Compute $\mathrm{DTW}_\Omega(\boldsymbol{\theta})$ and $\nabla\mathrm{DTW}_\Omega(\boldsymbol{\theta})$

**Input:** Distance matrix $\boldsymbol{\theta} \in \mathbb{R}^{N_A \times N_B}$

$\triangleright$ Forward pass

$v_{0,0} = 0$; $v_{i,0} = v_{0,j} = \infty$, $i \in [N_A], j \in [N_B]$

**for** $i \in [1, \ldots, N_A], j \in [1, \ldots, N_B]$ **do**

$$v_{i,j} = d_{i,j} + \min\nolimits_\Omega(v_{i,j-1}, v_{i-1,j-1}, v_{i-1,j})$$

$$\boldsymbol{q}_{i,j} = \nabla\min\nolimits_\Omega(v_{i,j-1}, v_{i-1,j-1}, v_{i-1,j}) \in \mathbb{R}^3$$

$\triangleright$ Backward pass

$\boldsymbol{q}_{i,N_B+1} = \boldsymbol{q}_{N_A+1,j} = \mathbf{0}_3$, $i \in [N_A], j \in [N_B]$

$e_{i,N_B+1} = e_{N_A+1,j} = 0$, $i \in [N_A], j \in [N_B]$

$\boldsymbol{q}_{N_A+1,N_B+1} = (0,1,0)$; $e_{N_A+1,N_B+1} = 1$

**for** $j \in [N_B, \ldots, 1], i \in [N_A, \ldots, 1]$ **do**

$$e_{i,j} = q_{i,j+1,1}\ e_{i,j+1} + q_{i+1,j+1,2}\ e_{i+1,j+1} + q_{i+1,j,3}\ e_{i+1,j}$$

**Return:** $\mathrm{DTW}_\Omega(\boldsymbol{\theta}) = v_{N_A,N_B}$

$\nabla\mathrm{DTW}_\Omega(\boldsymbol{\theta}) = (e)_{i,j=1}^{N_A,N_B}$