Swiss Institute of Banking and Finance

University of St.Gallen

1. Return and Risk

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7,150 Financial Markets

Agenda

Financial Markets

Returns

Returns with Cash Flows

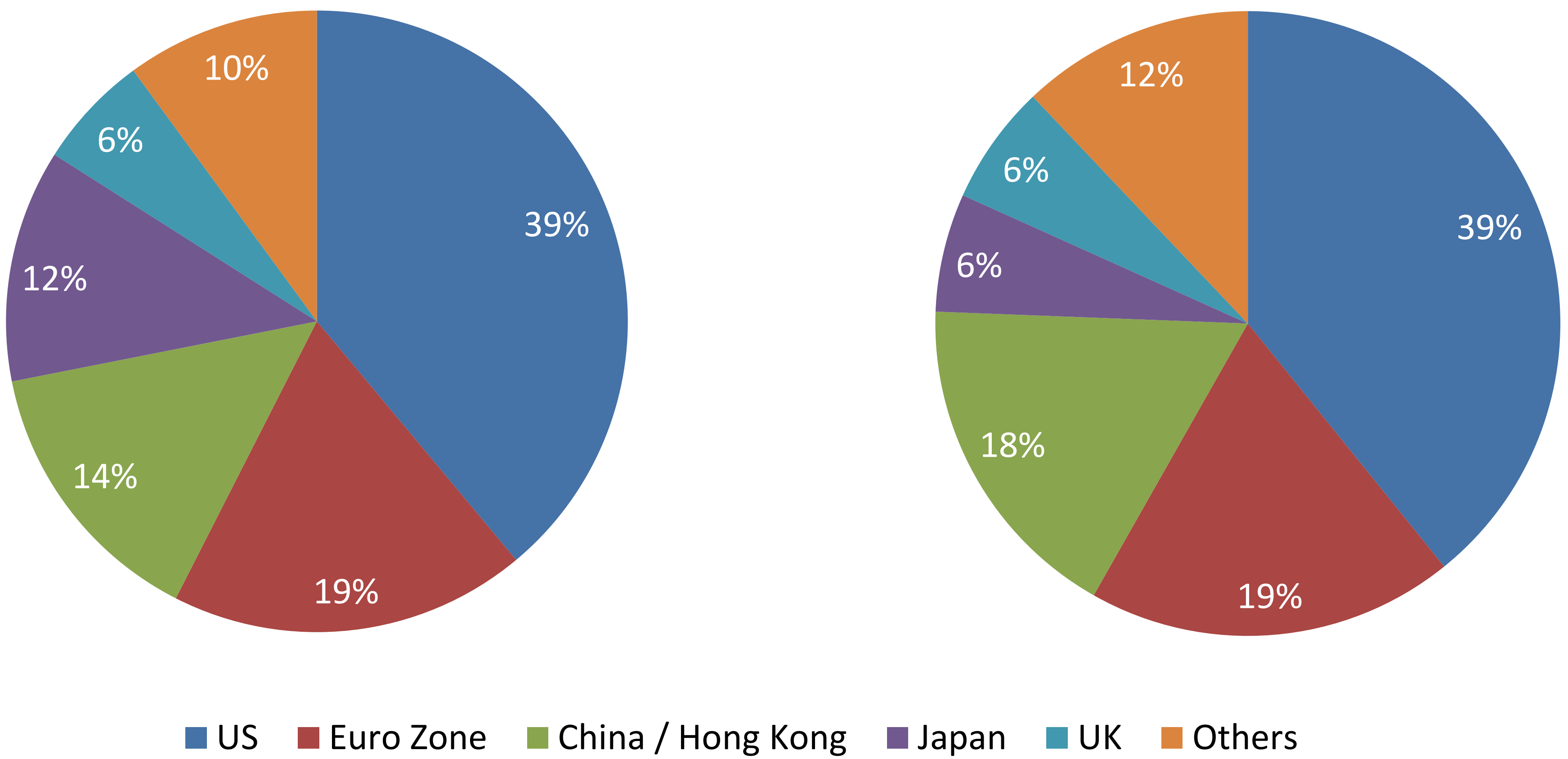
Return Distribution and Risk

Risk Measures

Global Debt Market Capitalization

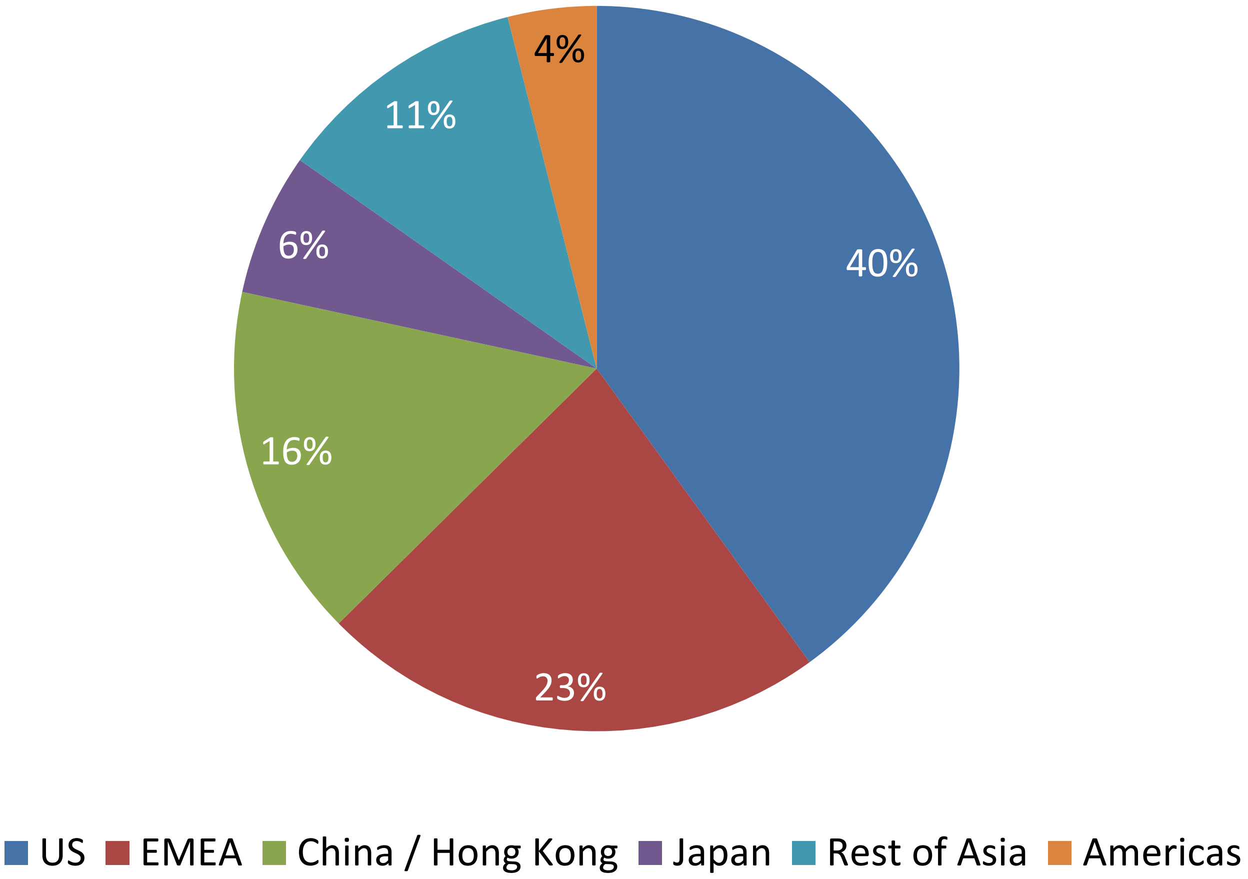
World *Total* Bond Market
(December 2019: 106 trillion USD)

World *Corporate* Bond Market
(December 2019: 54 trillion USD)



Global Equity Market Capitalization

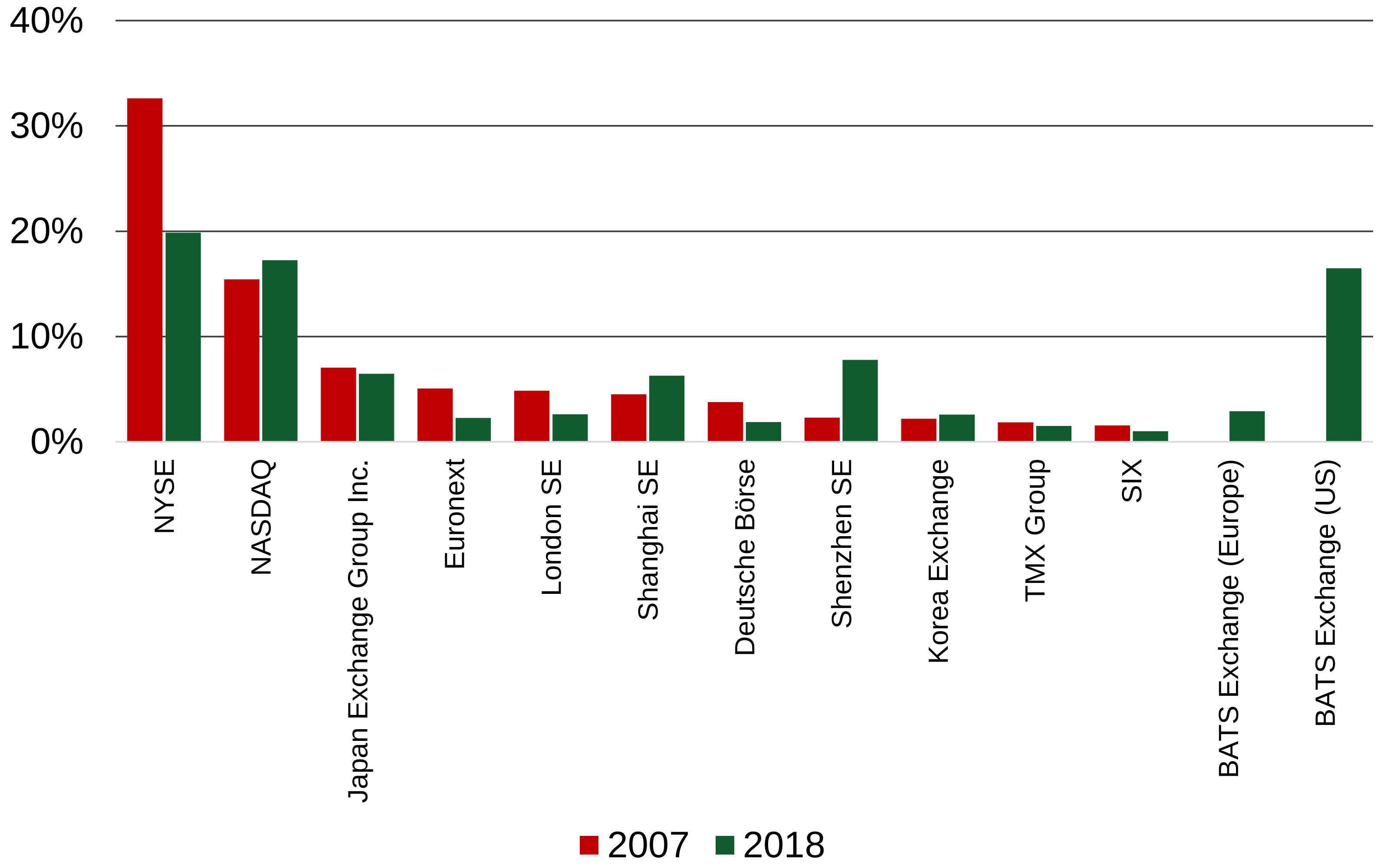
Global market capitalization
(June 2020: 89 trillion USD)



Source: World Federation of Exchanges

Global Stock Market Trading

Market share of stock exchanges in terms of trading volume



Trading volume (in Trn. USD p.a.)	
2009	**2019**
75.3	84.2

Source: World Federation of Exchanges

Types of Orders, Long- and Short-Selling and Transaction Costs

- When trading on financial markets, there are several ways to place an order: Market Orders, Limit Orders, Stop Orders.

- To perform market transactions, investors have to pay transaction costs (fees and commissions, bid-ask spread, price impact).

- An important difference when placing orders is between long- and short-selling.

- Short-selling is the practice of borrowing a security (e.g., from a broker), selling it immediately on the market, and buying it back at a later point in time in order to cover the short position.

- A short sale hence allows an investor to profit from a decline in a security's price.

Long and Short Positions

Purchase of a stock (= Long Position)			
Time		**Action**	**Cash Flow**
0		Buy security	- Initial price
1		Receive dividend	+ Dividend
2		Sell security	+ Ending price
Profit	=	Ending price + dividend - initial price	

Short sale of a stock (= Short Position)			
Time		**Action**	**Cash Flow**
0		Borrow stock; sell it	+ Initial price
1		Repay dividend and buy share to	-(Ending price + Dividend)
		replace the share originally borrowed	
Profit	=	Initial price - (ending price + dividend)	

Financial Market Infrastructure – Alternative Trading Systems

- An **Alternative Trading System** (ATS) is a non-exchange trading venue approved by regulatory authorities (e.g. SEC, BaFin, FINMA)

  Examples: NYSE Arca, Chi-X (by Instinet), Bloomberg Tradebook,
  The Brass Utility (BRUT), Tradegate.

- **Dark Pools** are bank- and exchange internal trading platforms that facilitate anonymous and intransparent trading (e.g. Euronext Block).

  Very popular among large institutional investors (e.g. Hedge fonds), because they facilitate the trading of large portfolios without public knowledge (front-running protection).

Financial Market Infrastructure - The Execution of a Trade

Investor → via phone, online banking etc. → **Broker**

Investor: Initiates an order and chooses a broker.

Broker: Looks at size and availability of the order. Then has diverse execution options ("order room").
Note: the *broker* decides how the trade is executed and has the duty of *best execution*.
(Examples: Charles Schwab, Scottrade, TD Ameritrade etc.)

Market Maker

For some markets, brokers direct orders to the MM that is responsible for the stock. Example: Nasdaq.

Exchange

If stock is listed on an exchange, brokers can direct orders to the exchange (floor, e.g. NYSE) or to a firm called "third market maker", a firm that stands ready to buy or sell a stock listed on an exchange at publicly quoted prices. Brokers are likely to use third MM when:
a) broker is not a member of exchange where stock is traded.
b) third MM pays the broker for the order ("payment for order flow").

ECN

An Electronic Communication Network connects major brokerages and individual traders so that they can trade directly between themselves without having to go through a middleman. It automatically matches buy and sell orders; is used especially for limit orders. Examples: Chi-X, Bloomberg TradeBook, Tradegate.

Internalization

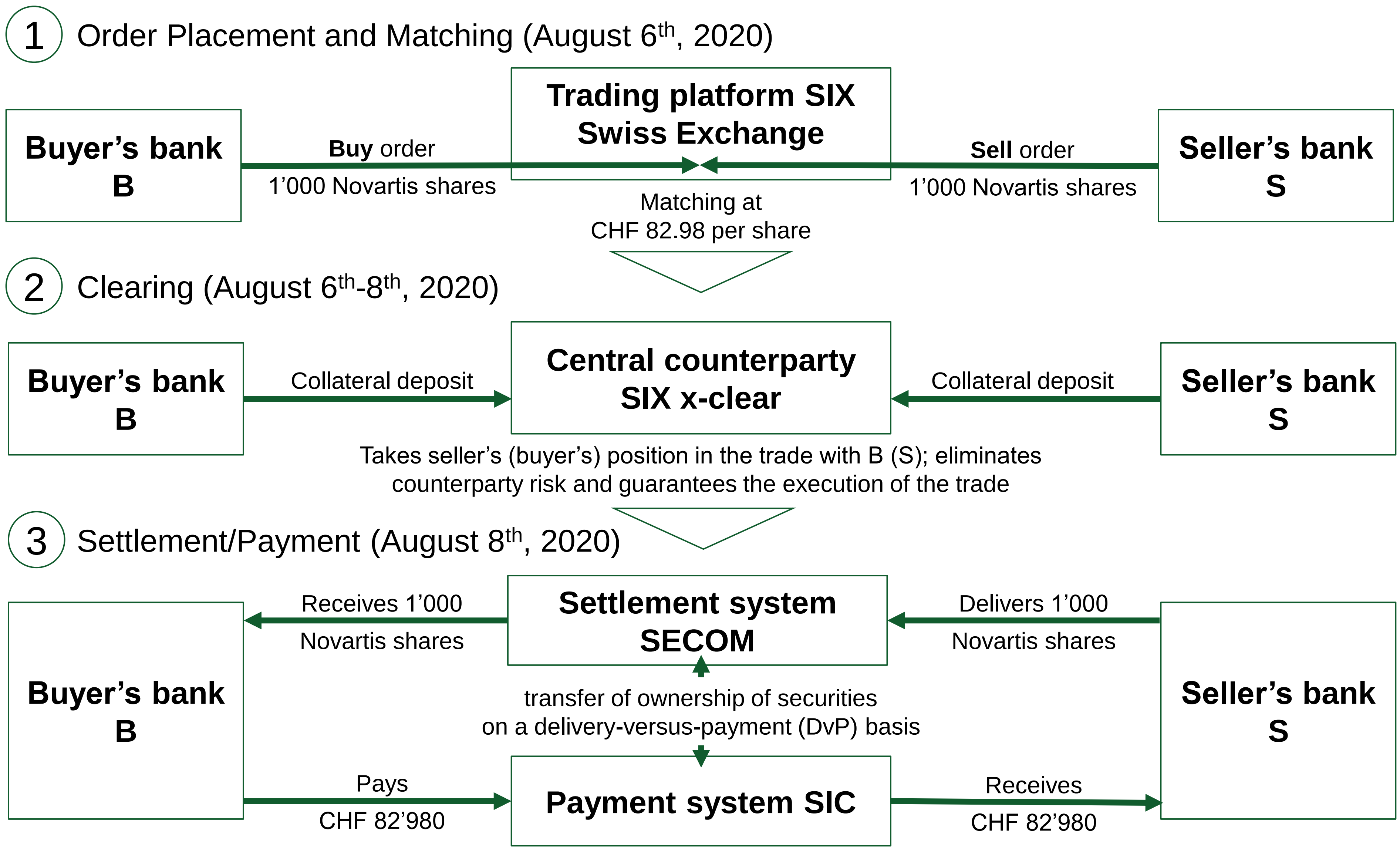
Broker uses inventory of own stocks (stock from brokerage company) to fulfil the order. Bid-Ask spread goes directly to broker.

Source: SEC

Financial Market Infrastructure - The Clearing of a Trade

- Brokerage firms, banks, and other financial institutions are members of Clearinghouses.
- Members are obliged to keep records of all the transactions during the trading day and subsequently send them to the Clearinghouse. The clearing institution checks all trades for consistency and then all transactions are netted out. Each member receives a list of the net amounts of securities to be delivered or received along with the net amount of money to be paid or collected.
- Every day, each **member settles with the Clearinghouse** instead of with various other firms.
- The clearing system eliminates the direct counterparty risk for the "final" counterparties of a transaction.
- In the US, clearing is accomplished by the Depository Trust and Clearing Corporation or Fedwire.
- In Europe, there are a number of national and international Clearinghouses. In Switzerland, SIX Swiss Exchange introduced a central clearing system in cooperation with SIX x-clear AG and the London-based LCH.Clearnet Group.

The Swiss Value Chain –
Example: Stock Trade of 1000 Novartis Shares



Quelle: SIX (2020)

Arbitrage

Definition:

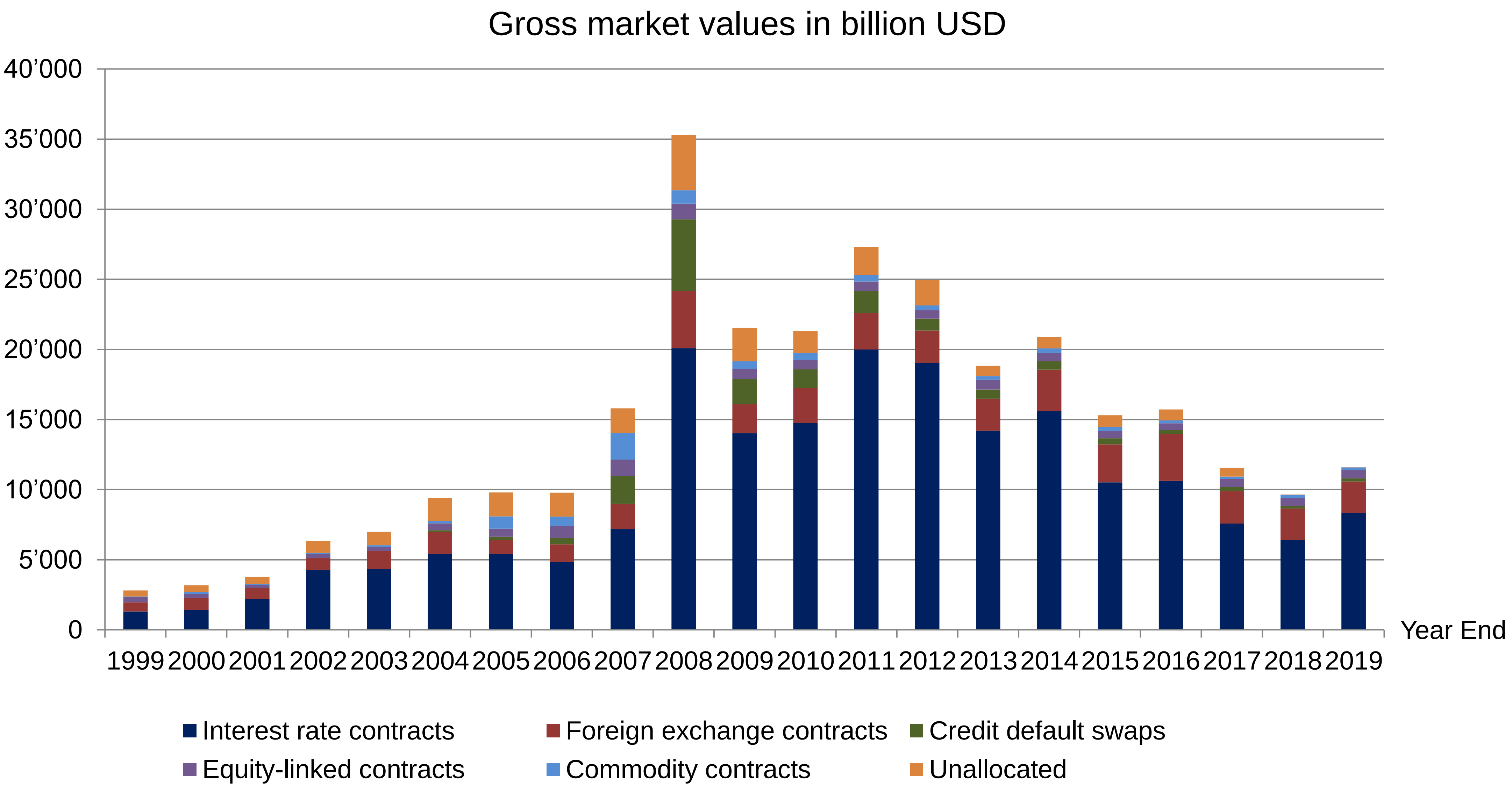
Arbitrage is a trading strategy
» leading to riskless profit
» without capital investment.

- Two assets with the same cash-flows must have the same price. Arbitrageurs would exploit potential price differences leading to a convergence of the prices.
- The concept of arbitrage is central to capital market theory.
- Arbitrage is the foundation of many pricing models in finance.
- Do we encounter arbitrage in the real world?

Traditional Market for Derivatives (1/2)

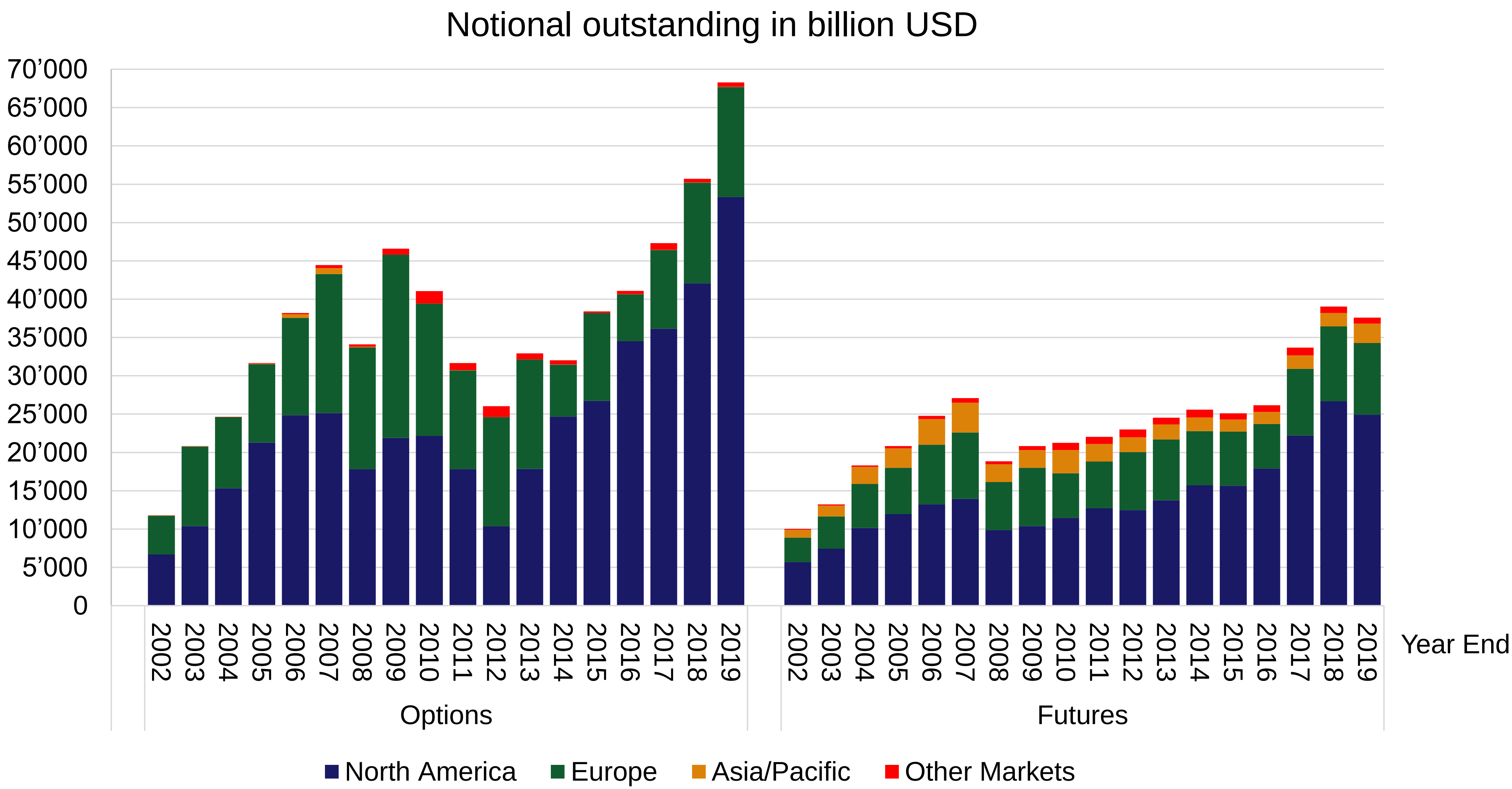
- Derivative securities markets
  - Exchange traded derivatives
    - Futures
    - Traded options
    - Futures Options
  - OTC traded derivatives
    - Forwards
    - Swaps
    - OTC Options
    - Warrants
    - Structured Products

OTC Derivatives



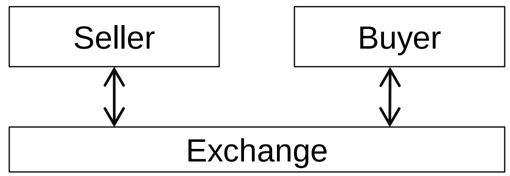
Source: BIS

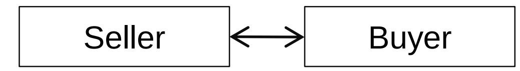
Exchange-Traded Options and Futures



* March 2019
Source: BIS

Traditional Market for Derivatives (2/2)



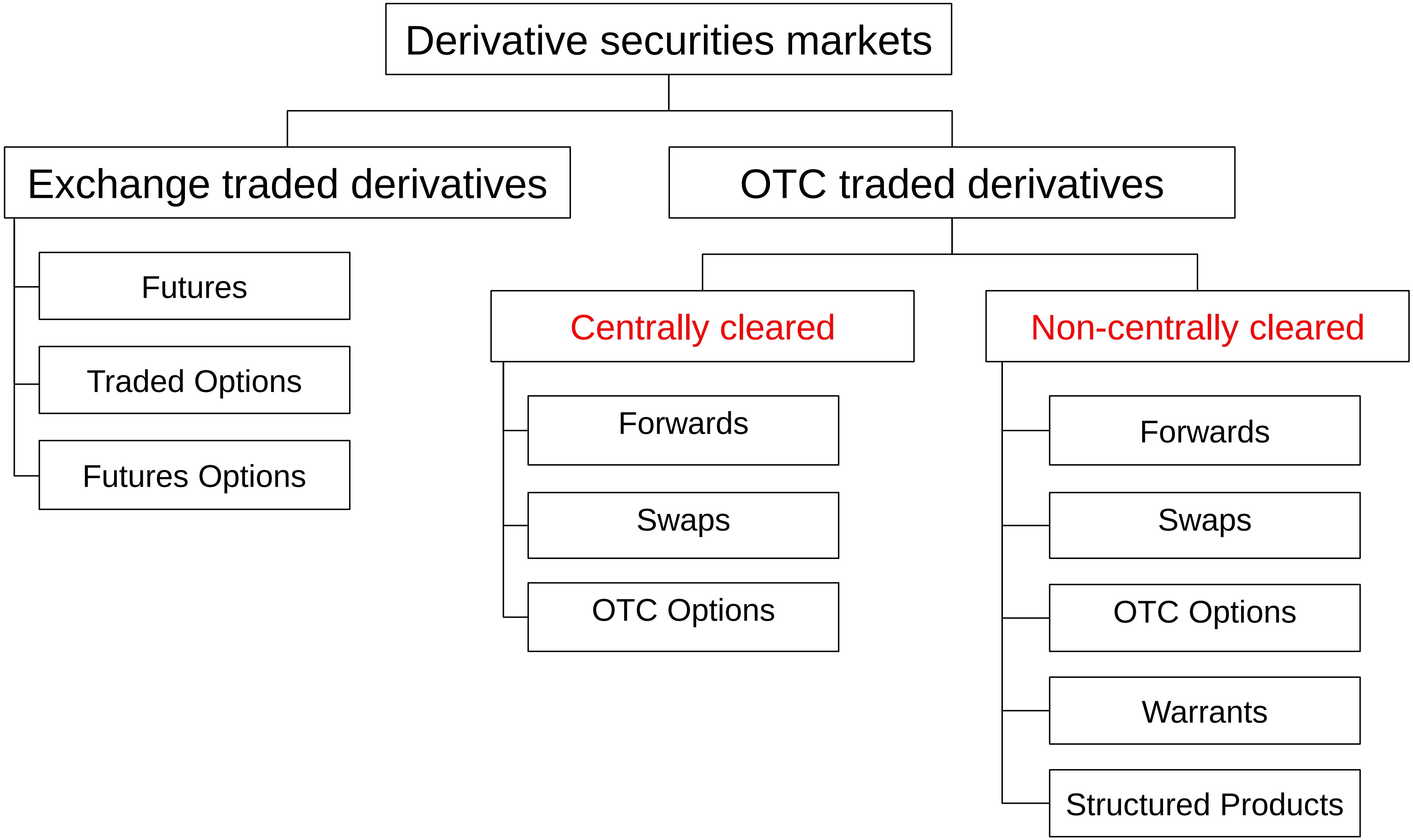


	Exchange-traded	**OTC**

Contract characteristics	• Exchange is trading partner (clearing)
	• Standardization
	• No individual counter-party risk (central clearing)
		• Specific terms defined exclusively by two counterparties (Customization/ tailored products)
		• General terms are defined in documentation
		    • Pre-Trade: Bilateral documentation and internal approvals
		    • Post-Trade: Trade verification, confirmation

Market characteristics	• Organized market with specific trading rules • Quotes and prices are quickly available • Transparency	• Dealer markets where brokers and dealers make a two-way market • Largely unregulated with respect to disclosure of information

Significant Change in OTC Market (1/2)



Significant Change in OTC Market (2/2)

- Financial crisis 2008 exposed weaknesses in OTC market
  - The build-up of large exposures which were not appropriately risk-managed
  - Limited transparency concerning level of activity and overall size
- Declaration of G20 in September 2009: central clearing for all standardized OTC derivatives

  → Nowadays, 70% of OTC derivatives are centrally cleared.[1]
- What drives the feasibility of derivatives to be centrally cleared?[2]
  - Legal, operational and economic standardization
  - Low complexity in terms of the difficulty of valuing a product economically
  - Liquidity due to economic terms (maturities, currency denominations, etc.)

[1] O'Malia, Scott. (2016, July 4). Non-cleared market is changing – not dying. *Risk.net,* Retrieved from http://www.risk.net
[2] Pirrong, C. (2011). *The economics of central clearing: theory and practice*. New York: International Swaps and Derivatives Association.

Example: OTC Derivatives Subject to EU Clearing Obligation

"In accordance with […] EMIR, ESMA shall maintain a Public Register to inform market participants on the clearing obligation."
(ESMA, Public Register for the Clearing Obligation under EMIR)

Extract of this public register:

Table 3: Forward Rate Agreement Classes

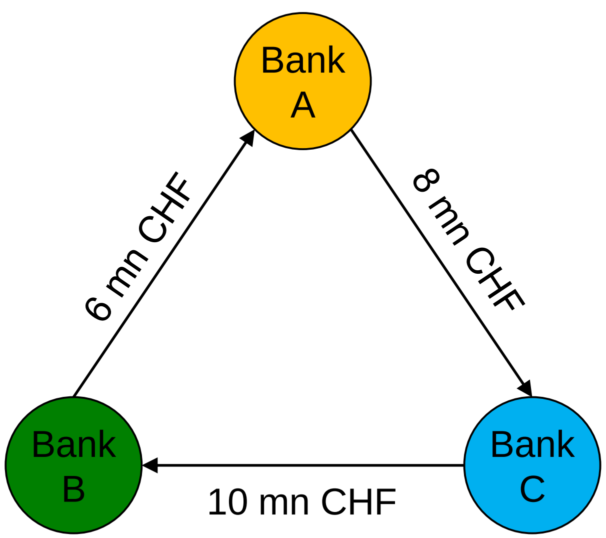
id	Type	Reference Index	Settlement Currency	Maturity	Settlement Currency Type	Optionality	Notional Type
A.3.1	FRA	EURIBOR	EUR	3D-3Y	Single currency	No	Constant or Variable
A.3.2	FRA	LIBOR	GBP	3D-3Y	Single currency	No	Constant or Variable
A.3.3	FRA	LIBOR	USD	3D-3Y	Single currency	No	Constant or Variable
C.2.1	FRA	NIBOR	NOK	3D-2Y	Single currency	No	Constant or Variable
C.2.2	FRA	WIBOR	PLN	3D-2Y	Single currency	No	Constant or Variable
C.2.3	FRA	STIBOR	SEK	3D-3Y	Single currency	No	Constant or Variable

→ All instruments contained in this list have to be centrally cleared

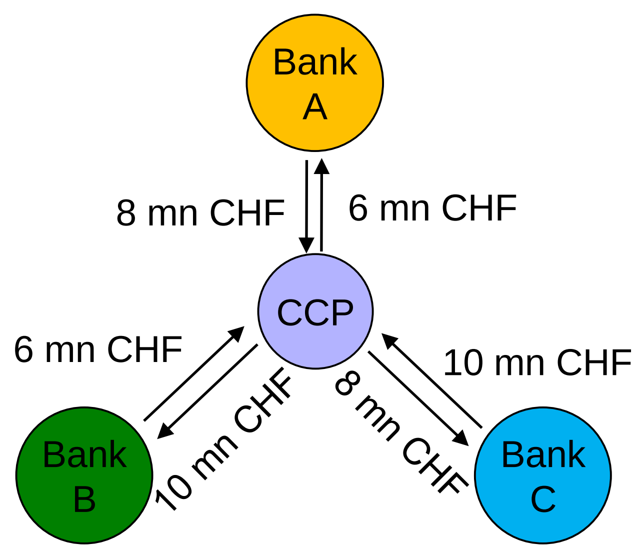
Objectives of Central Counterparties (CCP)

- Reduction of counterparty risk (no individual counterparty risk, margins)
- Improvement of market transparency: Mapping of market participants' exposures
- Efficiency increased due to better netting opportunities

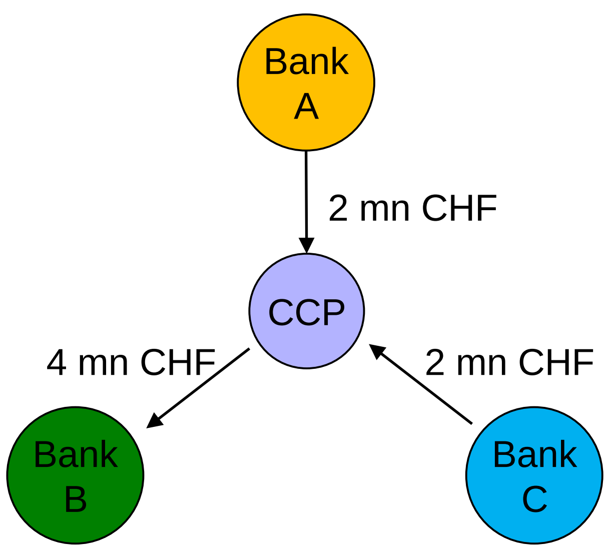
Non-cleared trades



Centrally cleared trades…



… allow exposure netting.



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Definition of Simple Returns

Definition:

A simple return is the percentage growth rate which leads an asset with starting value P_t to the terminal value P_T (incl. payout at the end of the period) with a **singular** payment of interest. That is, it shows what percentage gain (or loss) has been attained during the period in question in relation to the invested capital.

$$\textit{simple return} = \frac{\textit{change in value of the period}}{\textit{value at the start of the period}}$$

$$R_s = \frac{(P_T - P_t) + D_T}{P_t}$$

with R_s = simple return; P_T = price of the asset at time T; P_t = price of the asset at time t; D_T = dividend at time T.

The simple return is also called holding period return (HPR).

Different Time Horizons of Simple Returns

Depending on the situation, there are **different time horizons** for which returns are computed:

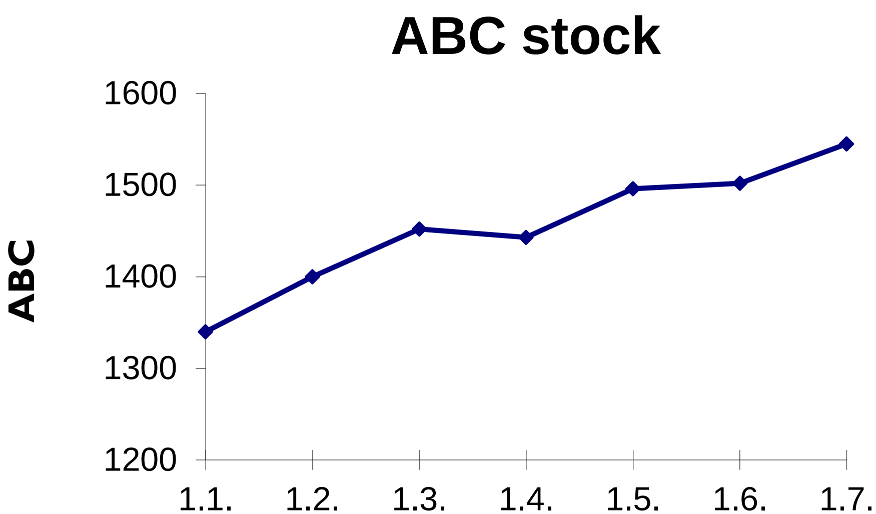
- Broker: minutely, hourly, daily
- Portfolio manager: daily, weekly, monthly
- Investor: up to several years

In order to be able to make comparisons, returns are **standardized:**

- as "simple, annualized returns" (without compound interest)
- as "effective, annualized returns" (with compound interest)

The investment horizon is **normalized to a year** and the period's return *R* computed accordingly.

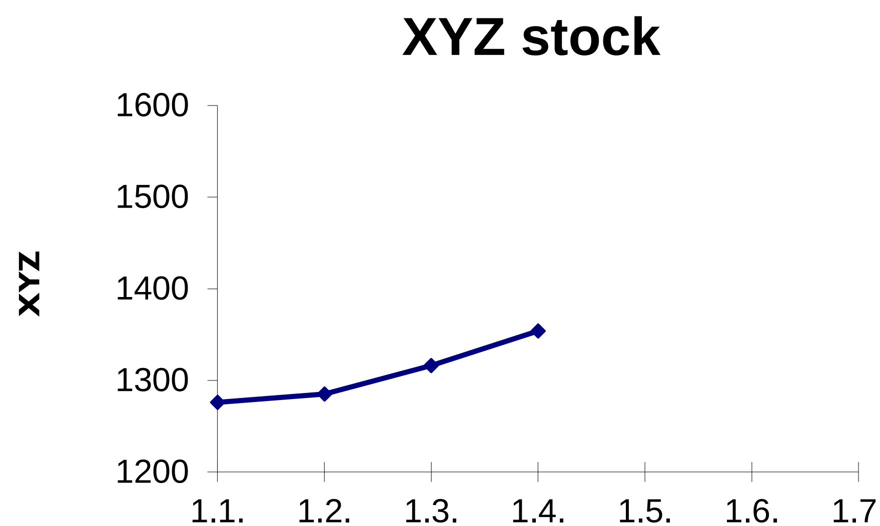
Example



Period (T-t): 6 months

$$R_s = \frac{(P_T - P_t) + D_T}{P_t}$$

$$= \frac{1545 - 1340}{1340} = 0.153 = 15.3\%$$



Period (T-t): 3 months

$$R_s = \frac{(P_T - P_t) + D_T}{P_t}$$

$$= \frac{1354 - 1276}{1276} = 0.061 = 6.1\%$$

(with $D_T = 0$)

Discrete Return: Annualization without Compounding

Which stock (portfolio) has the higher return over the time period considered?

→ Returns are standardized to make comparisons:

annualized return = return of period x periods per year

$$R = R_S \times n$$

ABC Stock	**XYZ Stock**
15.3% × 2 = 30.6%	6.1% × 4 = 24.4%

One refers to the simple annualized return *R* (also called annual percentage rate (APR)) and indicates the periods *n* ("semi-, quarter, etc. -annually ") with *R* = annualized return; R_s = return of the period; *n* = periods per year = 1 / period length [years].

Discrete Return: Annualization with Compounding

$$R_e = (1 + R_s)^n - 1$$ **effective, annualized return (including compound interest)**

ABC Stock

$(1+0.153)^2 - 1 = 32.9\%$

XYZ Stock

$(1+0.061)^4 - 1 = 26.7\%$

One refers to the effective annualized return R_e, whereas again R_s = simple return; n = number of periods. R_e is also called the effective annual rate (EAR).

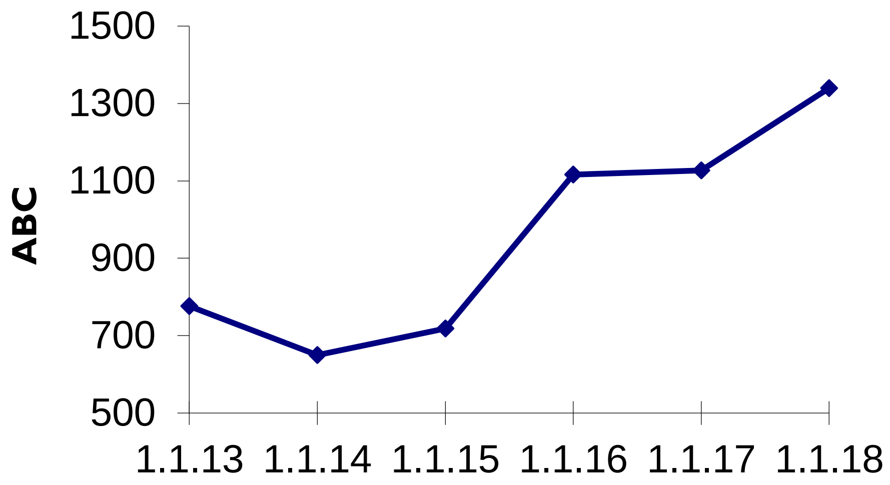
The Average Discrete Return of a Period

If an asset is considered over several periods, it is useful to compute the **average return** of **one** period.

Given:	n simple returns of i periods of the same length, or one return over the whole period
Required:	average return of one period
Computation:	as a geometric (**not** arithmetic) average

Discrete Returns: Geometric Average

		R_{Total}	$R_{Periods}$
1/1/13	776	↓	-
1/1/14	650		-16.2%
1/1/15	718		10.5%
1/1/16	1116		55.4%
1/1/17	1127		1.0%
1/1/18	1340	72.7%	18.9%



$$\bar{R} = \sqrt[5]{1.727} - 1 = 0.115 = 11.5\%$$

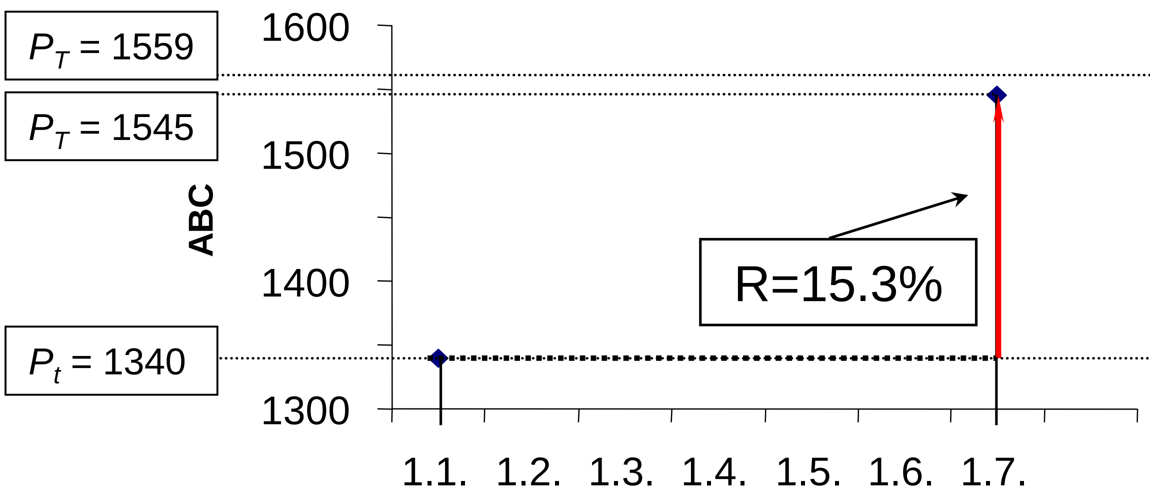
or

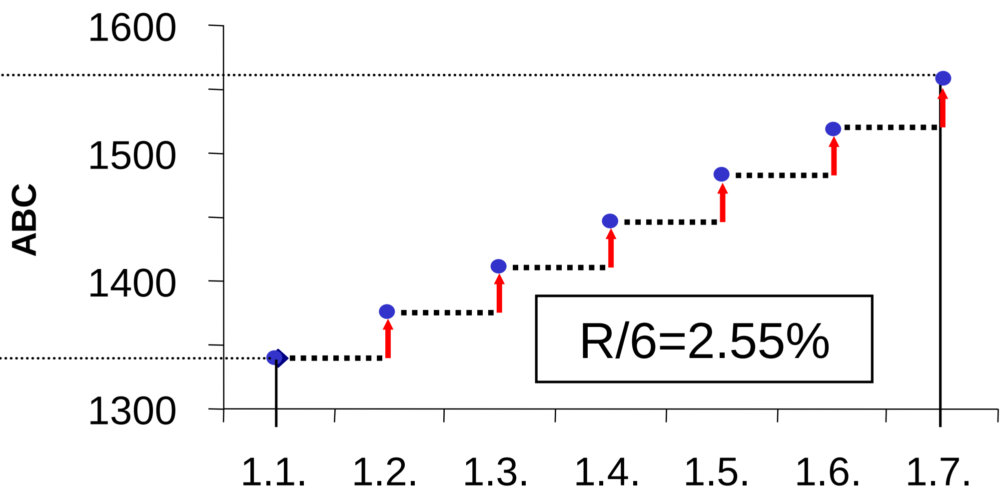
$$\bar{R} = \sqrt[5]{0.838 \times 1.105 \times 1.554 \times 1.01 \times 1.189} - 1 = 0.115 = 11.5\%$$

Transition to Continuously Compounded Returns

Simple return: **singular** interest (no compound interest)

Discrete return: **multiple** interest (incl. **compound interest**)





$$P_T = P_t \times (1 + R_s)$$

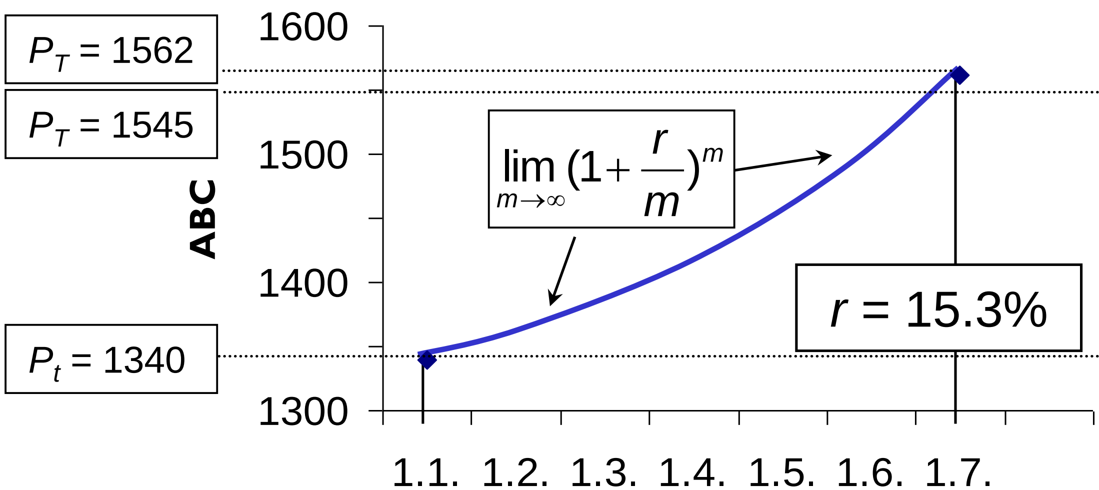
$$\underline{1545} = 1340 \cdot (1 + 0.153)$$

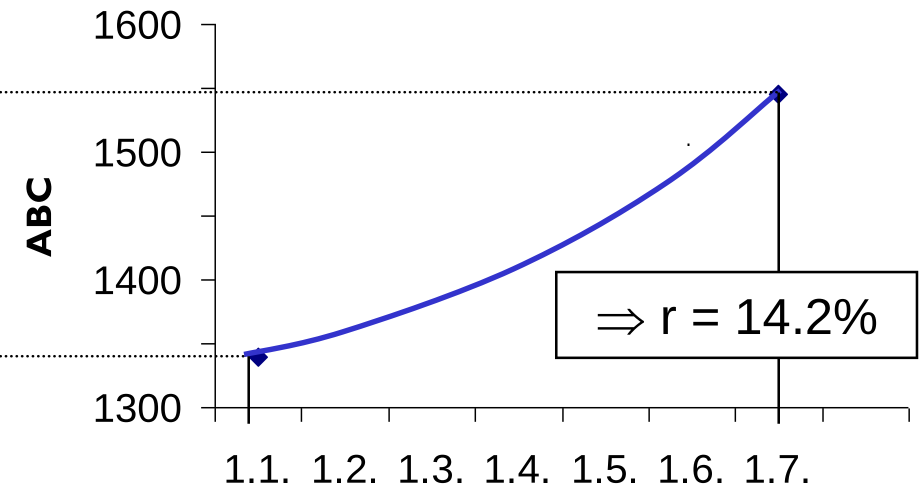
$$P_T = P_t \times \left(1 + \frac{R_s}{6}\right)^6$$

$$\underline{1559} = 1340 \cdot (1 + 0.0255)^6$$

Derivation of Continuously Compounded Returns

Continuously compounded return = **continuous** interest





$$P_T = \lim_{m\to\infty} P_t \times \left(1+\frac{r}{m}\right)^m \quad \Rightarrow \quad P_T = P_t \times e^r \longrightarrow P_T/P_t = e^r \quad \Leftrightarrow \quad r = \ln\left(P_T/P_t\right)$$

$$1562 = 1340 \cdot e^{0.153} \qquad 14.2\% = \ln\left(1545/1340\right)$$

$$\boxed{R = e^r - 1 \quad \Leftrightarrow \quad r = \ln(1+R)} \longleftarrow R = \frac{P_T - P_t}{P_t} \Leftrightarrow \frac{P_T}{P_t} = 1+R$$

$$15.3\% = e^{0.142} - 1 \quad \Leftrightarrow \quad 14.2\% = \ln(1+0.153)$$

Definition of Continuous Returns

Definition:

The percentage growth rate which leads an investment with an initial value of P_t to the terminal value P_T with continuously paid interest (incl. payout at the end of the period)

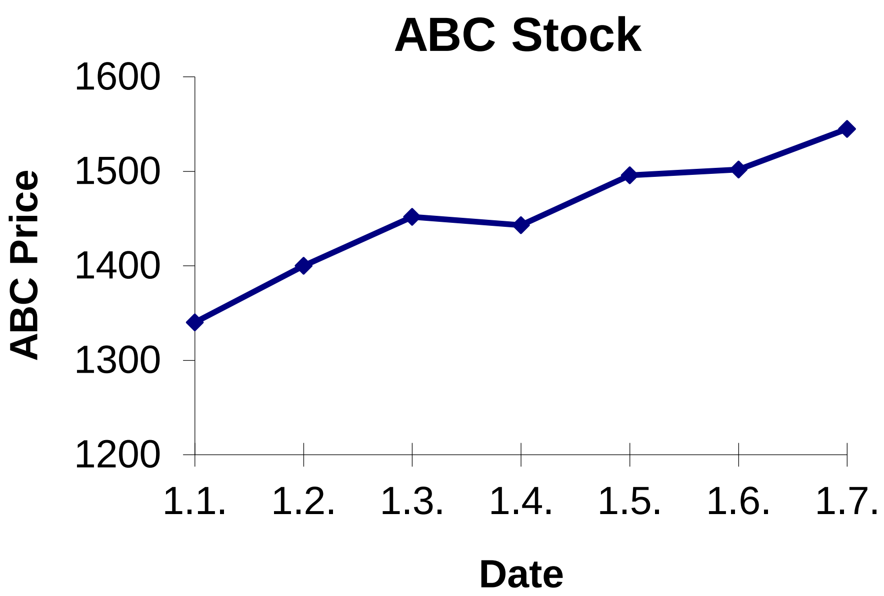
$$\textit{Continuous Return} = \ln\left[\frac{\textit{Value at the end of the period}}{\textit{Value at the beginning of the period}}\right]$$

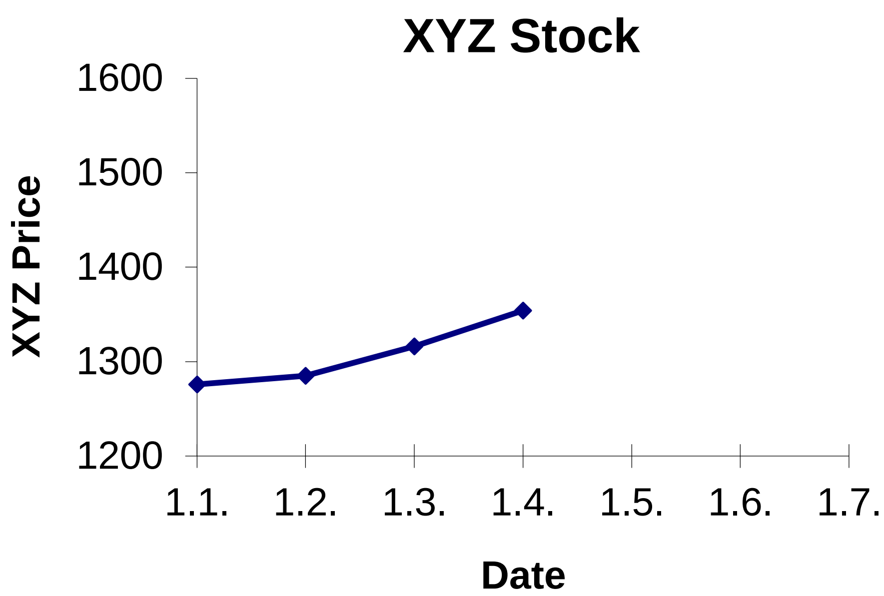
$$r_s = \ln\left(\frac{P_T + D_T}{P_t}\right)$$
$$= \ln(P_T + D_T) - \ln(P_t)$$

r_s = Continuous Return
P_T = Price of the asset at time T
P_t = Price of the asset at time t
D_T = Dividend at time T

Continuous Return over Different Time Horizons

Different time horizons for continuous returns





Period (T-t): 6 months

$$r_s = \ln\left(P_T + D_T\right) - \ln\left(P_t\right)$$

$$= \ln(1545) - \ln(1340) = 0.142 = \underline{14.2\%}$$

Period (T-t): 3 months

$$r_s = \ln\left(P_T + D_T\right) - \ln\left(P_t\right)$$

$$= \ln(1354) - \ln(1276) = 0.059 = \underline{5.9\%}$$

Annualization of Continuous Returns

Continuous returns are also standardized to allow for comparisons:

annualized return = return of the period x periods per year

$$r = r_s \times n$$

ABC Stock	**XYZ Stock**
14.2% × 2 = 28.4%	5.9% × 4 = 23.6%

Annualized continuous return r ,whereas r_s = continuous return of the period;
n = periods per year = 1 / period length [years].

Important: Since the continuous return is defined based on continuous interest payments, the simple annualization is always effective.

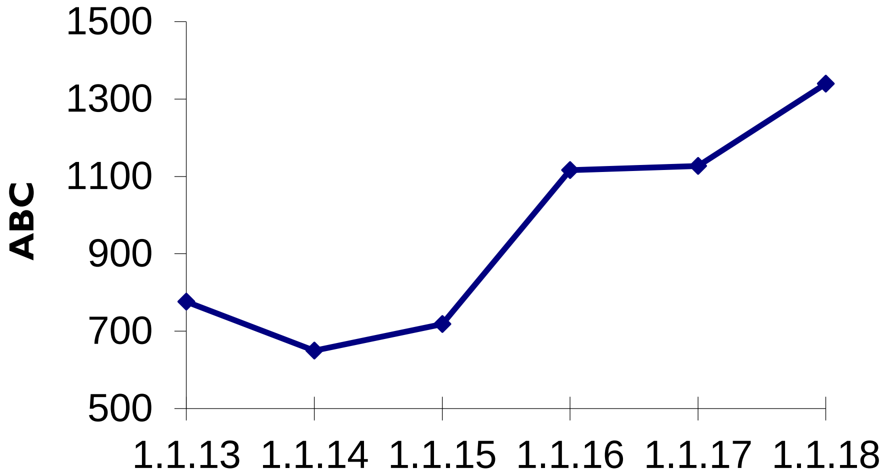
Average of Continuous Returns

If an asset is considered over several periods, it is useful to compute the average continuous return of one period.

Given:	*n* continuous returns of i periods of the same length, or one return over the whole period
Required:	average continuous return of a period
Computation:	as an arithmetic, thus simple average

Continuous Return: Arithmetic Average

		r_{Total}	$r_{Periods}$
1/1/13	776	↓	-
1/1/14	650		-17.7%
1/1/15	718		9.9%
1/1/16	1116		44.1%
1/1/17	1127		1.0%
1/1/18	1340	54.6%	17.3%



$$\bar{r} = \frac{0.546}{5} = 0.109 = 10.9\%$$ resp.

$$\bar{r} = \frac{-0.177+0.099+0.441+0.01+0.173}{5} = \frac{0.546}{5} = 0.109 = 10.9\%$$

Advantages of Continuous Returns

Continuous returns have the desirable property that

the conversion to different time horizons is easy:

$$r_{Total} = r_{Period} \times n$$

r_{Total} = Continuous return of the total period
r_{Period} = Continuous return of one period
n = Periods in total

- Annualized simply = annualized effectively
- Normal distribution as approximation
- Continuous returns are widely used for modeling purposes in practice

Average Interest in the Cross Section (1/2)

Discrete Compounding

$$E = w_1(1+R_1) + w_2(1+R_2) + ... + w_n(1+R_n)$$

E= terminal capital

Average return as a linear combination of single returns (weighted arithmetic mean)

$$\varnothing\text{-return} = \frac{E}{A} - 1 = \frac{w_1(1+R_1) + w_2(1+R_2) + ... + w_n(1+R_n)}{w_1 + w_2 + ... + w_n} - 1$$

$$= \frac{w_1}{A}R_1 + \frac{w_2}{A}R_2 + ... + \frac{w_n}{A}R_n$$

$w_1, w_2, ..., w_n$	portfolio values of each asset
$R_1, R_2, ..., R_n$	returns of the assets
$w_1 + w_2 + ... + w_n$	= A (initial capital)
E	= terminal capital

Average Interest in the Cross Section (2/2)

Continuous Compounding

$$E = w_1 \times e^{r_1} + w_2 \times e^{r_2} + \ldots + w_n \times e^{r_n}$$

Average return is *not* a linear combination of the continuous single returns

$$\varnothing\text{-return} = \ln\left(\frac{E}{A}\right) = \ln \frac{w_1 e^{r_1} + \ldots + w_n e^{r_n}}{w_1 + \ldots + w_n} \neq \frac{w_1}{A} r_1 + \frac{w_2}{A} r_2 + \ldots + \frac{w_n}{A} r_n$$

Why?

e^x and $\ln(x)$ are non-linear functions. The results follow directly from Jensen's inequality:

$$w_1 e^{r_1} + w_2 e^{r_2} + \ldots + w_n e^{r_n} \quad > \quad e^{w_1 r_1 + w_2 r_2 + \ldots + w_n r_n}$$

$$\underbrace{w_1 \ln E_1}_{r_1} + \underbrace{w_2 \ln E_2}_{r_2} + \ldots + \underbrace{w_n \ln E_n}_{r_n} \quad < \quad \ln(w_1 E_1 + w_2 E_2 + \ldots + w_n E_n)$$

Summary: Discrete vs. Continuous Return

	Time Series Average (Time)	**Cross-Section Average (Portfolio)**
Discrete returns	Geometric average	Arithmetic average (weighted)
Continuous returns	Arithmetic average	---

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Average return of portfolios with cashflows

Portfolio consisting of ABC stock:

Portfolio PF^1 (PF^2) with cashflows:
01.01.15: Outflow of -3000 (Inflow of 3000)
01.01.16: Inflow of 3000 (Outflow of -3000)

Date	ABC		$PF^{\text{no CF}}$
01.01.13	776		10000
		$R_0 = -16\ \%$	
01.01.14	650		8376
		$R_1 = 10\ \%$	
01.01.15	718		9253
		$R_2 = 55.4\%$	
01.01.16	1116		14381
		$R_3 = 1\%$	
01.01.17	1127		14523
		$R_4 = 19\%$	
01.01.18	1340		17268

$PF^1_{\text{pre-CF}}$	+	CFs	=	$PF^1_{\text{post-CF}}$
$K_0 = 10000$				10000
8376		$C_1 = 0$		8376
9253		$C_2 = -3000$		6253
9718		$C_3 = 3000$		12718
12844		$C_4 = 0$		12844
15271				$K_T = 15271$

How high is the average portfolio return?

$$\sqrt[5]{1+\frac{17268-10000}{10000}}-1 = 11.5\%$$

$$\sqrt[5]{1+\frac{15271-10000}{10000}}-1 \cancel{= 8.8\%}$$

Returns with Cash Flows – Time-Weighted Returns

Time-weighted returns of PF^1 und PF^2

The time-weighted return is adjusted by the cash flows and mirrors the average return earned on the changing amount of wealth.

$PF^1_{pre\text{-}CF}$		+ CFs	$= PF^1_{post\text{-}CF}$
$K_0 = 10000$			10000
	R_0		
8376		0	8376
	R_1		
9253		-3000	6253
	R_2		
9718		+3000	12718
	R_3		
12844		0	12844
	R_4		
15271			$K_T = 15271$

$PF^2_{pre\text{-}CF}$		+ CFs	$= PF^2_{post\text{-}CF}$
$K_0 = 10000$			10000
	R_0		
8376		0	8376
	R_1		
9253		+3000	12253
	R_2		
19044		-3000	16044
	R_3		
16203		0	16203
	R_4		
19265			$K_T = 19265$

$R_0 = -16\%$ $\quad R_1 = +10\%$ $\quad R_2 = \frac{9718-6253}{6253} = \frac{19044-12253}{12253} = 55.4\%$ $\quad R_3 = +1\%$ $\quad R_4 = +19\%$

$$\overline{R}_{ZR} = \sqrt[5]{(1-0.16)\cdot(1+0.1)\cdot(1+0.55)\cdot(1+0.01)\cdot(1+0.19)} - 1 = \underline{11.5\%}$$

Returns with Cash Flows – Money-Weighted Returns (1/2)

Money-weighted return of PF[1]

The initial wealth and all cash flows are compounded with the (to-be-determined) money-weighted rate of return so that the sum of these cash flows is equal to the terminal portfolio value.

$$K_0 \times \left(1+\bar{R}_{MWR}\right)^{T} + CF_1 \times \left(1+\bar{R}_{MWR}\right)^{T-1} + CF_2 \times \left(1+\bar{R}_{MWR}\right)^{T-2} + \ldots + CF_4 \times \left(1+\bar{R}_{MWR}\right)^{T-4} = K_T$$

Year	$PF^1_{\text{pre- CF}}$	CFs	$PF^1_{\text{post-CF}}$	FV at 9.3%
'13	$K_0 = 10000$			15605
'14		$C_1 = 0$		0
'15		$C_2 = -3000$		-3918
'16		$C_3 = 3000$		3584
'17		$C_4 = 0$		0
'18			$K_5 = 15271$	$\Sigma = 15271$

=

$$10000 \times (1+0.093)^5 - 3000 \times (1+0.093)^3 + 3000 \times (1+0.093)^2 = 15605 - 3918 + 3584 = 15271$$

Returns with Cash Flows – Money-Weighted Returns (2/2)

Money-weighted return of PF^2

With a different timing of the cash flows, another return results!

Example: Portfolio PF^2 with the following cash flows:

1/01/15: inflow of +3000; 1/01/16: outflow of -3000

Year	$PF^2_{\text{pre-CF}}$	CFs	$PF^2_{\text{post-CF}}$	PF^2: FV at 13.4%		PF^1: FV at 9.3%
'13	$K_0 = 10000$			18748		15605
'14		$C_1 = 0$		0	⟷	0
'15		$C_2 = 3000$		4374		-3918
'16		$C_3 = -3000$		-3857		3584
'17		$C_4 = 0$		0		0
'18			$K_5 = 19265$	$\Sigma = 19265$		$\Sigma = 15271$

$K_5 = \Sigma$ =

The return of PF^2 is higher than that of PF^1 because the cash flows are timed better.

Return Computation with Cash Flows

	Money-weighted rate of return (MWR)	Time-weighted rate of return (TWR)
Computation	Equals the **internal rate of return** of the cash flow stream	Equals the **geometric average** of the single period returns
Implication	**Dependent** on the timing of cash in- and outflows	**Independent** of the timing of cash in- and outflows
Use	Portfolio manager is (partly) **responsible** for the timing of the cash flows	Portfolio manager is **not responsible** for the timing of the cash flows

Agenda

- Financial Markets
- Returns
- Returns with Cash Flows
- **Return Distribution and Risk**
- Risk Measures

Quantification of Uncertainty

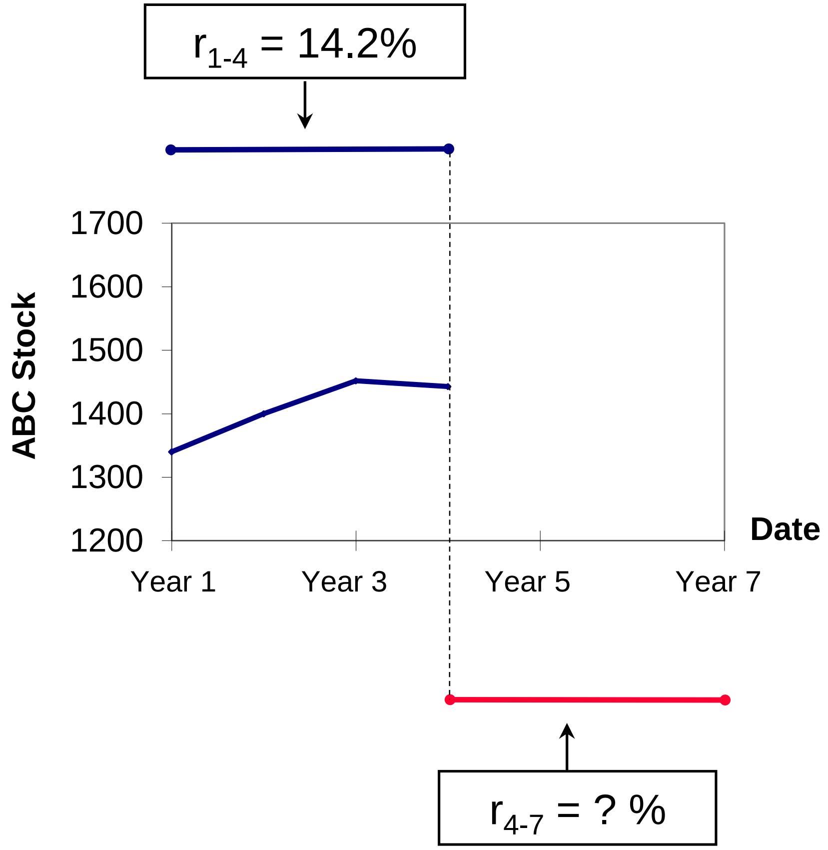
Up to now: calculation of returns on an ex-post basis.

Problem: often, we are interested in **future returns**. However, future returns are usually uncertain.

Approach:

1. Quantification of uncertainty
2. Quantified uncertainty (**risk**) is used for the investment decision

→Estimation of the **distribution of future returns** is necessary



Historical Distribution of Returns

Histogram of continuously compounded Swiss stock returns from 1926 to 2019

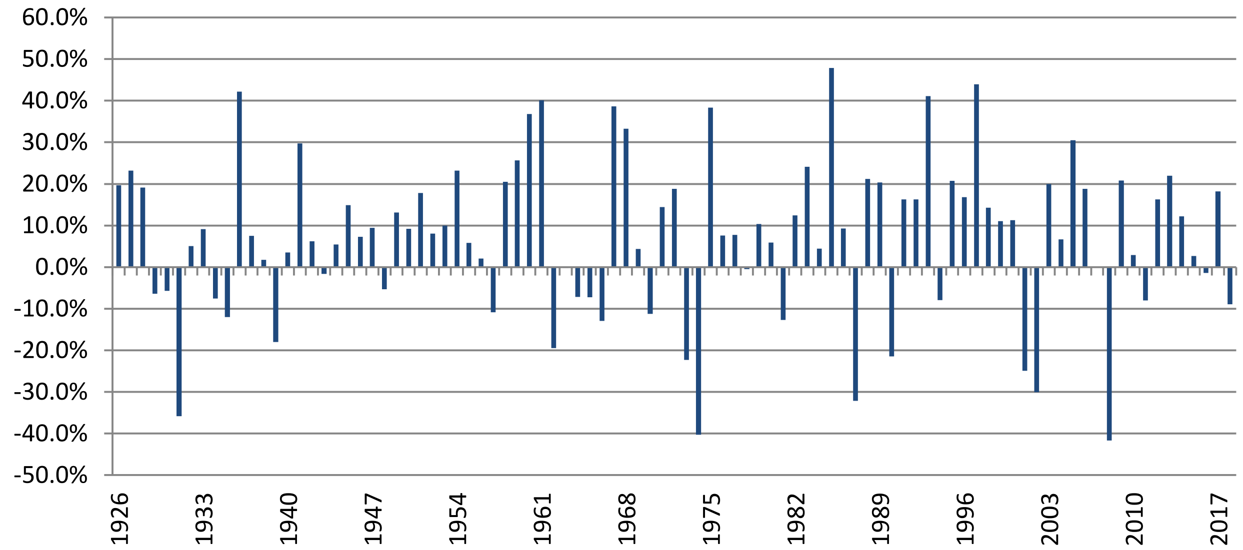
-50% bis -40%	-40% bis -30%	-30% bis -20%	-20% bis -10%	-10% bis 0%	0% bis 10%	10% bis 20%	20% bis 30%	30% bis 40%	40% bis 50%
					2015				
					2010				
					2004				
					1986	2017			
					1984	2014			
					1980	2012			
					1977	2006			
					1976	2003			
					1969	2000			
				2018	1956	1999			
				2016	1955	1998			
				2011	1953	1996			
				2007	1952	1992	2013		
				1994	1950	1991	2009		
				1978	1947	1982	1995	30.94%	
				1965	1946	1979	1989		
			1981	1964	1944	1972	1988		
			1970	1963	1942	1971	1983	**2019**	
			1966	1948	1940	1951	1959	2005	1997
			1962	1943	1938	1949	1958	1975	1993
	2002	2001	1957	1934	1937	1945	1954	1968	1985
2008	1987	1990	1939	1930	1933	1928	1941	1967	1961
1974	1931	1973	1935	1929	1932	1926	1927	1960	1936

Source: Pictet: The Performance Update 2020

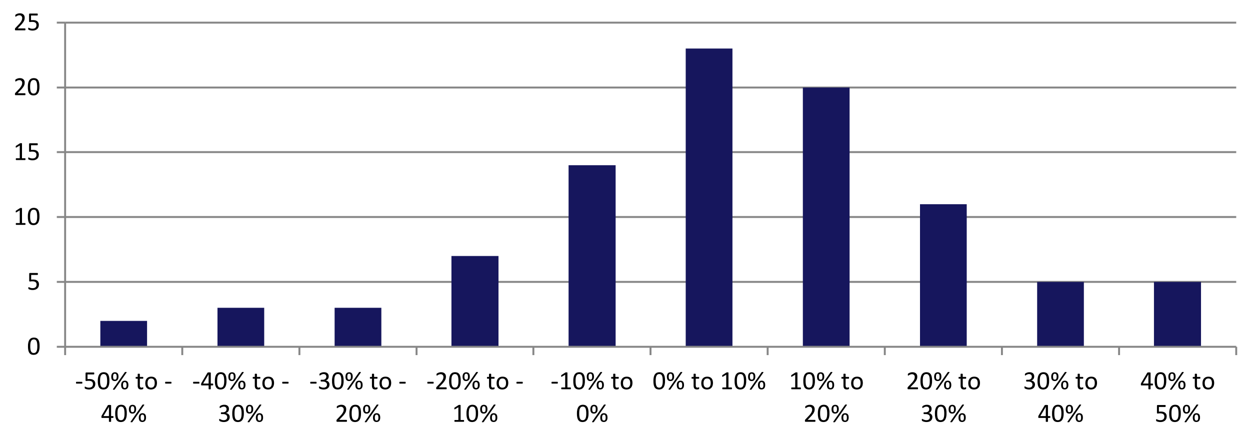
Distribution: Returns and their Frequency

Pictet Stock Index

Continuous annual returns from 1926-2018



Frequency distribution of returns



Source: Pictet: The Performance Update 2019

Reasons for Application of Normal Distribution

Empirical reasons:

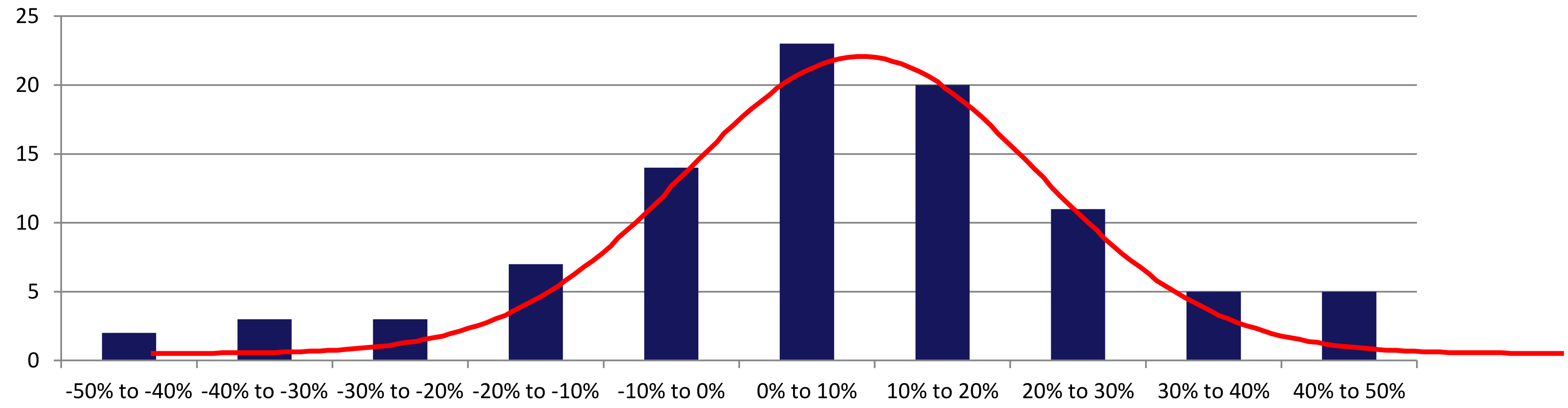
– Assumption of normal distribution often proves as good approximation

Theoretical reasons:

– Market efficiency, Random Walk, Central Limit Theorem

Practical reasons:

– Simplicity: complete characterization with first and second moment of distribution, that is mean and standard deviation



Alternatives to Normal Distribution

Stylized facts of asset returns

- Heavy tails
- Gain/loss asymmetry
- Leverage effect
- Volatility clustering

Problem: these facts are not adequately captured by the normal distribution

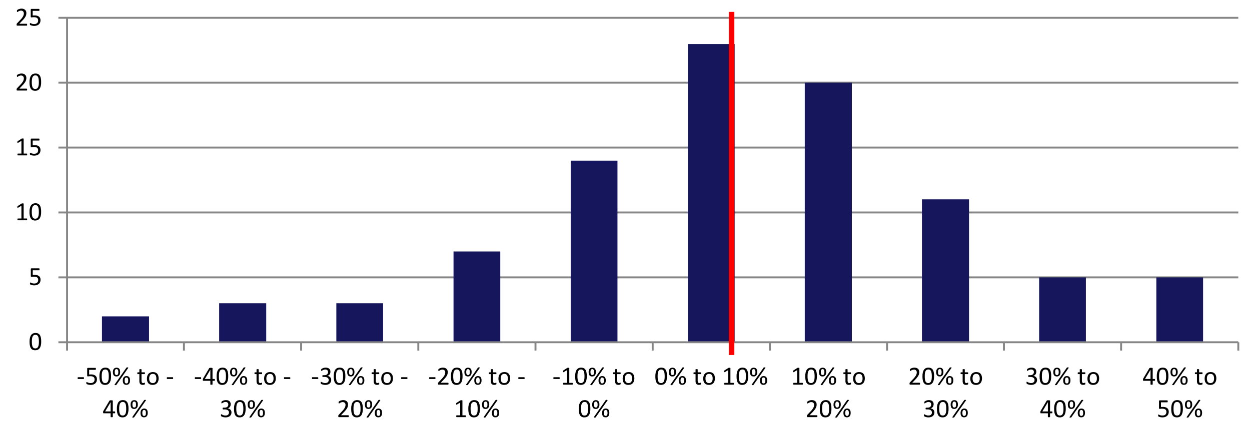
→ **Alternative distributions**

- Student's t
- Normal-inverse Gaussian
- Generalized hyperbolic, e.g. Variance-Gamma

The Mean as Estimator for the Expected Return

The average continuous return is calculated as the arithmetic average of the corresponding subperiod returns

	Year	P_t	$\ln(P_t)$	$\ln(P_t)-\ln(P_{t-1}) = r_t$	
	1925	100	4.6052		
n = 93	1926	121.69	4.8015	0.1963	19.63%
	1927	153.45	5.0334	0.2319	23.19%
	...	...	...	...	...
	2017	102913.45	11.5416	0.1817	18.17%
	2018	94093.76	11.4520	-0.0896	-8.96%



$\varnothing_{\text{arithm.}} = \mu = 7.36\%$

Statistical Expectation

(≠ Economic Expectation)

Source: Pictet: The Performance Update 2016

Variance and Standard Deviation as Risk Measures

The variance (σ^2) is the arithmetic average of the squared deviations of the returns from their mean. The standard deviation (σ) is the square root of the variance.

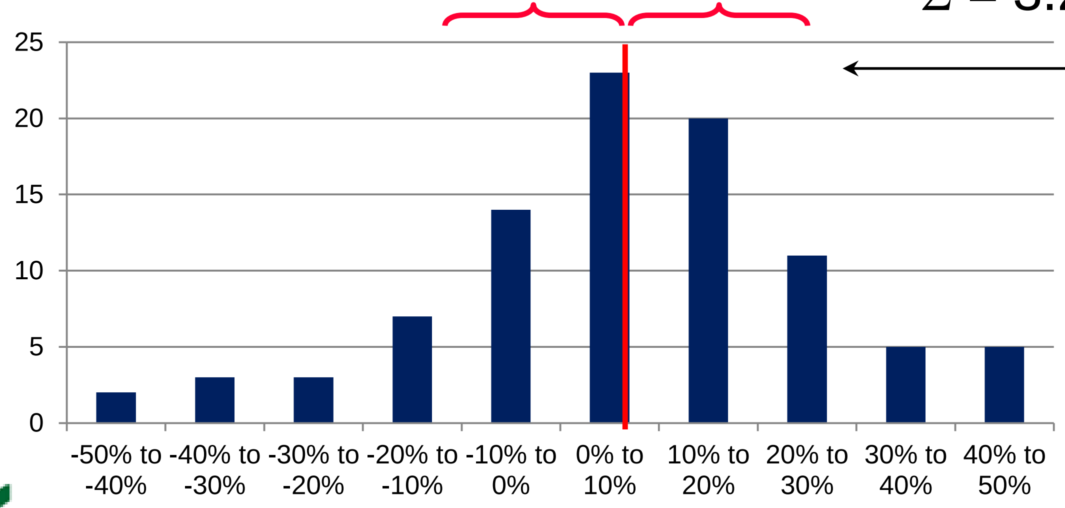
Year	P_t	r_t	$r_t - \mu$	$(r_t - \mu)^2$
1925	100			
1926	121.69	0.1963	0.1227	0.0151
1927	153.45	0.2319	0.1583	0.0251
...	...	...	...	...
2017	102913.45	0.1817	0.1080	0.0117
2018	94093.76	-0.0896	-0.1632	0.0266
				$\Sigma = 3.2559$

Variance:

$$\sigma^2 = \frac{1}{(93-1)} \cdot \underline{3.2559}$$

$$= 0.03539$$

σ σ



Standard Deviation:

$$\boxed{\sigma = \sqrt{Variance}}$$

$$= 0.1881 = \underline{18.8\%}$$

Source: Pictet: The Performance Update 2016

Variance and Standard Deviation Estimation

Simple approach assumes that volatility is constant → In practice it is not

Alternative Approaches:

- <u>**G**</u>eneralized <u>**A**</u>uto<u>**r**</u>egressive <u>**C**</u>onditional <u>**H**</u>eteroskedasticity (**GARCH**) model

    - Autoregressive: Tomorrow's volatility is a regressed function of today's variance
    - Conditional: Tomorrow's variance depends on the most recent variance
    - Heteroskedastic: Variances are not constant, but vary over time

- GARCH regresses on historical (lagged) terms: The generic GARCH (p, q) model regresses on (p) squared returns and (q) variances → The typical GARCH (1, 1) approximation "lags" on last period's squared return and last period's variance

$$\sigma_t^2 = \gamma V_L + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2$$

Weighted long run variance

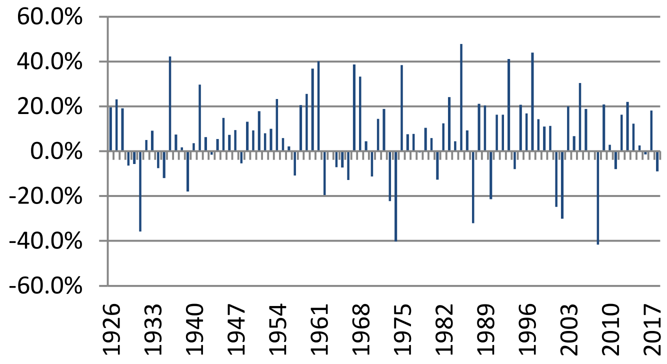
Weighted return^2 (one lag)

Weighted variance (one lag)

Distribution: Frequency vs. Probability

Frequency distribution

➡ Ex-post perspective



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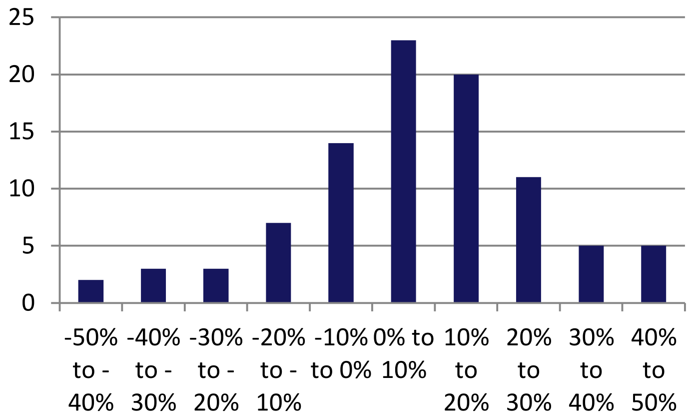
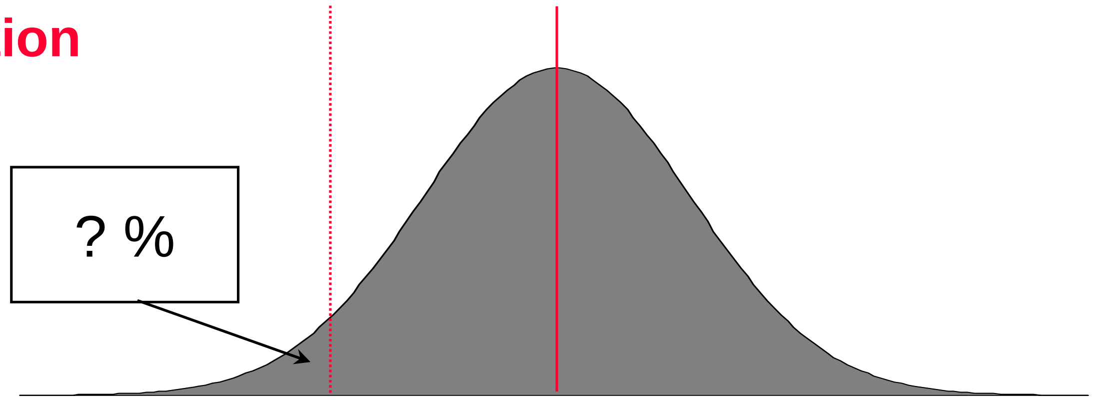


Probability distribution

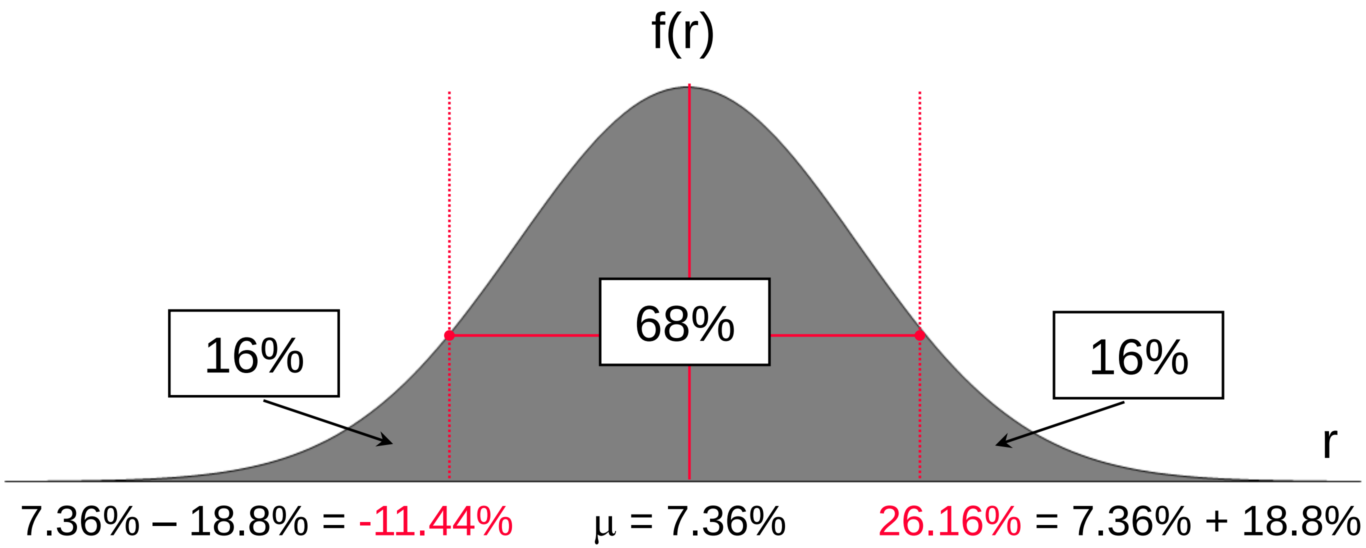
➡ Ex-ante perspective



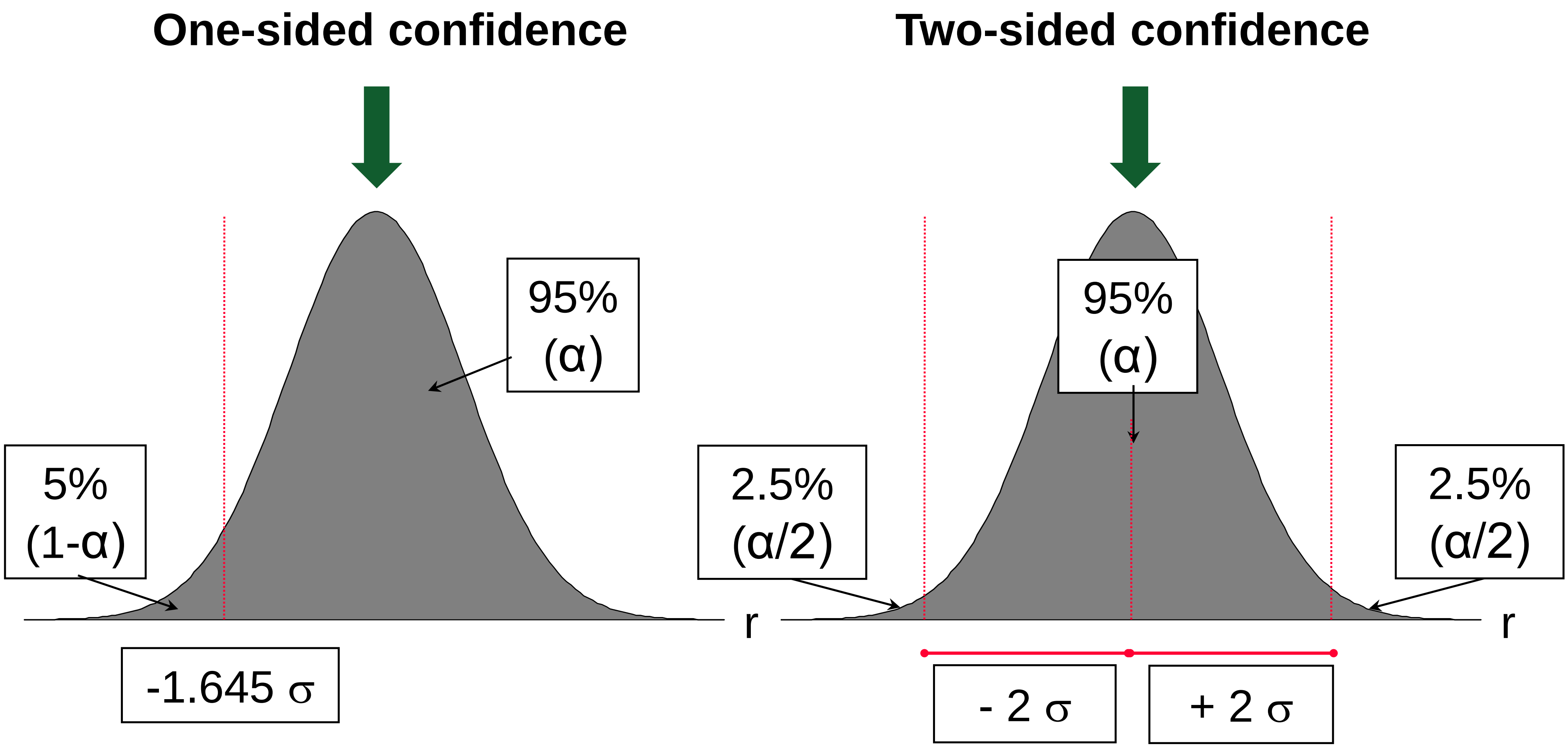
Assumed distribution for quantification of uncertainty

Normal Distribution: One-Sigma Confidence Interval

Theory:
68% of the mass of a normal distribution are within the boundaries of μ +/- 1σ, 32% are out of this area.



Normal Distribution: One- and Two-Sided Confidence



Applied at Value at Risk (see below)

The Economic Meaning of Risk

- Investors try to make **optimal investment** decisions. They optimize their future utility from their investment.

- In this context, they are interested in two aspects:
  - the **expected return**
  - the **expected risk** that results from the investment (within the μ–σ–world measured by the **standard deviation**)

- If investors base their investment decision exclusively on these two factors, they optimize in a μ–σ–world: Optimization between expected return (μ) and risk (σ).

Reward to Risk: Sharpe Ratio

- Measures the trade-off between reward (return in excess of risk-free rate) and risk (measured by standard deviation)

- Sharpe Ratio = $\dfrac{\text{Excess Return}}{\text{Standard Deviation of Excess Return}}$

- The **excess return** is also referred to as the **risk premium**.

Agenda

- Financial Markets
- Returns
- Returns with Cash Flows
- Return Distribution and Risk
- **Risk Measures**

Value at Risk: Definition

Definition of Value at Risk

- Highest possible loss that can occur over a certain time period at a certain confidence level.

Example

- The one-day VaR of the trading book of bank X is 2 million Euro at a confidence level of 99%.

Interpretation

- Bank X assumes that the maximum daily loss is 2 million Euro with a probability of 99% (price distribution, resp. value distribution of the portfolio is assumed).
- On average, the daily loss is bigger than 2 million Euro on 1 out of 100 days.

Computation

- VaR is a quantile-based risk measure
- Computation of quantile requires distribution
- Distribution can be estimated in different ways (Variance-covariance method (basic formula), simple historical VaR, etc.)

Why Value at Risk?

- Aggregation makes it possible to describe overall risk in one number
- The simplicity of the measure makes it suitable to communicate incurred risks to
  - Management
  - Shareholders
  - Supervisory bodies
  - Etc.
- Ratio for capital adequacy for banks:
  - VaR is the basis for the calculation of capital adequacy to cover market risks
  - Capital adequacy for market risks is added to capital adequacy for credit risks
  - Aggregation makes it possible to describe overall risk in one number
- As a quantile measure, VaR has limitations

The Basic Formula for Value at Risk

Value at Risk Basic Formula: $\text{VaR} = \mu\, t - z_{\alpha}\, \sigma \sqrt{t}$

Elements

1. Expected return (μ): Measured as the average of returns
2. Confidence factor (z_{α}): Measures the distance from the mean in number of standard deviations, e.g., 1.65 corresponds to a 95%-confidence level (1-α)
3. Volatility (σ) of the portfolio: Measured as the standard deviation of returns
4. Time horizon (t): VaR grows less than proportionally to time

Assumptions Underlying the Basic Formula

Assumptions

The following assumptions are implied by the VaR model on the previous page:

- Returns are normally distributed
- Returns are serially uncorrelated
- Payoffs are linear
- The underlying is the only risk factor

Note that the VaR concept in general does not require these assumptions.

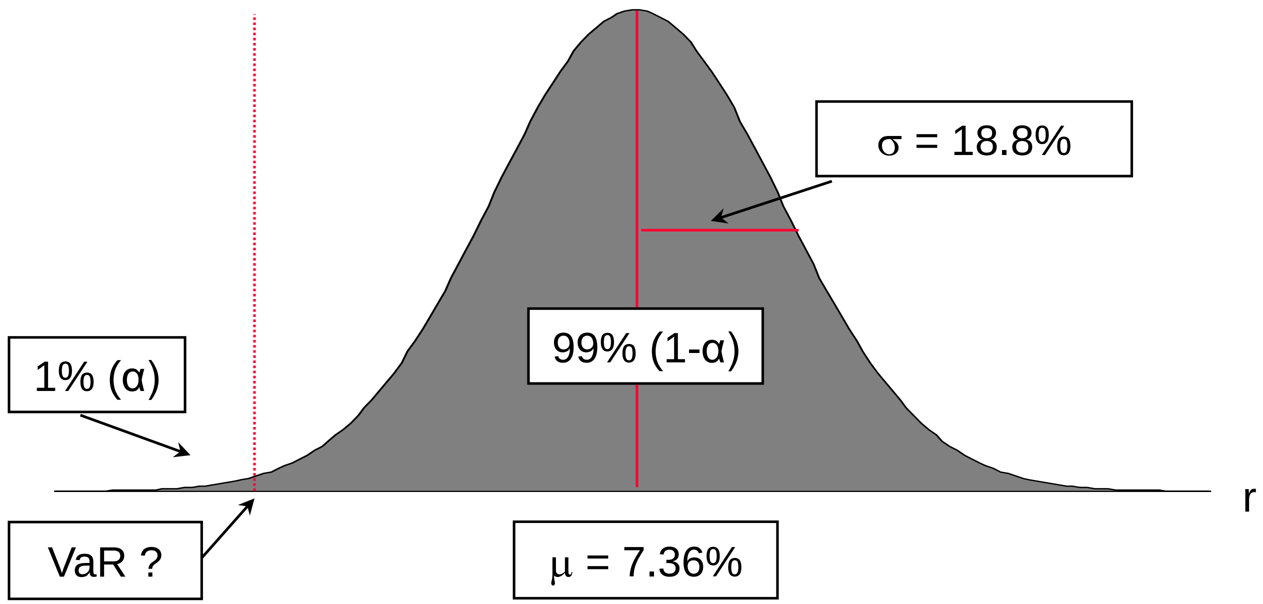
Basic Value at Risk: Example

Given

– A portfolio of 100’000 is invested in an Index with an expected return of 7.36% and a standard deviation (= volatility) of 18.8%.

Question

– Calculate the **Value at Risk** with a one-sided confidence level of 99% over a 1 year time horizon.



Simple Historical Value-at-Risk

Properties

- We do not need an assumption for the return distribution.
- Projects past distribution into future.

Example

- Stock position of USD 1 million.
- Confidence level = 90%
- Time horizon = 1 day
- Sorted Returns for two examples A & B:

	Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	Return (%)	**-2.4**	**-2.0**	-1.9	-0.8	-0.6	-0.4	-0.3	-0.3	-0.2	-0.2	-0.1	0.0	0.1	0.2	0.3	0.4	1.1	1.2	1.5	1.8
B	Return (%)	**-4.2**	**-2.0**	-1.9	-0.8	-0.6	-0.4	-0.3	-0.3	-0.2	-0.2	-0.1	0.0	0.1	0.2	0.3	0.4	1.1	1.2	1.5	1.8

lowest 10% of returns

→ VaR in example A = USD 20’000

→ VaR in example B = USD 20’000

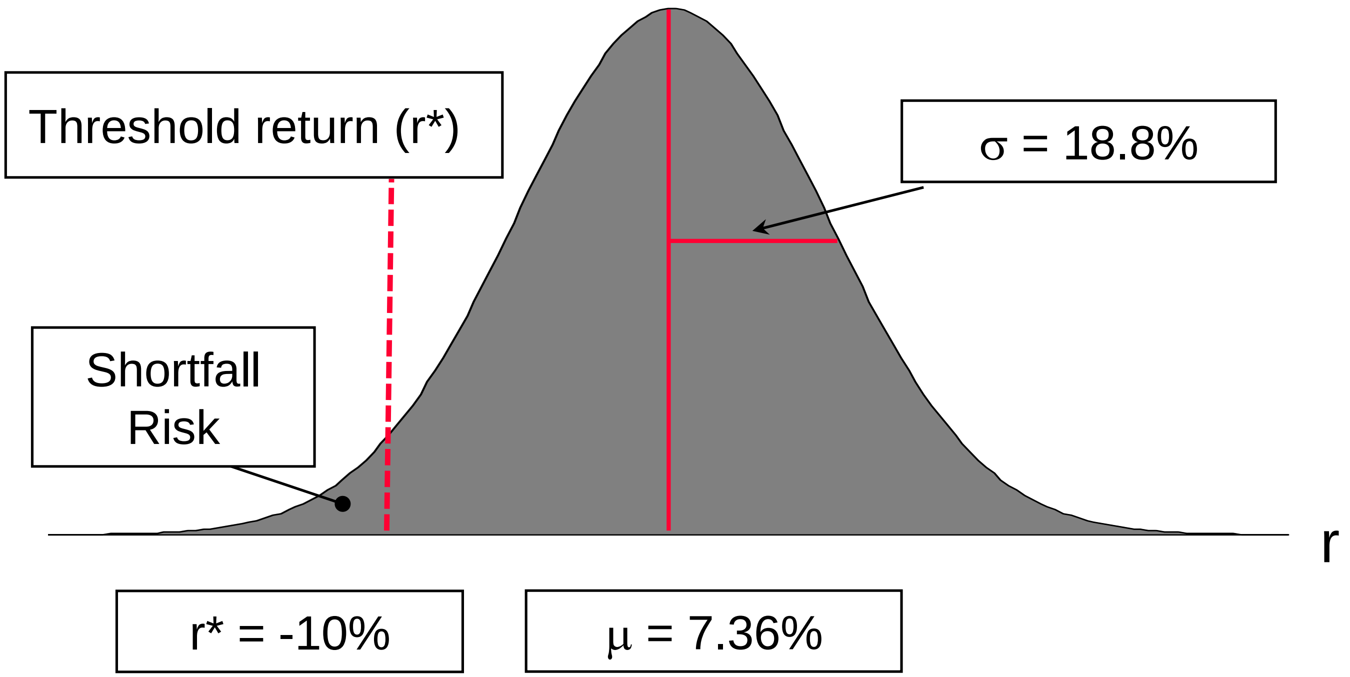
Shortfall Risk

Given

– Expected value and standard deviation have been estimated: μ = 7.36% and σ = 18.8%

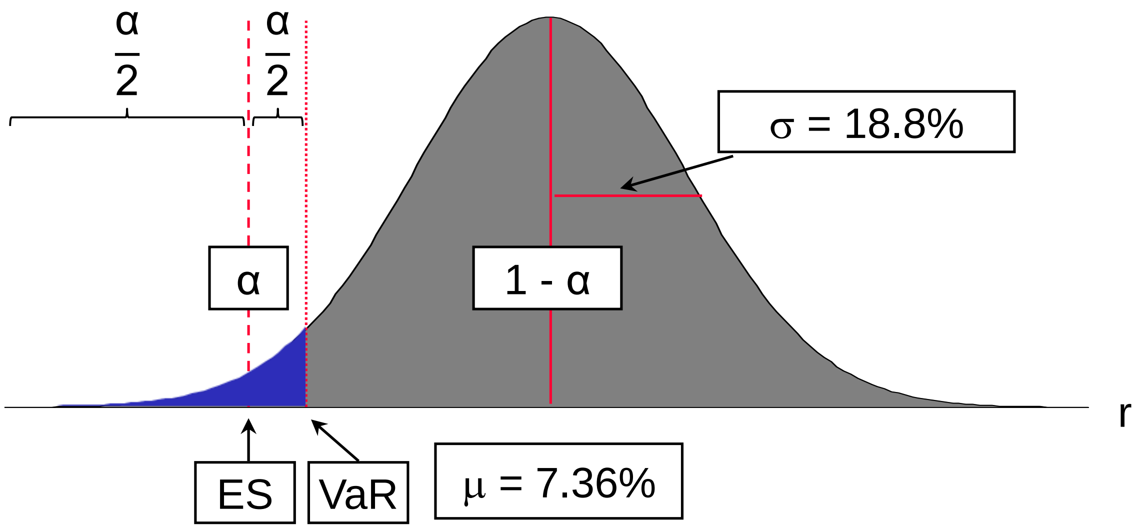
Question

– What is the probability of a **yearly loss greater than 10%**?



Expected Shortfall (ES)

- Also called conditional tail expectation (CTE)
- More conservative measure of downside risk than VaR
  - VaR takes the highest return from the worst cases: for 95% confidence interval, it is the return at the 95% quantile
  - ES takes an average return of the worst cases: for a 95% confidence interval, it is the average of the returns beyond the 95% quantile, i.e. returns in the lower 5% (α) probability mass
- Gained popularity in regulatory frameworks in the last years (e.g. Fundamental review of the trading book issued by BIS)



Lower Partial Standard Deviation (LPSD) and Sortino Ratio

Problems of “normal” standard deviation if returns are non-normal:

- Asymmetry of distribution → look at negative outcomes separately
- Alternative to risky portfolio = risk-free portfolio → need to consider deviations of returns from the risk-free rate

Lower Partial Standard Deviation (LPSD)

- Similar to usual standard deviation, but uses only negative deviations from r_f

Sortino Ratio

- Similar to Sharpe Ratio but uses LPSD in denominator