ENEL420 Assignment 2

Non-Parametric Spectral Density Estimation

Group 23

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11/10/2020

Introduction

Spectral density estimation plays a crucial role in many signal processing applications. The spectral density of a process contains all the information about the power of different frequency components of the process. This is useful information in signal processing as it helps to identify signal frequencies, noise frequencies, relative power between different frequency components, and other characteristics of the process. There are many different methods for estimating the spectral density of a process, two main categories can be defined as parametric and non-parametric. Parametric estimation assumes there is an underlying signal within the data, and so often the process can be interpreted as a periodic signal, distorted by noise. Non-parametric estimation makes no assumption on the components of the process and is used to estimate the spectral density of random processes. The effectiveness of an estimator is based on its resolution and variance. An ideal estimator would have low variance and high resolution, though in practice there is often a trade-off between the two characteristics. Reducing the variance with techniques such as windowing will often reduce the resolution of the estimator.

From the spectral density estimation, conclusions about the corresponding process can be made. Non-parametric techniques can be applied to electroencephalogram (EEG) signals to analyze and characterize brain cell activity [1]. Epilepsy is one of the most common brain disorders in humans, and patients often suffer from it their whole lives [2]. The spectral density analyses of EEG have been used to identify patients that suffer from epilepsy as well as develop understanding about how the disorder affects brain activity [3].

This paper investigates several non-parametric spectral density estimation techniques to extract information from raw EEG data and characterize them into several categories. The categories include healthy patients, patients that suffer from epilepsy, and patients that were undergoing a seizure when the data was recorded. The non-parametric techniques investigated in this paper are the Periodogram method, Welch's method, and Bartlett's method. A K-nearest neighbors (KNN) classifier was trained on data from each estimation technique and used to classify a test EEG dataset. The results of the classification were then used to compare each techniques ability to extract useful signal information from the raw EEG data.

Background

Non-parametric spectral density estimation methods

There are several different methods used to do non-parametric spectral density estimation. This project focuses on three of them; the periodogram method, Bartlett's method and Welch's method. These methods are discussed in [4]. The simplest of these methods is the periodogram method. In this method, the periodogram of a time domain signal is calculated and used as the spectral density estimate. The spectral density estimate of a discrete time signal x(n) can be calculated using the periodogram method as shown in Equation 1.

$$\hat{P}_{Per} = \frac{1}{N}|DFT(x(n))|^2 \tag{1}$$

There are several disadvantages to using the periodogram method. This is because since the discrete time signal is finite, it can be thought of as a multiplication of the signal with a rectangular window. Taking the periodogram of this results in a spectrum with unwanted side lobes. This tends to reduce the resolution of the estimate. The lobes may also hide smaller frequency components of the signal.

Bartlett's method improves on the periodogram method by using temporal and lag windowing to smooth the estimate of the spectrum. This is done by splitting the time domain signal into several segments and taking the periodogram of each. The periodograms are then averaged to give an estimate of the spectrum. Bartlett's method produces an estimate with lower variance than the periodogram method. However, Bartlett's windowing technique decreases the resolution of the estimate, and thus has a lower resolution than the periodogram method. The variance of the Bartlett estimate is inversely proportional to the number of periodograms that are averaged, so it can be reduced by increasing the number of segments. The resolution also decreases as the number of segments is increased.

Welch's method is similar to Bartlett's method, but the segments are allowed to overlap. Welch's method also applies a window function to each of the segments. These window functions usually weight the data points in the middle of the segments more heavily than those at the ends. This mitigates the effect of the data points on the ends of the segments being accounted for multiple times in the estimate due to the overlap between segments. The periodograms are then computed for each segment and averaged to give an estimate of the spectral density. The resolution and variance of the estimate depend on the amount of overlap between the segments and the window function that was used.

Application in EEG processing

A common application of non-parametric spectral density estimation is EEG processing. The spectrum of an EEG signal carries a significant amount of information about the brain activity of a patient. Some of this can be found from the power of the frequency components in the signal. The frequency range of an EEG signal is often split into frequency bands or wave patterns. Table 1 shows the wave patterns that can be observed from an EEG signal, and what information they hold. As seen in Table 1, high power in the gamma frequency band is thought to be linked to epileptic patients [5]. By applying a spectral density estimation technique, information about a patient's brain activity can be analyzed by looking at the power of the frequency components in each wave pattern. This information can then be used to get an idea of how the patient's brain is behaving, and whether to diagnose them with epilepsy [6].

Table 1: Wave patterns that can be observed in EEG signals.

Wave pattern	Frequency range	Where it is observed
Delta	0.4-4 Hz	Infants, adults in deep sleep
Theta	4-7 Hz	Young children, drowsy adults
Alpha	7-13 Hz	Adults with eyes closed
Beta	14-30 Hz	Alert or anxious adults, adults with their eyes open
Gamma	>30 Hz	Thought to increase cognitive function, memory and focus. Also typically found in epileptic patients.

KNN for classifying EEG spectra

Once the spectral density of an EEG signal has been computed using a non-parametric spectral density estimation method, it can be classified using an algorithm such as KNN. KNN is a type of neural network that can be trained to extract relevant features from a dataset to classify it. The technique used is called supervised training, where the model is first trained by passing sets of data that the correct category is known and adjusting weights within the model to minimize the prediction error. Once the model has been trained, it can be tested on more datasets to assess its ability to correctly classify datasets it has not 'seen' before. New spectra are classified by assigning them the category which is the smallest Euclidean distance away from the prediction. The accuracy of the KNN classification will depend on the estimator's ability to extract useful information from the raw EEG data.

Methods

Publicly available EEG data from [7] was used for this project. This contains five sets of data: one from healthy patients with their eyes open, one from healthy patients with their eyes closed, two from epileptic patients and one from epileptic patients undergoing seizures. These sets are denoted as A, B, C, D and E respectively. Each set contained 100 time-series EEG. The data was sampled at 173.61 Hz.

Periodogram method

The periodogram method was implemented using the Matlab periodogram function. The periodogram function returns the PSD estimate and the corresponding frequency vector.

Bartlett's method

Bartlett's method was implemented by splitting the data into non-overlapping segments of length n. The Matlab periodogram function was then used to compute a PSD estimate for each segment. These were then averaged using the Matlab mean function to find the PSD estimate using Bartlett's method. The effect of window size on the estimate quality was investigated by changing the segment length n.

Welch's method

Welch's method was implemented using the Matlab pwelch function. Like the periodogram function, this function returns the PSD estimate of a signal and the corresponding frequency vector. It has parameters that can be used to set the number of segments, amount of overlap and window function. Welch's method was applied using a Hamming window, and varying window sizes to investigate the effect of window size on the estimate quality.

KNN classification

The KNN model was used to determine how well the different estimation techniques were able to extract useful information from the raw EEG datasets. Our code creates a KNN model that can be used to determine which of the four classes a given spectral density estimate belongs to. This was done using the Matlab fitcknn function. The number of datasets that are used to create this model and the function used to estimate the PSD of each dataset can be changed. The datasets that have not been used for training can then be classified using the model using the Matlab predict function.

Our code uses this model to generate several metrics that can be used to evaluate how well the spectral density estimates form each method can be classified. The first metric is the resubstitution loss of the model. This shows what percentage of the training data would be classified by the model correctly. It gives an optimistic estimate of how accurately the model would classify new data. The code also calculates what percentage of the datasets that were not used for training are classified correctly by the model. This is done for each of the five sets of data so that the accuracy on each of these can be compared.

Results

Dataset comparison

PSD estimates for the different types of patients are shown in Figure 1. These estimates were calculated using Bartlett's method with 32 segments. The PSD estimate for the epileptic patient having a seizure is significantly different than those for the other patients. It has much higher power at all frequencies. The estimates for the other patients appear more similar. However, the estimates for the healthy patients have more power in the lower frequency components that those for the epileptic patients.

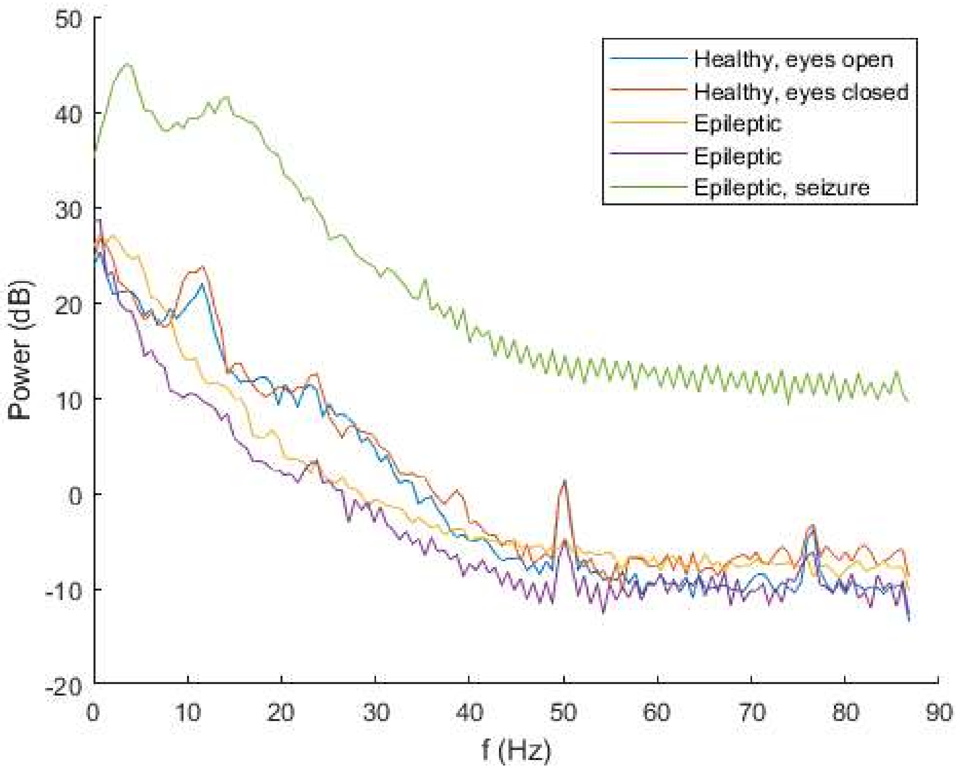

Figure 1: PSD estimates for data from each of the 6 dataset classes; A, B, C, D, E and F. Calculated using Bartlett's method.

Method comparison

A PSD estimate that was found using the periodogram method is shown in Figure 2. This is a PSD estimate for data from a healthy patient with their eyes open. Most of the power is contained in the lower frequencies. There is a spike visible at approximately 50 Hz which is likely due to mains-frequency noise. It is difficult to determine other features in the estimate by visual examination. Side lobes are also visible at frequencies above 45 Hz. These may be obscuring other features.

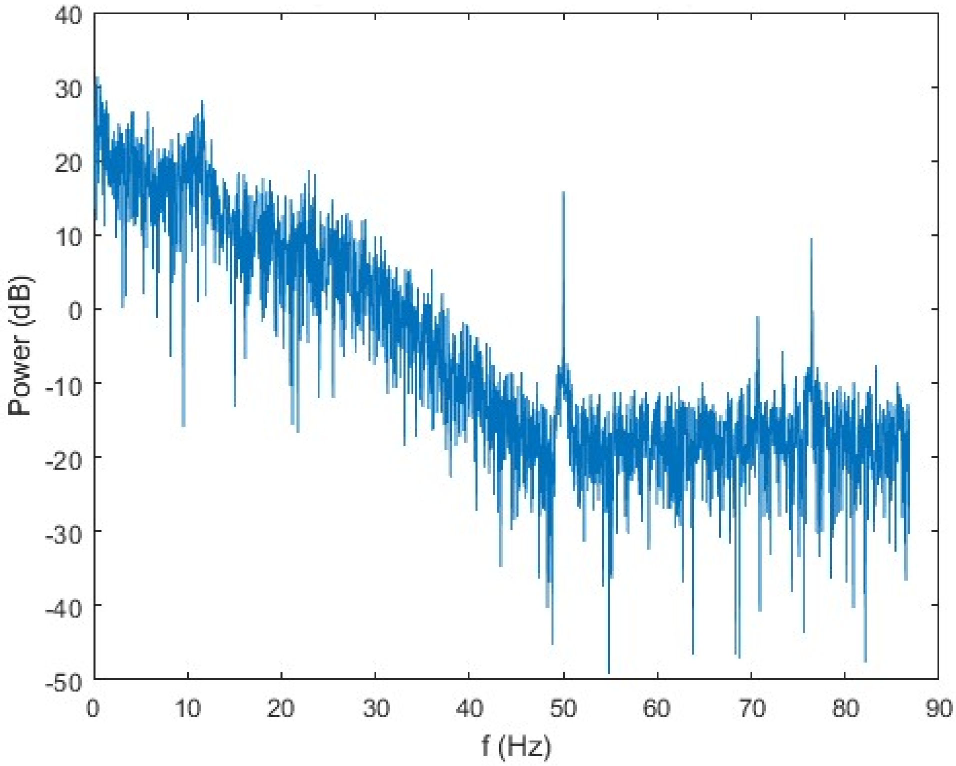

Figure 2: The PSD estimate that was found using the periodogram method

PSD estimates were also found using Bartlett's method. Estimates were found using 128, 64 and 32 segments to explore the effect of changing the number of segments. These are plotted in Figure 3. The PSD estimates shown are again for a healthy patient with their eyes open. As the number of segments increases, the resolution drops and the estimates become smoother. This is due to more segments being averaged. The segments are also shorter so there is less information contained in their periodograms. Because of this, the estimate is much easier to analyze by eye than the periodogram estimate. The increase in the number of segments also results in a loss of information. In the estimates that used 32 and 64 segments, the peak at 50 Hz is clearly visible. However, this information is lost when the number of segments is increased to 128.

There are also side lobes visible in the Bartlett's method estimates. This is because the segments used rectangular windows. The magnitude of the side lobes increases as the number of segments increases.

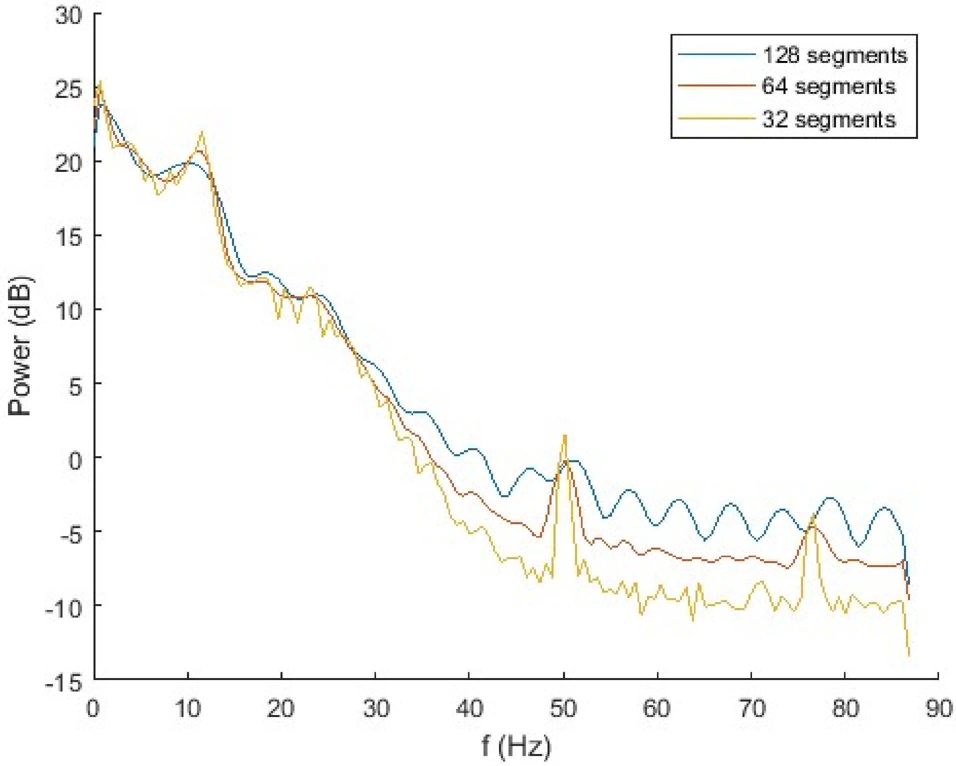

Figure 3: The PSD estimates that were found using Bartlett's method with 128, 64 and 32 segments.

Figure 4 shows the PSD estimates that were found using Welch's method with 128, 64 and 32 segments. The segments overlapped by 50% and a Hamming window was applied to each segment. The most notable difference between these estimates and the Bartlett's method estimates is that the Hamming window has eliminated the side lobes.

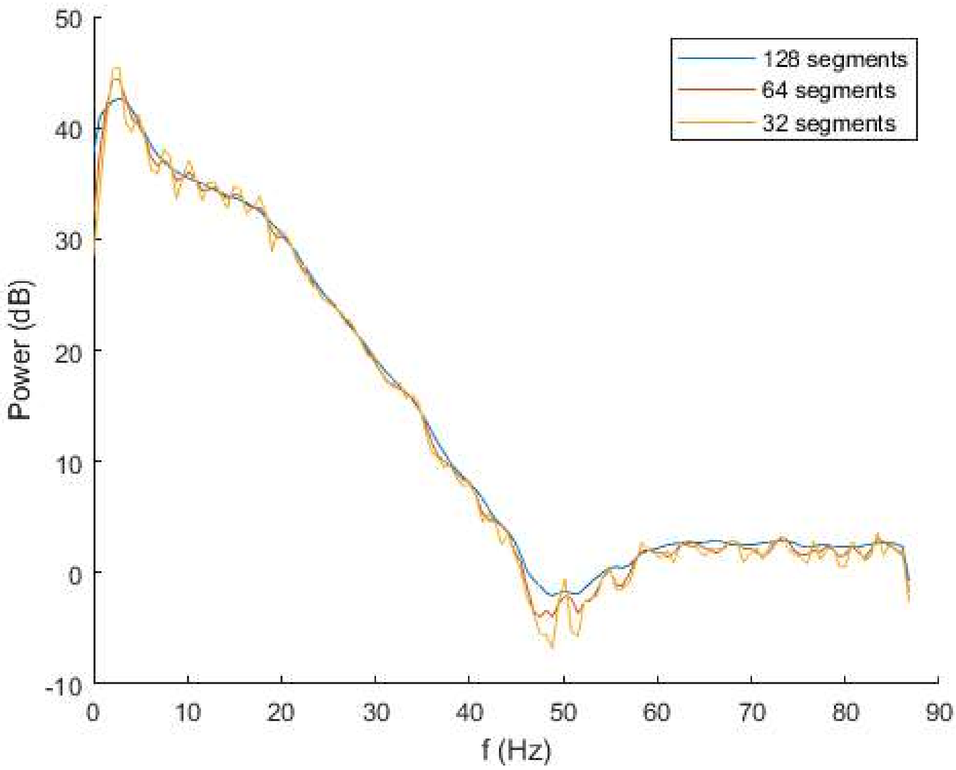

Figure 4: PSD estimates that were found using Welch's method with 128, 64 and 32 segments. A Hamming window function and an overlap of 50% were used.

KNN classification results

The resubstitution loss of the KNN models created using the different methods is shown in Table 2. This was found for models created using different numbers of training datasets for each class. The model created using the periodogram method tended to have a worse resubstitution loss than the other models. This may be because the features are difficult to distinguish due to the high variance and large side lobes. Decreasing the number of segments in both Bartlett's and Welch's method tended to decrease the resubstitution loss. This may be due to the increase in resolution resulting in more distinguishable features.

Table 2: The resubstitution loss for each power spectral density method for different numbers of training datasets.

Number of training datasets	**Periodogram**	**Bartlett's, 128 segments**	**Bartlett's, 64 segments**	**Bartlett's, 32 segments**	**Welch's, 128 segments**	**Welch's, 64 segments**	**Welch's, 32 segments**
50	0.0680	0.0560	0.0400	0.0480	0.0440	0.0480	0.0440
60	0.0500	0.0500	0.0400	0.0333	0.0400	0.0400	0.0367
70	0.0400	0.0429	0.0343	0.0286	0.0343	0.0371	0.0343
80	0.0400	0.0375	0.0375	0.0275	0.0350	0.0375	0.0400
90	0.0356	0.0400	0.356	0.0267	0.0356	0.0378	0.0378

The datasets that were not used for training the KNN model were then classified using the model. The percentage from each set of data that was classified correctly was calculated. These percentages are shown in Table 3. All of the models classified the patients having seizures relatively accurately. This was because the calculated PSD estimates for these patients were significantly different to the others. The patients with their eyes closed appeared to be the most difficult to classify. The models would often classify these as patients with their eyes open as the PSD estimates are fairly similar.

Both Welch's and Bartlett's method appeared to do better than the periodogram method as expected. Decreasing the number of segments also tended to improve Bartlett's and Welch's method as this caused the PSD estimates to have higher resolutions.

Table 3: The percentage of datasets that are correctly classified for each method. This is for models trained on 60 datasets.

Dataset	**Periodogram**	**Bartlett's, 128 segments**	**Bartlett's, 64 segments**	**Bartlett's, 32 segments**	**Welch's, 128 segments**	**Welch's, 64 segments**	**Welch's, 32 segments**
A	97.5	95	97.5	97.5	97.5	95	97.5
B	80	77.5	85.0	82.5	82.5	80	80
C	95	90	97.5	97.5	97.5	97.5	97.5
D	97.5	92.5	97.5	97.5	97.5	97.5	97.5
E	97.5	95	95.0	100	100	100	100

Conclusion

The PSD estimates that were found using each of the estimates were evaluated both by observing the graphs and using metrics from the KNN classification. It was found that both Welch's and Bartlett's method performed better than the periodogram method. It was difficult to observe features in the periodogram plots due to the high variance and large side lobes. The periodogram method also performed poorly in the KNN classification. It had a resubstitution loss of 6.80% when 50 training data sets from each class were used. This was much higher than the resubstitution losses for the other methods, which were often less than 5.00%.

It was also found that Bartlett's and Welch's methods performed better in the KNN classification when they had fewer segments. This is because increasing the number of segments would decrease both variance and resolution. Although having a lower variance can make a PSD easier to read visibly, using a neural network like the KNN algorithm performs much better with more information, i.e. a higher PSD resolution. This was also apparent when graphing the estimates. When the PSD estimate for a healthy patient with their eyes open was plotted using Bartlett's method with 32 segments, the peak at 50 Hz was clearly visible. However, as the number of segments was increased to 64 and 128, the peak became more difficult to see. It is clear to see that increasing the window size smooths the PSD estimation but results in a loss of information.

All the methods performed best when they were classifying the data from patients who were having seizures. Both Welch's and Bartlett's method were able to classify these correctly 100% of the time when they had 32 segments. This was because these spectra are very different when the patient is undergoing a seizure. They had much higher power in all of the frequency components. The most difficult patients to classify appeared to be the healthy patients with their eyes closed. These were only classified correctly at most 85% of the time. This was because these spectra were very similar to those for healthy patients with their eyes open.

It was found that both Bartlett's and Welch's methods made a significant improvement to the periodogram method, by applying windowing techniques to reduce bias and estimate variance. Bartlett's and Welch's method performed very similarly when using an optimal window length, and so it is hard to draw a conclusion whether one is better than the other from the KNN metrics. However, Welch's method does improve upon Bartlett's by removing side lobe affects due to the overlapping windows, and this makes the PSD estimate more easily interpreted, and results in less information being obscured by the lobes.

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