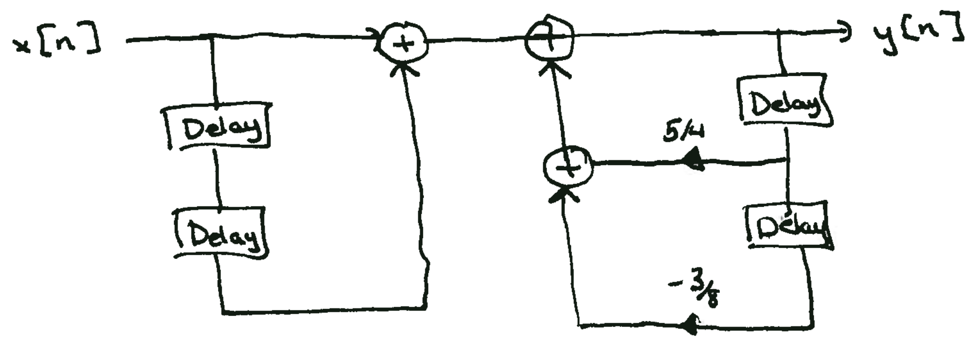
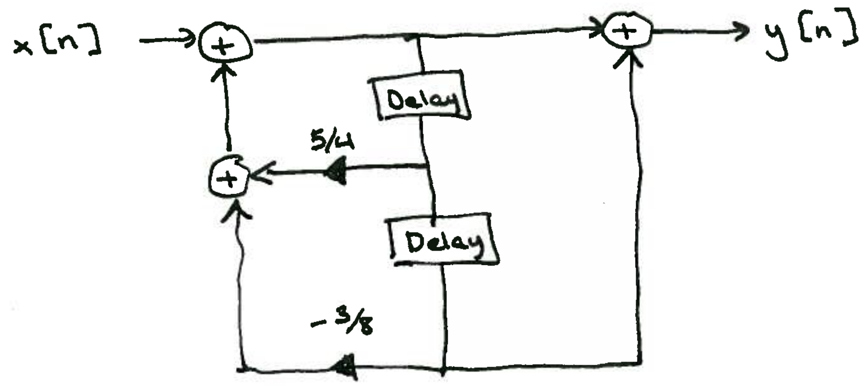
1) $y[n] - 5/4\, y[n-1] + 3/8\, y[n-2] = x[n] + x[n-2]$

$$H(z) = \frac{1+z^{-2}}{(1-\frac{1}{2}z^{-1})(1-\frac{3}{4}z^{-1})}$$

a) $y[n] = x[n] + x[n-2] + 5/4\, y[n-1] - 3/8\, y[n-2]$



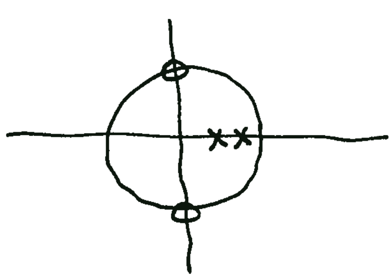
$\Rightarrow$



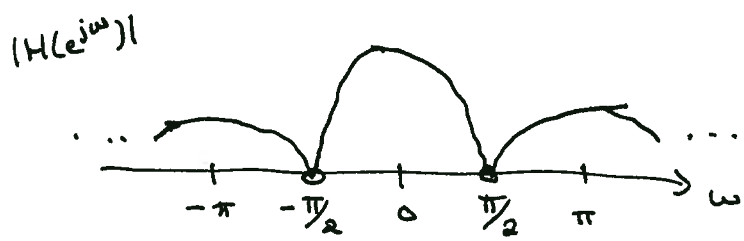
b) $$H(z) = \frac{z^2+1}{(z-\frac{1}{2})(z-\frac{3}{4})}$$ $\Rightarrow$ Poles: $\frac{1}{2}, \frac{3}{4}$

Zeros: $\pm j$

$\Rightarrow$



Since $|H(z)|$ is the ratio of the distances to $\pm j$ to the distances to $\frac{1}{2}$ and $\frac{3}{4}$, we can sketch $|H(e^{j\omega})|$:



c) Since

$$H(z) = \frac{(z-j)(z+j)}{(z-1/2)(z-3/4)}$$

ROC: $|z| > 3/4$
since system is causal

$$\Rightarrow H_{inv}(z) = \frac{(z-1/2)(z-3/4)}{(z-j)(z+j)}$$

Possible ROCs:

1) $|z| > |j| = 1$ causal, not BIBO-stable

2) $|z| < |j| = 1$ not causal, not BIBO-stable

Both ROCs are okay since both overlap $|z| > 3/4$.
(Neither is stable since neither includes the unit circle.)

d) $T = 0.01$ secs

First, we can check that there is no aliasing in the ideal sampler since $X_c(j\Omega) = 0$ for $|\Omega| \geq 2\pi \cdot 50$ and the sampling frequency is $\frac{2\pi}{T} = 2\pi \cdot 100 \geq 2 \cdot (2\pi \cdot 50)$

Since there is no aliasing, we have

$$H_{eff}(j\Omega) = H(e^{j\Omega T}) = H(e^{j0.01\Omega})$$

Since

$$H(e^{j\omega}) = H(z)\big|_{z=e^{j\omega}} = \frac{e^{j2\omega}+1}{(e^{j\omega}-1/2)(e^{j\omega}-3/4)}$$

we thus have

$$H_{eff}(j\Omega) = H(e^{j\omega})\big|_{\omega=0.01\Omega} = \frac{e^{j0.02\omega}+1}{(e^{j0.01\omega}-1/2)(e^{j0.01\omega}-3/4)}$$

2)

a) We are told that $N=2$, $\Omega_c = 1$.

To find the poles of this filter, solve

$$1 + \left(s/j\Omega_c\right)^{2N} = 0$$

$$\Rightarrow 1 + \frac{s^4}{j^4 1^4} = 0$$

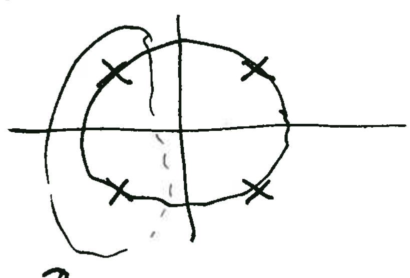
$$\Rightarrow 1 + s^4 = 0$$

$$\Rightarrow s^4 = -1 = e^{j\pi}$$

$$\Rightarrow |s| = 1 \qquad \measuredangle s = \pi/4 + 2\pi k/4 \qquad k = 0, 1, 2, 3$$

$$= \pi/4,\ 3\pi/4,\ 5\pi/4,\ 7\pi/4$$

Poles



Select just these 2 for causality and stability

$$H_c(s) = \frac{1}{(s - e^{j3\pi/4})(s - e^{j5\pi/4})} = \frac{1}{(s - e^{j3\pi/4})(s - e^{-j3\pi/4})}$$

b) Partial Fractions:

$$H_c(s) = \frac{A}{s - e^{j3\pi/4}} + \frac{B}{s - e^{-j3\pi/4}}$$

$$\Rightarrow A = \left.\frac{1}{s - e^{-j3\pi/4}}\right|_{s = e^{j3\pi/4}} = \frac{1}{e^{j3\pi/4} - e^{-j3\pi/4}} = \frac{1}{2j\sin(3\pi/4)}$$

$$= \frac{1}{2j \cdot \sqrt{2}/2} = \frac{1}{\sqrt{2}j}$$

$$B = \left.\frac{1}{s - e^{+j3\pi/4}}\right|_{s = e^{-j3\pi/4}} = \frac{1}{e^{-j3\pi/4} - e^{j3\pi/4}} = -A = -\frac{1}{\sqrt{2}j}$$

If $H_c(s) = \frac{A}{s-p_1} + \frac{B}{s-p_2}$, then after sampling $h_c(t)$

to get $h[n]$,

$$H(z) = \frac{A}{s-e^{p_1}} + \frac{B}{s-e^{p_2}}$$

$$\Rightarrow H(z) = \frac{\frac{1}{\sqrt{2}j}}{z-e^{e^{j3\pi/4}}} - \frac{\frac{1}{\sqrt{2}j}}{z-e^{e^{-j3\pi/4}}}$$

$$= \frac{\frac{1}{\sqrt{2}j}}{z-e^{(-1/\sqrt{2}+j/\sqrt{2})}} - \frac{\frac{1}{\sqrt{2}j}}{z-e^{(-1/\sqrt{2}-j/\sqrt{2})}}$$

c) Bilinear transformation:

$$H(z) = H_c(s)\Big|_{s=2\frac{1-z^{-1}}{1+z^{-1}}} = \frac{1}{\left(2\frac{(1-z^{-1})}{(1+z^{-1})} - e^{j3\pi/4}\right)\left(2\frac{(1-z^{-1})}{(1+z^{-1})} - e^{-j\frac{3\pi}{4}}\right)}$$

$$= \frac{(1+z^{-1})^2}{(2-2z^{-1}-e^{j3\pi/4}-e^{j3\pi/4}z^{-1})(2-2z^{-1}-e^{-j3\pi/4}-e^{-j3\pi/4}z^{-1})}$$

$$= \frac{(1+z^{-1})^2}{((2-e^{j3\pi/4})-(2+e^{j3\pi/4})z^{-1})((2-e^{-j3\pi/4})-(2+e^{-j3\pi/4})z^{-1})}$$

d) For the cts filter, we have

$$\Omega_p = .65$$
$$\Omega_s = 1.55$$

After bilinear transformation:

$$\omega_p = 2\arctan(\Omega_p/2) = 2\arctan(.65/2)$$
$$\omega_s = 2\arctan(\Omega_s/2) = 2\arctan(1.55/2)$$

$\Rightarrow$ Passband: $|\omega| \le 2\arctan(.65/2)$

Stopband: $|\omega| \ge 2\arctan(1.55/2)$