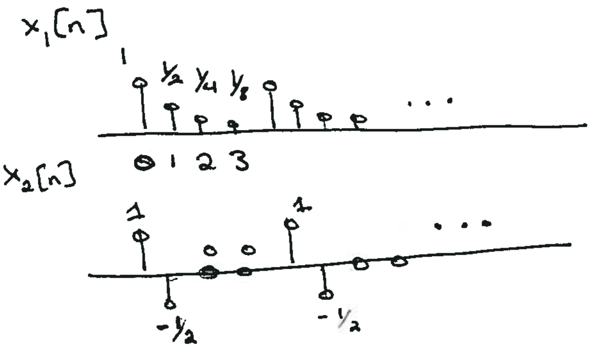
8.1-2)



1) Direct evaluation:

To simplify the calculation, we'll evaluate

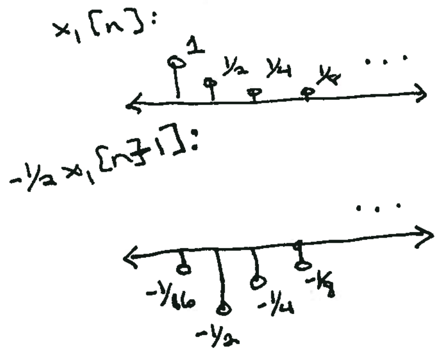
$$\sum_{m=0}^{N-1} x_2[m]\,x_1[n-m] = \sum_{m=0}^{3} x_2[m]\,x_1[n-m]$$

$$= x_2[0]\,x_1[n] + x_2[1]\,x_1[n-1] + x_2[2]\,x_1[n-2] + x_2[3]\,x_1[n-3]$$

$$= 1\cdot x_1[n] + (-1/2)\,x_1[n-1] + 0 + 0$$

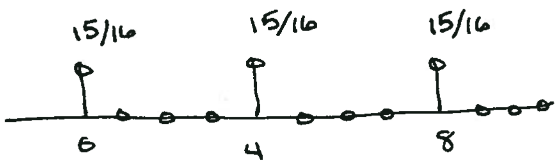
$$= x_1[n] - 1/2\,x_1[n-1]$$

Since we have



we note that

$x_1[n] - 1/2\,x_1[n-1]$ is:



2) Multiplying the Fourier series coefficients:

To find the Fourier series coefficients of $x_1[n]$:

$$\tilde{X}_1[k] = \sum_{n=0}^{3} x_1[n] e^{-j2\pi kn/4}$$

$$= 1e^{-j0} + \tfrac{1}{2}e^{-j2\pi k/4} + \tfrac{1}{4}e^{-j2\pi 2k/4} + \tfrac{1}{8}e^{-j2\pi 3k/4}$$

$$= 1 + \tfrac{1}{2}(-j)^k + \tfrac{1}{4}(-1)^k + \tfrac{1}{8}(j)^k$$

$$= \begin{cases} 1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} & k=0 \\ 1 - \tfrac{1}{2}j - \tfrac{1}{4} + \tfrac{1}{8}j & k=1 \\ 1 - \tfrac{1}{2} + \tfrac{1}{4} - \tfrac{1}{8} & k=2 \\ 1 + \tfrac{1}{2}j - \tfrac{1}{4} - \tfrac{1}{8}j & k=3 \end{cases}$$

$$= \begin{cases} 15/8 & k=0 \\ 3/4 - 3/8\,j & k=1 \\ 5/8 & k=2 \\ 3/4 + 3/8\,j & k=3 \end{cases}$$

To find the Fourier series coefficients of $x_2[n]$:

$$\tilde{X}_2[k] = \sum_{n=0}^{3} x_2[n] e^{-j2\pi kn/4}$$

$$= 1 \cdot e^{-j0} - \tfrac{1}{2}e^{-j2\pi k/4} + 0 + 0$$

$$= 1 - \tfrac{1}{2}e^{-j\pi/2\,k}$$

$$= 1 - \tfrac{1}{2}(-j)^k$$

$$= \begin{cases} 1 - \tfrac{1}{2} & k=0 \\ 1 + \tfrac{1}{2}j & k=1 \\ 1 + \tfrac{1}{2} & k=2 \\ 1 - \tfrac{1}{2}j & k=3 \end{cases}$$

$$= \begin{cases} 1/2 & k=0 \\ 1 + \tfrac{1}{2}j & k=1 \\ 3/2 & k=2 \\ 1 - \tfrac{1}{2}j & k=3 \end{cases}$$

Multiplying these together, we get:

$$\tilde{X}_1[k]\tilde{X}_2[k] = \begin{cases} 15/8 \cdot 1/2 & k=0 \\ (3/4 - 3/8 j)(1 + 1/2 j) & k=1 \\ 3/2 \cdot 5/8 & k=2 \\ (3/4 + 3/8 j)(1 - 1/2 j) & k=3 \end{cases}$$

$$= \begin{cases} 15/16 & k=0 \\ 3/4 - 3/8 j + 3/8 j - 3/16 j^2 & k=1 \\ 15/16 & k=2 \\ 3/4 + 3/8 j - 3/8 j - 3/16 j^2 & k=3 \end{cases}$$

$$= \begin{cases} 15/16 & k=0 \\ 15/16 & k=1 \\ 15/16 & k=2 \\ 15/16 & k=3 \end{cases}$$

To find the signal associated with these Fourier series coefficients:

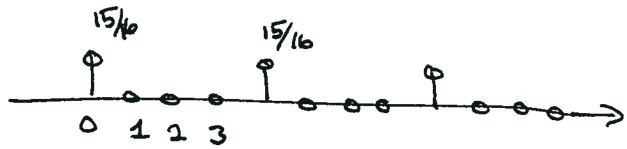
$$\frac{1}{4}\sum_{k=0}^{3} \tilde{X}_1[k]\tilde{X}_2[k]\, e^{j2\pi kn/4}$$

$$= \frac{1}{4}\cdot\frac{15}{16}\sum_{k=0}^{3} e^{j\pi kn/2}$$

$$= \begin{cases} 1/4 \cdot 15/16 \cdot 4 & n=0 \\ 0 & n=1,2,3 \end{cases}$$

(leads to equally spaced points on the unit circle in this case)

$$= \begin{cases} 15/16 & n=0 \\ 0 & n=1,2,3 \end{cases}$$



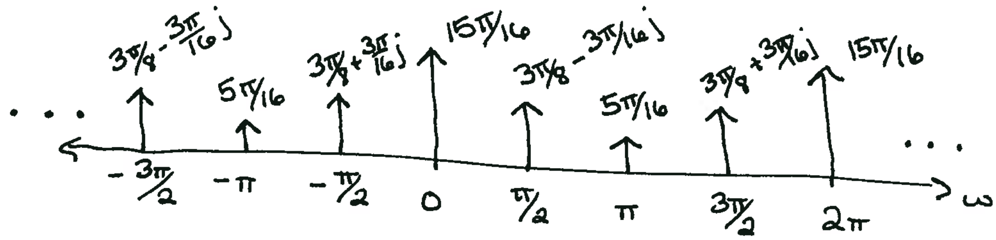
8.3)

From above, we found that the Fourier series coefficients of $x_1[n]$ were

$$\tilde{X}_1[k] = \begin{cases} 15/8 & k=0 \\ 3/4 - 3/8 j & k=1 \\ 5/8 & k=2 \\ 3/4 + 3/8 j & k=3 \end{cases}$$

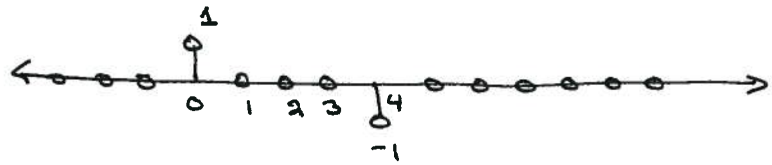
This means that the Fourier transform of $x_1[n]$ is a series of impulses at $2\pi k/4 = \pi/2 k$ for integers k with heights $2\pi/4 \, \tilde{X}_1[k] = \pi/2 \, \tilde{X}_1[k]$:

$X_1(e^{j\omega})$:

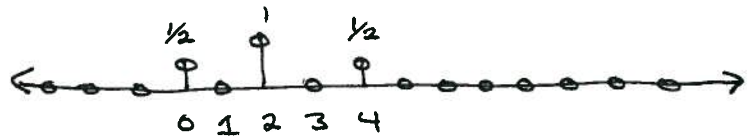


8.4)

$x_1[n]$:

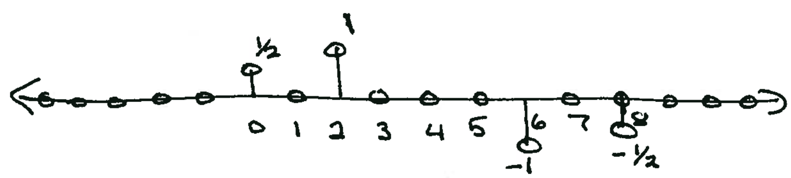


$x_2[n]$



a) The length of $x_1[n]$ is 5 and the length of $x_2[n]$ is also 5 so the total length of their convolution is $5+5-1=$ 9. Indeed, we can see that this convolution is:

$$\sum_{m=-\infty}^{\infty} x_1[m]x_2[n-m] = 1\,x_2[n] - 1\,x_2[n-4]$$



If $P \geq 9$, the circular convolution and the convolution will be the same on $n=0,\ldots,8$. However, if $P<9$, the circular convolution will involve the above, made periodic with period $P<9$, and there will be time-domain aliasing.

So $P=9$ is the minimum P.

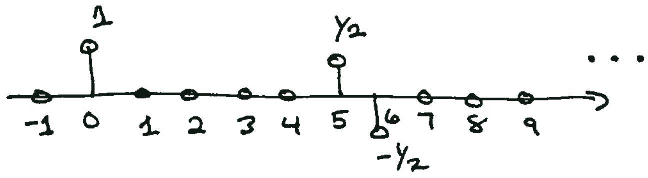
b) For this P, we can find the Fourier series coefficients of the resulting convolution by looking at the result above. We find that

$$= \tfrac{1}{2}e^{-j0} + 1e^{-j2\pi k 2/9} - 1e^{-j2\pi k 6/9} - \tfrac{1}{2}e^{-j2\pi k\cdot 8/9}$$

$$= \tfrac{1}{2} + e^{-j4\pi k/9} - e^{-j4\pi k/3} - \tfrac{1}{2}e^{-j16\pi k/9}$$

8.4)

x[n]



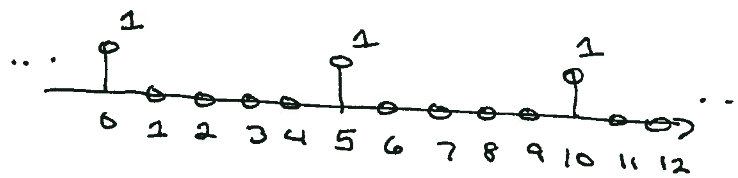
$$Y(e^{j\omega}) = X(e^{j\omega})\,P(e^{j\omega})$$

where $P(e^{j\omega}) = \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k/5)$

We know from class that multiplying with the signal

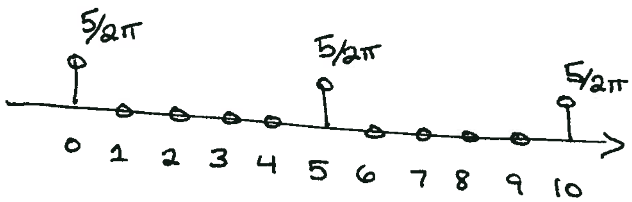
$$\frac{2\pi}{5} \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k/5)$$

in frequency leads to convolution w/ the impulse train



in time. Thus, multiplying with $P(e^{j\omega})$ as given is convolution with the signal

p[n]:



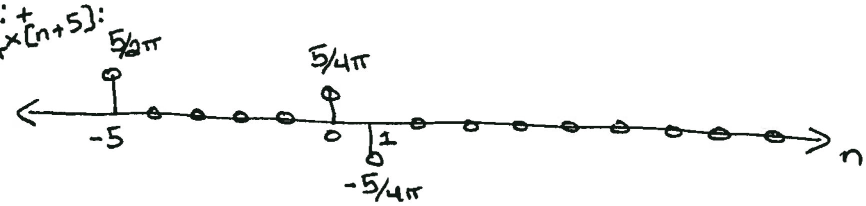
in time. Thus,

$$y[n] = 5/2\pi \sum_{m=-\infty}^{\infty} x[n+m5].$$

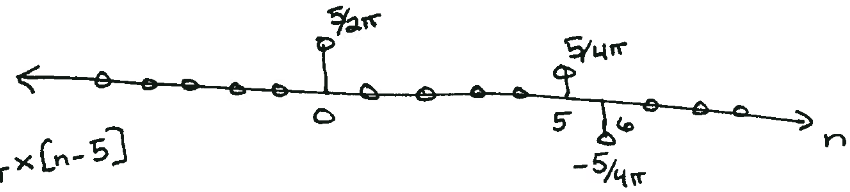
We therefore have a copy of $x[n]$ ~~spaced~~ shifted every multiple of 5 units all summed together and scaled by $5/2\pi$.

Looking at this, we have

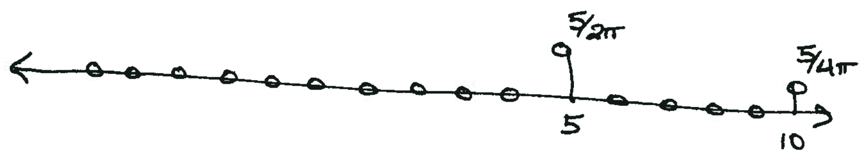
$y[n] = \vdots + \frac{5}{2\pi} x[n+5]$:



$+ \frac{5}{2\pi} x[n]$:

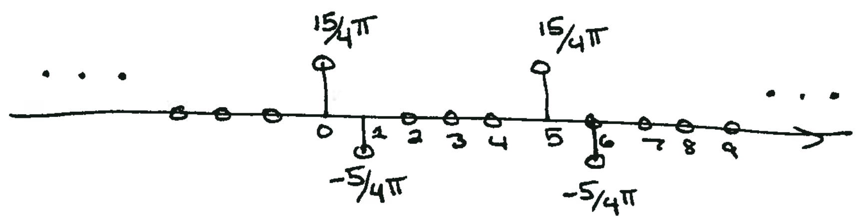


$+ \frac{5}{2\pi} x[n-5]$



$+ \ldots$

$=$



($y[n]$ is periodic w/ period 5)