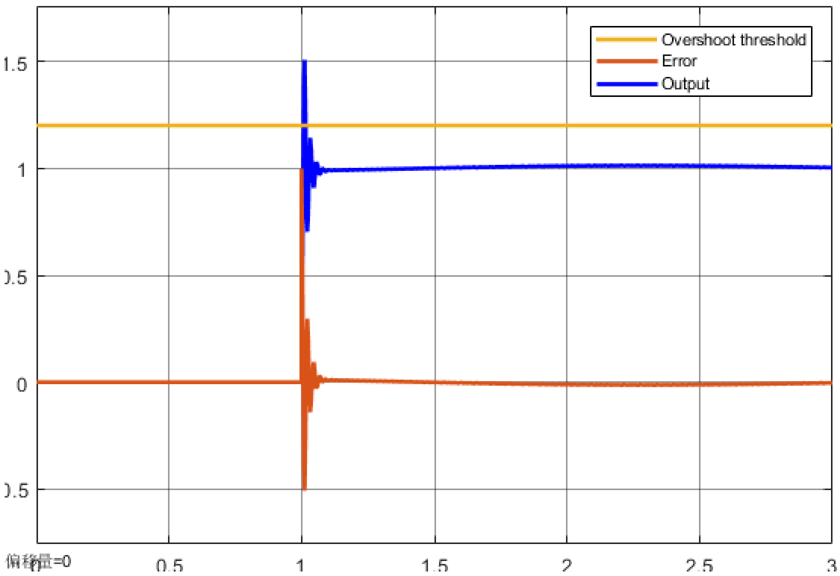


Figure 9: PID response

1.2.2. Polar and Bode Plot

The Polar Plot and Bode plot:

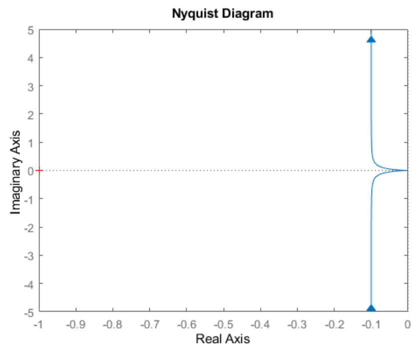


Figure 10: Nyquist for P

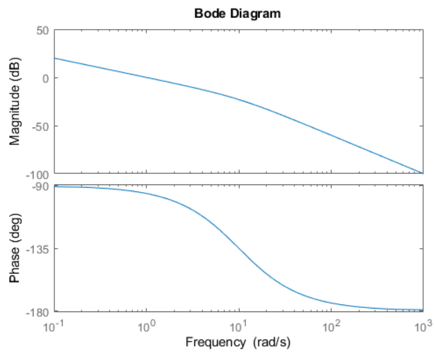


Figure 11: Bode for P

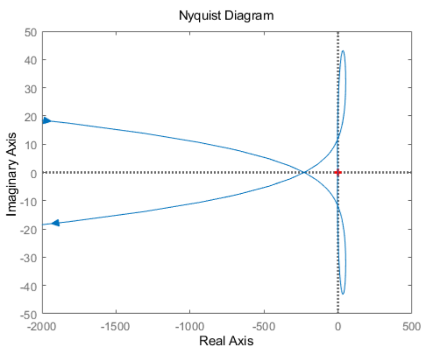


Figure 12: Nyquist for CP

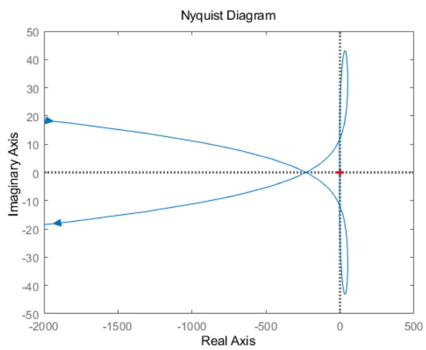


Figure 13: Bode for CP

1.2.3. Bode plot of TF_{ru} and TF_{du}

The results:

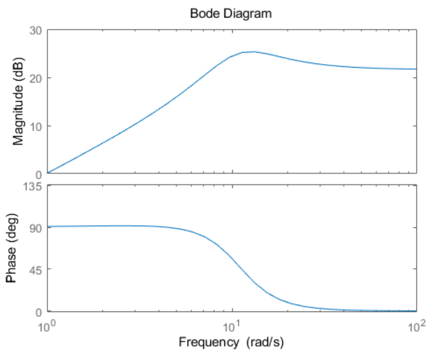


Figure 14: Bode Diagram for TF from r to u

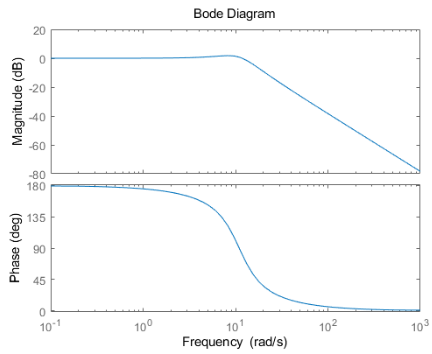


Figure 15: Bode Diagram for TF from r to u

1.2.4. System response

Results:

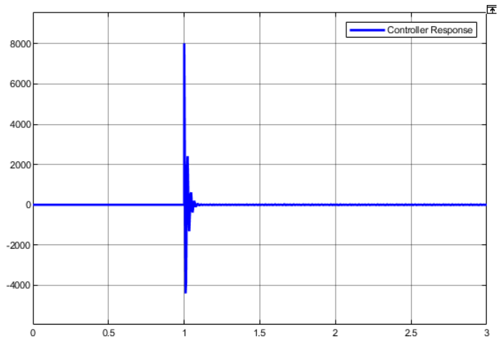


Figure 16: Controller Response for r=1

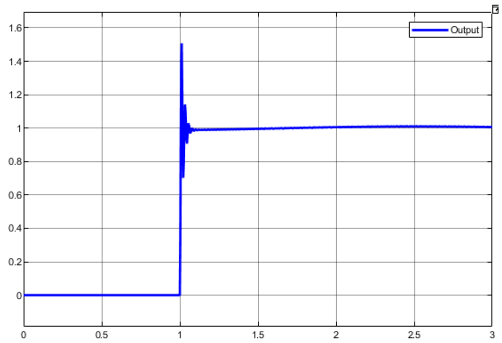


Figure 17: Output for r=1

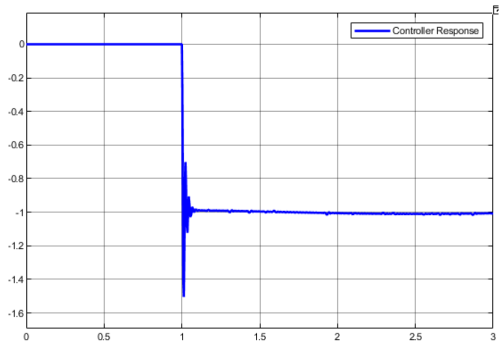


Figure 18: Controller Response for d=1

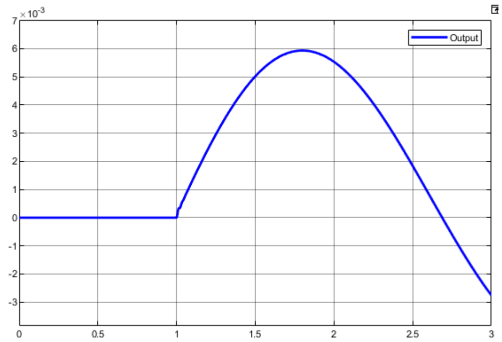


Figure 19: Output for d=1

2 Question 2

Question 2 (30%)

Consider a transfer function of the open loop $L(s) = C(s)P(s)$ in Figure 1. Assume that the closed loop is stable, and $|L(j\omega)|$ is a monotonically decreasing function of ω. Let ω_c be crossover frequency ($|L(j\omega)|_{\omega=\omega_c} = 1$).

1. How does controller $C(s)$ need to change (increase or decrease) ω_c in order to attenuate the effect of mesurement noise $n(t)$ on $y(t)$ (the spectrum of measurement noise is concentrated in high frequencies)?
2. How does controller $C(s)$ need to change (increase or decrease) ω_c in order to decrease the rise time of system response $y(t)$ for a step in $r(t)$?

2.1 ω_c design attenuate noise

The decision of crossover frequency is made by two hand sides:

- High enough to get fast response
- Low enough to avoid amplify noise too much

If our target is to attenuate the effect of measurement noise n(t) on y(t), we need to decrease ω_n. However, this action will lost some speed of response, but it isn't mentioned in this question.

2.2 ω_c design decrease rise time

As mentioned before, decreasing rise time means faster response. So we need to increase the crossover frequency ω_c. Again, there are some side effects, depends on the design target.

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