Using Reinforcement Learning to solve Quadratic Assignment Problems

Final Talk

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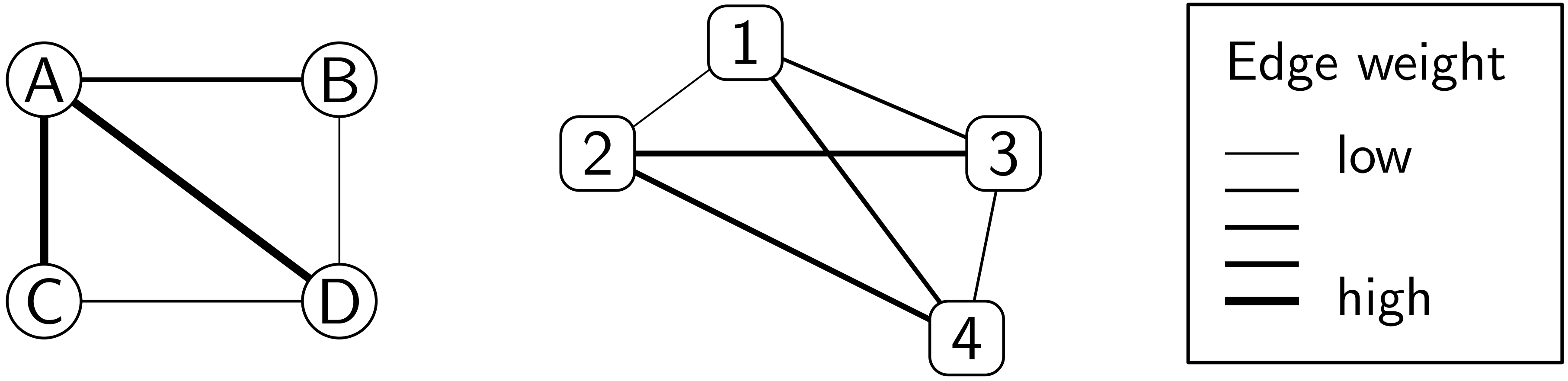
The Quadratic Assignment Problem



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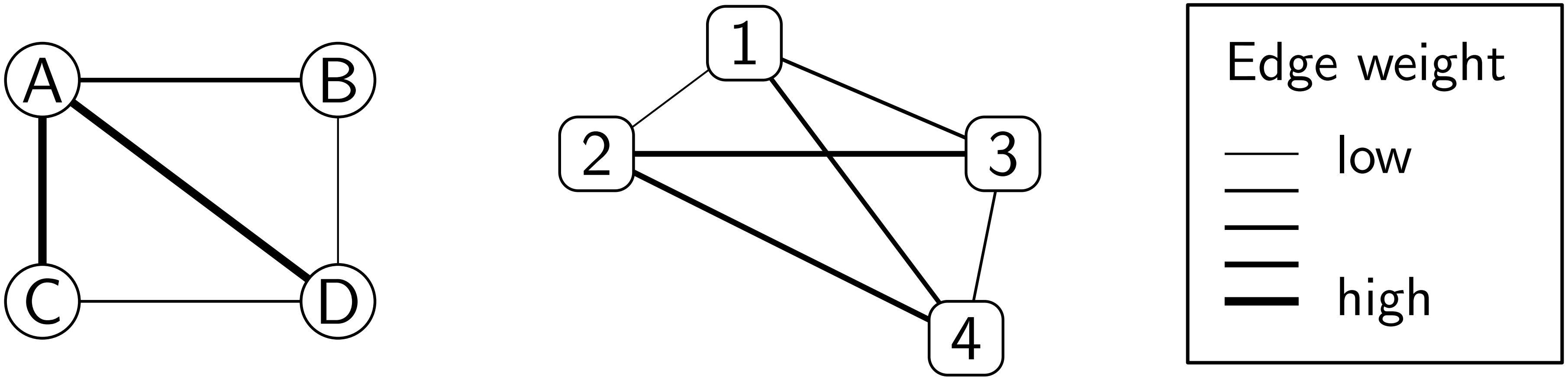
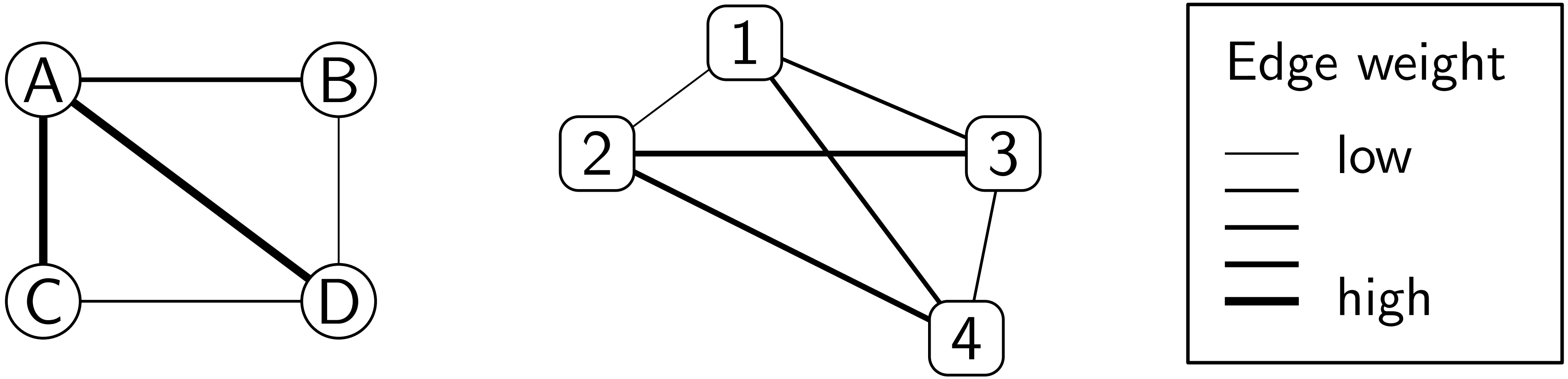


Example: Economics

Transport volume between facilities

Cost per unit between locations

The Quadratic Assignment Problem



Example: Economics

Transport volume
between facilities

Cost per unit
between locations

Example: Keyboard layout

Letter pair frequency

Travel time between keys

Mathematical formulation

$$\sum_{i,j} a_{i,j} b_{f(i),f(j)}$$

Mathematical formulation

edge weights

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$$\sum_{i,j} a_{i,j} b_{f(i),f(j)} + \sum_i c_{i,f(i)}$$

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Goal

Find assignment f that minimizes cost

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Find assignment f that minimizes cost

$\rightarrow$ How can RL be applied?

Transition function

$$(\text{state}, \text{action}) \rightarrow (\text{new state}, \text{reward})$$

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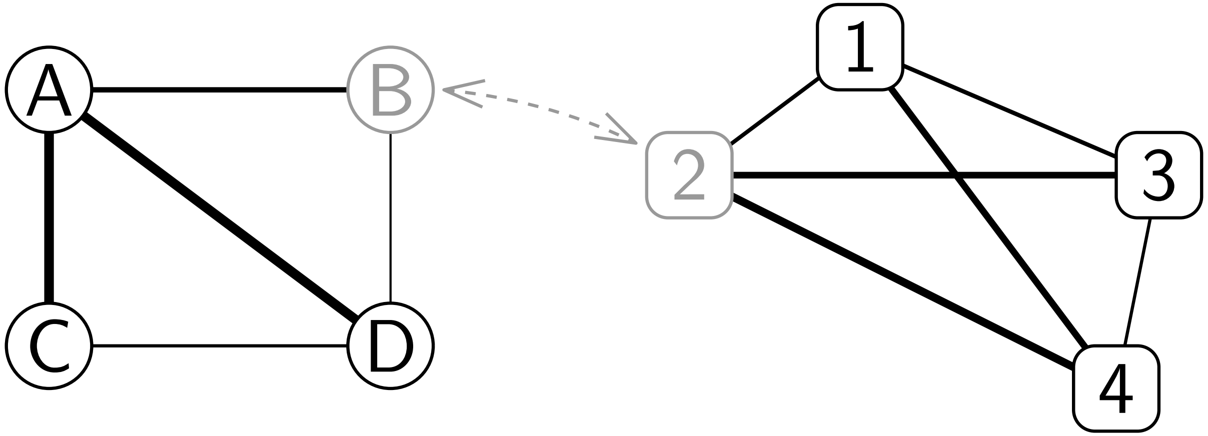
Transition function

$$(\text{state}, \text{action}) \rightarrow (\text{new state}, \text{reward})$$

$$T : (\mathrm{QAP}_n, \{1, \ldots, n\}^2) \rightarrow (\mathrm{QAP}_{n-1}, \mathbb{R})$$

Reduced subproblem representation

Can we remove the assigned nodes from the graph?



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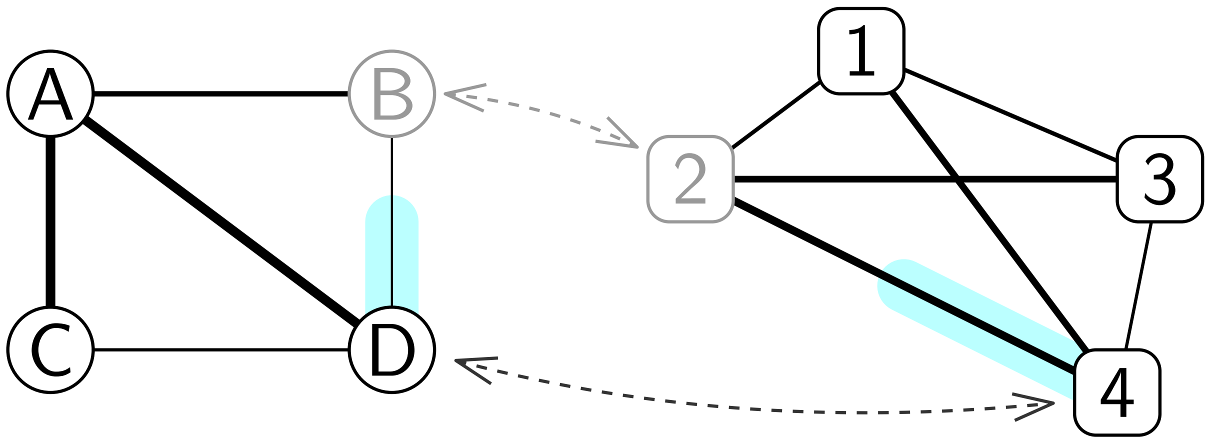

Assignment cost based on half assigned edges

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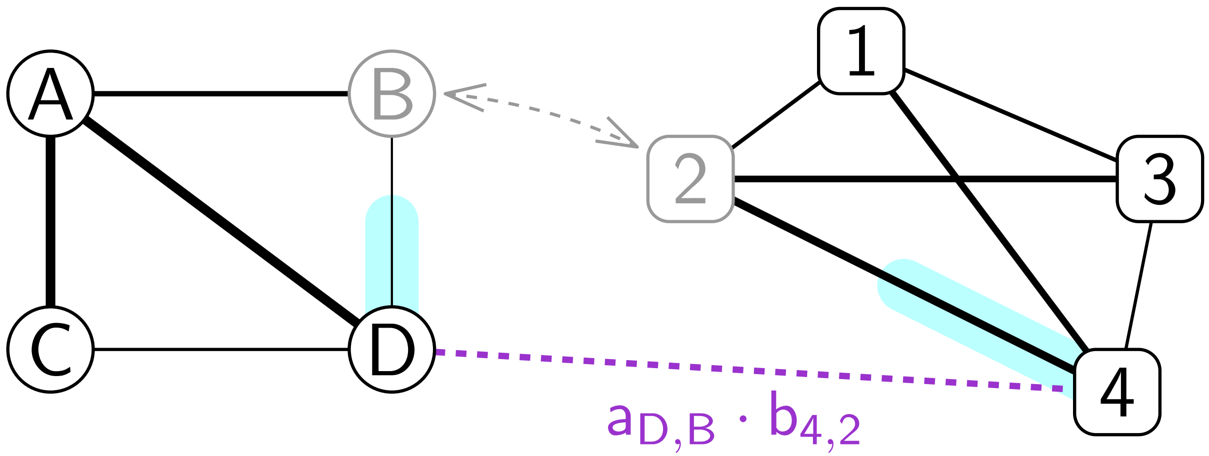

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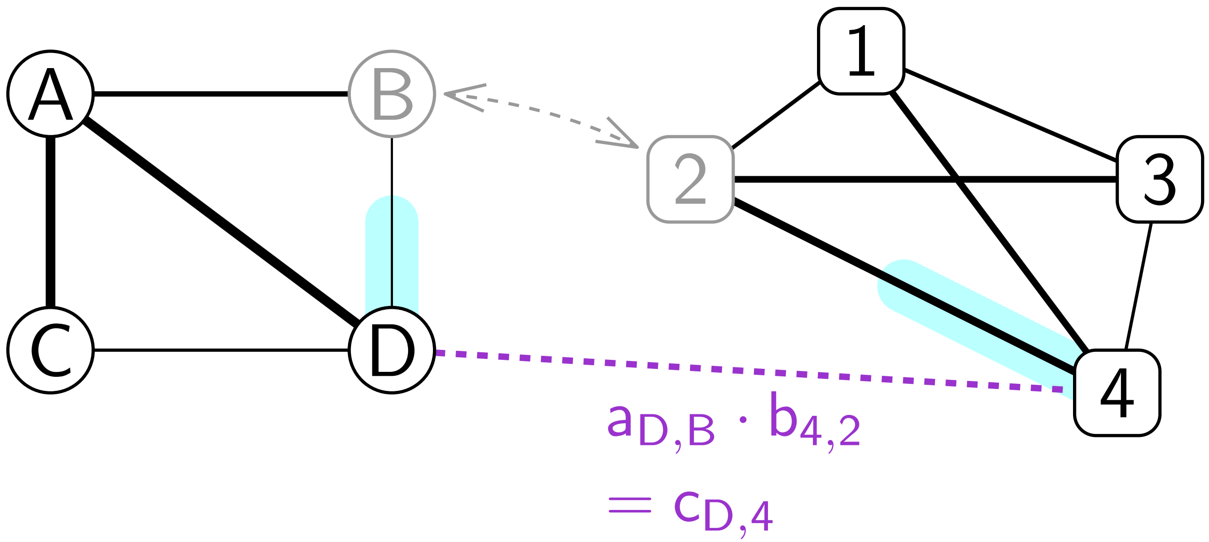

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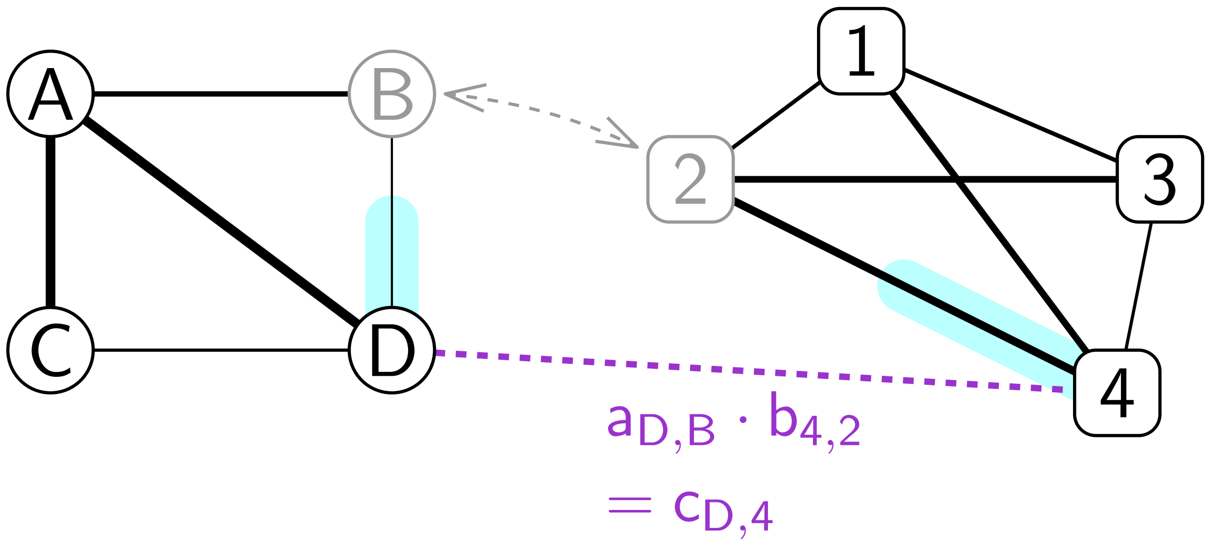

Assignment cost based on half assigned edges

$\rightarrow$ Encode in new edge type between graphs

Reduced subproblem representation

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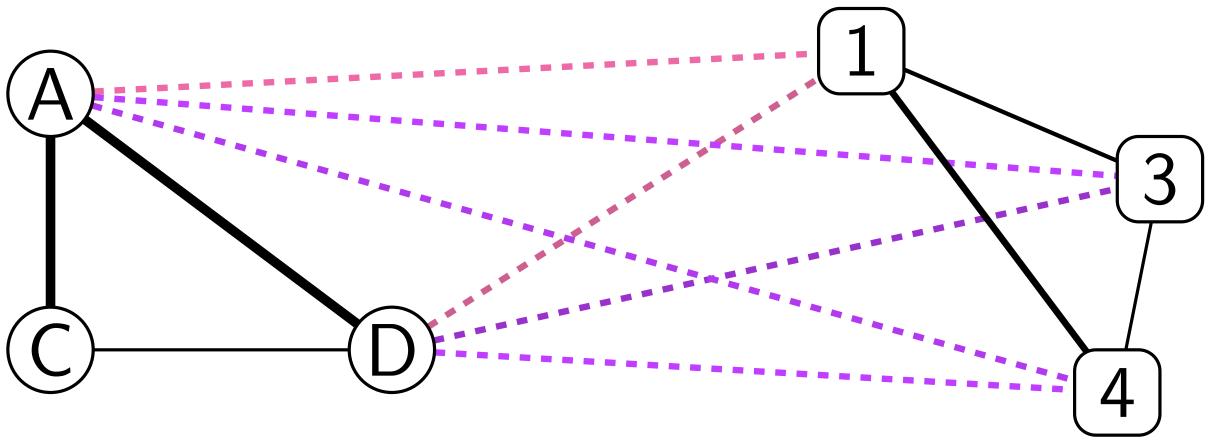

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Advantages of closed transitions

$$8 \to 7 \to 6 \to 5 \to 4 \to 3 \to 2$$

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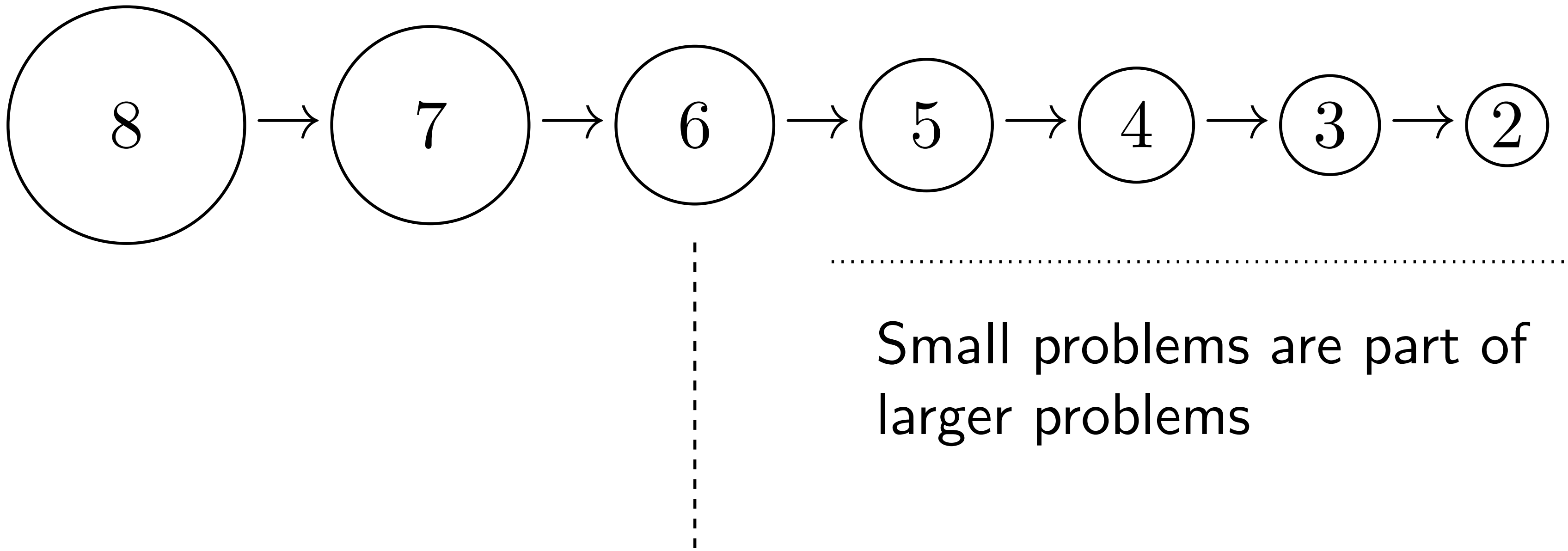
Small problems are part of larger problems

Advantages of closed transitions



Small problems are part of larger problems

No artificial states

Advantages of closed transitions

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Small problems are part of larger problems

No artificial states

$\longleftarrow$

incremental learning

Policy network architecture

Compute value for each action

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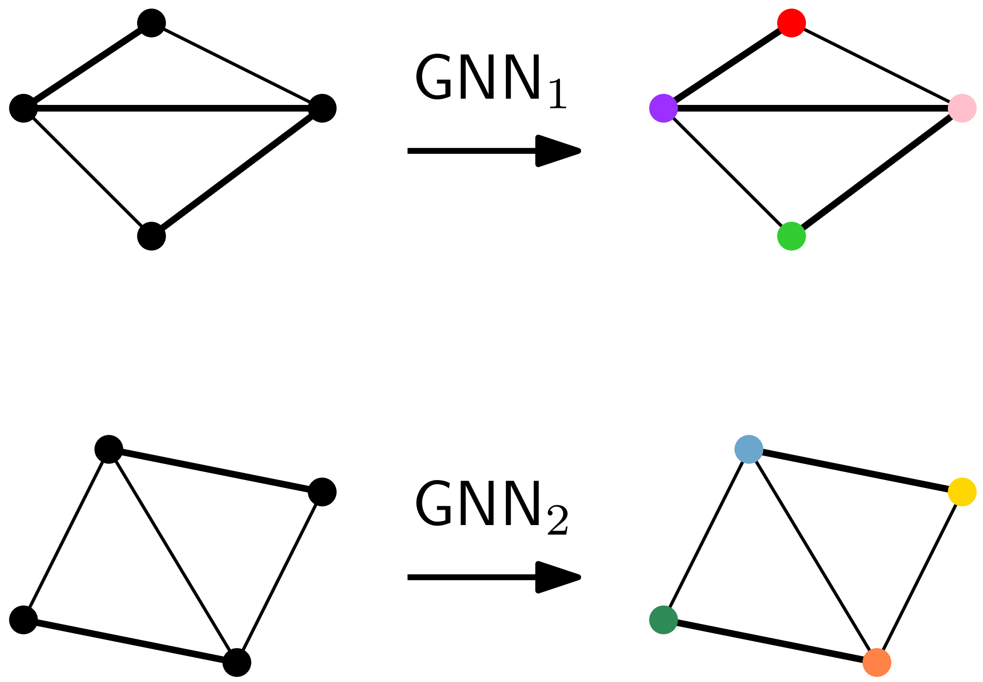
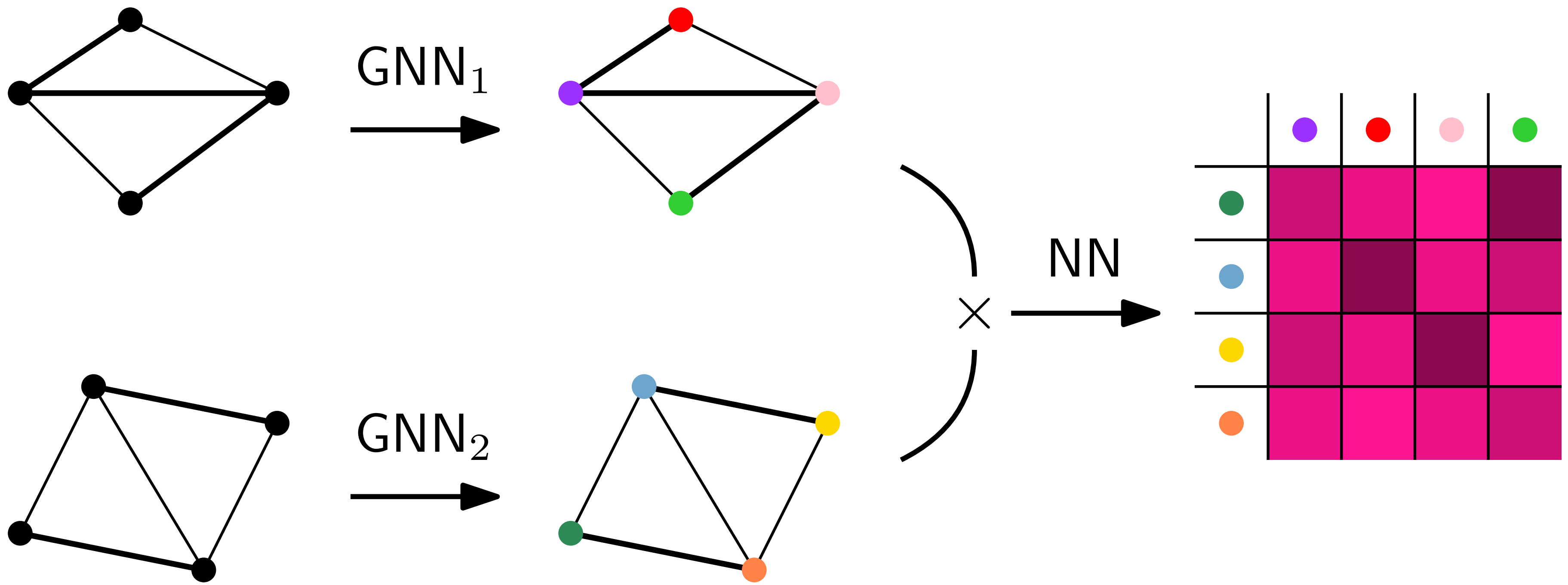


Encode graph structure

Policy network architecture

Compute value for each action

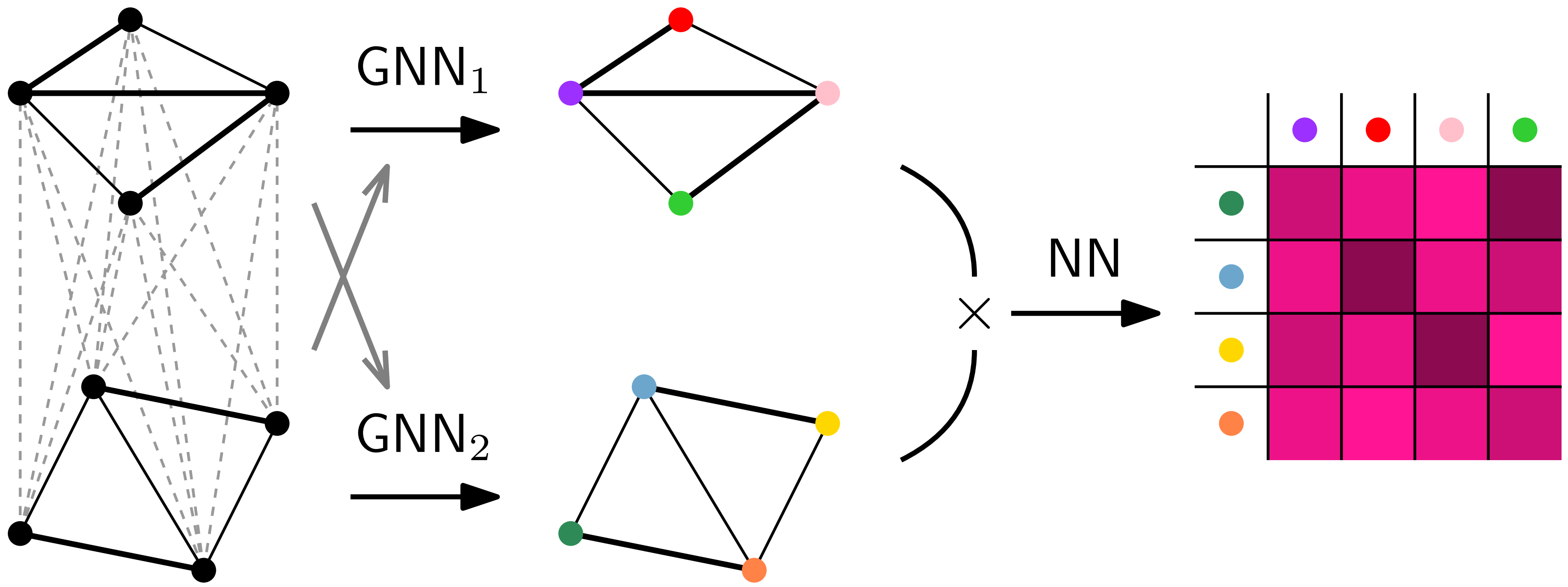


Encode graph structure

Predict pair values

Policy network architecture

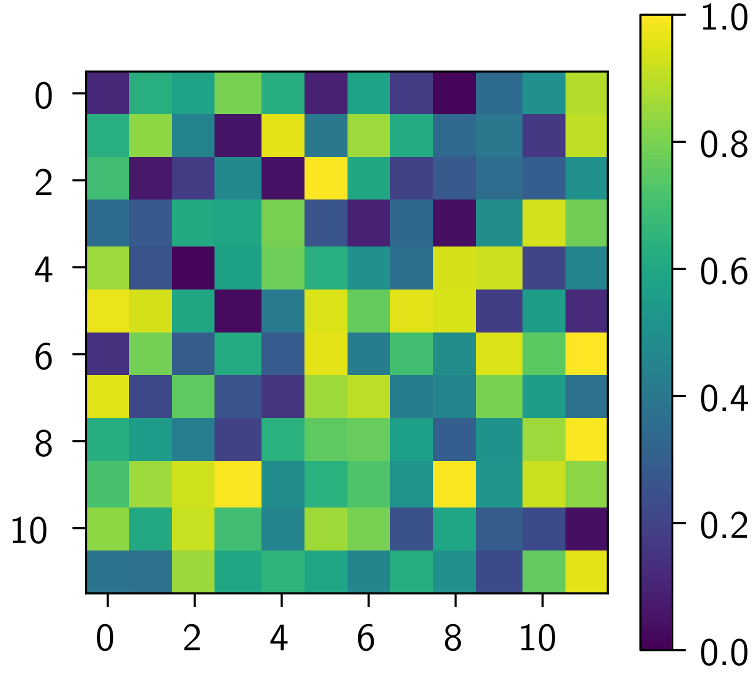
Compute value for each action



Encode graph structure

Predict pair values

Separation experiment: Setup



Input connectivity matrix

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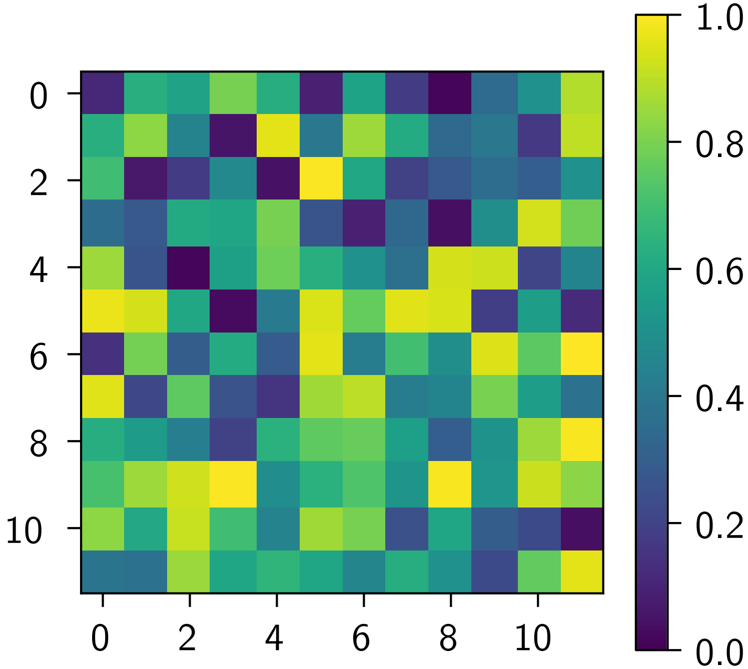

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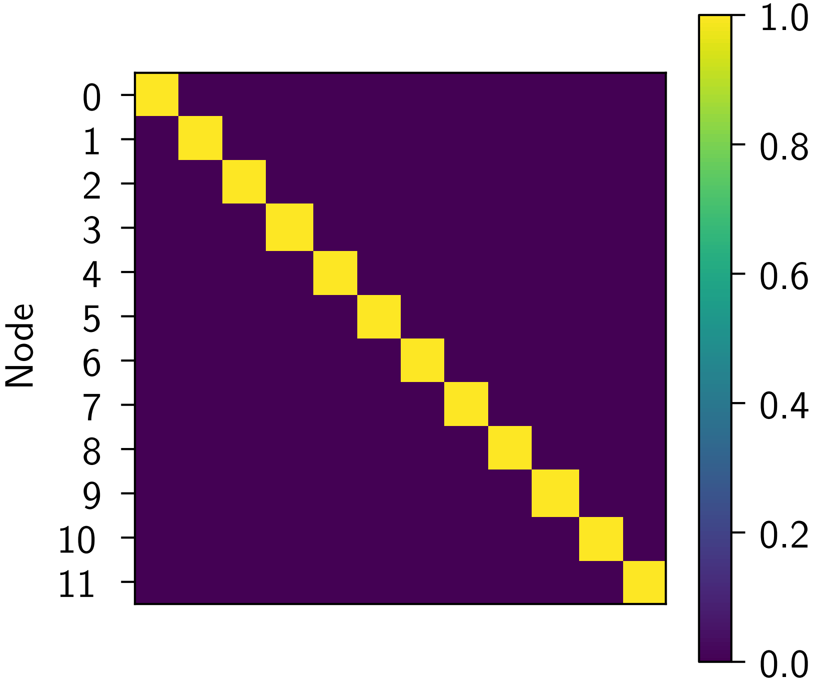
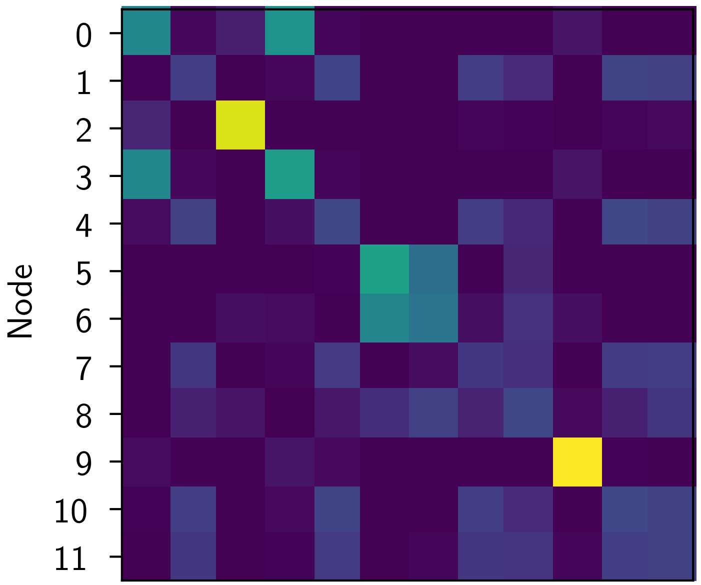


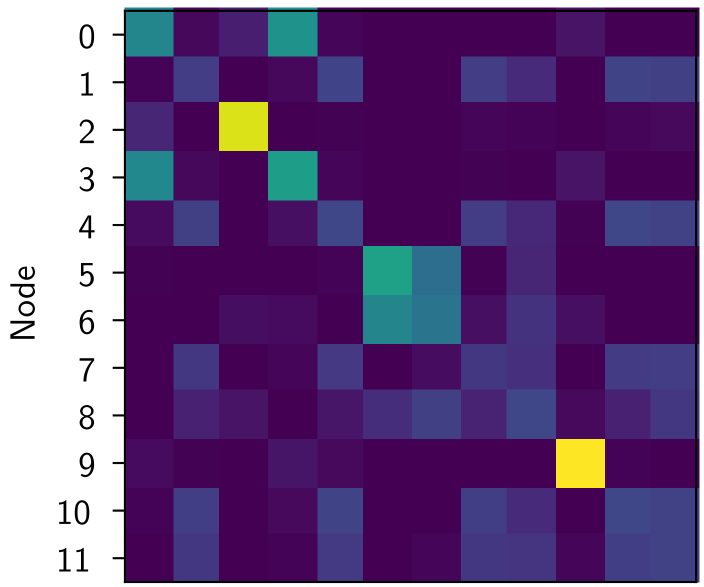
Target node embeddings

Separation experiment: Results



Output of trained GNN

Separation experiment: Results



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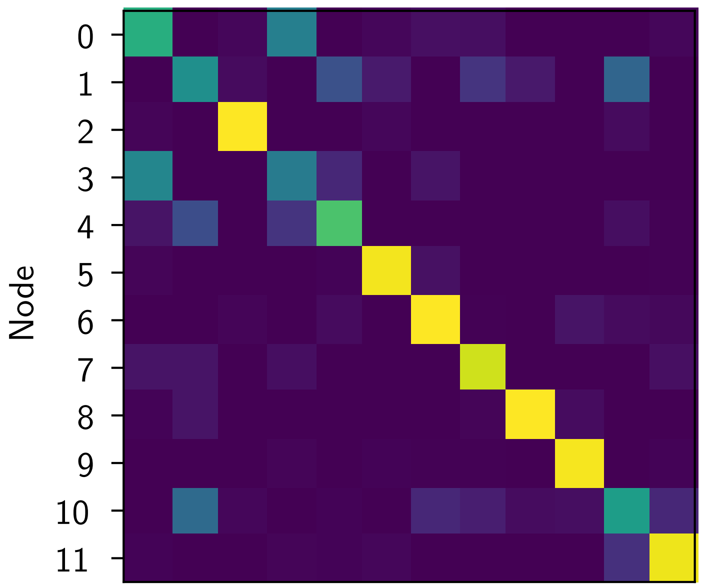
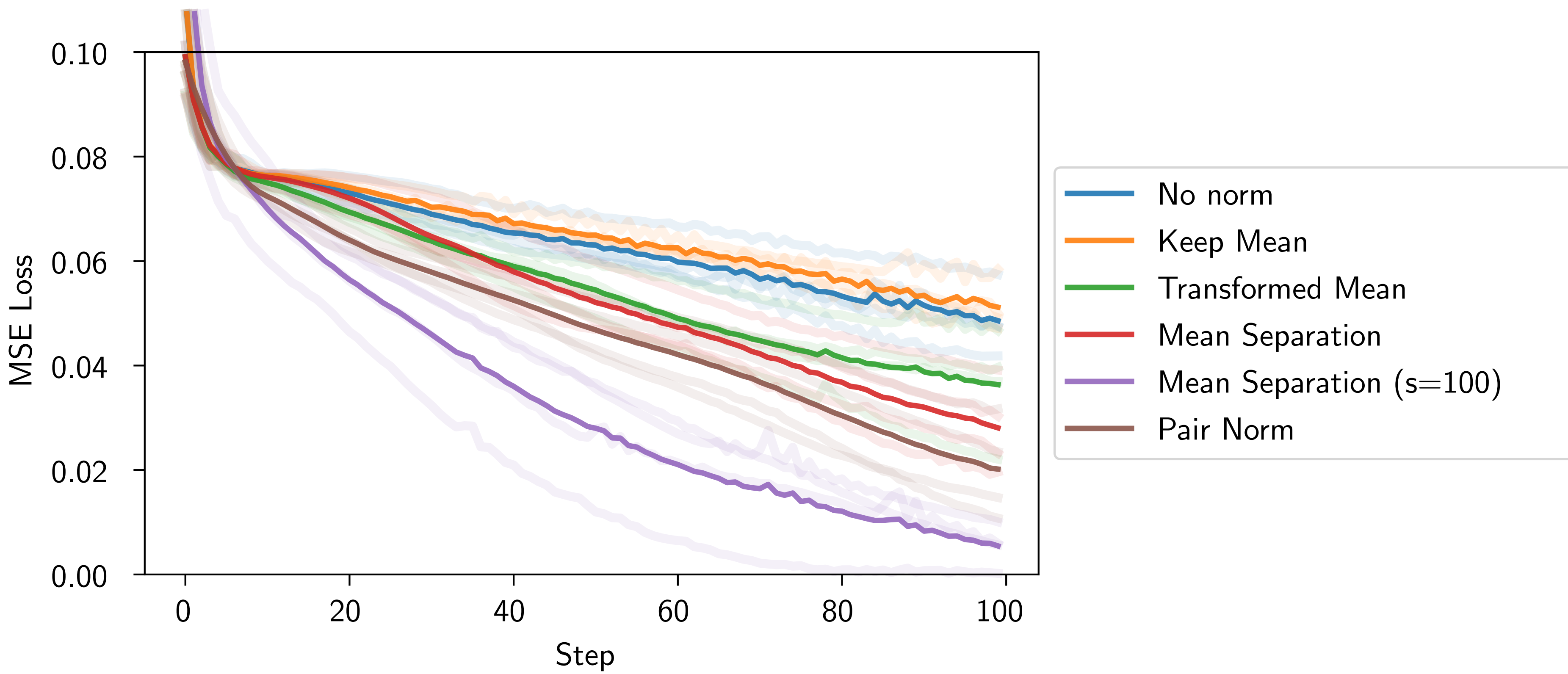


PairNorm

Separation experiment: Training progress comparison



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Problem generator

Undirected 8 node graphs with random weights

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$8! = 40320$ possible assignments

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RL algorithms

	Value-based	Policy gradient
Monte Carlo		
Temporal difference		

Experiment setup

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Undirected 8 node graphs with random weights

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RL algorithms

	Value-based	Policy gradient
Monte Carlo	MCQ	REINFORCE
Temporal difference	DQN	A2C

Performance indicators

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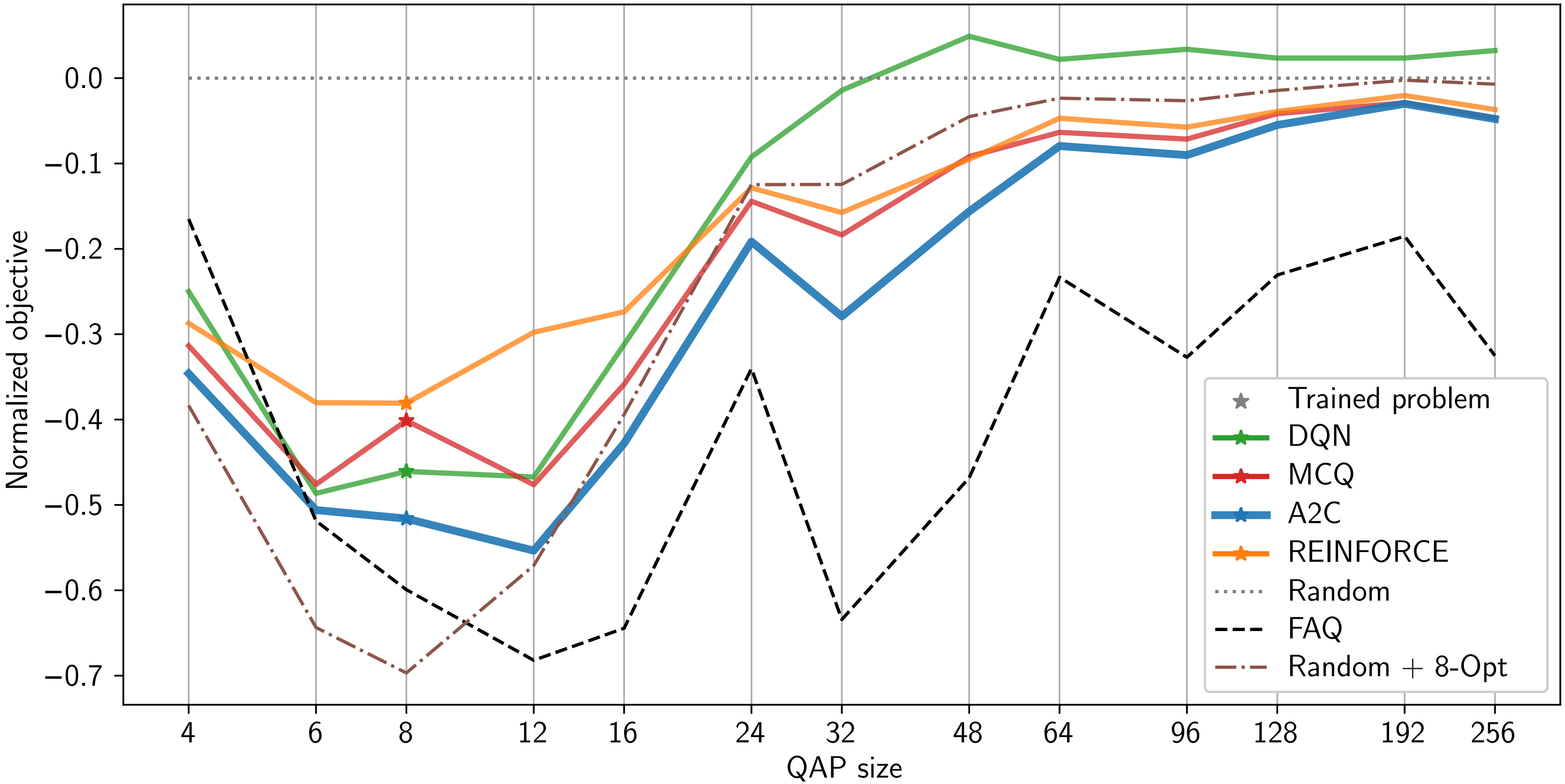
Gap to best known value

$$\frac{v - v_{\text{known}}}{v_{\text{known}}}$$

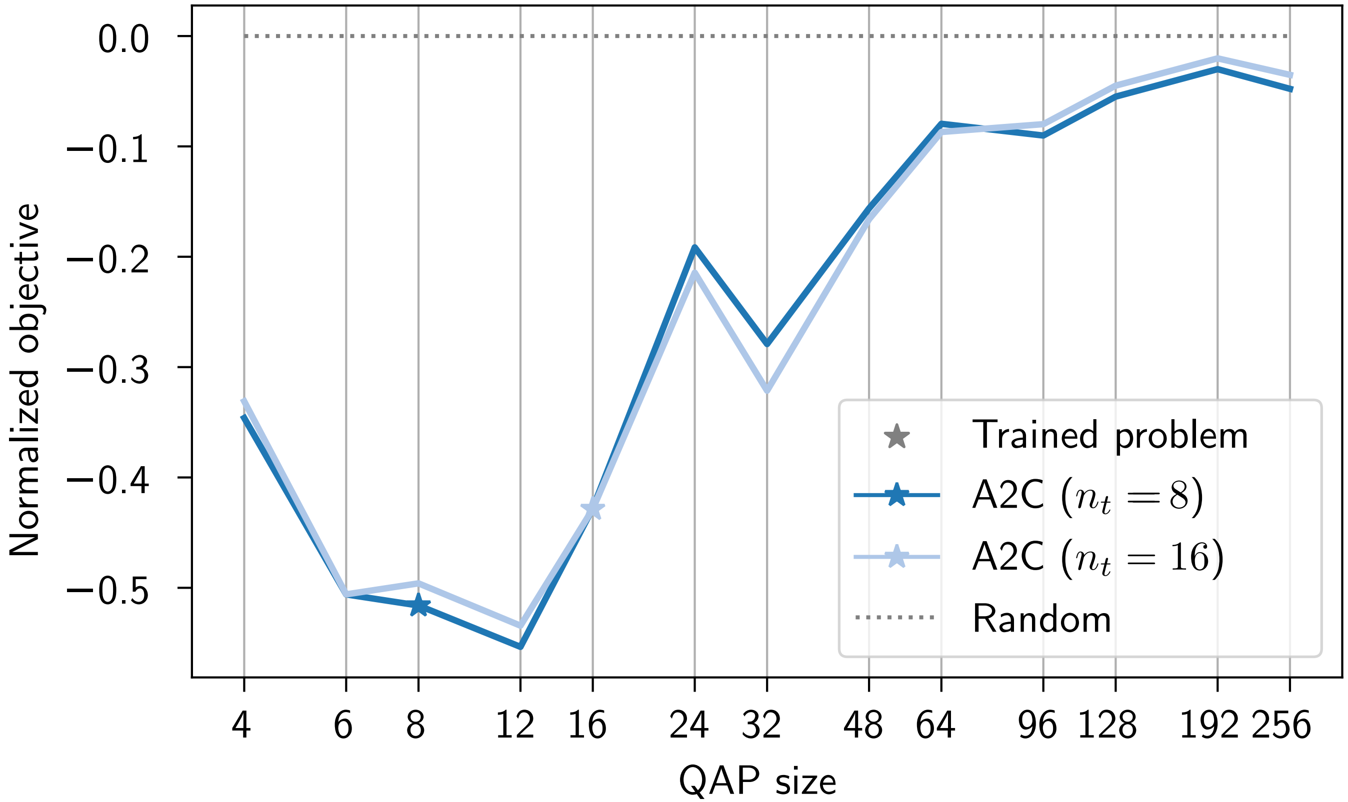
Performance indicators

Gap to best known value	$\frac{v - v_{\text{known}}}{v_{\text{known}}}$
Normalization to randomness	$\frac{\text{mean}(v) - \text{mean}(v_{\text{random}})}{\text{variance}(v_{\text{random}})}$

Generalization results



Larger training size does not improve results



QAPLIB results

	FAQ	MCQ	DQN	RF	A2C	NGM	10k rand
bur (26-26)	**0.2**	<u>3.0</u>	14.4	4.0	3.8	3.4	3.1
chr (12-25)	**54.9**	125.3	<u>70.4</u>	87.9	74.5	121.3	130.5
els (19-19)	23.7	**<u>9.7</u>**	132.2	132.2	98.9	57.0	56.5
esc (16-128)	32.0	22.1	**<u>12.6</u>**	19.8	17.8	32.0	28.8
had (12-20)	**0.8**	8.0	9.4	4.0	<u>2.9</u>	4.4	4.5
kra (30-32)	**5.7**	<u>20.5</u>	28.9	21.7	22.0	31.4	31.4
lipa (20-90)	**2.5**	14.4	14.2	13.4	<u>12.0</u>	14.1	14.1
nug (12-30)	**3.0**	18.9	20.6	12.3	<u>12.2</u>	16.3	15.4
rou (12-20)	**3.8**	18.7	14.0	14.3	14.2	13.1	<u>11.2</u>
scr (12-20)	17.4	36.9	21.1	28.1	**<u>16.9</u>**	30.2	30.2
sko (42-100)	**1.3**	15.2	21.5	14.0	<u>11.8</u>	18.1	17.3
ste (36-36)	**7.0**	61.0	141.1	<u>51.8</u>	66.6	102.2	108.0
tai (12-150)	**7.0**	<u>18.8</u>	63.4	45.5	40.9	22.2	22.9
tho (30-150)	**2.4**	24.7	32.8	22.8	<u>19.0</u>	25.3	26.0
wil (50-100)	**0.8**	10.0	9.9	<u>6.6</u>	6.9	9.4	8.8
mean	**10.8**	<u>27.2</u>	40.4	31.9	28.0	33.4	33.9

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