EASC 605 Project 3: Modelling glacier drainage
Due: 2 December 2021

NOTE: Katie should skip question 3. Tim should start with question 3 and see how the model progresses; it might suffice to make Question 3 the whole assignment and skip Questions 1–2.

1. **Hydraulic potential**
Calculate the spatially distributed subglacial hydraulic potential using the surface and bed DEMs of Glacier 1 for (a) $P_\mathrm{w} = P_\mathrm{atm}$, (b) $P_\mathrm{w} = 0.5P_\mathrm{i}$, and (c) $P_\mathrm{w} = P_\mathrm{i}$ without using in-built software functions. Perform an initial quality check of the DEMs, including, for example, scanning for negative values of ice thickness. Plot the hydraulic potential distributions in some useful way with contours, or with vectors/streamlines that indicate the hydraulic potential gradient. Use units of MPa. Plot a longitudinal profile and several transverse profiles of the glacier surface and bed, along with the height to which water would rise in boreholes drilled to the glacier bed along these profiles for cases (a)–(c) above. Units should be in m above sea level. Create one figure per profile, with surface, bed and borehole water levels for (a)–(c) plotted together. Describe your methods (including equations) and explain your results.

Symbol	Description	Value	Units
ρ_i	Density of ice	917	$\mathrm{kg\,m^{-3}}$
ρ_w	Density of water	1000	$\mathrm{kg\,m^{-3}}$
g	Acceleration due to gravity	9.81	$\mathrm{m\,s^{-2}}$
L	Latent heat of fusion (water/ice)	3.34×10^5	$\mathrm{J\,kg^{-1}}$
c_w	Specific heat capacity of water	4.217×10^3	$\mathrm{J\,kg^{-1}\,K^{-1}}$
c_t	Pressure-melting coefficient	7.5×10^{-8}	$\mathrm{K\,Pa^{-1}}$
A	Flow law coefficient for isothermal ice	2.4×10^{-24}	$\mathrm{Pa^{-3}\,s^{-1}}$
n	Flow law exponent	3	
f_R	Darcy-Weisbach friction coefficient	0.15	

Table 1: Useful constants and parameters.

2. **Röthlisberger channels**
In a 1-D flow-following coordinate system defined by s, conservation of ice mass leads to

$$\frac{\partial S}{\partial t} = \frac{m_\mathrm{i}}{\rho_\mathrm{i}} - 2\,S\,A\left(\frac{p_\mathrm{i} - p_\mathrm{w}}{n}\right)^n, \tag{1}$$

and water mass to

$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial s} = \frac{\dot{m}_\mathrm{i} + \dot{b}}{\rho_\mathrm{w}}, \tag{2}$$

with conduit cross-sectional area S, time t, conduit wall melting rate $\dot{m}_\mathrm{i}$, ice overburden pressure p_i, water pressure p_w, ice flow-law rate factor A, Glen's n, discharge Q and external water-source term $\dot{b}$. The conduit wall melting rate is

$$m_\mathrm{i} = \frac{Q}{L}\left[-\rho_\mathrm{w}\, g\, \frac{\partial z_\mathrm{b}}{\partial s} - (1-\gamma)\,\frac{\partial p_\mathrm{w}}{\partial s}\right], \tag{3}$$

where $\gamma = c_t\, \rho_\mathrm{w}\, c_\mathrm{w}$ (see Table 1), z_b is the bed elevation and turbulent flow velocity

$$\bar{v} = -\left(\frac{r}{f_r\, \rho_\mathrm{w}}\right)^{1/2} \frac{\partial \phi}{\partial s}\left|\frac{\partial \phi}{\partial s}\right|^{-1/2}, \tag{4}$$

with conduit radius r, friction factor f_R, fluid potential gradient

$$\frac{\partial \phi}{\partial s} = \frac{\partial p_\mathrm{w}}{\partial s} + \rho_\mathrm{w}\, g\, \frac{\partial z_\mathrm{b}}{\partial s}, \tag{5}$$

and discharge

$$Q = \bar{v}\, S. \tag{6}$$

See constants in Table 1.

(a) Use the equations above to solve analytically for the steady-state discharge in a subglacial channel as a function of pressure- and elevation gradients along the channel ($\frac{\partial p_\mathrm{w}}{\partial s}$ and $\frac{\partial z_\mathrm{b}}{\partial s}$). Eliminate S and v from the analytical equation and present your analytical expression for Q in as simple a form as possible. Check with at least one other student to see that you arrived at equivalent results. Pencil and paper is fine for this question.

(b) Now assume the channel lies along a flat bed. Using your result in (a), what is the relationship between discharge and the pressure gradient? Plot discharge versus pressure for a position along the channel 2 km from the terminus under 200 m of ice. Explain the significance of your results.

(c) Imagine an empty Röthlisberger channel of uniform diameter equal to 3 m under an outlet glacier of 600 m uniform thickness. This channel connects to a moulin ~15 km upglacier, where a field book was dropped earlier in the season. You are currently at the glacier terminus. Can you retrieve the field book and return safely before the channel gets too small? State all of your assumptions and show your calculations. Pencil and paper is fine.

3. **Jökulhlaups**

Use the governing equations in (2) to code up a one-dimensional model of an evolving R-channel that transmits a glacier outburst flood. Assume a simple glacier geometry (e.g. flat glacier bed, parabolic glacier surface) with an ice-dammed reservoir at the upstream boundary. Attempt to simulate the conduit discharge and conduit cross-sectional area as a function of time. The simplest approach is to apply a prescribed discharge function, representing the lake drainage into the conduit, at the upstream end of the domain. A more realistic approach would introduce another governing equation to describe the lake level as a function of discharge into the lake, discharge out of the lake into the subglacial conduit, and lake cross-sectional area. Once this model is working, investigate its sensitivity to, for example, the Darcy-Weisbach roughness parameter, the flow-law coefficient, reservoir geometry and glacier geometry. Describe your methods and explain your results.