ECEN 4632: Quiz 3

Spring 2012

This quiz is out of 25 total points. You have 50 minutes to complete it. No study aids are allowed except for three 8.5" x 11" sheets of paper containing any handwritten information that you would like. No calculators or other electronic devices are allowed.

1) Consider the causal system described by the difference equation

$$y[n] - \frac{5}{4}y[n-1] + \frac{3}{8}y[n-2] = x[n] + x[n-2]$$

which has system function

$$H(z) = \frac{1 + z^{-2}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{3}{4}z^{-1}\right)}$$

(a) **(3 pts)** Draw an implementation of this system using at most two delay elements as well as addition and scaling elements.

(b) **(3 pts)** Find the poles and zeros of $H(z)$ and use these to sketch $|H(e^{j\omega})|$. Label any important quantities in your sketch.

(c) **(3 pts)** Find the system function and all possible ROCs for inverse system(s) for this system. *Say whether each is causal and/or BIBO-stable.*

(d) **(3 pts)** Suppose that we construct a system consisting of an ideal sampler that samples every $T = 0.01$ seconds, followed by the discrete-time system above, followed by an ideal reconstruction system using the same T. Assume that the input $x_c(t)$ has a Fourier transform such that $X_c(j\Omega) = 0$ for all $|\Omega| \geq 2\pi 50$. Find the effective continuous-time frequency response of the resulting system, i.e. find $H_{eff}(j\Omega)$ such that $Y_c(j\Omega) = H_{eff}(j\Omega)X_c(j\Omega)$.

2) Suppose that we want to design a discrete-time filter meeting the design specifications

$$\begin{aligned} |H_1(e^{j\omega})| &> 0.9 \text{ for } |\omega| \leq 0.65 \\ |H_1(e^{j\omega})| &< 0.4 \text{ for } 1.55 \leq |\omega| \leq \pi \end{aligned}$$

via impulse invariance from a continuous-time Butterworth filter. We already found that Butterworth filter with order $N = 2$ and $\Omega_c = 1$ would satisfy our needs. I.e. we know that

$$\begin{aligned} \left| \frac{1}{1 + \left(\frac{\Omega}{1}\right)^4} \right| &> (0.9)^2 \text{ for } |\Omega| \leq 0.65 \\ \left| \frac{1}{1 + \left(\frac{\Omega}{1}\right)^4} \right| &< (0.4)^2 \text{ for } |\Omega| \geq 1.55 \end{aligned}$$

(a) **(3 pts)** Find $H_c(s)$ for this continuous-time Butterworth filter.

(b) **(4 pts)** Use impulse invariance and the continuous-time filter above to find a discrete-time filter satisfying the desired specifications above. What is the system function $H_1(z)$ for the resulting filter? *(Notes: You should not have to find any impulse responses to find $H_1(z)$ from $H_c(s)$. You may leave your result as two separate fractions.)*

(c) **(3 pts)** Suppose that we had instead applied the bilinear transformation to $H_c(s)$ to get a discrete-time filter. What would the system function $H_2(z)$ be for the resulting filter?

(d) **(3 pts)** What are the passband and stopband of the filter $H_2(z)$ obtained via bilinear transformation? (You do not need to evaluate any trigonometric expressions in your answer.)