ENEL420

Assignment 2

Non-parametric Spectral Density Estimation

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Group 23

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Topic Description

Spectral density estimation is an essential part of signal processing. The spectral density of a process contains all the information relating to the power of different frequency components of the process. This is useful information in signal processing as it helps to identify signal frequencies, noise frequencies, relative power between different frequency components, and other characteristics of the process. There are many different methods for estimating the spectral density of a process, two main categories can be defined as parametric and non-parametric. Parametric estimation assumes there is an underlying signal within the data, and so often the process can be interpreted as a periodic signal, distorted by noise. Non-parametric estimation makes no assumption on the components of the process and is used to estimate the spectral density of random processes.

The effectiveness of an estimator is based on its resolution and variance. An ideal estimator would have low variance and high resolution, though in practice there is often a trade-off between the two characteristics. Reducing the variance with techniques such as windowing will often reduce the resolution of the estimator.

Spectral Density Estimation Methods

Several non-parametric spectral density estimation methods are described in [1]. One of the simplest methods for estimating the spectral density of a signal is the periodogram method. In the periodogram method, the periodogram is calculated and used as the spectral density estimate. The estimate of the spectral density of a discrete time signal x(n) of length N can be calculated as shown in Equation 1.

$$\hat{P}_{Per} = \frac{1}{N}|DFT(x(n))|^2 \tag{1}$$

Unfortunately, the periodogram method often does not give an accurate estimate of the spectral density of a signal. Because the discrete time signal is finite, it can be thought of as a multiplication of the signal with a rectangular window. Taking the DFT of this results in the spectral density estimate having unwanted lobes. This tends to reduce the resolution of the estimate. The lobes may also hide other, smaller frequency components of the signal.

There are several other methods that improve upon the periodogram method. These often smooth the result from the periodogram method using temporal and lag windowing. These methods for estimating the spectral density include Bartlett's method, Welch's method, the Daniell method and the Black-Tukey method.

In Bartlett's method, the signal data is split into several segments that do not overlap. The periodogram of each of these segments is then calculated. The estimate of the spectrum is then found by taking the average of all the periodograms. Doing this reduces the variance of the spectral density estimate. The variance of the estimate is also inversely proportional to the number of periodograms that are averaged, so can be decreased by increasing the number of segments. However, Bartlett's method also has a lower resolution than the periodogram method. The resolution is proportional to the number of segments used, so becomes worse as the number of segments increases.

Welch's method is similar to Bartlett's method. However, in Welch's method segments may overlap. A window function is also applied to each of the segments. These window functions often give more weight to data points in the middle of the segments. This mitigates the effect of data points on the ends of the segments being accounted for multiple times in the estimate due to the overlap between segments. The periodograms are then calculated for each segment, then averaged as in Bartlett's method. The resolution and variance of an estimate found using Welch's method is dependent on the amount of overlap between segments and the window functions that were used.

In the Daniell method, the squared magnitude of the DFT of the time-series data is calculated. Then, groups of points in the calculated spectrum are average together. This gives a frequency resolution that is the same as that from Bartlett's method.

The Blackman-Tukey method smooths an estimate found using the periodogram method by convolving it with an autocorrelation window as shown in Equation 2. This reduces the variance of the periodogram as it reduces the effect of unreliable estimates. Unfortunately, the resolution also reduced. The resolution of the estimate depends on the window.

$$\hat{P}_{BT} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \hat{P}_{Per}\left(e^{ju}\right)W\left(e^{j(\omega-u)}\right)du \tag{2}$$

Chosen Application

The chosen application is to apply non-parametric density estimation methods to analyze bio-potential signals, with a focus on electroencephalogram (EEG) signals. [2] proposes a method for analyzing EEG signals to characterize brain cell activity. The aim of the research is to analyze epileptic activity using non-parametric density estimation methods on EEG signals.

There is a publicly available EEG dataset [3] online that contains five sets of EEG data, they are labeled as sets A, B, C, D, and E. Sets A and B contain EEG data from healthy patients with their eyes open (set A) and eyes closed (set B). Sets C, D and E are taken from patients who suffer from epilepsy. Sets C and D are taken during seizure-free intervals, while set E is taken during epileptic activity. Our aim is to apply different spectral density estimation techniques to analyze signals from the EEG dataset and compare their ability to extract useful information that could help predict which set the data came from (A, B, C, D or E).

Remaining Tasks

In the next stage of this project, several non-parametric spectral density methods will be used to obtain the spectra of EEG datasets. We plan to use the EEG dataset from [3]. Matlab code will be written to estimate the spectral density of this data. Several methods will be used to do this, and the results from each will be compared. The first method that will be used is the periodogram method. It was decided to use this method as all the other methods discussed in this report are based off it. Therefore, it will provide an interesting comparison as we will be able to observe how each of the methods has improved upon the periodogram method. We also plan to use both Bartlett's and Welch's methods as they are popular method of spectral density estimation. It may also be interesting to observe the effect of overlapping the periodograms. We will also be able to investigate the effect of using different window functions in Welch's method. Once we have obtained spectra using these methods, we will compare them. This will give us an opportunity to observe the different characteristics of each method, including the variance and the resolution.

In [2], several benchmarks are set out for evaluating the performance of spectral density estimation of an EEG signal. These revolve around if the estimates are good enough to be able to classify them into different categories. We would like to see if we are able to use our estimates to classify the data into either being from a healthy patient or from a patient with epilepsy. A harder benchmark to meet would be to see if we could use our estimated spectra to differentiate between patients with their eyes open and with their eyes closed. This may be difficult for us to do since we are not medical experts, but it would be a good way to evaluate each of the different methods.

References

[1] *Spectrum Estimation,* Uppsala Universitet, Uppsala, Sweden, 2006. Available: http://www.signal.uu.se/Courses/CourseDirs/SignalbehandlingIT/forelas06.pdf

[2] Tianbo Chen, Ying Sun, Mehdi Maadooliat, *Nonparametric collective spectral density estimation with an application to clustering the brain signals*, 2018

[3] Andrzejak, R. G., Lehnertz, K., Mormann, F., Rieke, C., David, P., & Elger, C. E. *Indications of nonlinear deterministic and finite-dimensional structures in time series of brain electrical activity: dependence on recording region and brain state,* 2001.

[4] Hangfang Zhao, Lin Gui, *Nonparametric and parametric methods of spectral analysis,* 2019.