EASC 605 Assignment 1: Energy-balance modelling
Due: 20 October 2021

Introduction

This project was designed by Regine Hock (U Oslo), but makes use of data from the Donjek Range field project (SFU Glaciology Group). You are asked to use the bulk aerodynamic approach (Munro, 1990) to calculate the point-scale surface energy balance at Glacier 1 from the data (refer to Hock (2005) for methodology) and then to determine the coefficients (degree-day factors) for a classical degree-day model.

Data

Several automatic weather stations were operational on and near our study glaciers in the St. Elias Mountains, southwest Yukon, from 2006–2015. These stations are located at approximately 2300 m above sea level. You will receive data that were collected from May through August, 2008 at two sites called "GL1 off ice" and "GL1 on ice". The purpose of this assignment is to calculate the surface energy balance for summer 2008 for Glacier 1 (GL1) using the meteorological data from the station called "GL1 on ice", and then to determine the degree-day factors for snow and ice for this site. The "GL1 off ice" dataset is only used to supply the precipitation record for Glacier 1. Andrew MacDougall has concatenated the data in ASCII format with a "readme" file that lists the names of the data fields (see MacDougall and Flowers, 2011).

At "GL1 on ice" we measure the following variables at 5-minute intervals: wind speed and direction, air temperature, net radiation, relative humidity, incoming shortwave, outgoing shortwave; at 30 minute intervals we measure: distance to surface (ablation or accumulation), barometric pressure, and rainfall. The temperature, wind and radiation sensors are all nominally mounted at ~2m above the ice surface. Because we do not have independent measurements of outgoing longwave radiation, from which to calculate the surface temperature with the Stefan-Boltzmann law, we shall assume the glacier surface is it at the melting point.

Theory

The energy-balance equation can be written

$$Q_N + Q_H + Q_L + Q_R + Q_G + Q_M = 0, \tag{1}$$

with net radiation Q_N, sensible heat flux Q_H, latent heat flux Q_L, heat flux due to rain Q_R, ground heat flux Q_G and the energy available for melt Q_M. Fluxes directed toward the surface are defined to be positive.

Table 1: Useful constants and parameters.

Variable	Description	Value	Units
g	Acceleration due to gravity	9.81	$\mathrm{m\,s^{-2}}$
ρ_i	Density of ice	910	$\mathrm{kg\,m^{-3}}$
ρ_w	Density of water	1000	$\mathrm{kg\,m^{-3}}$
ρ_0	Density of air at P_0	1.29	$\mathrm{kg\,m^{-3}}$
P_0	Standard atmospheric pressure	101.325	kPa
c_p	Specific heat capacity of air	1.005×10^3	$\mathrm{J\,kg^{-1}\,K^{-1}}$
c_w	Specific heat capacity of water	4.180×10^3	$\mathrm{J\,kg^{-1}\,K^{-1}}$
L_f	Latent heat of fusion	3.34×10^5	$\mathrm{J\,kg^{-1}}$
L_v	Latent heat of evaporation	2.514×10^6	$\mathrm{J\,kg^{-1}}$
L_s	Latent heat of sublimation	2.848×10^6	$\mathrm{J\,kg^{-1}}$
k	von Karman's constant	0.4	
		Assume for Glacier 1	
z_0	Roughness length for wind speed	2.4	mm
z_{0T}	Roughness length for temperature	0.24	mm
z_{0e}	Roughness length for water vapour	0.24	mm

Net radiation

The net radiation can be written

$$Q_N = G - R + L\downarrow + L\uparrow, \tag{2}$$

with global radiation G, reflected shortwave radiation R, incoming longwave radiation $L\downarrow$ and outgoing longwave radiation $L\uparrow$.

Turbulent heat fluxes

The turbulent heat fluxes can be calculated using the bulk aerodynamic approach (Munro, 1991) in which

$$Q_H = c_p \frac{\rho_0 P}{P_0} \frac{k^2}{\left[\ln\left(\frac{z}{z_0}\right) - \Psi_M(z/L)\right]\left[\ln\left(\frac{z}{z_{0T}}\right) - \Psi_H(z/L)\right]} u_z(T_z - T_\mathrm{s}) \tag{3}$$

$$Q_E = L_v \frac{0.623\,\rho_0}{P_0} \frac{k^2}{\left[\ln\left(\frac{z}{z_0}\right) - \Psi_M(z/L)\right]\left[\ln\left(\frac{z}{z_{0e}}\right) - \Psi_E(z/L)\right]} u_z(e_z - e_\mathrm{s}), \tag{4}$$

for wind speed u_z ($\mathrm{m\,s^{-1}}$) at height z, air temperature T_z (K) at height z, surface temperature T_s assumed to be 273.15 K, $z = 2$ m the instrument height above the surface, vapour pressure e_z at height z and vapour pressure at the surface e_s.

Vapour and atmospheric pressure

Vapour pressure e is related to relative humidity RH (%) as

$$RH = e/E \times 100, \tag{5}$$

with the saturation vapour pressure (Pa)

$$E = 610.78 \exp\left(\frac{17.08085\,T}{234.15 + T}\right), \tag{6}$$

for air temperature T in °C. The vapour pressure of the surface (e_s) is calculated as above but with the surface temperature. Air pressure decreases with elevation according to

$$P = P_0 \exp(-0.0001184\,z), \tag{7}$$

for elevation z in m and pressure in Pa.

Atmospheric stability

Atmospheric stability must be accounted for in calculating the turbulent fluxes. There are multiple methods for doing this. Parameters Ψ_M, Ψ_H and Ψ_E are the Monin-Obukhov stability functions for momentum, heat and humidity, respectively. During the melt season when the glacier surface reaches 0° C, air near the surface of the glacier tends to become stably stratified and produces a shallow inversion (cold near the surface, warmer above). This is due to the fact that the surface temperature of the ice cannot exceed the melting point. Turbulent heat fluxes are suppressed during stable conditions (defined by $z/L > 0$) and enhanced for unstable conditions ($z/L < 0$). We will assume the following form for the stability functions (Beljaars and Holtslag, 1991):

$$\Psi_M(z/L) = -(az/L + b(z/L - c/d)\exp(-dz/L) + bc/d), \tag{8}$$

$$\Psi_H(z/L) = \Psi_E(z/L) = -((1 + 2az/3L)^{1.5} + b(z/L - c/d)\exp(-dz/L) + bc/d - 1), \tag{9}$$

for $a = 1$, $b = 2/3$, $c = 5$, $d = 0.35$ and Monin-Obukhov length

$$L = \frac{\rho_0 \, c_p \, u_*^3 \, T}{k \, g \, Q_H}, \tag{10}$$

for T in Kelvin and frictional velocity

$$u_* = \frac{k \, u}{\ln\left(\frac{z}{z_0}\right) - \Psi_M(z/L)}. \tag{11}$$

Calculation of the stability length scale L requires an iteration, because it depends on Q_H and u_*:

- compute Q_H assuming neutral conditions ($\Psi_M = \Psi_H = \Psi_E = 0$)
- compute the friction velocity u_*
- compute the length scale L
- recompute Q_H using u_* and L from above
- repeat until the change in Q_H is negligible.

When $z/L < 0$ (unstable conditions), we neglect the stability correction and assume neutral conditions ($z/L = 0$): $\Psi_M = \Psi_H = \Psi_E = 0$.

Heat flux due to rain

Sensible heat flux from rain is calculated as

$$Q_R = \begin{cases} \rho_\mathrm{w} \, c_\mathrm{w} \, R \, (T_r - T_s) & T > 0 \\ 0 & T \leq 0, \end{cases} \tag{12}$$

where R is the precipitation rate and T_r the temperature of the rain, usually assumed equal to air temperature. Check your units! $Q_R = 0$ for sub-freezing air temperatures.

Ground heat flux

Here we assume that the heat flux into and out of the ice $Q_G = 0$. This is not generally a valid assumption for Glacier 1, but an iterative multi-layer thermal model would be required to include this component of the energy balance.

Energy available for melt

Any melt energy Q_M must be converted to water-equivalent melt in some sensible units such as $\mathrm{mm\,h^{-1}}$ or $\mathrm{cm\,d^{-1}}$. This should only be done when the energy balance is positive and the surface is at the melting temperature.

Methodology

- Make sure to inspect the data before doing any calculations. Plot all the timeseries you will be using, and check the data for obvious errors and gaps. Amend these if necessary, making a careful note of your procedures. You may find a significant jump in the SR50 record, corresponding to an instrumental reset; this will have to be adjusted.
- Plot hourly timeseries of air temperature, relative humidity, incoming shortwave radiation, outgoing shortwave radiation, net longwave radiation (determined as the difference between the net all-wave and net shortwave radiation measurements), barometric pressure and rainfall. This will require averaging the 5-minute and half-hourly data into hourly values. Provide a qualitative interpretation of the data. For example, when was it clear versus cloudy and how do you know?
- Calculate the hourly energy balance at the site of the Glacier 1 met station and plot each of the energy balance components as a timeseries. Sum the energy balance components for the summer season and determine the partitioning of melt energy between each of the components. How important is the radiative balance as compared to the turbulent heat fluxes? Can you identify periods with distinct energy partitioning? Given the noisy nature of the SR50 record, it is recommended that daily means rather than hourly means are used.
- Calculate and plot the albedo and identify when the surface was snow covered vs. snow free. Recall that valid values of albedo do not exceed 1. It is important to identify these periods in order to estimate melt from the SR50 record, and to calculate the degree-day factors for snow and ice.
- Calculate the observed melt (in water equivalent) from the surface lowering data assuming a snow density of 500 $\mathrm{kg\,m^{-3}}$ and an ice density as in the table. Compare your timeseries of modelled and measured melt.
- Calculate degree-day factors for snow and ice using positive degree-days calculated from the GL1–onice temperature record. Plot the melt timeseries obtained with the degree-day model and compare it to the timeseries of observed melt and the timeseries of melt produced with your energy-balance model.

Results

Write up your results as a brief report that includes your computer code (the calculations, not the plotting) in an appendix. Structure the report like a research paper with an abstract, short introduction, methods, results, discussion, conclusion and references. You may be making different choices about precisely how you go about this assignment, so be sure to document your procedure thoroughly in the methods section. In your discussion you should describe and interpret the variation of the energy balance components through the melt season. You may want to investigate the sensitivity of your results to the assumed density of snow, the means by which the surface type (snow vs. ice) is defined, and the values of roughness length z_0, z_{0T} and z_{0e}.

References

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Hock, R., 2005. Glacier melt: A review of processes and their modelling. *Prog. Phys. Geogr.*, 29, 362–391.

MacDougall, A.H., and G.E. Flowers. 2011. Spatial and temporal transferability of a distributed energy-balance glacier melt model. *J. Clim.* 24, 1480–1498.

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