Tim Flannagan

Problem 1 [15]. Widgets are manufactured in three factories: A B and C. The proportion of defective widgets from each factory are as follows... What is the likelihood that a given widget is defective?

<u>Givens:</u>
P(A), P(B) = 0.3
P(C) = 0.4
P(Defect | A) = 0.01
P(Defect | B) = 0.04
P(Defect | C) = 0.02
Bayes' Theorem: $\frac{P(B|A)P(A)}{P(B|A)P(A)+P(B|\neg A)P(\neg A)}$

The probability that a widget has defected can be found by adding up the sums of all possibilities and their respective probability. Here, we have three different factories and using the givens above we can calculate the probability that a widget has defected.

$$P(defected) = P(defected \mid A)P(A) + P(defected \mid B)P(B) + P(defected \mid C)P(C)$$
$$P(defected) = (0.01)(0.3) + (0.04)(0.3) + (0.02)(0.4) = 0.023$$

The probability that a widget has defected is 2.3%.

Problem 2 [20]. Suppose there are two full bowls of cookies. Bowl #1 has 10 chocolate chip and 30 plain cookies, while bowl #2 has 20 of each. Our friend Stacy picks a bowl at random, and then picks a cookie at random. We may assume there is no reason to believe Stacy treats one bowl differently from another, likewise for the cookies. The cookie turns out to be a plain one. How probable is it that Stacy picked it out of Bowl #1?

<u>Givens:</u>
Bowl #1 has 10 chocolate chip, 30 plain
Bowl #2 has 20 chocolate chip, 20 plain

<u>Varialbles</u>:
Let b1 represent bowl #1
Let b2 represent bowl #2

The probability that Stacy picks bowl #1 is 0.50, and bowl #2 is 0.50
P(b1), P(b2) = 0.50

The probability that Stacy picks a plain cookie from bowl #1 is 30/40 = 0.75; we can represent this as P(plain cookie | b1) = 0.75.

Likewise, the probability that Stacy picks a plain cookie from bowl #2 is 20/20 = 0.50; we can represent this as P(plain cookie | b2) = 0.50.

Using Bayes' theorem, we can find out the probability that Stacy picked a plain cookie out of bowl #1, which is represented as P(b1 | plain cookie).

$$P(b1 \mid plain\ cookie) = \frac{P(plain\ cookie \mid b1)P(b1)}{P(plain\ cookie \mid b1)P(b1) + P(plain\ cookie \mid b2)P(b2)} = \frac{(0.75)(0.5)}{(0.75)(0.5)+(0.5)(0.5)} = 0.6$$

The probability that we chose a plain cookie from bowl #1 is 60%.

Problem 3[15]. The blue M&M was introduced in 1995. Before then, the color mix in a bag of plain M&Ms was (30% Brown, 20% Yellow, 20% Red, 10% Green, 10% Orange, 10% Tan). Afterward it was (24% Blue, 20% Green, 16% Orange, 14% Yellow, 13% Red, 13% Brown). A friend has two bags of M&Ms, and tells me that one is from 1994 and one from 1996. My friend won't tell me which is which, but gives me one M&M from each bag. One is yellow and one is green. What is the probability that the yellow M&M came from the 1994 bag?

Givens:
P(1994), P(1996) = 0.50
P(yellow | 1994) = 0.20
P(yellow | 1996) = 0.14
We know that both M&M's are from different bags.'

We want to know the probability that the yellow M&M we chose is from the 1994 bag. We can represent this as P(1994 | yellow). Using Bayes' Theorem we get the following equation:

$$P(1994 \mid yellow) = \frac{P(yellow \mid 1994)P(1994)}{P(yellow \mid 1994)P(1994) + P(yellow \mid 1996)P(1996)} = \frac{(0.2)(0.5)}{(0.2)(0.5)+(0.14)(0.5)} = \frac{0.10}{0/17} = 0.588$$

The probability that we chose a yellow M&M from the 1994 bag is 58.8%.