Swiss Institute of Banking and Finance

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3. Derivatives

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7,150 Financial Markets

Economic Rationale for Derivatives

1. Risk transfer and risk management

2. Cost

3. Standardization and liquidity

4. Price discovery and information

Agenda

Forwards

Swaps

Futures

Options

Forward (1/3)

- **A forward contract is an**
  - unconditional agreement
  - to buy or sell
  - a specific amount
  - of a certain asset
  - at a certain time in the future
  - for a negotiated price specified today (the delivery price)
- In contrast, a spot contract is an agreement to buy or sell immediately.
- **The transaction underlying a forward contract is defined today, but is executed in the future.**

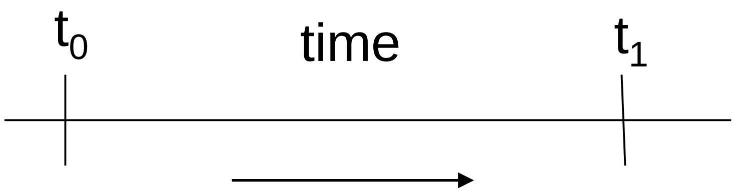
Forward (2/3)

Agreement to buy or sell	**Long/short**
a specific amount	**Contract size**
of a certain asset	**Underlying**
at a certain point in time	**Maturity**
for a negotiated price	**Delivery price**

Forward (3/3)

Long

- Commit at t_0
- Receive underlying at t_1
- Pay the negotiated price at t_1

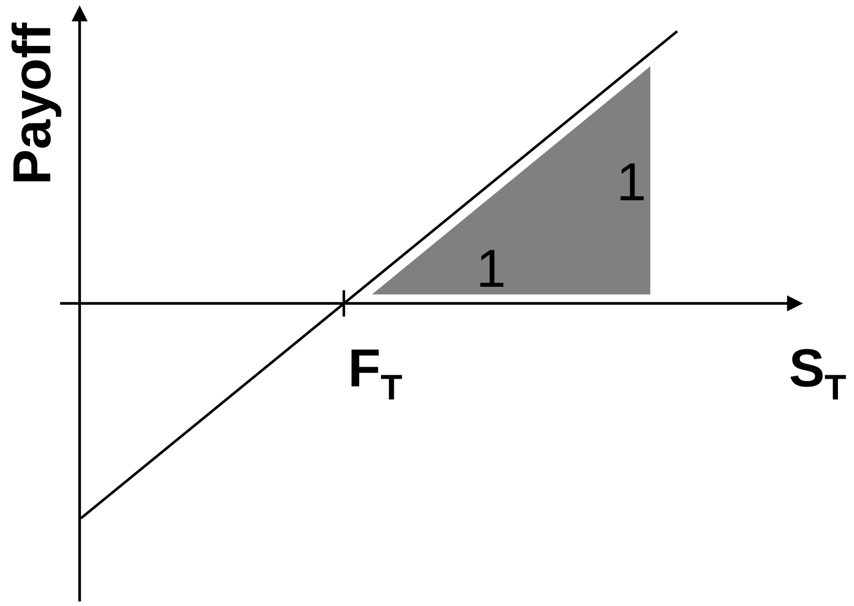


Short

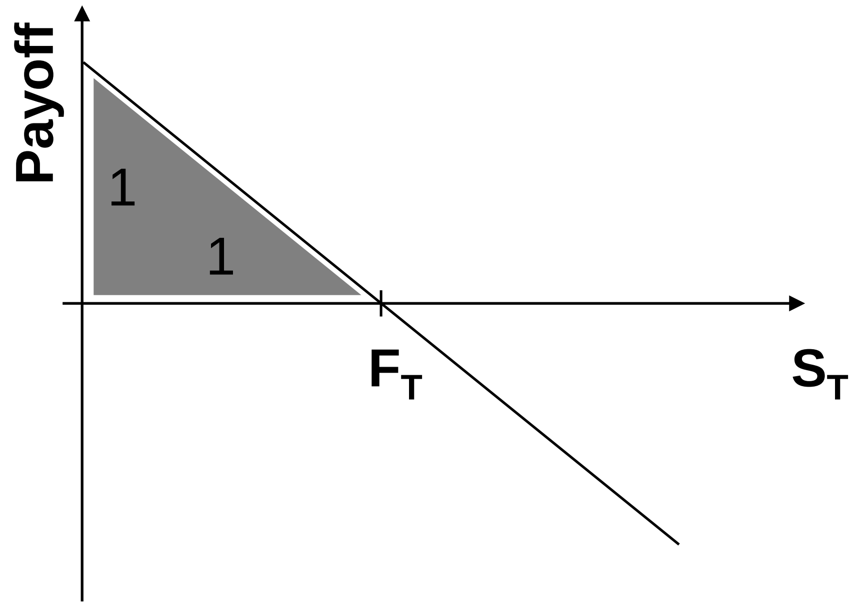
- Commit at t_0
- Deliver underlying at t_1
- Receive the negotiated price at t_1

Payoff of a Forward at Maturity

Long forward position



Short forward position



F_T = forward price
S_T = underlying

Forward Contracts: Hedging (1/2)

- **Hedging foreign currency risk with forward contracts**:

  On February 18th, the treasurer of a Swiss corporation knows that the corporation will have to pay USD 1m in 6 months (i.e., on August 18th) and wants to hedge against changes in the exchange rate.

  The quotes on the exchange rate between the Swiss Franc (CHF) and US Dollar (USD) as of February 18 are (illustrative figures):

Swiss Francs per dollar	*Bid*	*Ask*
Spot	0.9047	0.9050
1-month forward	0.9020	0.9025
3-month forward	0.8970	0.8976
6-month forward	0.8920	0.8924

  The treasurer can agree to buy USD 1m 6 months forward at an exchange rate of 0.8924. This forward contract specifies that the corporation will buy USD 1m for CHF 892,400 on August 18.

Forward Contracts: Hedging (2/2)

What are possible outcomes?

1) The spot exchange rate rises to **0.9500** at the end of the 6-month period:

The value of the forward contract to the company is:

950,000 CHF – 892,400 CHF = **57,600 CHF**

2) The spot exchange rate falls to **0.8500** at the end of the 6-month period:

The value of the forward contract to the company is:

850,000 CHF – 892,400 CHF = **-42,400 CHF**

⇒ Because it costs nothing to enter into a forward contract, the payoff from the contract is also the trader's total gain or loss from the contract.

Pricing of Forwards / Futures

- The relation between the current spot and the current forward/future price
  - does not depend on expectations of market participants
  - but on interest rates and time.
- Forwards/futures can be replicated.

➔ **Arbitrage-free pricing**

F = S (spot price) + CC (cost of carry)

- Cost of carry = interest – income resulting from the underlying + costs associated with holding the underlying (for stocks: interest – dividend).
- Remark: When pricing commodity futures, storage costs and convenience yields have to be taken into account.

➔ **without any costs and income**

$$F_t = S_t (1+r)^T$$ discrete compounding

$$F_t = S_t e^{rT}$$ continuous compounding

Cash-and-Carry Arbitrage

Assumption		
$F > S + CC$	Example: $F = 110$, $S = 100$, $CC = 5$	
Today	**Cash Position**	
Short forwards	no cash-flow	
Buy underlying spot	$- S$	$- 100$
Borrow money (credit)	$+S$	$+100$
At maturity		
Deliver underlying, get forward price	$+F$	$+110$
Pay back credit	$- S - CC$	$- 105$
Profit	$\mathbf{= F - S - CC}$	$\mathbf{= 5}$

Reverse Cash-and-Carry Arbitrage

Assumption		
$F < S + CC$	Example: $F = 100$, $S = 100$, $CC = 5$	
Today	**Cash Position**	
Long forward	no cash-flow	
Short underlying	$+S$	+100
Invest money	$- S$	- 100
At maturity		
Receive underlying, pay price	$- F$	- 100
Receive invested money	$+S + CC$	+105
Profit	$= S + CC - F$	**= 5**

Forward Rate Agreement (FRA) (1/2)

- A forward rate agreement (FRA) is an agreement that a certain rate will apply to a certain principal L during a certain future time period.
- The assumption underlying the contract is that the borrowing or lending would normally be done at LIBOR.
- Swaps are packages (or series) of FRAs.

- **<u>Notation</u>**:

  R_F Forward rate

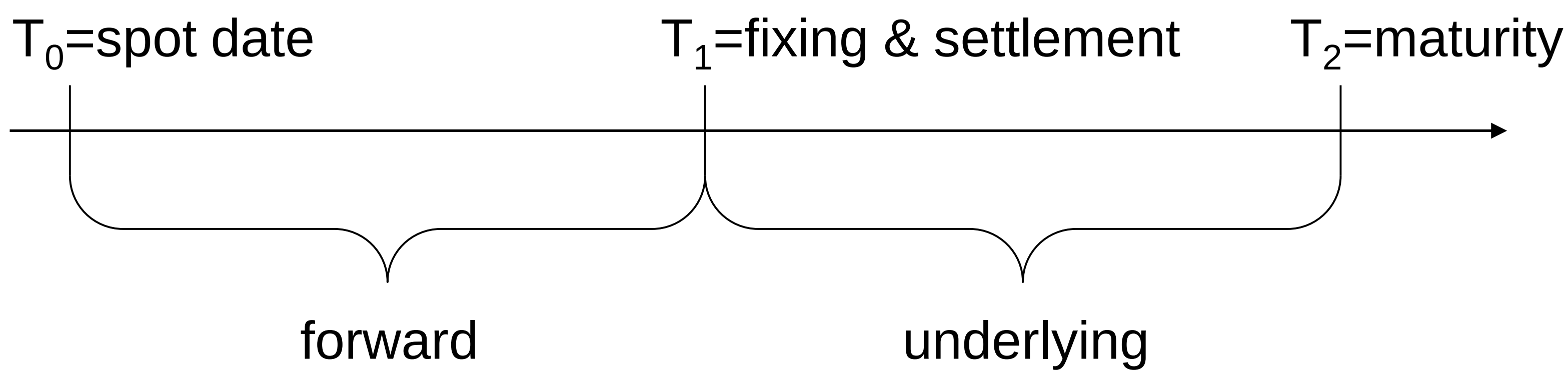
  R_K Reference rate (e.g., LIBOR)

  L Principal underlying the contract

  DCC "Day Count Convention" (e.g., 360, 365, etc.)

- FRAs are quoted on a AxB basis, where (A) is the number of months until the loan begins and (B) is the number of months until the loan ends

Forward Rate Agreement (FRA) (2/2)



$$\text{Cash settlement} = \frac{(R_K - R_F)\dfrac{(T_2 - T_1)}{DCC}L}{1 + \left(R_K \dfrac{(T_2 - T_1)}{DCC}\right)} = \frac{(R_K - R_F)L}{\dfrac{DCC}{(T_2 - T_1)} + R_K}$$

Forward Rate Agreement (FRA) – Example

- On April 12 Company X decides to issue a commercial paper on May 14 with a time to maturity of three months and a volume of USD 1m.
- Using FRAs, the company is going to hedge against rising interest rates.
- The forward rate on a 1x4 FRA (1 month from now for 3 months) is 6.25%.
- The maturity of the forward is from April 14 to May 14, the maturity of the underlying is from May 14 to August 16.
- On May 14 the LIBOR rate is 7%.

1. What is the cash settlement of the FRA?

2. Compare the cash settlement with the interest payments on the commercial paper. What is the difference?

3. Explain the difference.

Value of an FRA

Present Value Formula:

Discrete Case

$$V_{FRA} = \frac{L(R_K - R_F)(T_2 - T_1)}{(1 + R_2 T_2)}$$

Continuous Case

$$V_{FRA} = L(R_K - R_F)(T_2 - T_1)e^{-R_2 T_2}$$

- R_2 is the risk-free interest rate for maturity T_2 from the current spot curve.
- R_K is the forward rate of the reference rate.
- R_F is the contractual forward rate.
- T_1, T_2 are measured in years

Value of an FRA – Example

- Suppose that
  - the three-month LIBOR rate is 5%,
  - the six-month LIBOR rate 5.5%, and
  - the FRA will receive a rate of 7%
  - on a principal of USD 1m
  - between the end of month 3 and the end of month 6.

- First, calculate the forward rate using the arbitrage relation

- Second, what is the value of the FRA?

Agenda

- Forwards
- **Swaps**
- Futures
- Options

Swaps

- Swaps are contracts between two parties who agree to exchange future cash-flows.
- Typically one party pays fixed cash-flows and the other side pays floating cash-flows.
- In a receiver swap the owner gets fixed und pays floating cash-flows.
- In a payer swap the owner gets floating und pays fixed cash-flows.
- A major part of swaps are either based on currencies or interest rates.

Swap Trades (1/2)

- Swaps are traded directly between counterparties **Over-The-Counter** (OTC). The International Capital Market Association (ICMA) developed standard terms and conditions of swaps.

- Swaps are traded without upfront payments. The **Swap Rate** (fix payments) is set so that the initial value of the contract is zero.

- The fixed rate reflects changes in the spot curve and the floating rate spreads reflect changes in the credit quality.

- Swaps are rarely redeemed early. Instead parties engage in counter trades that offset the payments.

Swap Trades (2/2)

Central Counterparties (CCP)

- A Central Counterparty for OTC derivatives is an independent legal entity (often provided by an exchange, e.g., EurexOTC Clear) that interposes itself between the buyer and the seller of a derivative security
- CCPs shall improve market resilience by lowering counterparty risk and increase transparency on the OTC markets[1]
- Pursuant the *European Financial Market Infrastructure* Directive (EMIR) all standardized OTC swaps have to be cleared via a CCP since mid-2014 (part of the application of the U.S. Dodd-Frank Act).

[1]For more detailed information about CCPs, see Cecchetti, Gyntelberg, and Hollanders (2009). Central Counterparties for OTC Derivatives. *BIS Quarterly Review September 2009.*

Swap Contracts (1/3): Interest Rate Swap Structure

Interest rate swap

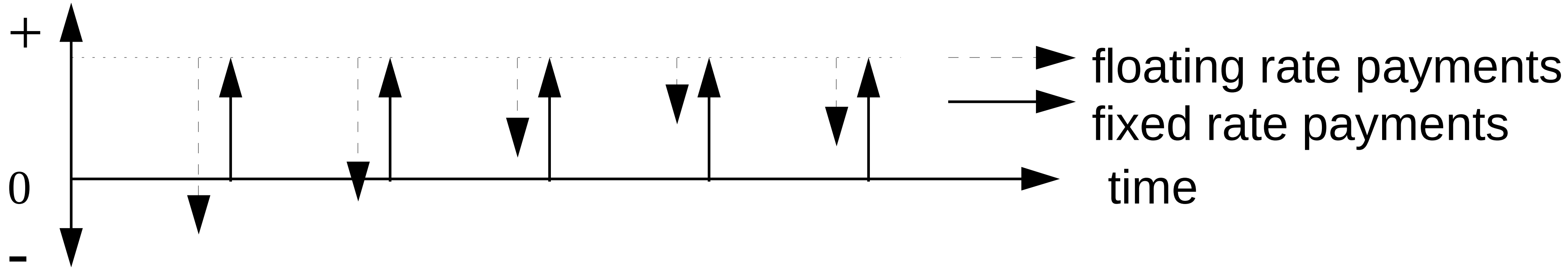
- Exchange of coupon payments.
- Fixed coupon against floating coupon payments of two hypothetical par-bonds with equal maturity and equal coupon payment dates.
- No exchange of nominal values at maturity.

Example

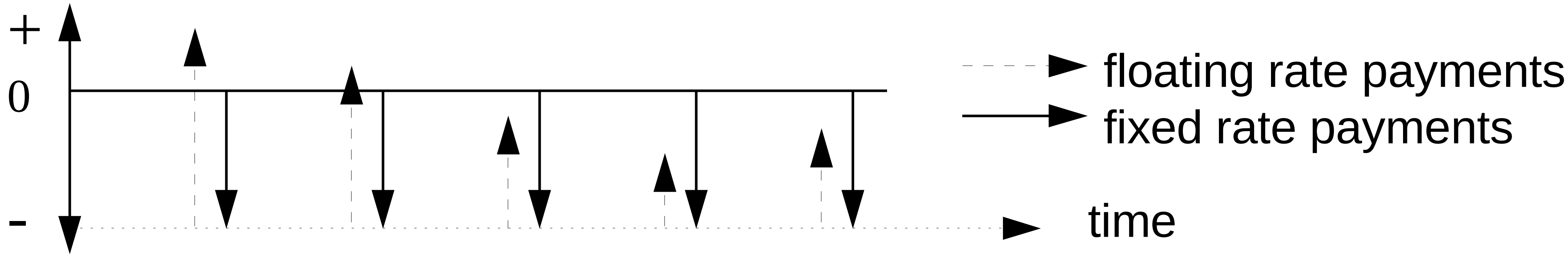
- Suppose, a company can raise money at a fixed interest rate.
- However, the company prefers a floating exposure, because it expects lower interest rates in the future.

Swap Contracts (2/3): Interest Rate Swap Cash Flows

„Receiver“ pays floating interest rate, receives fixed interest rate



„Payer“ pays fixed interest rate, receives floating interest rate



Swap Contracts (3/3): Cross-Currency Swap

- Similar principle as for the interest rate swap.

Difference

- The currencies of swapped interest rates are different.
- Possible combinations:
  - fix against fix
  - floating against floating
  - fix against floating
- Nominal value is exchanged at maturity.
- The interest rate swap is a special case of the currency swap.

Advantages of Swaps

- **Reducing interest rate risk at low cost**:
  - If one party receives fixed payments but has floating liabilities, it can reduce its exposure by entering a payer swap.

- **Credit arbitrage /**
  capitalizing on comparative advantage (David Ricardo, 1817)

Credit Arbitrage: Example

We have two companies X and Y that can refinance according to the following conditions:

	Company X	Company Y
Fixed Interest Rate	2.50 %	3.25 %
Floating Interest Rate	Euribor	Euribor + 25BP

Difference
75BP
25BP
50BP

- Company X wants to refinance at a floating rate for 5 years
- Company Y wants to refinance at a fixed rate for 5 years

If the companies refinance according to their demand the interest rates are:

- Company X floating at Euribor
- Company Y fixed at 3.25 %

What is the arbitrage profit?

Pricing of Swaps: Example

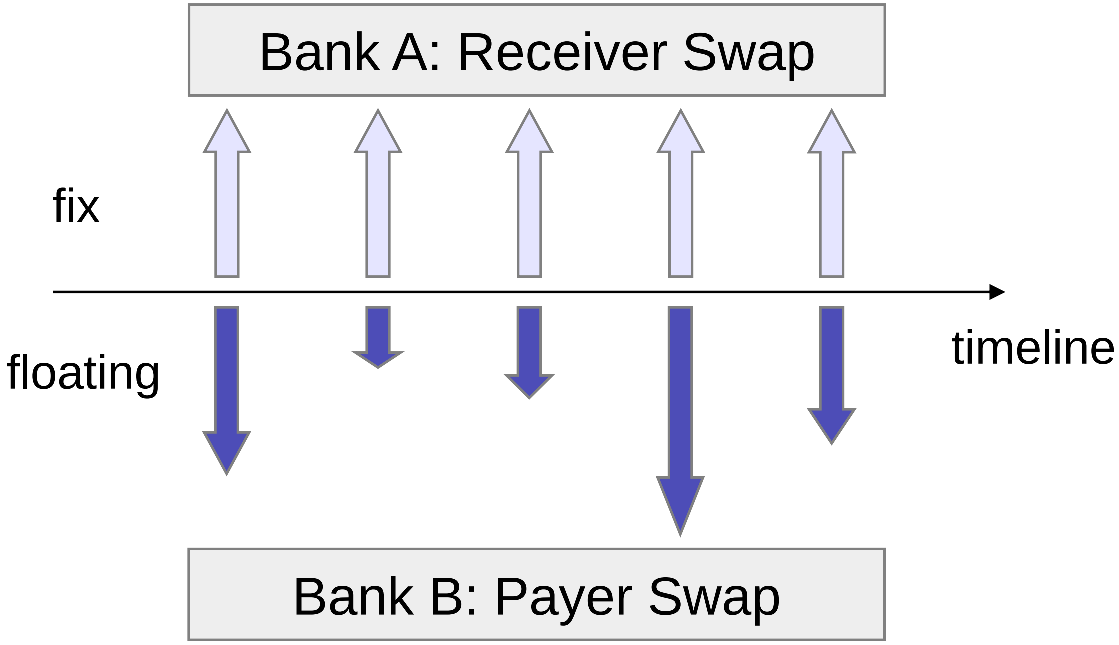
We consider an interest rate swap with a maturity of 5 years, a notional value of CHF10m and a swap rate of 5%.

The notional value is not exchanged at redemption.

1. Determining the market price of a swap at an arbitrary point in time after inception

2. Deriving the fixed swap rate at inception so that no payments are made at t = 0

   → The value of the swap is 0.



Pricing of Swaps: Market Value of a Swap (1/2)

Step 1: Pricing floating payments

Step 1:

Time t	1	2	3	4	5
Spot Rates: r_t	0.47%	0.92%	1.33%	1.68%	1.99%
Discount Rates	0.9953	0.9819	0.9611	0.9355	0.9062
Forward Rates: $f_{t,t+1}$	1.37%	2.16%	2.74%	3.24%	-
Cash Flows in 1000 * CHF (Expected)	47.00	137.20	215.50	273.73	323.95
Present Value in 1000 * CHF	46.78	134.71	207.13	256.08	293.55
PV(float) in 1000 * CHF	938.25 (≈ 938)				

Pricing of Swaps: Market Value of a Swap (2/2)

Step 2: Pricing fixed payments

$$PV(fix) = N \cdot \left(C \underbrace{\frac{1}{1+r_1}}_{d_1} + C \underbrace{\frac{1}{(1+r_2)^2}}_{d_2} + \ldots \right) = N \cdot C \cdot (d_1 + d_2 + \ldots) = \sum_{t=1}^{T} C \cdot N \cdot d_t = C \cdot N \cdot \sum_{t=1}^{T} d_t$$

$$PV(fix) = 10m \cdot 0.05 \cdot 4.780 = 2.390m$$

Value of the swaps = PV(fix) – PV(float) = 2.390m – 0.938m = 1.452m

Pricing of Swaps: Deriving the Swap Rate

Step 3: Setting PV(float) equal to PV(fix) we can derive the fixed coupon C

$$PV(\text{fix}) \overset{!}{=} PV(\text{float}) \qquad C \times 10m \times \sum_{t=1}^{5} d_t \overset{!}{=} 0.938m$$

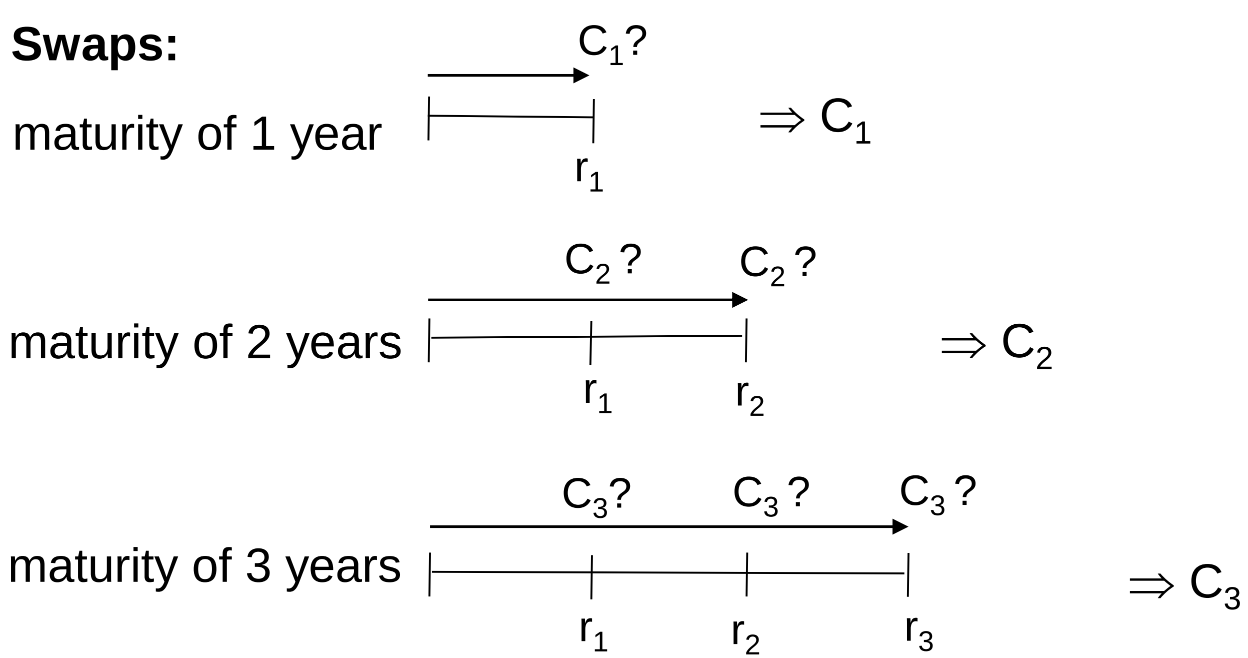
$$\Rightarrow \quad C = \frac{0.938m}{10m \cdot 4.780} = 1.96\%$$

Deriving Swap Rates from Spot Rates (1/2)

- Swap rates can directly be derived from spot rates.
- The underlying nominal value for the floating and the fixed part is set at 100 (N = 100)

EXAMPLE

- We calculate the swap rates C_t for three swaps with maturities of 1, 2 and 3 years. We use the yield curve from the previous example.



Deriving Swap Rates from Spot Rates (2/2)

$$\frac{C_1 \cdot N + N}{1+r_1} = 100 \Rightarrow C_1 = \frac{100(1+r_1)}{N} - 1 \Rightarrow C_1 = r_1 = 0.47\%$$

$$\frac{C_2 \cdot N}{1+r_1} + \frac{C_2 \cdot N + N}{(1+r_2)^2} = 100 \Rightarrow C_2 = \left(1 - \frac{1}{(1+r_2)^2}\right)\left(\frac{(1+r_1)(1+r_2)^2}{(1+r_1)+(1+r_2)^2}\right) = 0.918\%$$

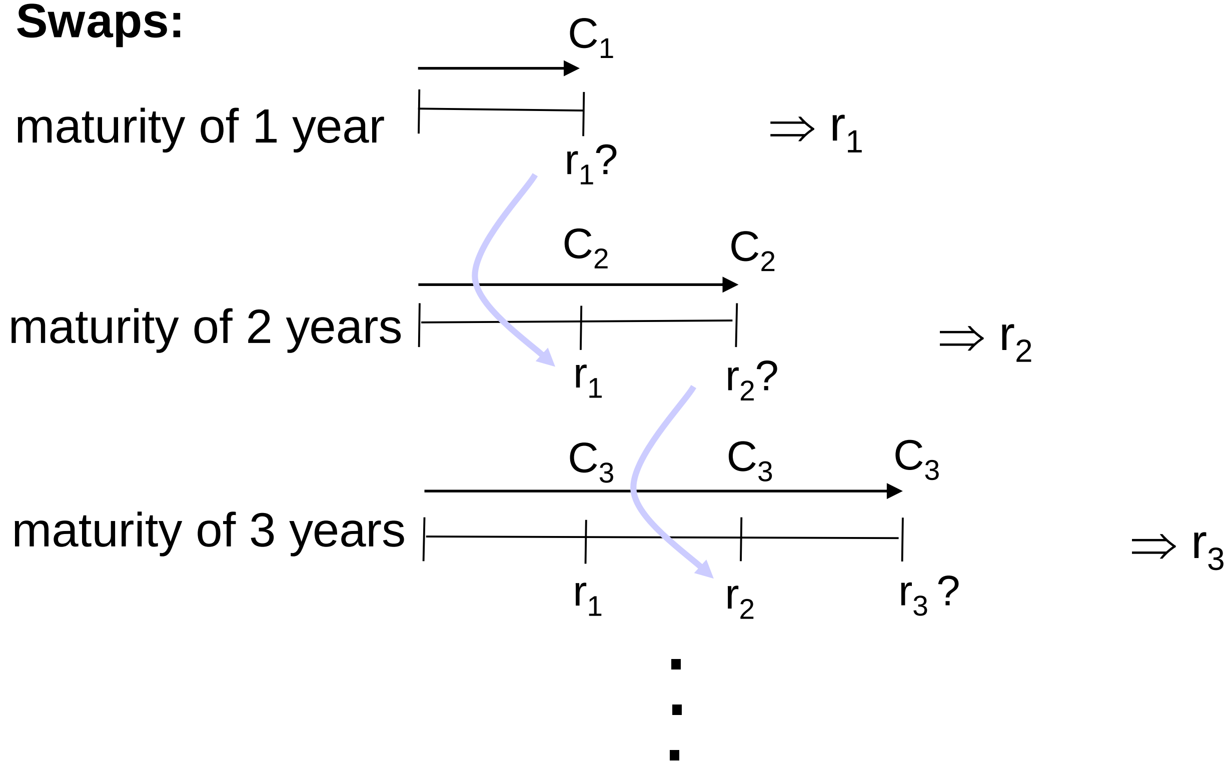
$$\frac{C_3 \cdot N}{1+r_1} + \frac{C_3 \cdot N}{(1+r_2)^2} + \frac{C_3 \cdot N + N}{(1+r_3)^3} = 100 \Rightarrow C_3 = \left(1 - \frac{1}{(1+r_3)^3}\right) \div \left(\frac{1}{1+r_1} + \frac{1}{(1+r_2)^2} + \frac{1}{(1+r_3)^3}\right) = 1.323\%$$

- The swap rates are C_1 = 0.47%, C_2 = 0.918% and C_3 = 1.323%.

Deriving the Spot Rates from Swap Rates (1/2)

Obviously, the exchange of rates is also possible into the other direction.

Swaps:



Deriving the Spot Rates from Swap Rates (2/2)

EXAMPLE

- The swap rates for maturities 1, 2 and 3 years are C_1 = 0.47%, C_2 = 0.918% und C_3 = 1.323%. What are the corresponding spot rates ?

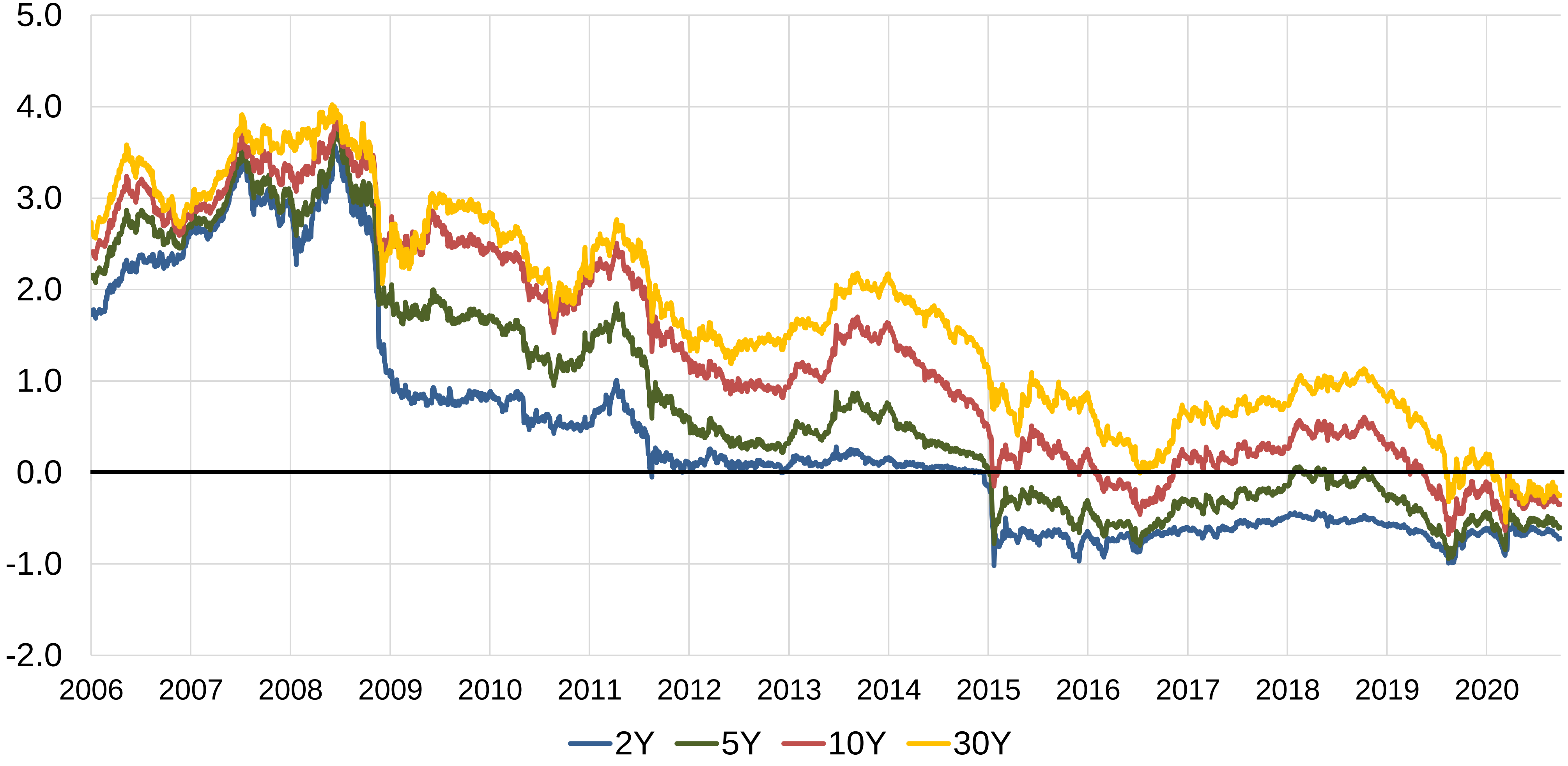
$$\frac{C_1 N + N}{1+r_1} = 100 \Rightarrow r_1 = C_1 = 0.47\%$$

$$\frac{C_2 N}{1+r_1} + \frac{C_2 N + N}{(1+r_2)^2} = 100 \Rightarrow r_2 = \sqrt{\frac{1+r_1}{1+r_1-C_2}(1+C_2)} - 1 = 0.92\%$$

$$\frac{C_3 N}{1+r_1} + \frac{C_3 N}{(1+r_2)^2} + \frac{C_3 N + N}{(1+r_3)^3} = 100 \Rightarrow r_3 = \sqrt[3]{\frac{1+C_3}{1-\frac{C_3}{1+r_1}-\frac{C_3}{(1+r_2)^2}}} - 1 = 1.33\%$$

- **Note: Swap rates are not spot rates of zero coupon bonds.**

Swap Rates on the CH - Market



Source: Bloomberg

Agenda

Forwards

Swaps

Futures

- **Futures Contracts**
- Commodity Futures
- Contango & Backwardation

Options

Futures Contracts

- Agreement to buy or sell an asset for a certain price at a certain time in the future.
- Similar to forward contracts.
- Whereas forward contracts are traded OTC, futures contracts are traded on an exchange.

Forward Contracts vs. Futures Contracts

Non-centrally cleared Forward contract	Centrally cleared Forward contract	Futures contract
Tailor-made	Standardized	Standardized
Not exchange traded	Not exchange traded	Exchange traded
No cash-flows until Maturity (other than collateral)	No cash-flows until Maturity (other than collateral)	Marking to market
Held until maturity	Held until maturity	Closing-out with offsetting transactions
Individual counterparty risk	No individual counterparty risk	No individual counterparty risk

Mechanics of Futures Contracts

Marking to market

- A margin is the cash balance (or security deposit) required from a futures or options trader/broker.
- Daily settlement of profits and losses on a margin account.
- Margins avoid or at least minimize the possibility of contract defaults.

Closing transactions

- Open positions can be closed at any point in time by reversing the original transaction: An investor being short in a contract goes long in the same contract to close his position (and vice versa).
- Physical settlement of specific futures contracts.

Operation of Margins (1/3)

- The amount that must be deposited at the time a futures contracts is entered into is known as **initial margin**.
- To ensure that the balance in the margin account never becomes negative, a **maintenance margin** is set.
- If the balance in the margin account falls below the maintenance margin, the investor receives a **margin call** and is expected to top up the margin account to the initial margin level.
- The extra funds deposited are known as a **variation margin**.

Example

- Long position in 1 gold futures contract: contract is entered on September 5 and closed out on September 15.
- Current futures price is USD 1325 per ounce, contract size is 100 ounces.
- Initial margin of USD 2,000 per contract, maintenance margin is USD 1’500 per contract.

Operation of Margins (2/3)

Development of margin account balance **with maintenance margin**:

	Futures price	Daily loss/gain	Cum. Loss/gain	Margin account balance	Margin call
Sep 05	1340	-	-	2000	
Sep 06	1342	200	200	2200	
Sep 07	1335	-700	-500	1500	0
Sep 08	1337	200	-300	1700	0
Sep 09	1333	-400	-700	1300	700
Sep 10	1332	-100	-800	1900	0
Sep 11	1325	-700	-1500	1200	800
Sep 12	1319	-600	-2100	1400	600
Sep 13	1327	800	-1300	2800	0
Sep 14	1333	600	-700	3400	0
Sep 15	1342	900	200	4300	0

Operation of Margins (3/3)

Development of margin account balance **without maintenance margin**:

	Futures price	Daily loss/gain	Cum. Loss/gain	Margin account balance
Sep 05	1340	-	-	2000
Sep 06	1342	200	200	2200
Sep 07	1335	-700	-500	1500
Sep 08	1337	200	-300	1700
Sep 09	1333	-400	-700	1300
Sep 10	1332	-100	-800	1200
Sep 11	1325	-700	-1500	500
Sep 12	1319	-600	-2100	-100
Sep 13	1327	800	-1300	700
Sep 14	1333	600	-700	1300
Sep 15	1342	900	200	2200

Open Interest

- **Open interest** is the total number of open or outstanding futures contracts that exist at a given time
- When a futures contract is traded, the open interest can either:

  - increase by one: both sides enter into new contract
  - decrease by one: both sides offset old position
  - stay the same: one side enters a new contract, one side offsets old position

Example

Time	**Trading Activity**	**Open Interest**
1	**A** buys 5 futures and **B** sells 5 futures contracts	5
2	**C** buys 3 futures and **D** sells 3 futures contracts	8
3	**A** sells 2 futures to **D** who buys 2 futures contracts	6
4	**E** buys 3 futures from **C** who sells 3 futures contracts	6

Futures Contracts

Futures on short-term interest rates

- Eurodollar, Euroyen, T-Bill, EURIBOR

Bond futures

- T-Bond, British long gilt, Deutscher Bund, CONF

Stock index futures

- S&P500, Nikkei 225, DAX, SMI, CAC40

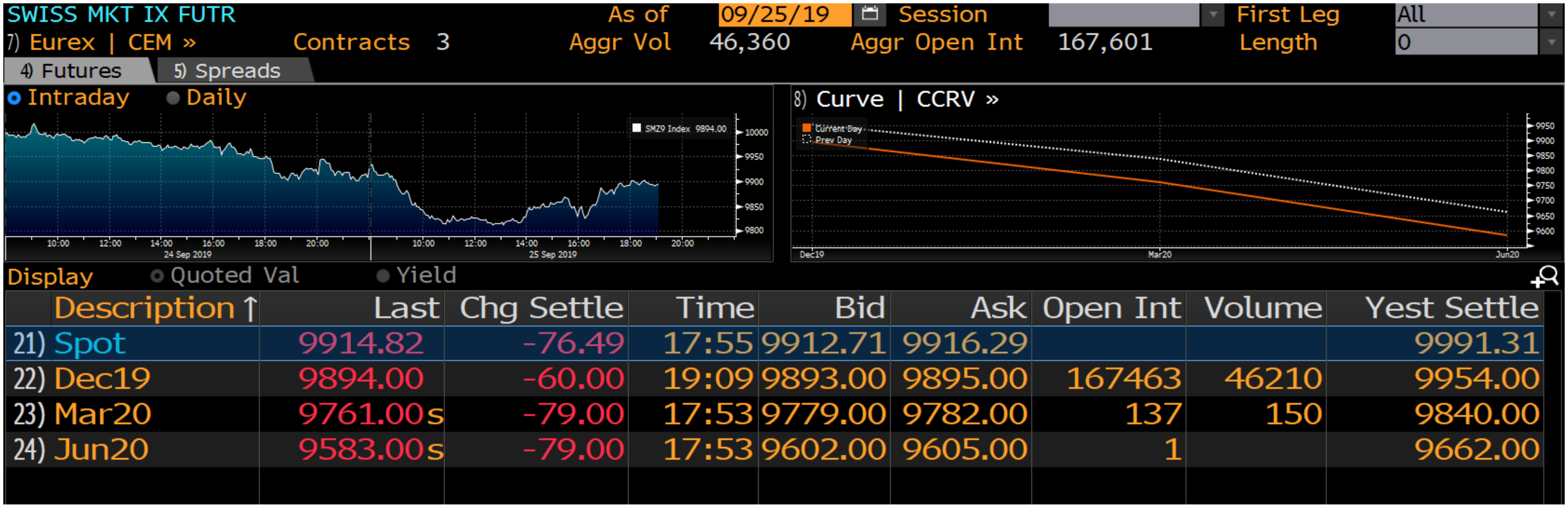
Currency futures

- Swiss Franc, Euro

Commodity futures

- Oil Futures, Corn Futures

Futures price quotation in Bloomberg



Description	Last	Chg Settle	Time	Bid	Ask	Open Int	Volume	Yest Settle
21) Spot	9914.82	-76.49	17:55	9912.71	9916.29			9991.31
22) Dec19	9894.00	-60.00	19:09	9893.00	9895.00	167463	46210	9954.00
23) Mar20	9761.00s	-79.00	17:53	9779.00	9782.00	137	150	9840.00
24) Jun20	9583.00s	-79.00	17:53	9602.00	9605.00	1		9662.00

Last Price

Ask Price

Traded Volume

Expiration Month

Bid Price

Open Interest

Example – SMI Futures (1/2)

Contract value

- CHF 10 per SMI index point.

Settlement

- Cash settlement, payable on the first exchange day following the final settlement day.

Quotation

- In points, with no decimal places.

Minimum price movement

- 1 point, representing a value of CHF 10.

Contract terms

- The three nearest quarterly months of the March, June, September, December cycle.

Last trading day and final settlement day

- Last trading day is the final settlement day.
- Final settlement day is the third Friday of each maturity month if this is an exchange day; otherwise the exchange day immediately preceding that day.

Daily settlement price

- The Daily Settlement Prices for the current maturity month are derived from the volume-weighted average of the prices of all transactions during the minute before 17:20 CET (reference point), provided that more than five trades transacted within this period.

Source: Eurex

Example – SMI Futures (2/2)

Short position in 50 SMI futures contracts with maturity December 2019:

Description	Last	Chg Settle	Time	Bid	Ask	Open Int	Volume	Yest Settle
21) Spot	9914.82	-76.49	17:55	9912.71	9916.29			9991.31
22) Dec19	9894.00	-60.00	19:09	9893.00	9895.00	167463	46210	9954.00
23) Mar20	9761.00s	-79.00	17:53	9779.00	9782.00	137	150	9840.00
24) Jun20	9583.00s	-79.00	17:53	9602.00	9605.00	1		9662.00

Source: Bloomberg

- The daily settlement price of the December 2019 SMI futures contract on September 25, 2019 was 9,954.00.
- Assuming a settlement price of the SMI futures contract of 9,856.00 the day after, the daily gain on the **short** futures position is:

$$-50 \cdot \text{CHF } 10 \cdot (9{,}856.00 - 9{,}954.00) = \text{CHF } 49{,}000$$

Example – CONF Futures (1/2)

Contract standard

- A notional long-term debt instrument issued by the Swiss Federal Government with a term of 8 to 13 years and an interest rate of 6 percent.

Contract size

- CHF 100,000

Settlement

- A delivery obligation arising out of a short position in a CONF Futures contract may only be fulfilled by the delivery of specific debt securities - namely, long-term Swiss Federal Government Bonds with a minimum issue amount of CHF 500m and a remaining term upon delivery of 8 to 13 years. In the case of callable bonds, the first and last call dates must be between 8 and 13 years.

Quotation

- In a percentage of the par value, carried out two decimal places.

Minimum price movement

- 0.01 percent, representing a value of CHF 10.

Delivery day

- The 10th calendar day of the respective delivery month, if this day is an exchange trading day; otherwise, the immediately following exchange trading day.

Delivery months

- The three successive months within the cycle March, June, September and December.

Source: Eurex

Example – CONF Futures (2/2)

- Long position in 100 CONF futures contract with maturity December 2019:

Dec 2019 ▼

Opening price	High	Low	Bid price	Bid vol	Ask price	Ask vol	Diff. to prev. day last	Last price	Date	Time	Daily settlem. price	Traded contracts	Oper interes (adj.)
162.99	163.20	162.81	162.81	n.a.	162.95	n.a.	-0.04%	162.90	09/25/2019	17:10:00	162.90	119	2,567

Source: Eurex

- The daily settlement price of a Dec 19 CONF futures contract is 162.90.
- Assuming a settlement price of the CONF futures contract of 162.73 the day after, the daily loss on the **long** futures position is:

#Contracts Contract price change per basis point Change in BP

$$100 \cdot 10\ \text{CHF} \cdot (16{,}273 - 16{,}290) = -17{,}000\ \text{CHF}$$

$$100 \cdot 1000\ \text{CHF} \cdot (162.73 - 162.90) = -17{,}000\ \text{CHF}$$

Example – CME Eurodollar Future (1/3)

Underlying

- 3-month Eurodollar Time Deposit (interest rate earned on Eurodollars deposited by one bank with another bank).

Contract value

- USD 1m

Price Quotation

- IMM price points: 100 points minus the three-month London interbank offered rate for spot settlement on the 3rd Wednesday of contract month. E.g., a price quote of 97.45 signifies a deposit rate of 2.55 percent per annum. One interest rate basis point = 0.01 price points = $25 per contract.

Settlement

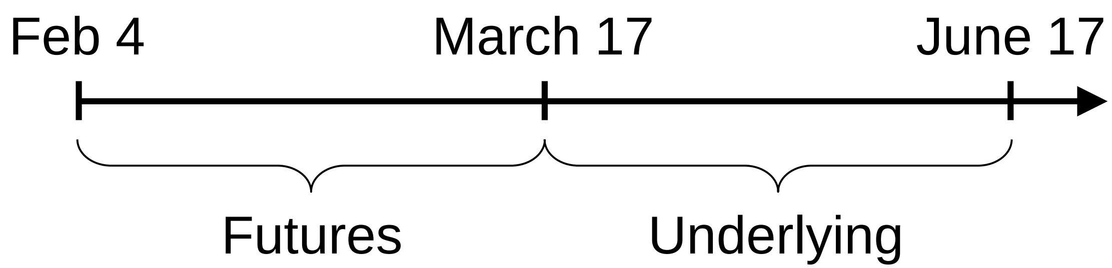
- Cash settlement: the settlement price is set equal to 100-R, where R is the actual 3-month Eurodollar interest rate, expressed with quarterly compounding and an actual 360 day count convention.
- Contracts end on the third Wednesday of the delivery/contract month.

Contract month

- March, June, September and December for up to 10 years into the future.
- Short-maturity contracts trade also for other months.

Example – CME Eurodollar Future (2/3)

- On February 4, an investor wants to lock in the interest rate that will be earned on USD 5m for 3 months starting on March 17:



- The investor goes long 5 March Eurodollar futures contracts at 97.63 (implies an interest rate of 2.37%).
- On March 16, the futures 3-month LIBOR interest rate is 2% (final settlement price). The investors gains on the long futures position:

$$5 \cdot \text{USD } 25 \cdot (9{,}800 - 9{,}763) = \text{USD } 4{,}625$$

- Because the futures quote is 100 minus the futures interest rate, an investor who is long gains when the interest rates fall.

Example – CME Eurodollar Future (3/3)

- The interest earned on the USD 5m for 3 months at 2% is:

$$\text{USD } 5\text{m} \cdot 0.25 \cdot 0.02 = \text{USD } 25{,}000$$

- The gain on the futures contracts brings this up to USD 29,625. This corresponds to the interest rate that would have been earned if the interest rate had been 2.37%:

$$\text{USD } 5\text{m} \cdot 0.25 \cdot 0.0237 = \text{USD } 29{,}625$$

- The futures trade has the effect of locking in an interest rate equal to 2.37%, or (100-97.63).

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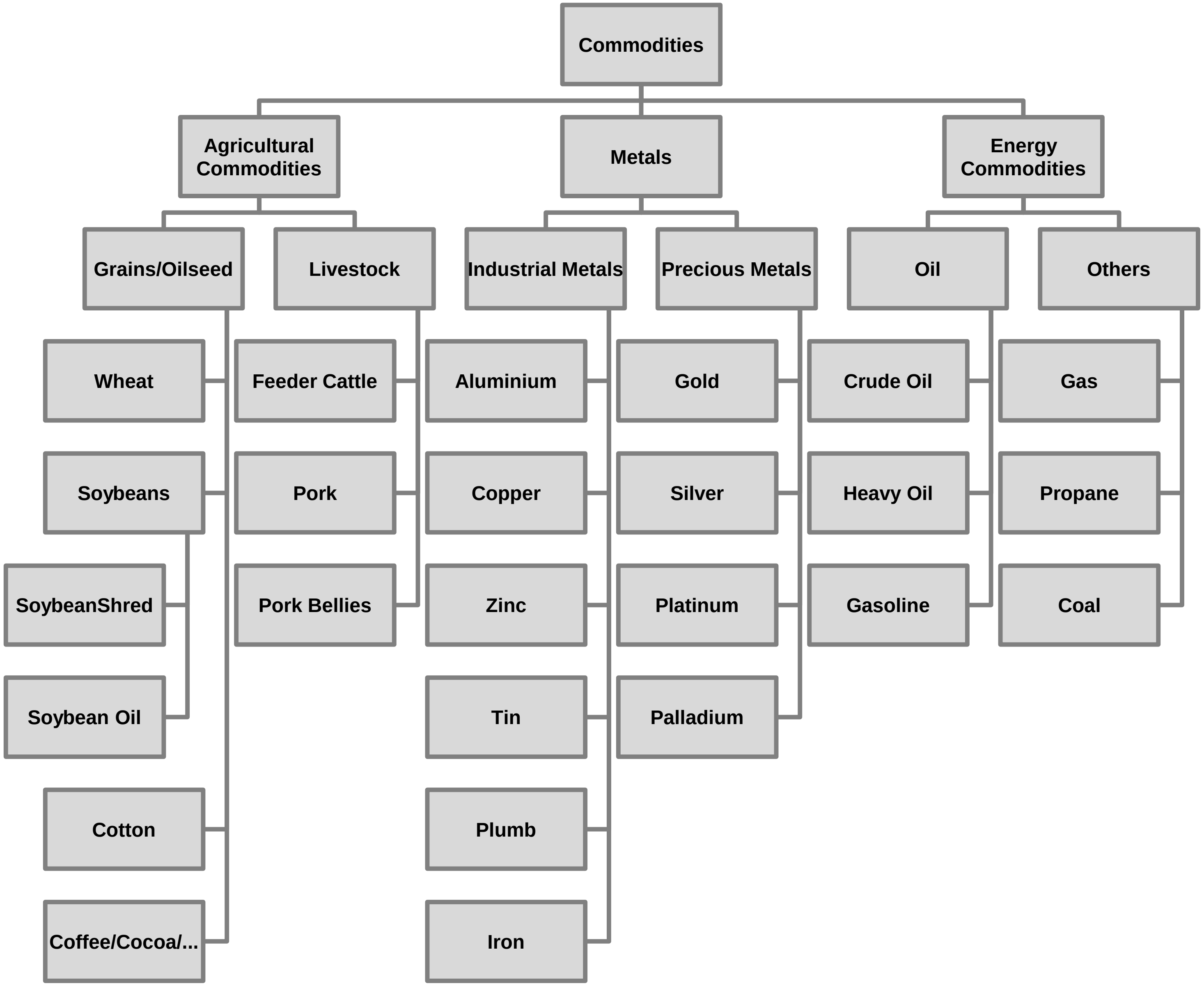
Futures

- Futures Contracts
- **Commodity Futures**
- Contango & Backwardation

Options

What are Commodities?



Commodities vs. Classical Investments

Significantly higher costs

- Storage cost
- Transportation cost
- Transaction cost

Many commodities can get spoiled or have a low value per unit of volume

Heterogeneous products

- Different types of wheat, etc. are traded
- Agricultural goods are harvested several times a year
- Oil is different from well to well

Seasonal effects

- E.g. during winter demand is higher for heating oil and in summer more gasoline is used

Commercial and Non-Commercial Traders

Net position of Commercials, Non-commercials and Non-reportable
(in thousands of standardized contracts, net long: +, net short: -)

Futures only	Commercials	Non-commercials	Non-reportable
	Net positions		
Corn	-181.1	81.7	99.4
Live cattle	-92.1	88.3	3.8
Crude Oil*	26.7	-39.9	13.2
Gold	-159.6	123.8	35.8
Silver	-62.6	41.0	21.6

* Crude Oil, Light Sweet - Nymex
Source: Commodity Futures Trading Commission, Commitments of Traders Report, September 17th, 2019

Hedging Pressure Hypothesis:

– Producers and consumers seek to protect themselves from price volatility
– Producers lock in sale prices (short), consumers lock in purchase prices (long)

Commodity Futures Prices

Basic rule for determination of futures prices as starting point for commodity futures pricing:

$$F = S + \text{cost of carry}$$

Theory of Storage:

- **Reasons for a higher cost of carry of commodities:**
  - Storage cost
  - Insurance Cost
  - Spoilage

Storage costs (in the broader sense)

- **Reasons for lower cost of carry of commodities:**
  - Security: Commodity is available
  - Possibility to take advantage of other transactions in the meanwhile

Convenience yield

Commodity Futures Prices – Investment Assets (1/2)

- Commodities such as gold or silver are **investment assets**:
  - Held as financial investment by many investors
  - Ownership of physical commodity provides no benefits that are not obtained by holders of the futures contract.
- **Determination of futures prices** on commodities that are investment assets based on arbitrage-free pricing:

$$F_0 = S_0 + \text{cost of carry} \Leftrightarrow F_0 = S_0 \cdot (1 + r_{t,T}) + SC_{t,T}$$

where r represents the interest rate and SC the storage costs.

- With **continuous compounding** and the storage costs expressed as a percentage of the spot price (u):

$$F_0 = S_0 e^{(r+u)T}$$

Commodity Futures Prices – Investment Assets (2/2)

- **Cash-and-carry arbitrage** if $F_0 > S_0 \cdot (1 + r_{t,T}) + SC_{t,T}$
  - Short the futures contract
  - Borrow an amount of
  - Purchase the commodity (pay storage costs at T)

	t	T
Short the futures contract	0	F
Borrow an amount of	S	-S(1+r)
Purchase the commodity (pay storage costs at T)	-S	-SC
	0	>0

➔ Tendency for S to increase and for F to decrease until the pricing equation holds again.

- **Reverse cash-and-carry arbitrage** if $F_0 < S_0 \cdot (1 + r_{t,T}) + SC_{t,T}$
  - Sell the commodity (and save the storage costs)
  - Invest the proceeds at the risk-free rate
  - Take a long position in the futures contract

	t	T
Sell the commodity (and save the storage costs)	S	SC
Invest the proceeds at the risk-free rate	-S	S(1+r)
Take a long position in the futures contract	0	-F
	0	>0

➔ Tendency for S to decrease and for F to increase until the pricing equation holds again.

Commodity Futures Prices – Consumption Assets (1/3)

- Commodities such as copper or oil are **consumption assets**:
  – Held because of the commodities' consumption value.
  – Ownership of physical commodity provides benefits that are not obtained by holders of the futures contract.
  – The benefits form holding the physical asset are referred to as the convenience yield or convenience value provided by the commodity.

- The **convenience yield** is defined such that:

$$F_0 = S_0 \cdot (1 + r_{t,T}) + SC_{t,T} - CV_{t,T}$$

where CV represents the convenience yield or value.

- If the convenience yield is equal or close to 0, a market is said to be at **full-carry**, otherwise a market is called **non-full-carry market**.

Commodity Futures Prices – Consumption Assets (2/3)

- Alternatively, with continuous compounding, the **convenience yield** is defined such that:

$$F_0 = S_0 e^{(r+u-y)T}$$

where u represents the storage costs and y the convenience yield.

Commodity Futures Prices – Consumption Assets (3/3)

- **Cash-and-carry arbitrage and reverse cash-and-carry arbitrage limited**
  - Reluctance of commodity holder to sell the commodity and buy futures contracts because futures contracts cannot be consumed.
  - Number of possible arbitrageurs is limited.
  - Difficulties to short commodities.

- Concept of **quasi-arbitrage**: the major difference between pure and quasi-arbitrage is that the pure arbitrageur has no initial net position, while the quasi-arbitrageur moves from an initial to a synthetic position.

Investment vs. Consumption Assets: Example (1/3)

Gold

Month	Last
AUG 2017	1318.20
SEP 2017	1321.50
OCT 2017	1322.00
DEC 2017	1325.90
FEB 2018	1330.60
APR 2018	1333.60
JUN 2018	1335.90

Specifications:

- Contract = 100 troy oz.
- Quoted prize = USD per troy oz.

WTI Crude Oil

Month	Last
OCT 2017	46.44
NOV 2017	46.84
DEC 2017	47.17
JAN 2018	47.43
FEB 2018	47.64
MAR 2018	47.84
APR 2018	48.27
MAY 2018	48.36
JUN 2018	48.22

Specifications:

- Contract = 1.000 barrels per lot
- Quoted prize = USD per barrel

Source: Bloomberg & CME

Investment vs. Consumption Assets: Example (2/3)

- Gold: 100 ounces/contract
- Oil: 1000 barrels/contract (1 bbl = 158.98 Liter)
- Interest rate: 1.00 %

- Storage costs: October 2017 → June 2018 = 9 months

  – Gold (OCT17 – JUN18): $15/100 troy oz./month + $30 per contract

  Storage cost Gold = $15*9+$30 = $165 per contract = $1.65 per ounce

---

  – Oil (OCT17 – JUN18): $0.55/barrel/month

  Storage cost Oil = $0.55*1000*9 = $4950 per contract = $4.95 per barrel

Investment vs. Consumption Assets: Example (3/3)

- **Futures price gold:**
  - Gold: Futures price (JUN18) = Futures price (OCT17) * (1 + Interest) + Storage Cost

    = $1’322.00 * (1 + 0.01 * 9 / 12) + $1.65 = $1’333.57

    ... and thus very close to the real JUN18 price $1’335.90

    difference = 0.17%

    ⇒ **Full-carry market**
- **Futures price oil:**
  - Oil: Futures price (JUN18)=Futures price (OCT17) * (1 + Interest) + Storage Cost

    = $46.44 * (1 + 0.01 * 9 / 12) + $4.95 = $51.74

    ... and thus far away from the real JUN18 price $48.22

    difference = 7.30%

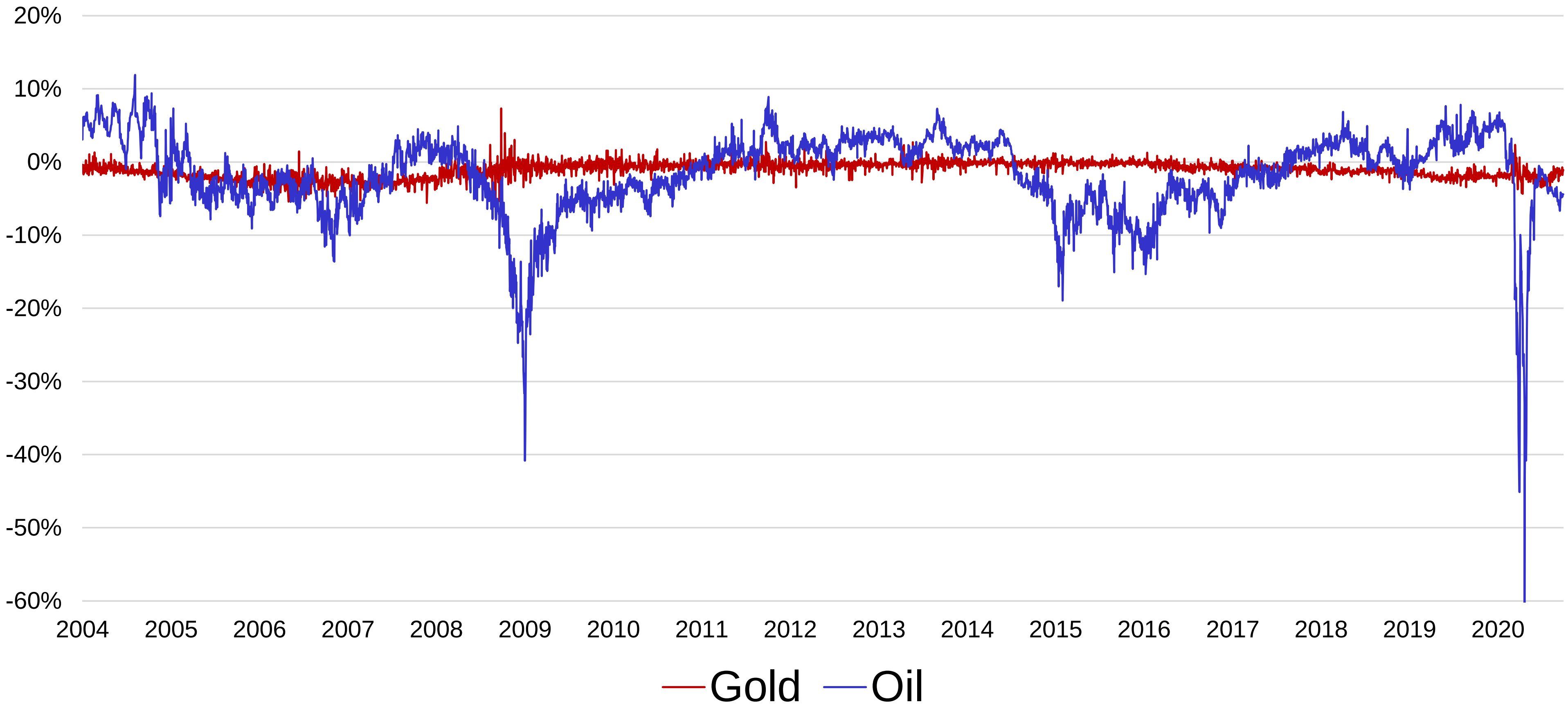
    ⇒ Convenience value of $3.52 per barrel for 9 months

    ⇒ Convenience value of $3.52 * 1’000 barrels = $3’520 per contract

    ⇒ **Non-full-carry market**

The Implied Convenience Yield of Crude Oil and Gold

Implied convenience yield net of storage costs (in %): $y - u = \dfrac{\ln\left(S_0/F_0\right)}{T} + r$



Convenience Yield Depends on Supply and Demand

- The greater the possibility that shortages will occur, the higher the convenience yield
  - Low inventories
  - High demand

- The lower the possibilities that shortages will occur, the lower the convenience yield
  - High inventories
  - Low demand

Contango and Backwardation

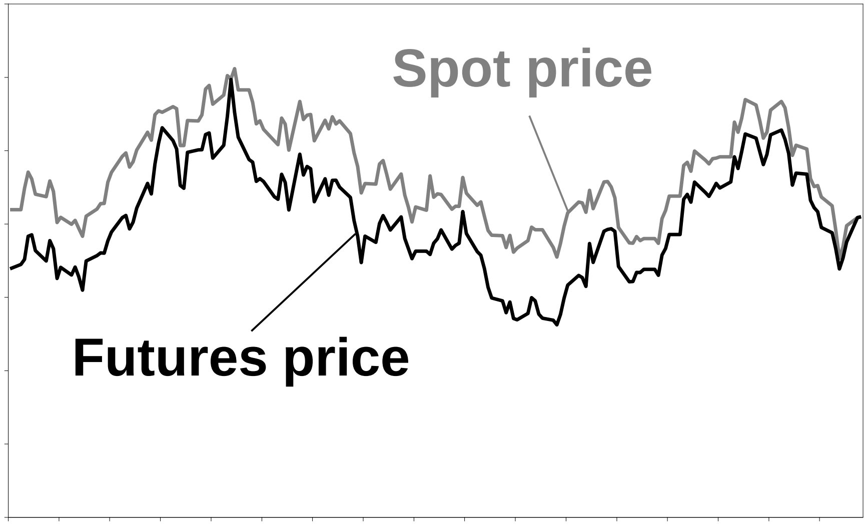
Theoretical:

- When the **futures price is below the expected future spot price**, the situation is known as **normal backwardation**.
- When the **futures price is above the expected future spot price**, the situation is known as **contango**.

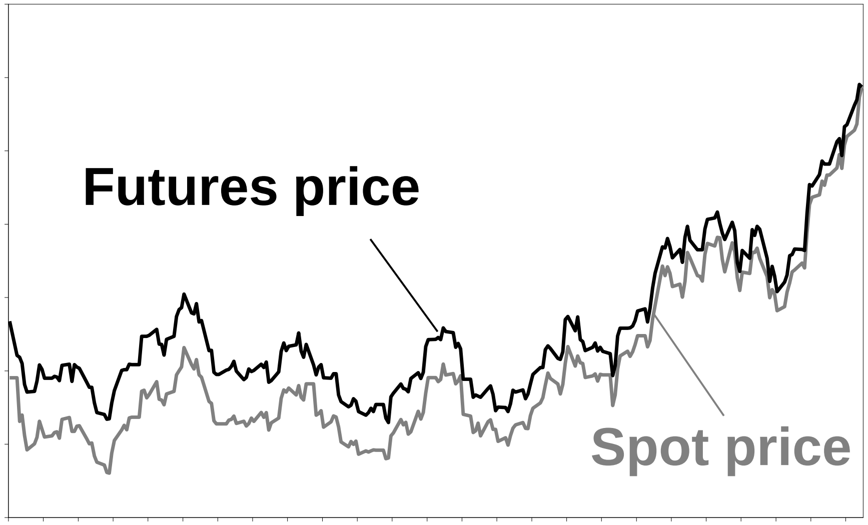
Practical:

- Since the expected future spot price is not observable, it is usually replaced by the **current spot price**.

Backwardation



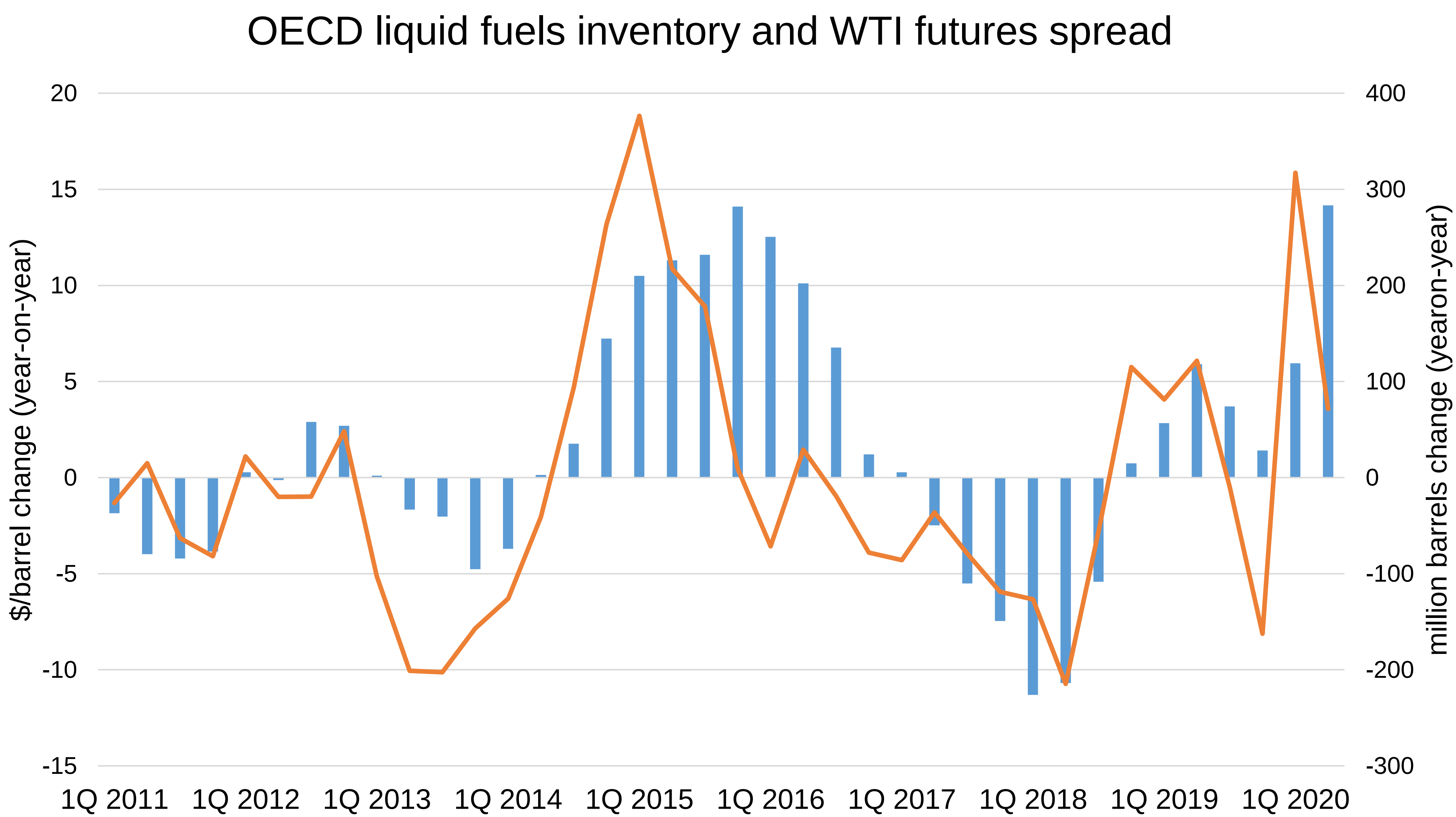
Contango



Contango and Backwardation Depend on Storage Costs and Convenience Yield

- Markets with **high storage costs** are in general more in **contango**:
  - Agricultural goods
  - …
- Markets with **high convenience yield** are in general more in **backwardation**:
  - Oil markets (approx. 60% of the time)
  - Energy
  - Metals
  - …
- Storage cost and convenience yield are subject to fluctuations over time.

Inventory Influence on Contango and Backwardation



Source: U.S. Energy Information Administration

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Swaps

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Options

- **Introduction**
- Option Payoffs
- Elementary Option Strategies
- Distribution-Free Characteristics

What is an Option?

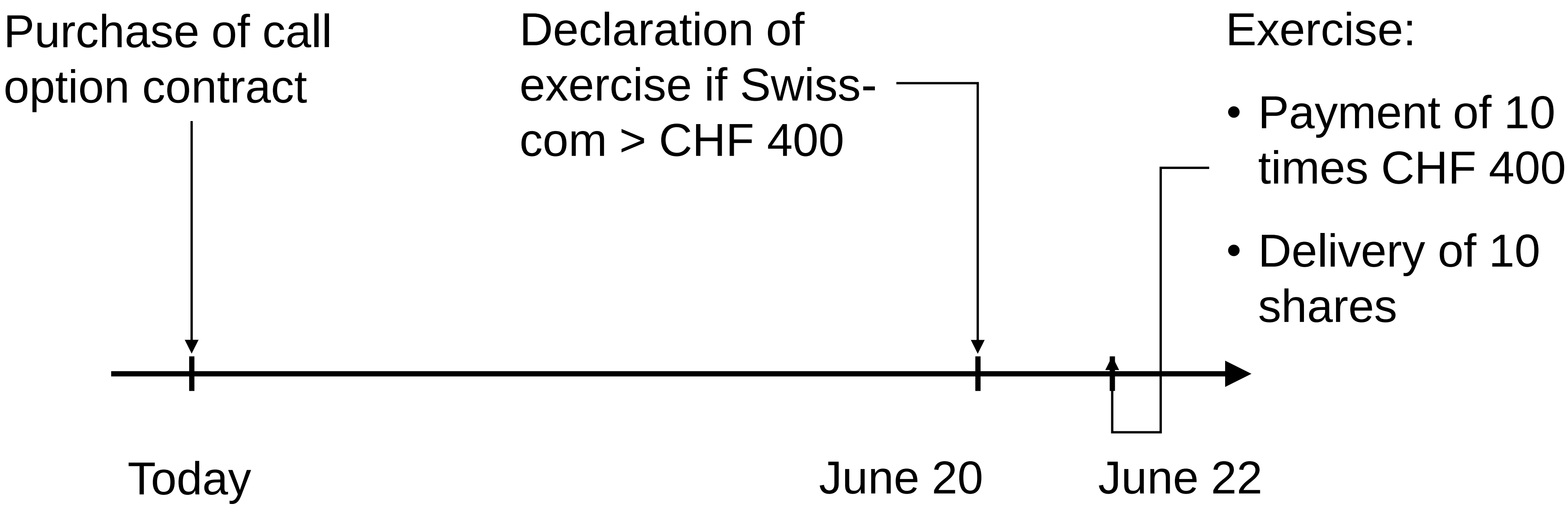
The holder of an option has…

... the *right* to buy or sell
... a predefined amount
... of a predefined asset
... for a predefined price
... on or until a predefined date in the future.

Example

- The holder of one call option contract (one contract consists of 10 options) on Swisscom with expiration date June 20 and a strike price of CHF 400...

  ➔ ...has the right to buy 10 shares of Swisscom CHF 400 each until June 20.



Purchase and Sale of an Option

Purchase of an option

The purchaser goes **long** in the option.

Sale of an option

The seller goes **short** in the option

=

The seller (writer) **writes** (sells) the option

Purchase and Sale of a Call Option

Long call

- The **buyer** of a call option **acquires the right to buy.**
  ➔ Purchaser acquires **right, but no commitment**.

Short call

- The **seller** of a call option **gives the buyer the right to buy.**
  ➔ The **seller is committed to sell** if the buyer decides to buy.

Purchase and Sale of a Put Option

Long put

- The buyer of a put option **acquires the right to sell.**

  ➔ Purchaser acquires **right, but no commitment.**

Short put

- The **seller** (writer) of a put option gives the buyer **the right to sell.**

  ➔ The **seller** is **committed** to buy if the buyer decides to sell.

Terminology (1/2)

Option price (premium)

- Price for the right to sell or buy an asset (paid by the buyer to the seller).

Strike price, exercise price

- The price at which the asset may be bought or sold in an option contract.

Exercise

- Holder of option communicates to writer the intention to exercise.

Expiration, expiry, maturity

- Last possible day to exercise (sometimes defined as day after last day).

European option

- Option that can be exercised only at maturity.

American option

- Option that can be exercised at any time during its life.

Underlying

- Asset on which the price of an option depends (e.g. stock, currency, index, etc.)

Terminology (2/2)

Intrinsic value

- Value of option if exercised immediately: for a call option, this is the greater of the excess of the asset price over the strike price or zero. For a put option, this is the greater of the excess of the strike price over the asset price or zero.

Time value

- Value of an option arising from the time left to maturity (equals the difference between the option price and the intrinsic value).

In-the-money

- Option with intrinsic value > 0

Out-of-the-money

- Option with intrinsic value = 0

At-the-money

- Value of underlying equals strike price

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Payoff and Profit of a Call Option (1/2)

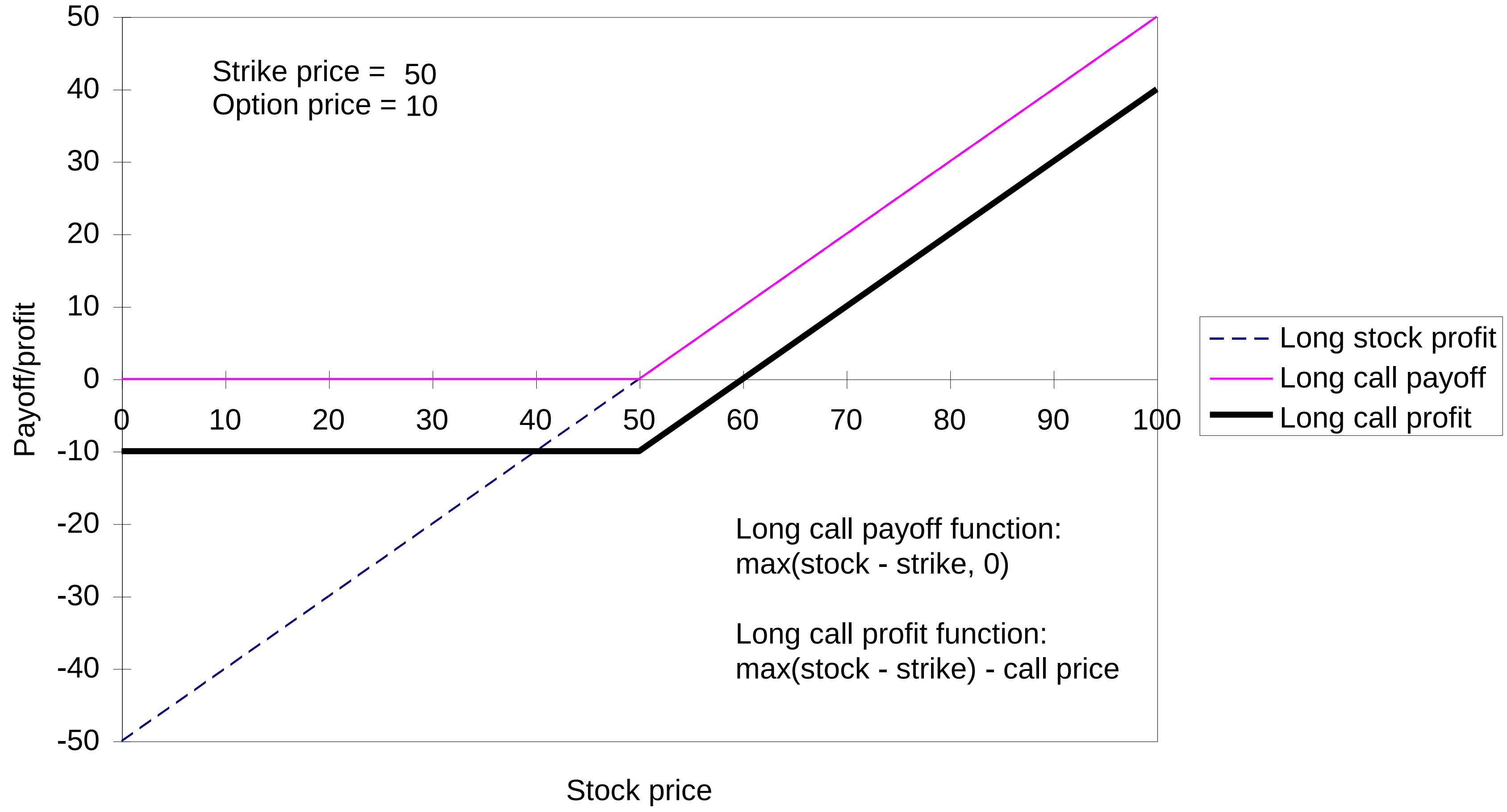
Initial data

- Underlying 50
- Strike price 50
- Option price 10

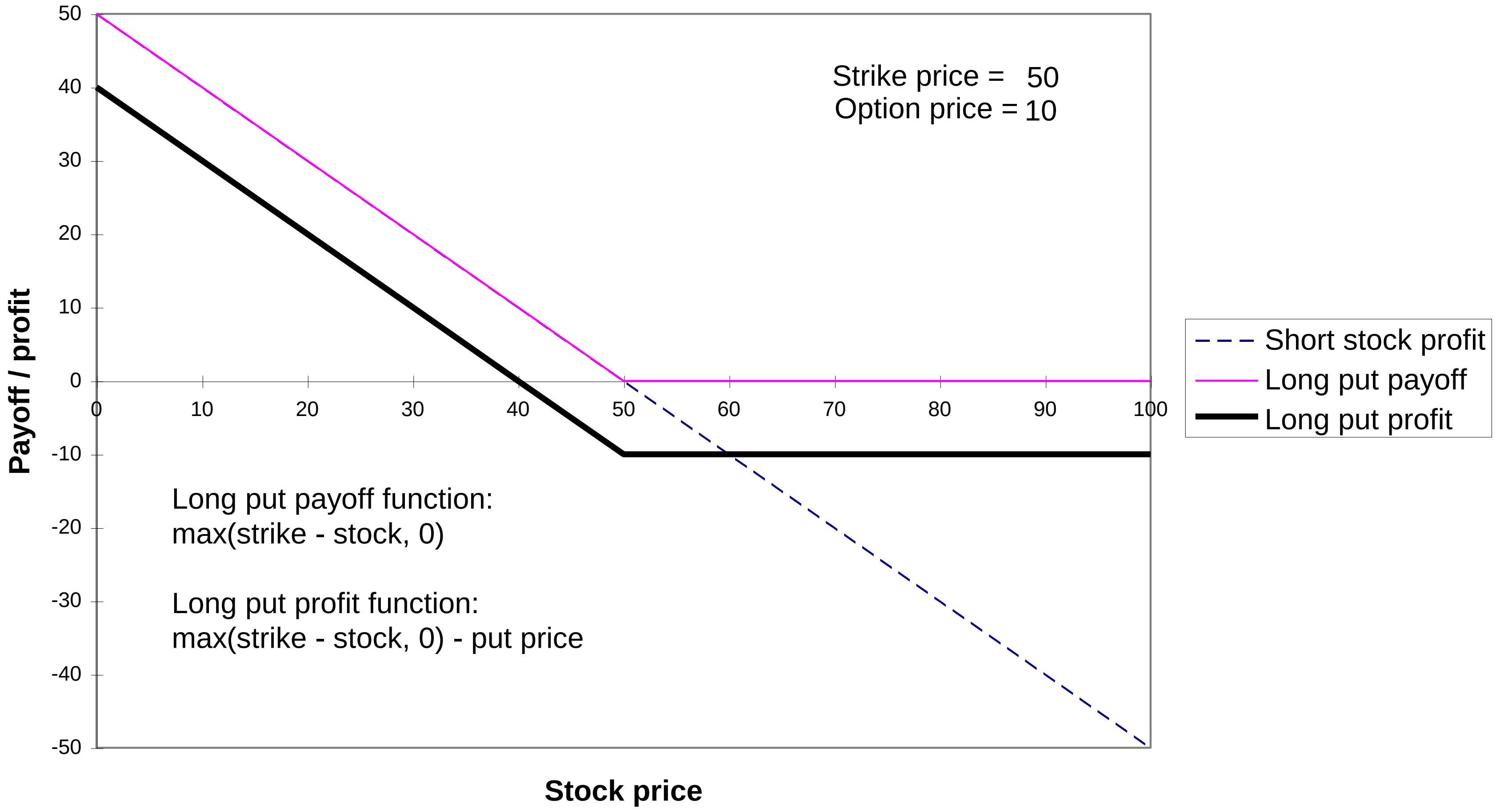
Payoff and profit of option at maturity

Scenarios for underlying	30	40	50	60	70	80	90
Strike price	50	50	50	50	50	50	50
Payoff (max(S-K,0))	0	0	0	10	20	30	40
Profit (payoff - price)	-10	-10	-10	0	10	20	30

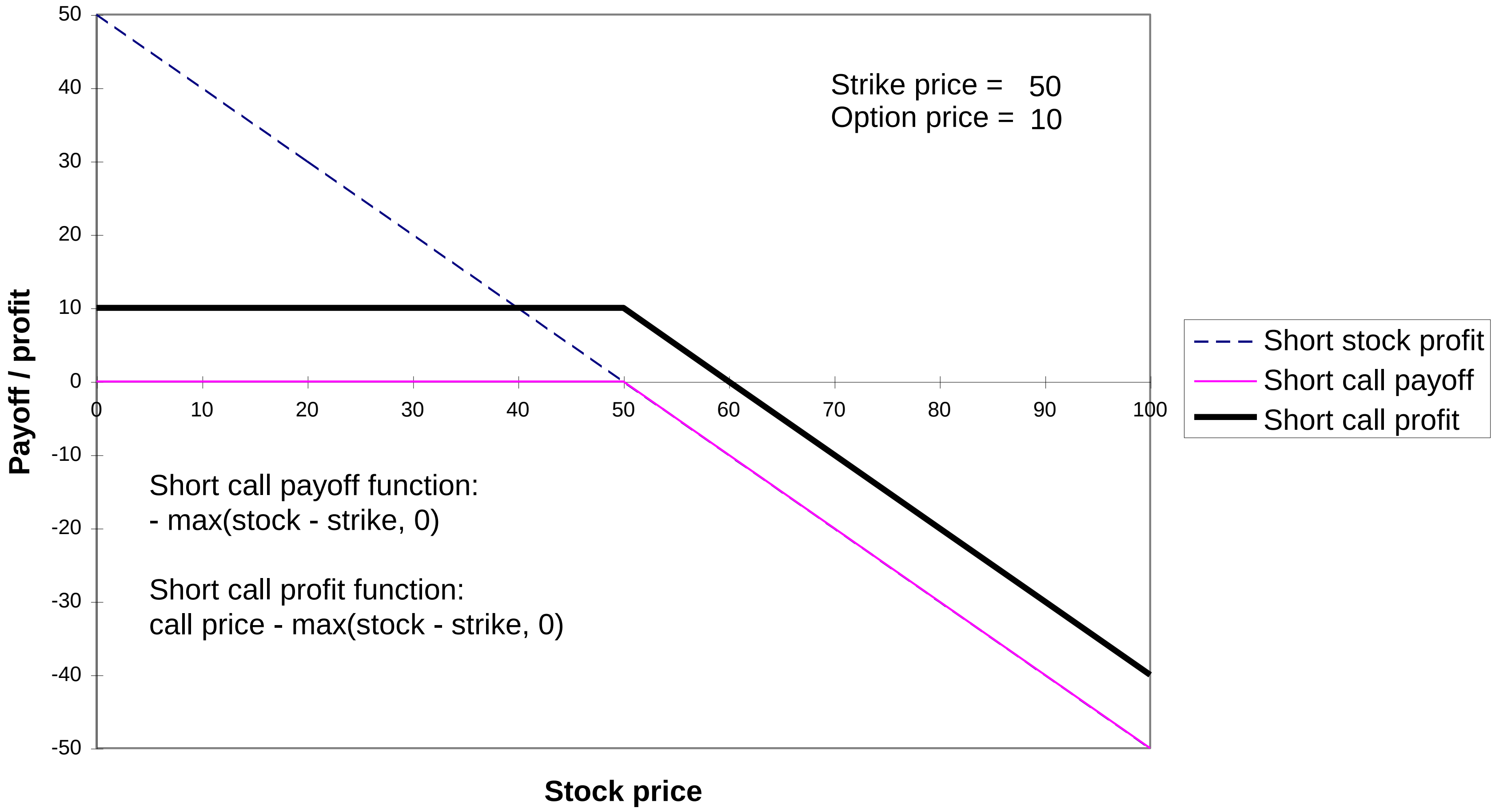
Payoff and Profit of a Call Option (2/2)



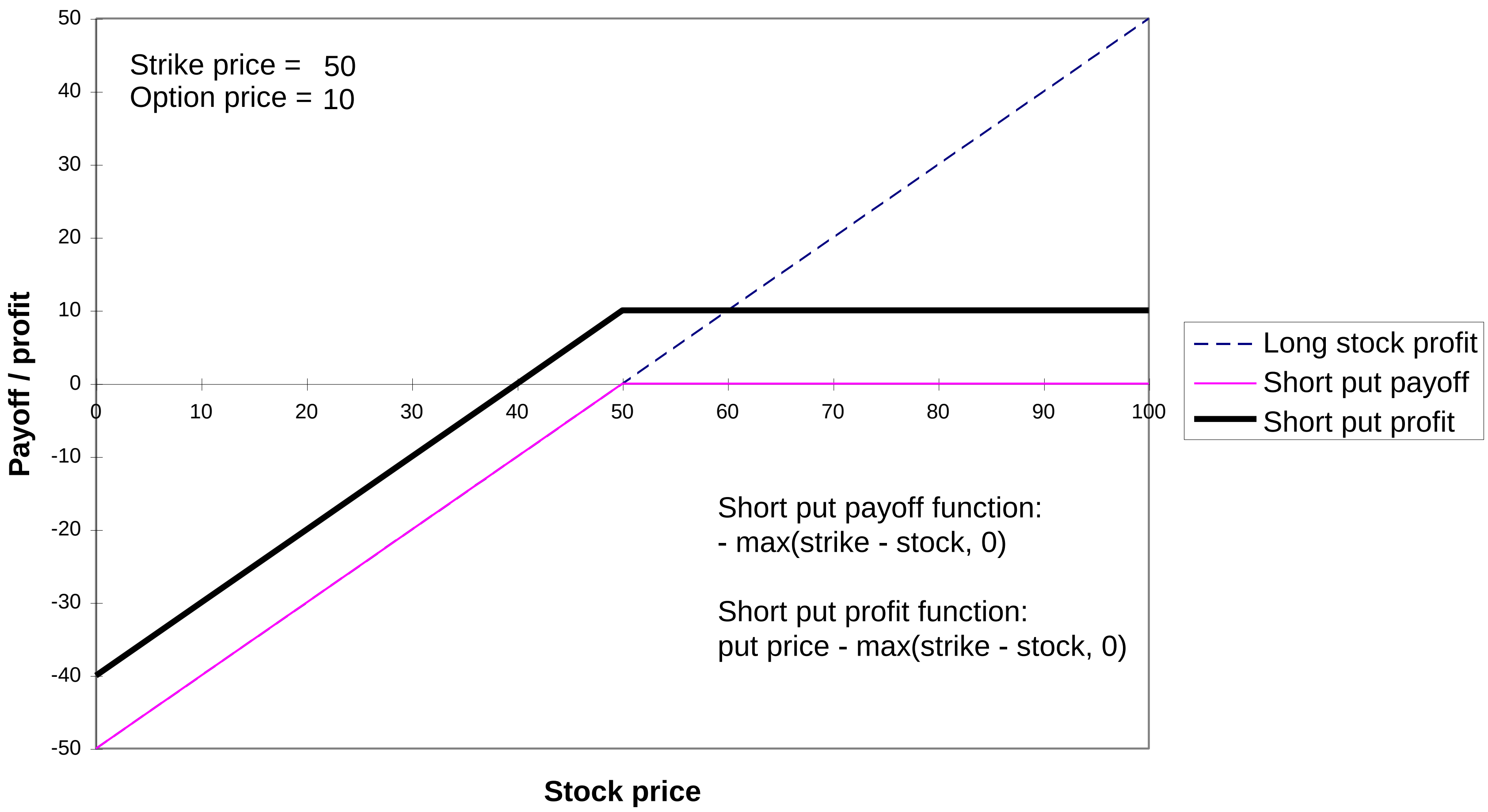
Payoff and Profit of a Put Option



Profit of a Short Call (Naked Call Writing)



Profit of a Short Put



Using Options: Speculation (1/2)

- Assume an investor wants to buy XYZ stock in September. He is expecting a price increase and wants to limit his exposure to 100 shares. The stock price as of today is at $100. A December at-the-money call option (strike X = $100) costs $5.

Comparison of the profit and loss with the two alternative strategies:

- Investment in 100 XYZ shares
- Investment in 100 options on XYZ stock

Using Options: Speculation (2/2)

	Strategy 1: Buy 100 shares for $10,000		Strategy 2: Buy 100 stock options for $500	
Stock price scenarios	*Profit / Loss*	*Return*	*Profit / Loss*	*Return*
Stock price drops to $80	-$2,000	-20%	-$500	-100%
Stock price remains constant at $100	$0	0%	-$500	-100%
Stock price rises to $120	$2,000	20%	$1500	300%

Using Options: Hedging (1/2)

An investor owns 500 ABC shares in August. The current ABC stock price is at $100. The investor is worried that the stock price might drop, yet still wants to benefit from share price appreciations

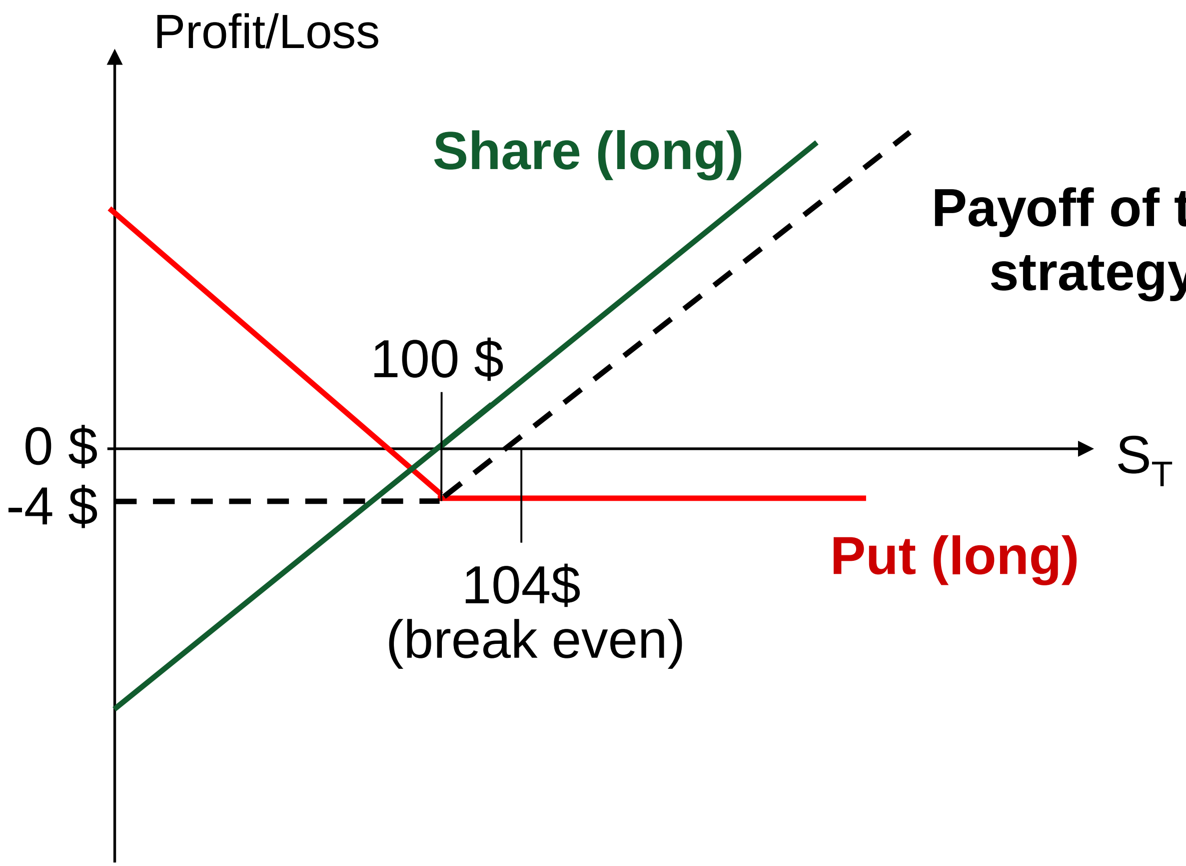
Strategy:

- Purchase of put options (e.g., October maturity) at the CBOE to sell 500 shares at a strike of $100.
- Since every put option on the CBOE requires the sale of 100 shares, the investor needs 5 contracts
- One put option costs $4
- Contract costs = 100 x $4 = $400
- Total hedging costs: 5 x $400 = $2,000

→ Protective put strategy!

Using Options: Hedging (2/2)

Profit and loss per share



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- **Elementary Option Strategies**
- Distribution-Free Characteristics

Option Strategies

Examples

- Covered call
- Protective put
- Bull spread
- Bear spread
- Straddle
- Strangle
- Butterfly spread

Covered Call

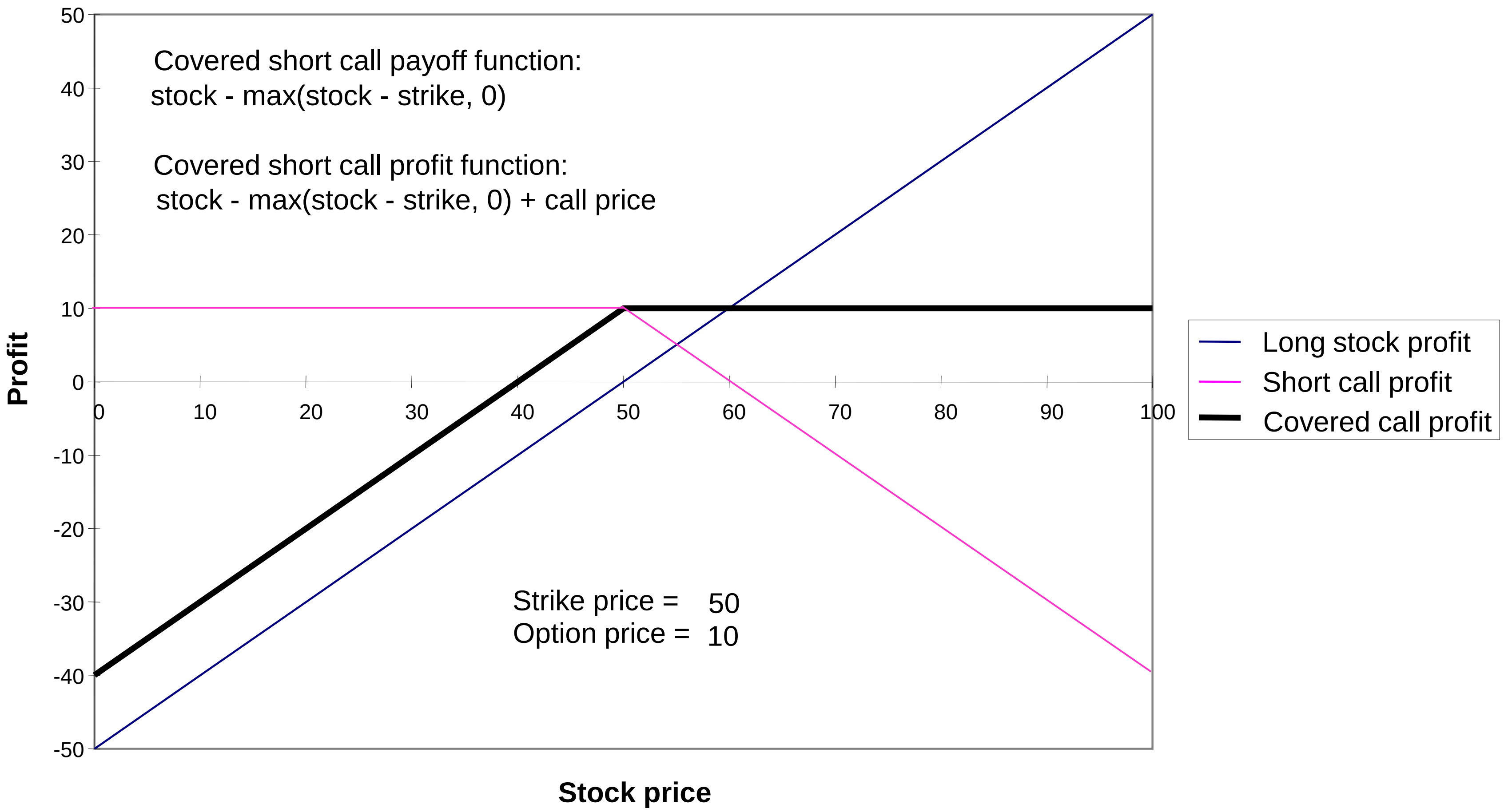
Composition

- Underlying + short call.

Characteristics

- Upward potential is given up in favor of option premium income. Upside potential is limited by the strike price of the short call option.
- Downside potential remains.

Covered Call Profit



Protective Put

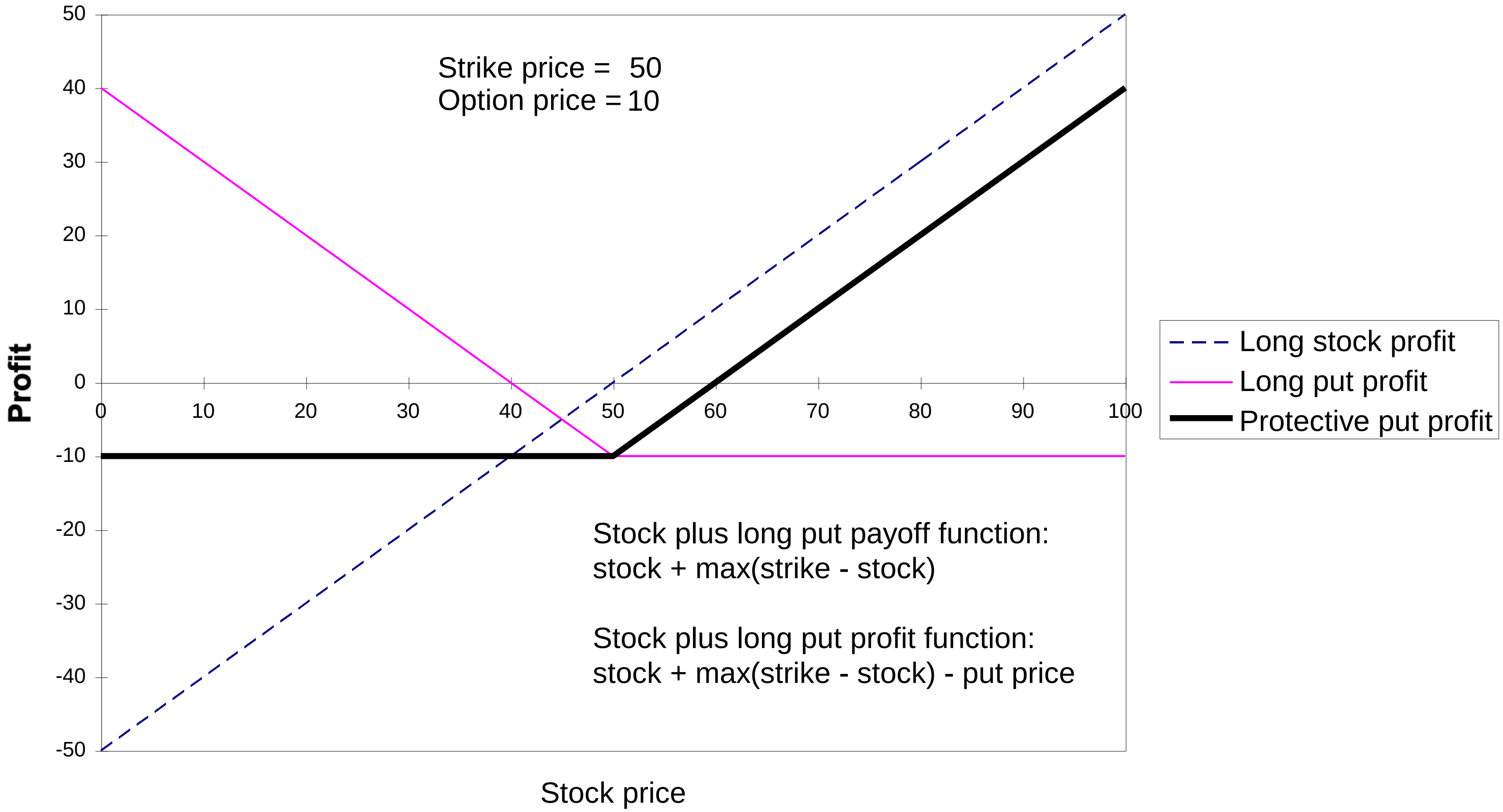
Composition

- Underlying + long put

Characteristics

- Downside potential eliminated by long put.
- Costs of this insurance: put premium.
- Beginning (in terms of the price of the underlying) of insurance is determined by strike price.

Protective Put Profit



Bull Spread

Composition

- Long call (low strike) + short call (high strike)

or

- Long put (low strike) + short put (high strike)

Characteristics

- Long position with cap and floor
- Caps and floors are determined by strike price
- Risk of position increases with distance between the two strike prices

Bull Spread Example

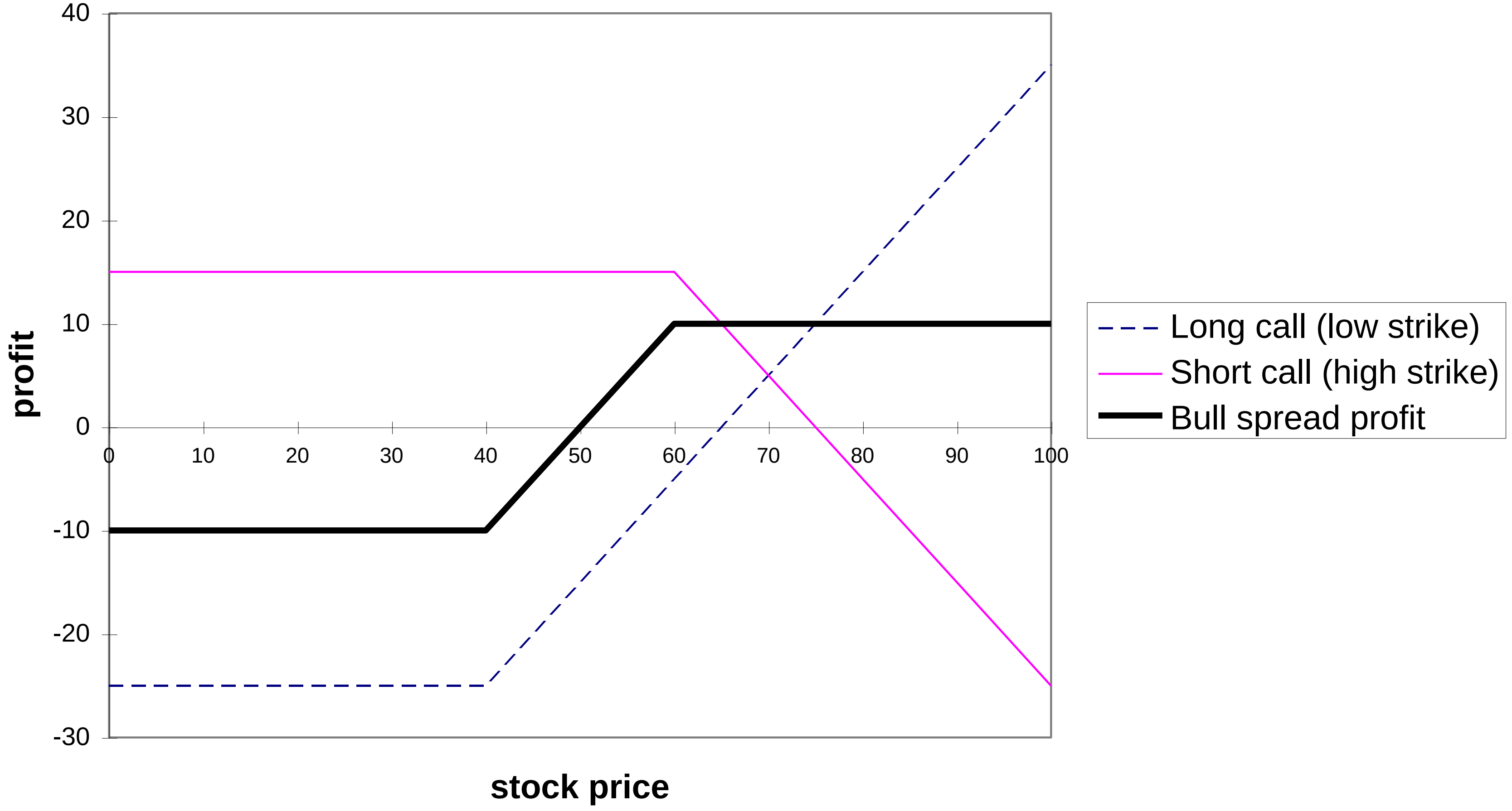
Initial data

• Underlying		50	
• Call	1	Strike	40
		Price	25
• Call	2	Strike	60
		Price	15

Bull Spread at maturity

Underlying	30	40	45	50	55	60	70	80
Payoff call 1	0	0	5	10	15	20	30	40
Profit call 1	**-25**	**-25**	**-20**	**-15**	**-10**	**-5**	**5**	**15**
Payoff short call 2	0	0	0	0	0	0	-10	-20
Profit short call 2	**15**	**15**	**15**	**15**	**15**	**15**	**5**	**-5**
Spread	**-10**	**-10**	**-5**	**0**	**5**	**10**	**10**	**10**

Bull Spread Profit with Calls



Bear Spread

Composition

- Short call (low strike) + long call (high strike)

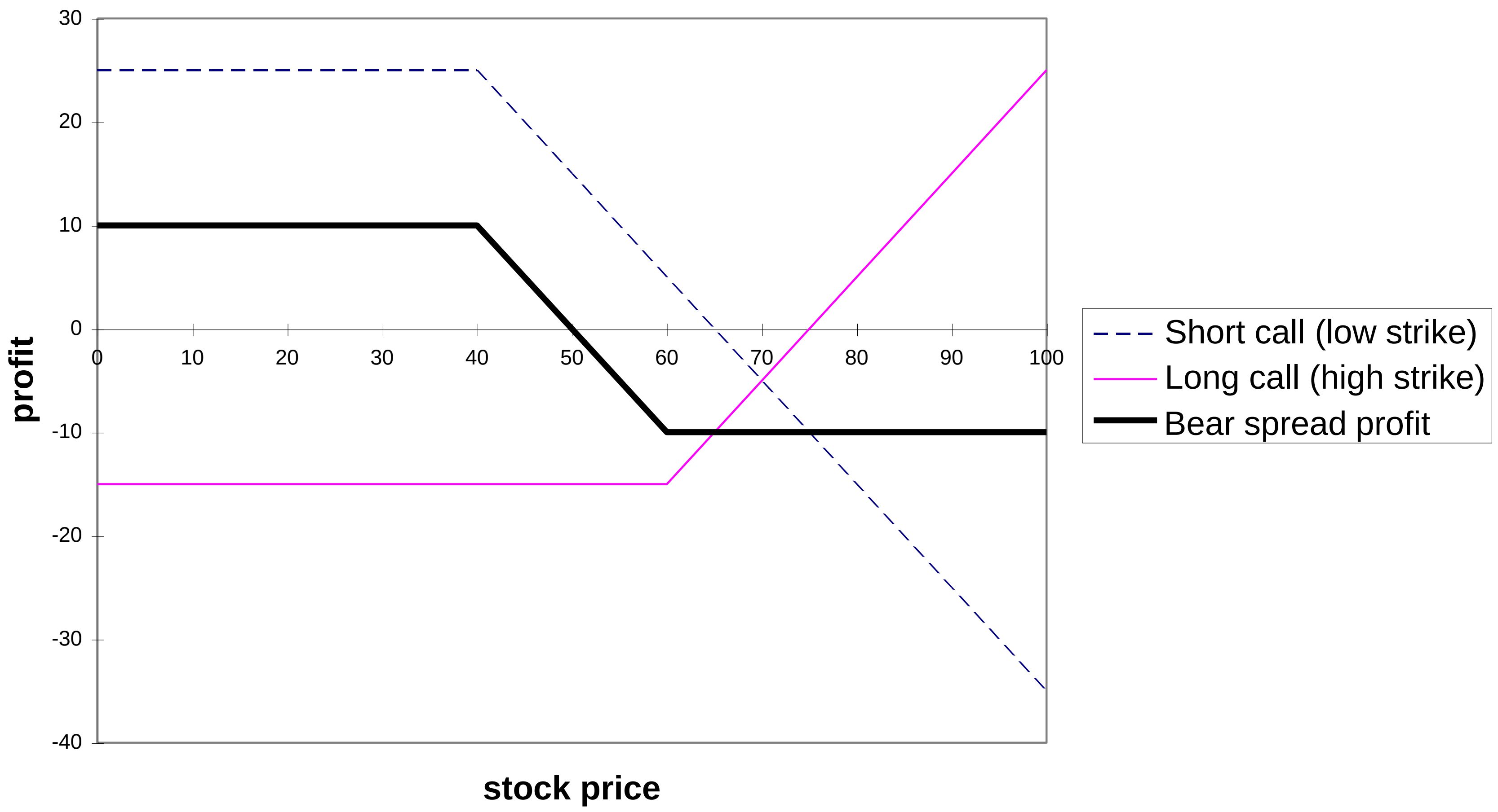
or

- Short put (low strike) + long put (high strike)

Characteristics

- Short position with cap and floor.
- Caps and floors are determined by the strike prices.
- Risk of position increases with distance between the two strike prices.

Bear Spread Profit with Calls



Straddle

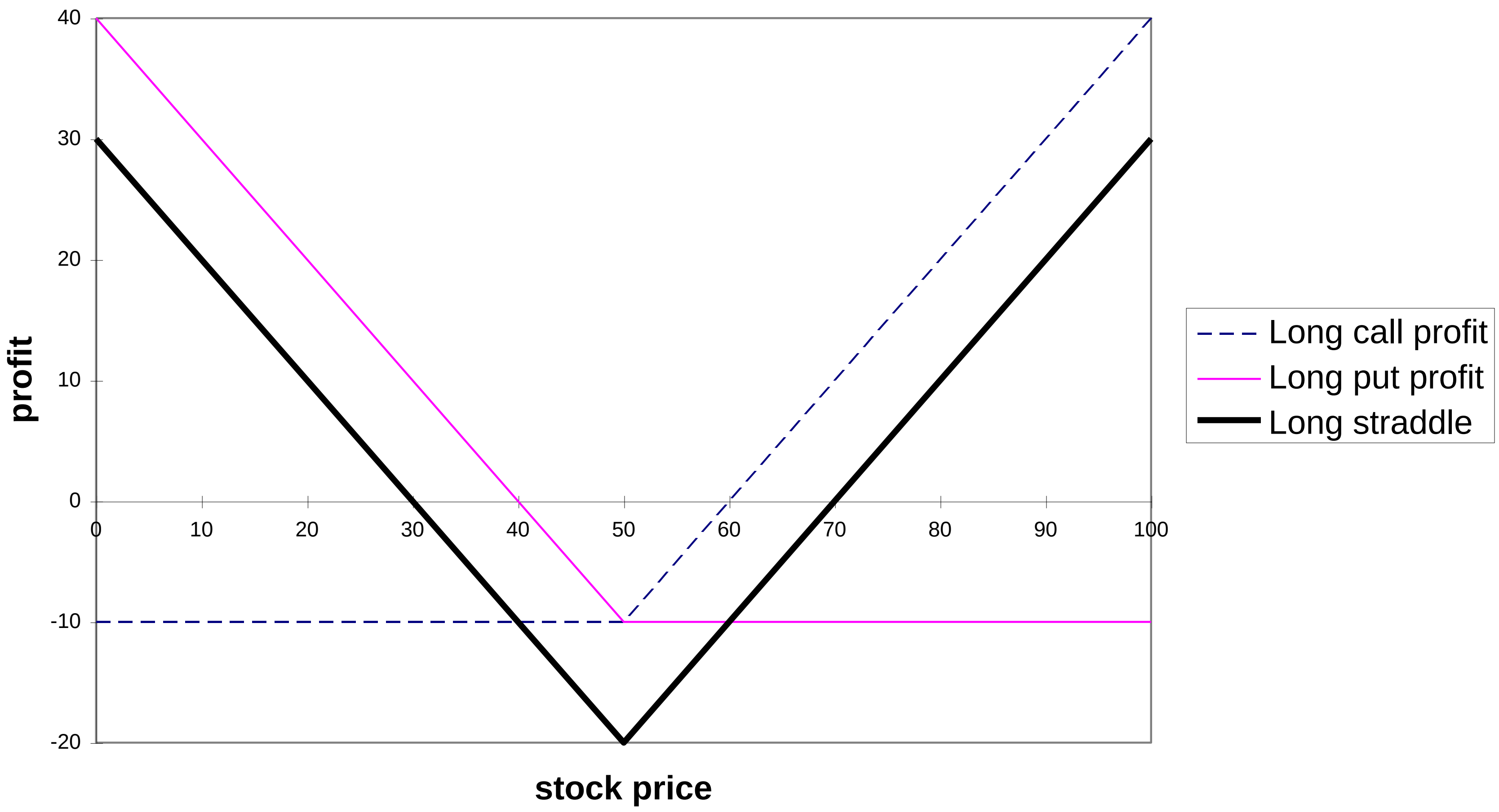
Composition

- Call + put with same strike price

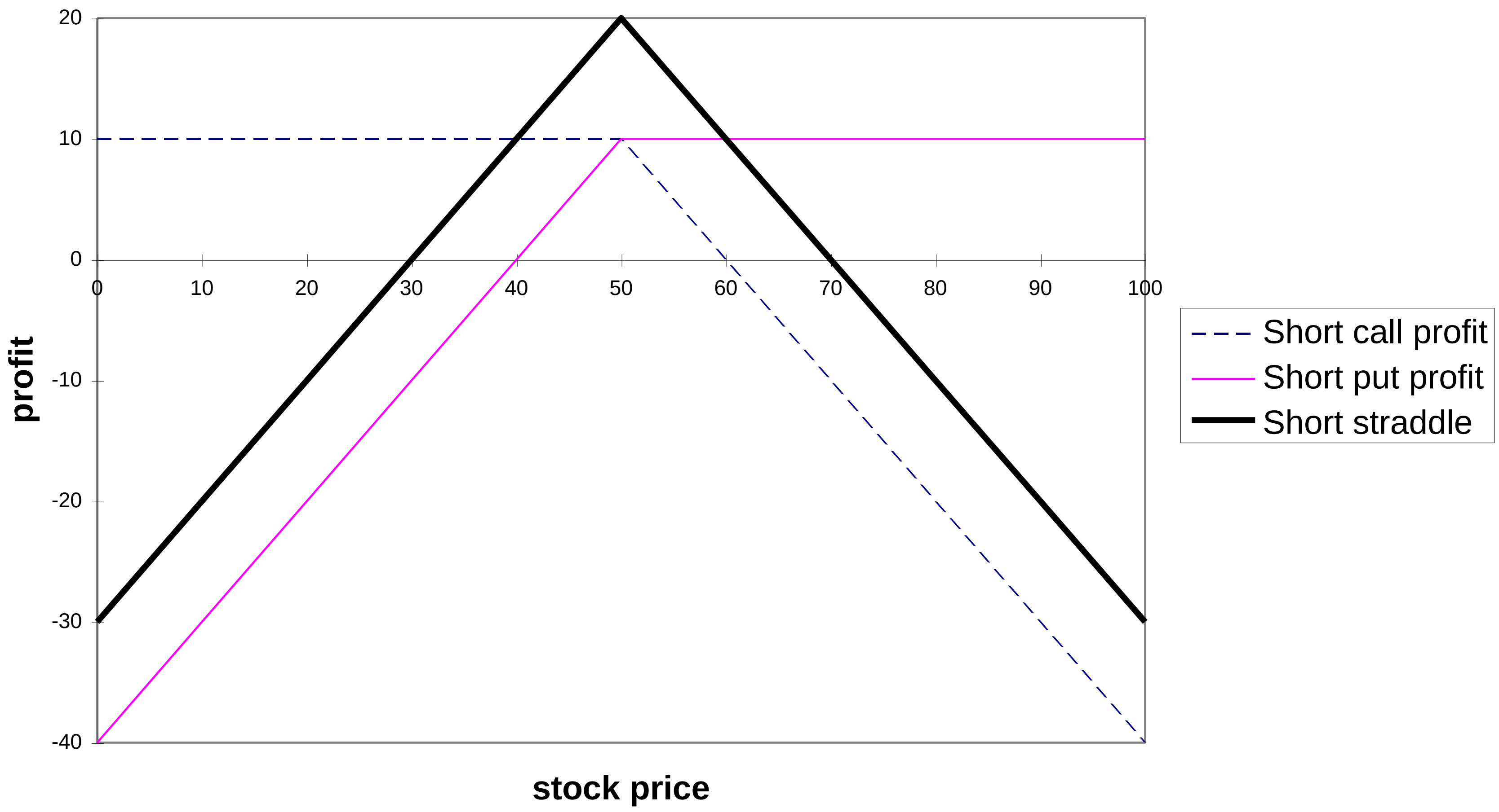
Characteristics

- Upside and downside potential ➔ speculation on volatility.
- Short: Profit equal to option prices when stock price remains close to strike price (speculation on (very) low volatility).
- Long: costly, since call and put have to be bought.
- Short: premium income from sale of call and put.
- Long: loss cannot be higher than investment in position.
- Short: unlimited loss potential.

Long Straddle Profit



Short Straddle Profit



Long Butterfly Spread

Composition

- 1 call (high strike) + 1 call (low strike) + 2 short calls (middle strike)

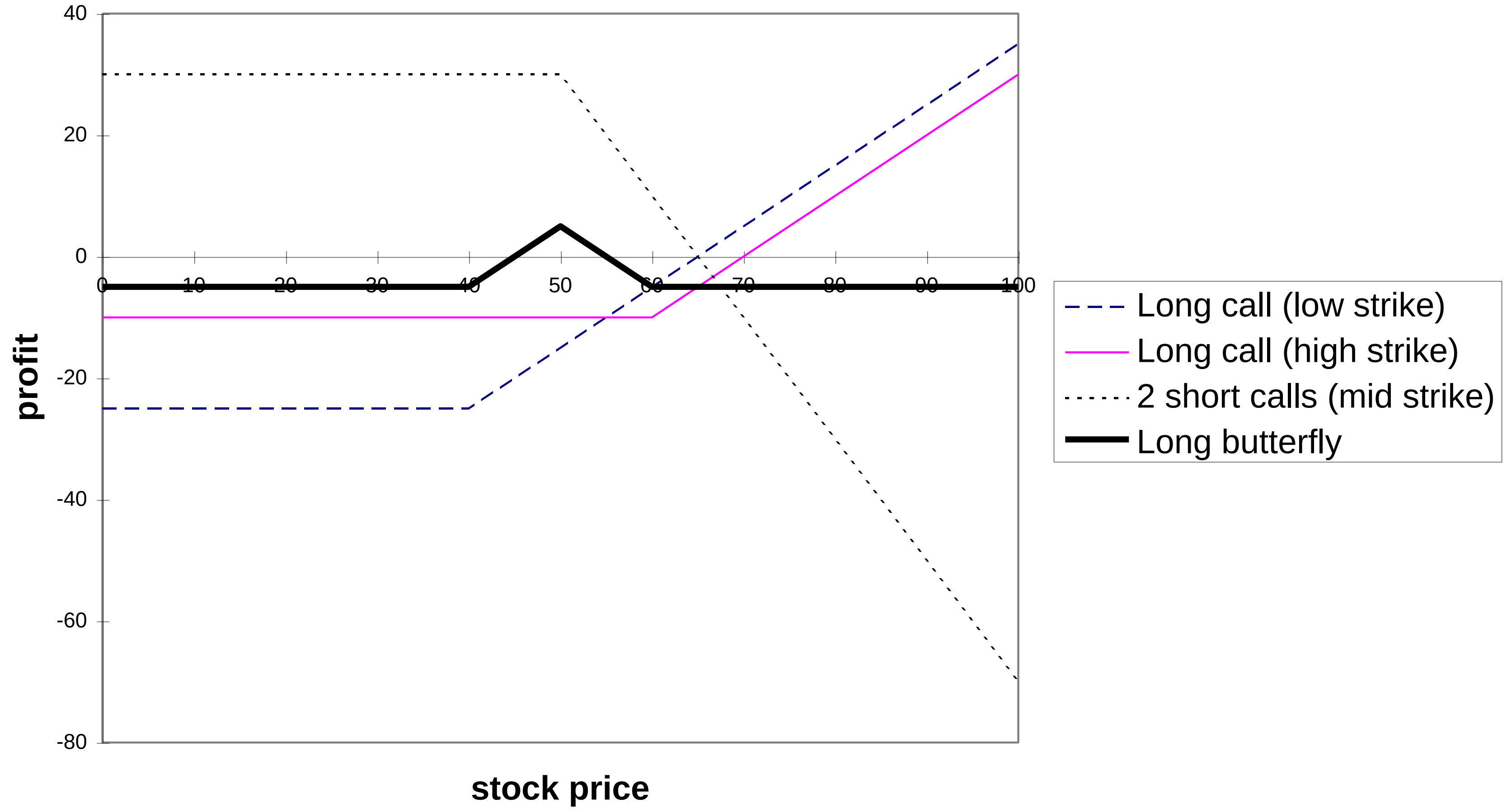
or

- 1 put (high strike) + 1 put (low strike) + 2 short puts (middle strike)

Characteristics

- Profit in some interval.
- Profit interval determined by strike prices.
- Loss potential limited.
- The strategy is relatively cheap to implement, except for transaction costs.

Long Butterfly Spread



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Put-Call Parity for European Options

Put-call parity formulas

$$c = S - \frac{K}{R} + p - D \qquad \text{or} \qquad c = \frac{F - K}{R} + p$$

c Price of European call
p Price of European put
R Interest factor
F Forward price

S Price of underlying
K Strike price
D Present value of dividend on underlying

➔ Price of put follows from price of call.
➔ Price of call follows from price of put.

Proof of Put-Call Parity

Value at expiration

- Stock + European put

Stock	S
Put	max (K - S, 0)
Stock + put	max (K, S)

- Money market investment + European call

Money market	K
Call	max (S - K, 0)
Money + call	max (S, K)

➔ max (K, S) = max (S, K)

➔ Both portfolios have an equal value on and before maturity.

Put-Call Parity – Example

Initial data

S = 1,000 R = 1.05 (Interest rate r = 5%)

K = 900 p = 57.14

$$c = S - \frac{K}{R} + p = 1{,}000 - \frac{900}{1.05} + 57.14 = 200$$

At maturity

Scenarios for S	800	900	1000	1100	1200
Call (K=900)	0	0	100	200	300
Put (K=900)	100	0	0	0	0
Bank account	900	900	900	900	900
Underlying + put	**900**	**900**	**1000**	**1100**	**1200**
Bank account + call	**900**	**900**	**1000**	**1100**	**1200**

Put-Call-Forward Parity (1/4)

$$c(K=F)=p(K=F)$$

c	Price of European call	p	Price of European put
K	Strike price	F	Forward price

$$c(K=F)-p(K=F)=\text{long forward}$$

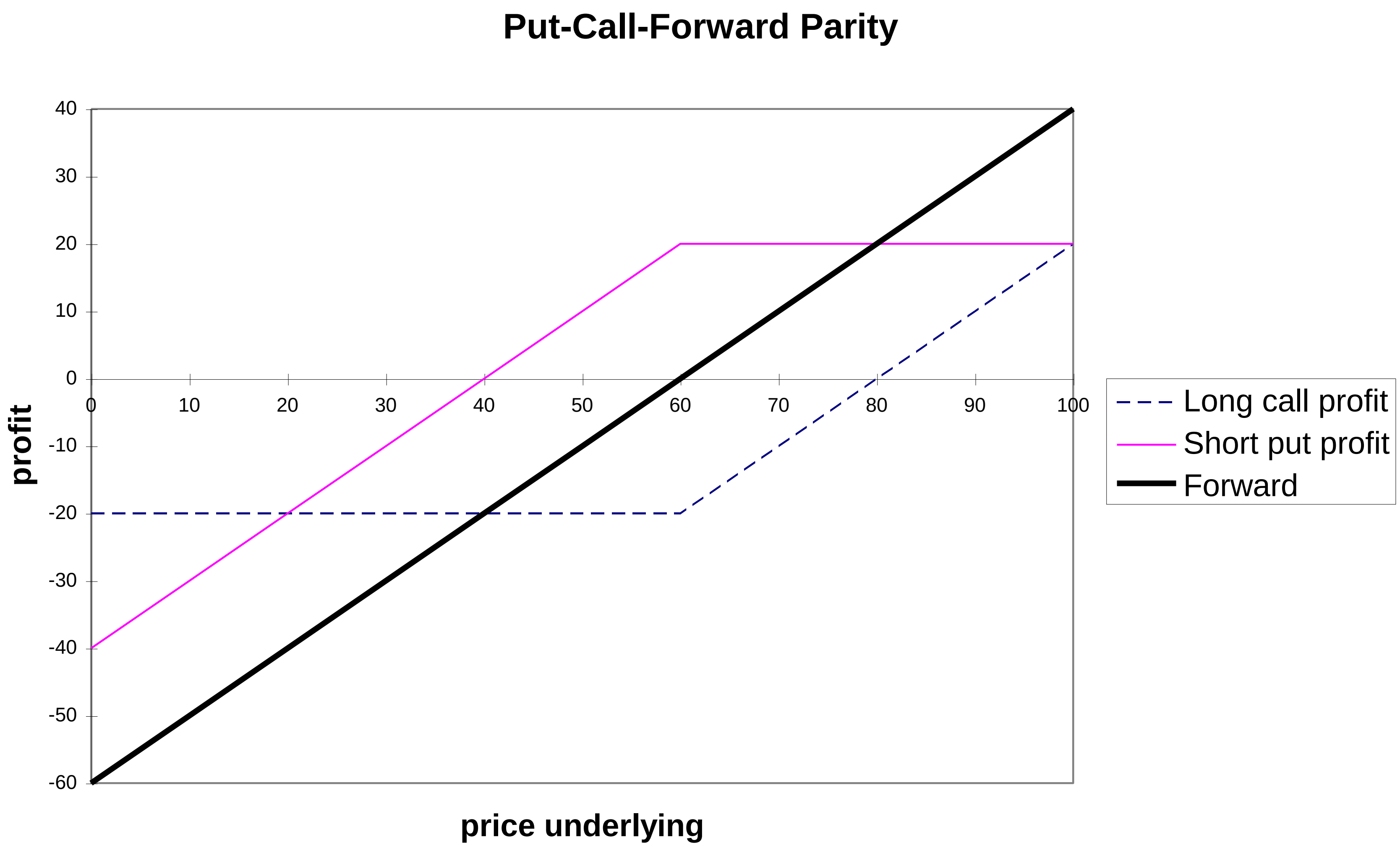
➔ Call - put = synthetic long forward contract

➔ - Call + put = synthetic short forward contract

➔ Call - long forward = call + short forward = synthetic put option

➔ Forward + put = synthetic call

Put-Call-Forward Parity (2/4)



Put-Call-Forward Parity (3/4)

Portfolio consisting of

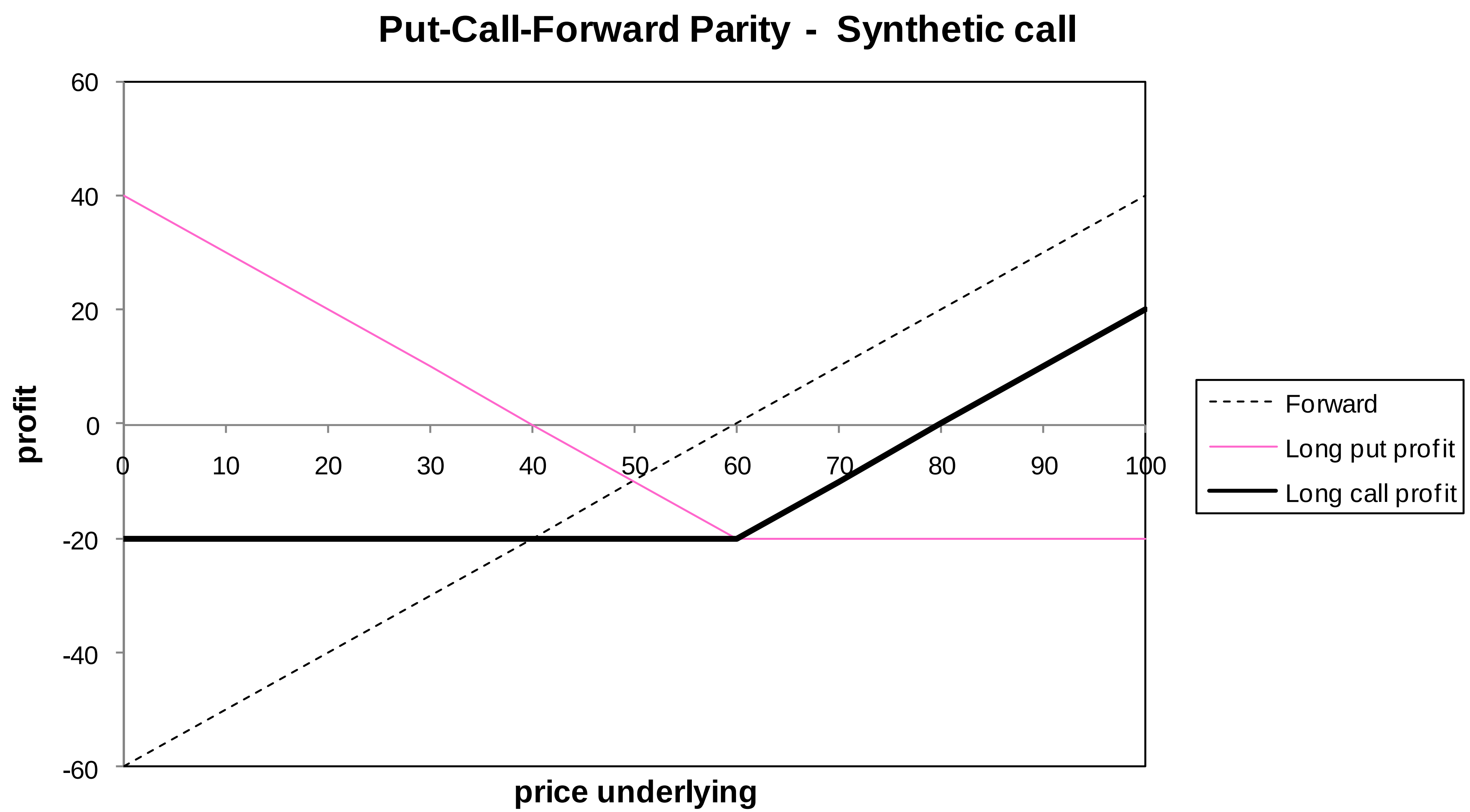
	Strike	Price
Forward	60	0
Put	60	20

At maturity

Underlying	30	40	50	60	70	80	90	100
Forward profit	-30	-20	-10	0	10	20	30	40
Put profit	10	0	-10	-20	-20	-20	-20	-20
Total	-20	-20	-20	-20	-10	0	10	20

➔ **The “Total” corresponds to the profit from a call priced at 20 with a strike price of 60.**

Put-Call-Forward Parity (4/4)



Example of Arbitrage Transaction

Market prices	Strike	Price
Forward	60	0
Put	60	20
Call	60	25

Underlying	30	40	50	60	70	80	90	100
Long synthetic call	-20	-20	-20	-20	-10	0	10	20
Short call	25	25	25	25	15	5	-5	-15
Total	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**

➔ Profit of 5 regardless of underlying price at maturity.

➔ Riskless profit of 5 (arbitrage profit).

Price Behavior of Options

- Time value is highest when option is at-the-money.

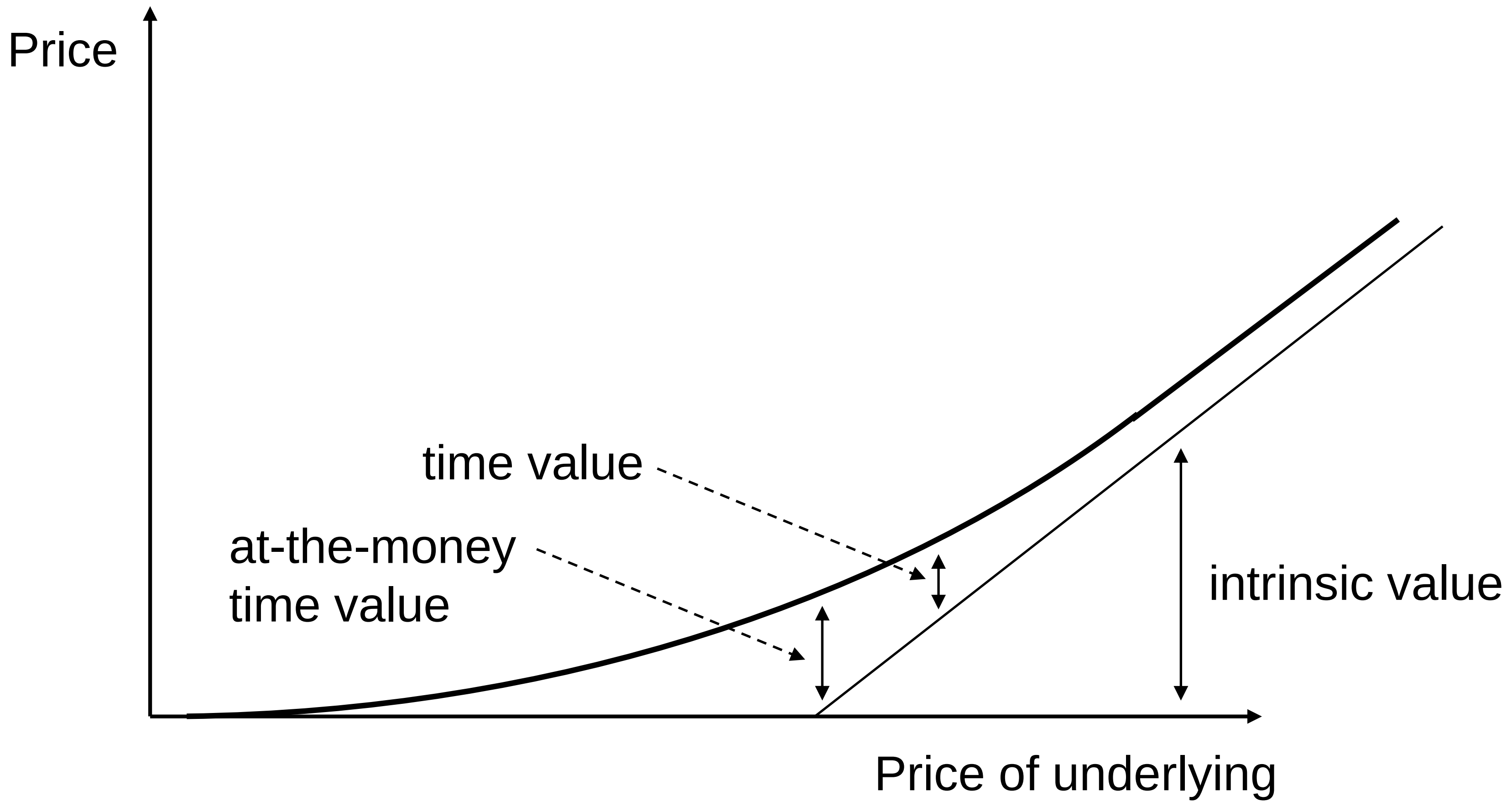
Time value decreases as option goes out-of-the-money

- Probability of exercise decreases.

Time value decreases as option goes in-the-money

- The more in-the-money, the more similar to the underlying.
- Advantages of option (limited risk, small initial capital) disappear.

Price Behavior of a Call

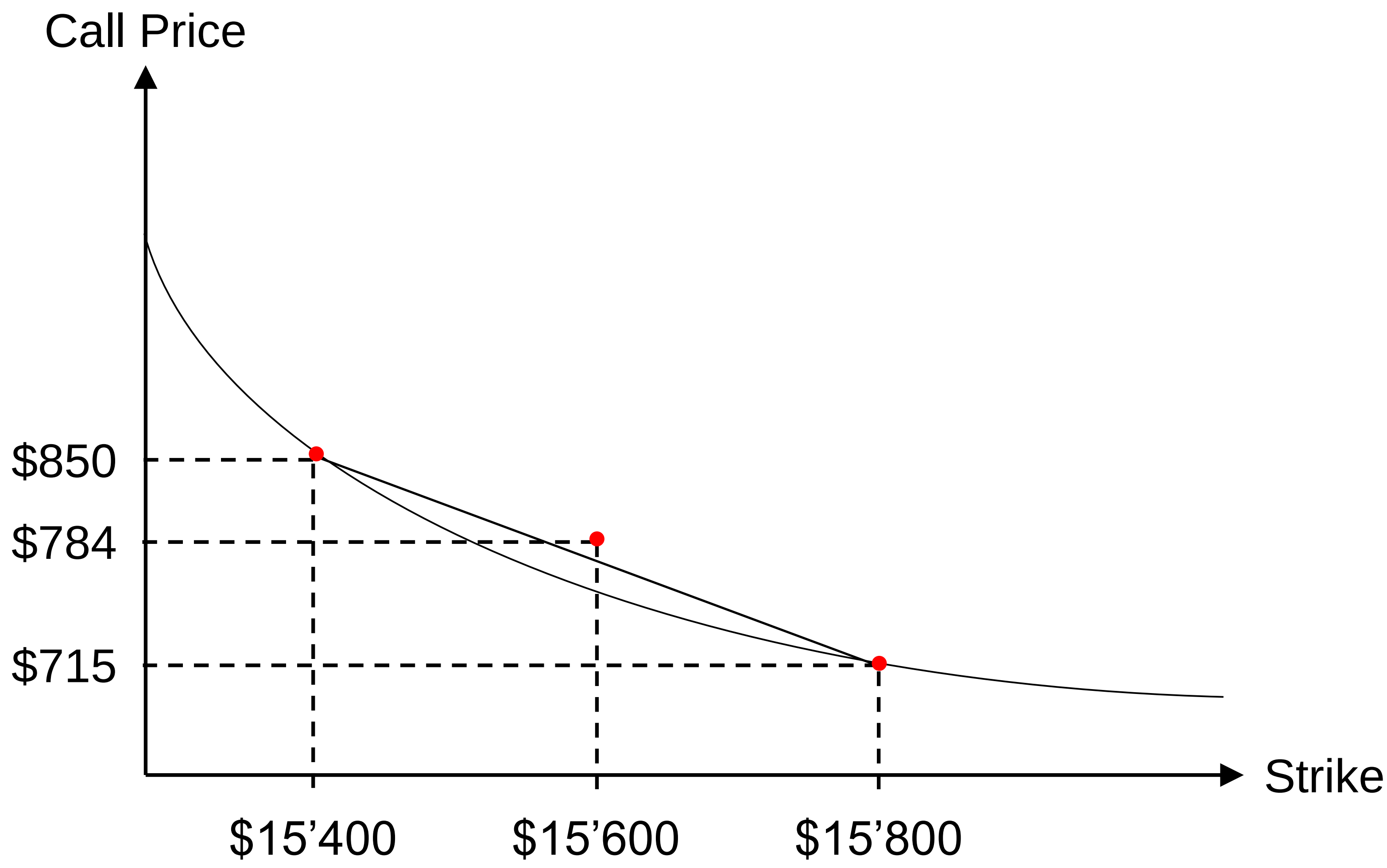


Factors Affecting Option Prices

	European call	European put	American call	American put
Price of underlying	+	-	+	-
Strike price	-	+	-	+
Dividends	-	+	-	+
Interest rate	+	-	+	-
Time to maturity	+ (no dividends)	depends	+	+
Volatility	+	+	+	+

Convexity of Option Prices

What if option prices are not convex?



American Call Options: Early Exercise (1/2)

Early exercise of an American call on stock without dividends

- American calls on a non-dividend-paying stock are never exercised prior to maturity. Why? Consider the following cases:
- Exercise option and keep stock?
  - Instead of exercising, keep option, exercise at maturity and pay strike price then (money earns interest in the meantime).
  - If option later goes out-of-the-money, not exercising is clearly superior.
- Exercise option and sell stock?
  - Instead, short stock and keep option, and earn interest on proceeds from shorting the stock.
  - Or sell a forward on stock and keep option.

American Call Options: Early Exercise (2/2)

Conclusion (non-dividend-paying underlying)

➔ American call options on stock without dividend payments during the lifetime of the option are never exercised prior to maturity.

➔ American call (C) = European call (c) (on a non-dividend-paying underlying).

Early exercise of an American call on underlying with dividends

- American calls on a dividend-paying stock may be exercised prior to maturity. Under what conditions?

➔ Early exercise if expected price drop from dividend payment exceeds time value of the call option.

➔ Call options are exercised only immediately *before* dividend payments.

American Put Options: Early Exercise

Early exercise of an American put on stock without dividends

- In contrast to American calls, it can be optimal to exercise an American put on a non-dividend-paying underlying prior to maturity.
- Pricing model necessary to determine exact exercise time.

➔ American put (P) > European put (p) (on a non-dividend-paying underlying)

Early exercise of an American put on stock with dividends

- Dividends decrease the value of the stock and thus increase the value of a put option.

➔ If early exercise not optimal without dividends, then not optimal with dividends either.

➔ Put options are exercised only immediately *after* dividend payments.

Equity and Debt are Options

Setting

- V_t is the value of a firm's total assets (Equity + Debt).
- All debt (D = face value) will terminate at t=1.
- Therefore at t=1:

If $V_1 \geq D$: The firm pays off all debt at t=1.
The remaining value of equity is $V_1 - D$.

If $V_1 < D$: Debtholders get the firm (all assets).
The value of equity is 0.

Resulting values at t=1:

- Value of equity $= \overbrace{\max(V_1 - D, 0)}^{=c}$
- Value of debt $= \min(D, V_1)$

$$= D - \underbrace{\max(D - V_1, 0)}_{=p}$$

- The pay-off of equity equals the pay-off of a call option on V with strike price D (=c).
- The pay-off of debt equals the face value of Debt (=D) minus the pay-off of a put option (=p).

Implications on Firm

Equity owners hold long call position

- Higher volatility increases the value of equity. A diversification across industries or markets (e.g. a conglomerate) is unfavorable for equity investors («diversification discount»).
- The value of equity is linked to the value of debt via put-call-parity.
- Unexpected cash-flows to shareholders are favorable for equity holders. The value of the call option decreases by less than the cash-flow.

Debtholders hold short put position

- Higher volatility decreases the value of debt. A debtholder wants the firm to take as little risk as possible.
- The value of debt is linked to the value of equity via the put-call-parity.
- Option pricing models can be used to determine the default risk of debt («Merton-Model»)
- Unexpected cash-flows to shareholders increase the value of the put and thus decrease debtholders' value.