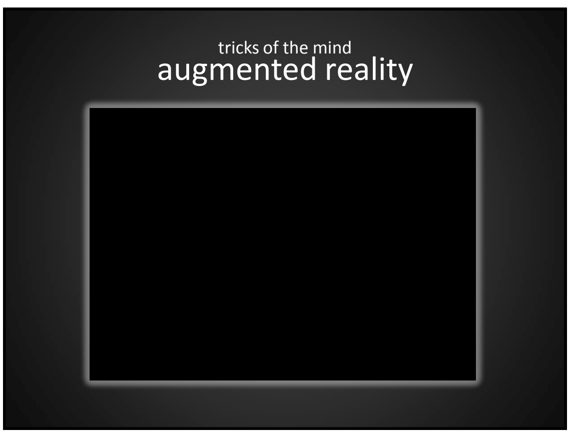
Računalniška grafika

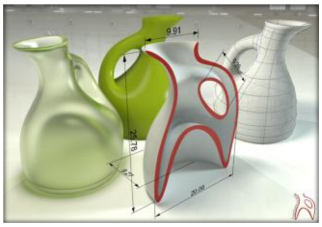
krivulje in ploskve



KRIVULJE

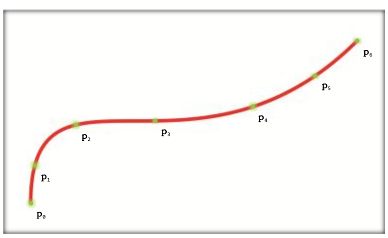
uporaba krivulj

risbe in skice
grafi
tipografije
računalniško podprto načrtovanje
modeliranje realnih predmetov



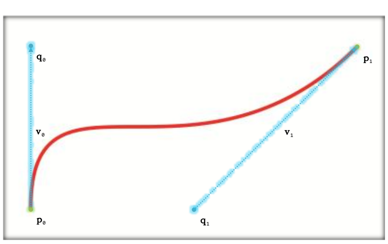
podajanje krivulj

množica točk
kontrolne točke
interpolacija



podajanje krivulj

množica točk
kontrolne točke
aproksimacija



enačba krivulje

eksplicitno
implicitno
parametrično

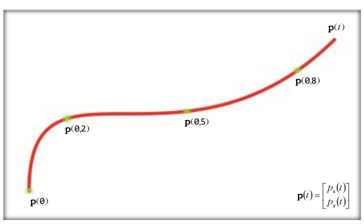
$$p_y = f(p_x)$$
$$p_y = kp_x + m$$
$$p_y = \cos p_x$$

$$f(p_x, p_y) = 0$$
$$ap_x + bp_y + c = 0$$
$$p_x^{\,2} + p_y^{\,2} - r^2 = 0$$

$$\mathbf{p}(t) = \begin{bmatrix} p_x(t) \\ p_y(t) \end{bmatrix}$$
$$p_x(t) = f(t)$$
$$p_y(t) = g(t)$$

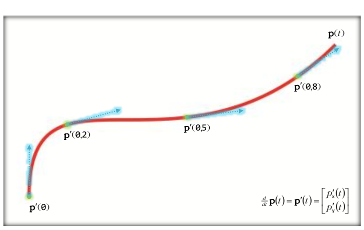
parametrične krivulje

točka kot funkcija ene spremenljivke



parametrične krivulje

tangenta krivulje



parametrične krivulje

krog
elipsa
hiperbola
parabola
premica

$$\mathrm{p}(t)=\begin{bmatrix} p_x(t) \\ p_y(t) \end{bmatrix}$$

$$p_x(t)=r\cos t,\quad p_y(t)=r\sin t$$

$$p_x(t)=a\cos t,\quad p_y(t)=b\sin t$$

$$p_x(t)=a\sec t,\quad p_y(t)=b\tan t$$

$$p_x(t)=at^2,\quad p_y(t)=2at$$

$$p_x(t)=a_x+(b_x-a_x)t,\quad p_y(t)=a_y+(b_y-a_y)t$$

$$f(t)=\sum_{i=0}^{n} c_i t^i$$

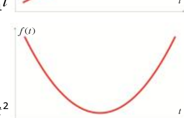
POLINOMSKE FUNKCIJE

polinomske funkcije

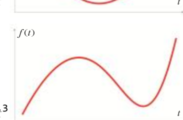
linearne
kvadratne
kubične

$$f(t)=c_0+c_1t$$

$$f(t)=c_0+c_1t+c_2t^2$$



$$f(t)=c_0+c_1t+c_2t^2+c_3t^3$$



$$\mathrm{p}(t) = \sum_{i=0}^{n} \mathrm{c}_i t^i$$

POLINOMSKE KRIVULJE

polinomske krivulje

- linearne
- kvadratne
- kubične

$$\mathrm{p}(t) = \mathrm{c}_0 + \mathrm{c}_1 t$$

$$\mathrm{p}(t) = \begin{bmatrix} p_x(t) \\ p_y(t) \\ p_z(t) \end{bmatrix}, \quad \mathrm{c}_0 = \begin{bmatrix} c_{0x} \\ c_{0y} \\ c_{0z} \end{bmatrix}, \quad \mathrm{c}_1 = \begin{bmatrix} c_{1x} \\ c_{1y} \\ c_{1z} \end{bmatrix}$$

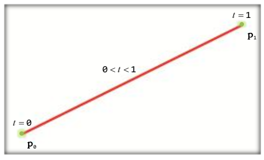
$$p_x(t) = c_{0x} + c_{1x} t$$
$$p_y(t) = c_{0y} + c_{1y} t$$
$$p_z(t) = c_{0z} + c_{1z} t$$

$$\mathrm{p}(t) = \mathrm{c}_0 + \mathrm{c}_1 t + \mathrm{c}_2 t^2$$
$$\mathrm{p}(t) = \mathrm{c}_0 + \mathrm{c}_1 t + \mathrm{c}_2 t^2 + \mathrm{c}_3 t^3$$

polinomske krivulje

- krivulja prvega reda
- linearna interpolacija dveh točk
- tri ekvivalentne predstavitve
  - utežena vsota dveh točk
  - polinom
  - matrični zapis

$$\mathrm{p}(t) = \mathrm{lerp}(t, \mathrm{p}_0, \mathrm{p}_1) = \mathrm{p}_0(1 - t) + \mathrm{p}_1 t$$
$$\mathrm{p}(t) = \mathrm{p}_0 + (\mathrm{p}_1 - \mathrm{p}_0) t$$
$$\mathrm{p}(t) = \begin{bmatrix} \mathrm{p}_0 & \mathrm{p}_1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ t \end{bmatrix}$$

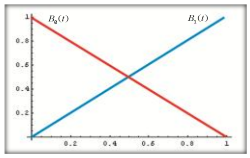


krivulja prvega reda

utežena vsota dveh točk

$$p(t) = p_0(1-t) + p_1 t$$

$$B_0(t) = 1-t, \quad B_1(t) = t$$
$$p(t) = p_0 B_0(t) + p_1 B_1(t)$$

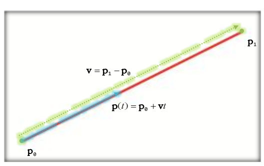


krivulja prvega reda

polinom

$$p(t) = p_0 + (p_1 - p_0)t$$

$$v = p_1 - p_0$$
$$p(t) = p_0 + vt$$



krivulja prvega reda

matrični zapis

$$p(t) = \begin{bmatrix} p_0 & p_1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ t \end{bmatrix}$$

$$G = \begin{bmatrix} p_0 & p_1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \quad T(t) = \begin{bmatrix} 1 \\ t \end{bmatrix}$$
$$p(t) = GBT(t)$$

$$p(t) = (GB)T(t) = CT(t) \begin{bmatrix} p_0 & p_1 - p_0 \end{bmatrix} \begin{bmatrix} 1 \\ t \end{bmatrix} = p_0 + (p_1 - p_0)t$$
$$p(t) = G(BT(t)) = GB(t) = \begin{bmatrix} p_0 & p_1 \end{bmatrix} \begin{bmatrix} 1-t \\ t \end{bmatrix} = p_0(1-t) + p_1 t$$

krivulja prvega reda

izračun tangente krivulje

$$\tfrac{d}{dt}\,\mathrm{p}(t) = \mathrm{p}'(t) = \mathrm{p}_1 - \mathrm{p}_0$$

$$\mathrm{p}(t) = \mathrm{p}_0(1-t) + \mathrm{p}_1 t$$
$$\mathrm{p}'(t) = \mathrm{p}_0(-1) + \mathrm{p}_1 1$$

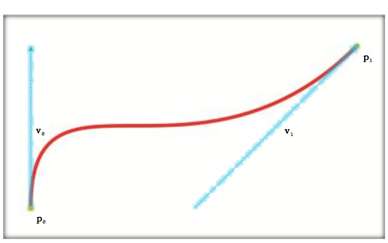
$$\mathrm{p}(t) = \mathrm{p}_0 + (\mathrm{p}_1 - \mathrm{p}_0)t$$
$$\mathrm{p}'(t) = \mathrm{p}_0 0 + (\mathrm{p}_1 - \mathrm{p}_0)1$$

$$\mathrm{p}(t) = \begin{bmatrix}\mathrm{p}_0 & \mathrm{p}_1\end{bmatrix}\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1\\ t\end{bmatrix}$$
$$\mathrm{p}'(t) = \begin{bmatrix}\mathrm{p}_0 & \mathrm{p}_1\end{bmatrix}\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}$$

HERMITSKE KRIVULJE

hermitske krivulje

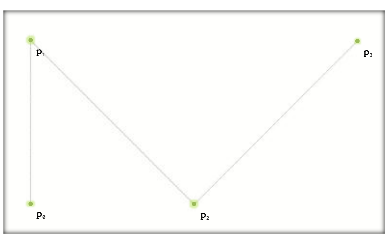
začetna in končna točka
tangenti na krivuljo



BÉZIEROVE KRIVULJE

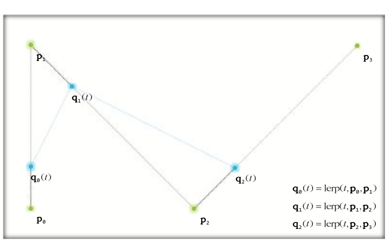
Bézierove krivulje

De Castejau konstrukcija



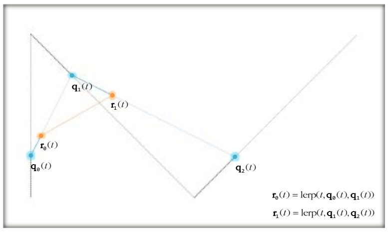
Bézierove krivulje

De Castejau konstrukcija



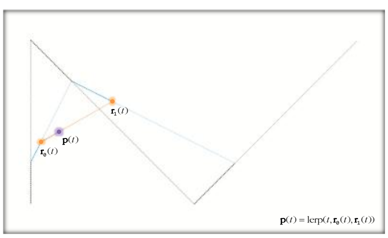
Bézierove krivulje

De Castejau konstrukcija



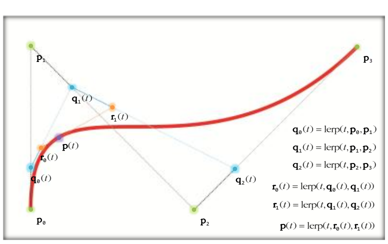
Bézierove krivulje

De Castejau konstrukcija



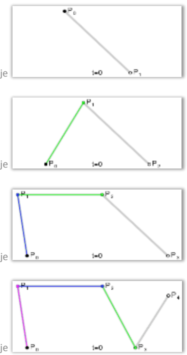
Bézierove krivulje

De Castejau konstrukcija



Bézierove krivulje

De Castejau konstrukcija



prve stopnje

druge stopnje

tretje stopnje

četrte stopnje

Bézierove krivulje

rekurzivna linearna interpolacija

$$\mathbf{p}(t) = \mathrm{lerp}(t, \mathbf{r}_0(t), \mathbf{r}_1(t))$$

$$\mathbf{r}_0(t) = \mathrm{lerp}(t, \mathbf{q}_0(t), \mathbf{q}_1(t))$$

$$\mathbf{r}_1(t) = \mathrm{lerp}(t, \mathbf{q}_1(t), \mathbf{q}_2(t))$$

$$\mathbf{q}_0(t) = \mathrm{lerp}(t, \mathbf{p}_0, \mathbf{p}_1)$$

$$\mathbf{q}_1(t) = \mathrm{lerp}(t, \mathbf{p}_1, \mathbf{p}_2)$$

$$\mathbf{q}_2(t) = \mathrm{lerp}(t, \mathbf{p}_2, \mathbf{p}_3)$$

$\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$

Bézierove krivulje

utežena vsota kontrolnih točk

$$\mathbf{q}_0(t) = \mathrm{lerp}(t, \mathbf{p}_0, \mathbf{p}_1) = \mathbf{p}_0(1-t) + \mathbf{p}_1 t$$

$$\mathbf{q}_1(t) = \mathrm{lerp}(t, \mathbf{p}_1, \mathbf{p}_2) = \mathbf{p}_1(1-t) + \mathbf{p}_2 t$$

$$\mathbf{q}_2(t) = \mathrm{lerp}(t, \mathbf{p}_2, \mathbf{p}_3) = \mathbf{p}_2(1-t) + \mathbf{p}_3 t$$

$$\mathbf{r}_0(t) = \mathrm{lerp}(t, \mathbf{q}_0(t), \mathbf{q}_1(t)) = (\mathbf{p}_0(1-t) + \mathbf{p}_1 t)(1-t) + (\mathbf{p}_1(1-t) + \mathbf{p}_2 t)t$$

$$\mathbf{r}_1(t) = \mathrm{lerp}(t, \mathbf{q}_1(t), \mathbf{q}_2(t)) = (\mathbf{p}_1(1-t) + \mathbf{p}_2 t)(1-t) + (\mathbf{p}_2(1-t) + \mathbf{p}_3 t)t$$

$$\mathbf{p}(t) = \mathrm{lerp}(t, \mathbf{r}_0(t), \mathbf{r}_1(t)) = ((\mathbf{p}_0(1-t) + \mathbf{p}_1 t)(1-t) + (\mathbf{p}_1(1-t) + \mathbf{p}_2 t)t)(1-t) + ((\mathbf{p}_1(1-t) + \mathbf{p}_2 t)(1-t) + (\mathbf{p}_2(1-t) + \mathbf{p}_3 t)t)t$$

$$\mathrm{p}(t) = \mathrm{p}_0(1-t)^3 + \mathrm{p}_1 3(1-t)^2 t + \mathrm{p}_2 3(1-t)t^2 + \mathrm{p}_3 t^3$$

$$B_0(t) = (1-t)^3 = 1 - 3t + 3t^2 - t^3$$

$$B_1(t) = 3(1-t)^2 t = 3t - 6t^2 + 3t^3$$

$$B_2(t) = 3(1-t)t^2 = 3t^2 - 3t^3$$

$$B_3(t) = t^3$$

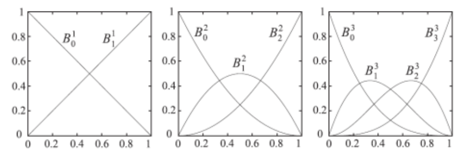
Bézierove krivulje

Bernsteinovi polinomi

$$\mathrm{p}(t) = \sum_{i=0}^{n} \mathrm{p}_i B_i^n(t), \quad B_i^n(t) = \binom{n}{i}(1-t)^{n-i} t^i$$

$$\binom{n}{i} = \frac{n!}{i!(i-n)!}, \quad n! = \prod_{k=1}^{n} k$$

$$\sum_{i=0}^{n} B_i^n(t) = 1$$



Bézierove krivulje

kubični polinom

$$\mathbf{p}(t) = \mathbf{p}_0(1-t)^3 + \mathbf{p}_1 3(1-t)^2 t + \mathbf{p}_2 3(1-t)t^2 + \mathbf{p}_3 t^3$$

$$\mathbf{p}(t) = \mathbf{p}_0\left(1 - 3t + 3t^2 - t^3\right) + \mathbf{p}_1\left(3t - 6t^2 + 3t^3\right) + \mathbf{p}_2\left(3t^2 - 3t^3\right) + \mathbf{p}_3 t^3$$

$$\mathbf{p}(t) = \mathbf{p}_0 + \left(-3\mathbf{p}_0 + 3\mathbf{p}_1\right)t + \left(3\mathbf{p}_0 - 6\mathbf{p}_1 + 3\mathbf{p}_2\right)t^2 + \left(-\mathbf{p}_0 + 3\mathbf{p}_1 - 3\mathbf{p}_2 + \mathbf{p}_3\right)t^3$$

$$\mathrm{p}(t) = \mathrm{c}_0 + \mathrm{c}_1 t + \mathrm{c}_2 t^2 + \mathrm{c}_3 t^3$$

$$\mathrm{c}_0 = \mathrm{p}_0$$

$$\mathrm{c}_1 = \left(-3\mathrm{p}_0 + 3\mathrm{p}_1\right)$$

$$\mathrm{c}_2 = \left(3\mathrm{p}_0 - 6\mathrm{p}_1 + 3\mathrm{p}_2\right)$$

$$\mathrm{c}_3 = \left(-\mathrm{p}_0 + 3\mathrm{p}_1 - 3\mathrm{p}_2 + \mathrm{p}_3\right)$$

$$\mathbf{p}'(t) = \mathbf{c}_1 + \mathbf{c}_2 2t + \mathbf{c}_3 3t^2$$

$$\mathbf{p}(0) = \mathbf{p}_0, \quad \mathbf{p}(1) = \mathbf{p}_3, \quad \mathbf{p}'(0) = 3(\mathbf{p}_1 - \mathbf{p}_0), \quad \mathbf{p}'(1) = 3(\mathbf{p}_3 - \mathbf{p}_2)$$

Bézierove krivulje

matrični zapis

$$\mathbf{p}(t) = \mathbf{C}\mathbf{T}(t) = \begin{bmatrix}\mathbf{c}_0 & \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3\end{bmatrix}\begin{bmatrix}1 \\ t \\ t^2 \\ t^3\end{bmatrix}$$

$$\mathbf{c}_0 = \mathbf{p}_0, \quad \mathbf{c}_1 = -3\mathbf{p}_0 + 3\mathbf{p}_1, \quad \mathbf{c}_2 = 3\mathbf{p}_0 - 6\mathbf{p}_1 + 3\mathbf{p}_2, \quad \mathbf{c}_3 = -\mathbf{p}_0 + 3\mathbf{p}_1 - 3\mathbf{p}_2 + \mathbf{p}_3$$

$$\mathbf{p}(t) = \mathbf{G}\mathbf{B}(t) = \begin{bmatrix}\mathbf{p}_0 & \mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3\end{bmatrix}\begin{bmatrix}B_0(t) \\ B_1(t) \\ B_2(t) \\ B_3(t)\end{bmatrix}$$

$$B_0(t) = 1 - 3t + 3t^2 - t^3, \quad B_1(t) = 3t - 6t^2 + 3t^3, \quad B_2(t) = 3t^2 - 3t^3, \quad B_3(t) = t^3$$

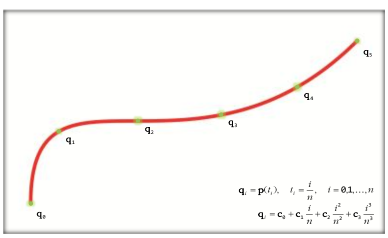
$$\mathrm{p}(t) = \mathrm{G}\mathrm{B}_b\mathrm{T}(t) = \begin{bmatrix}\mathrm{p}_0 & \mathrm{p}_1 & \mathrm{p}_2 & \mathrm{p}_3\end{bmatrix}\begin{bmatrix}1 & -3 & 3 & -1 \\ 0 & 3 & -6 & 3 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 \\ t \\ t^2 \\ t^3\end{bmatrix}$$

Bézierove krivulje

risanje

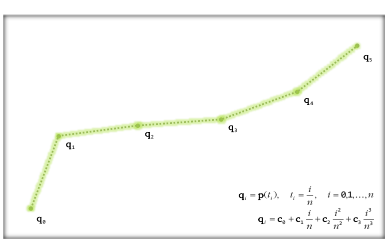
Bézierove krivulje

risanje



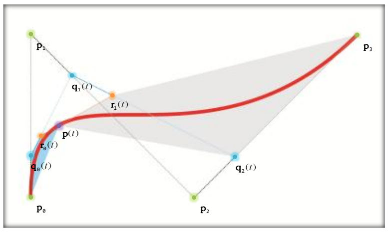
Bézierove krivulje

risanje



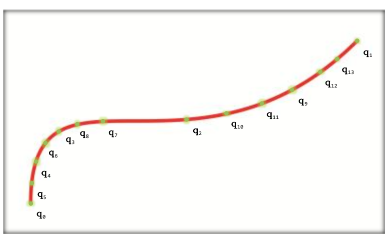
Bézier krivulje

risanje



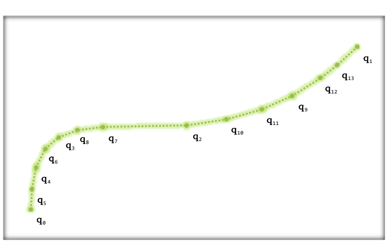
Bézierove krivulje

risanje



Bézierove krivulje

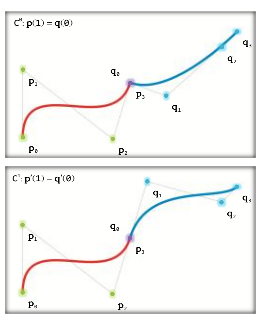
risanje



SESTAVLJENE KRIVULJE

sestavljanje krivulj

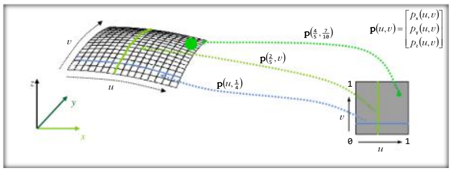
zveznost C^k
zveznost G^k



PARAMETRIČNE PLOSKVE

parametrične ploskve

neodvisna parametra u in v



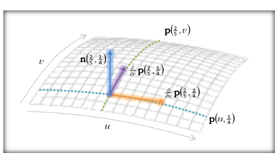
parametrične ploskve

tangente

$$\frac{\partial}{\partial u}\mathbf{p}(u,v)=\begin{bmatrix} \frac{\partial}{\partial u} p_x(u,v) \\ \frac{\partial}{\partial u} p_y(u,v) \\ \frac{\partial}{\partial u} p_z(u,v) \end{bmatrix}, \quad \frac{\partial}{\partial v}\mathbf{p}(u,v)=\begin{bmatrix} \frac{\partial}{\partial v} p_x(u,v) \\ \frac{\partial}{\partial v} p_y(u,v) \\ \frac{\partial}{\partial v} p_z(u,v) \end{bmatrix}$$

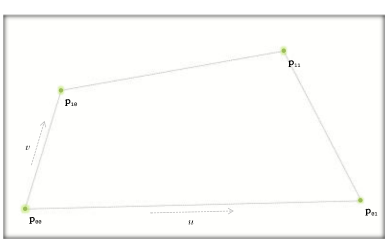
$$\mathbf{n}(u,v)=\frac{\partial}{\partial u}\mathbf{p}(u,v)\times\frac{\partial}{\partial v}\mathbf{p}(u,v)$$

$$\mathbf{n}_u(u,v)=\frac{\mathbf{n}(u,v)}{\|\mathbf{n}(u,v)\|}$$



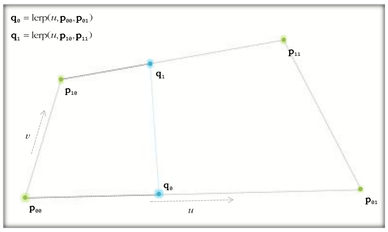
bilinearna krpa

kontrolna mreža štirih točk



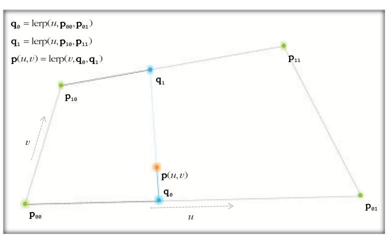
bilinearna krpa

prvi korak



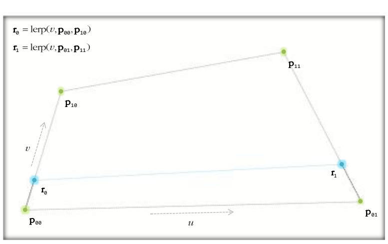
bilinearna krpa

drugi korak



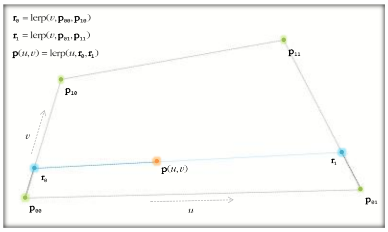
bilinearna krpa

prvi korak



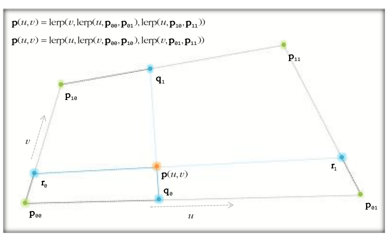
bilinearna krpa

drugi korak



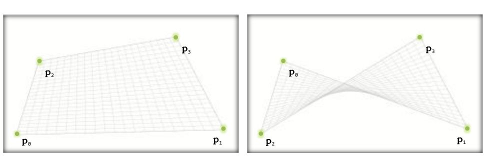
bilinearna krpa

Drugi korak



bilinearna krpa

lastnosti



bilinearna krpa

utežena vsota točk
bilinearni polinom
matrični zapis

$$\mathbf{p}(u,v) = \mathbf{p}_{00}(1-u)(1-v) + \mathbf{p}_{01}u(1-v) + \mathbf{p}_{10}(1-u)v + \mathbf{p}_{11}uv$$

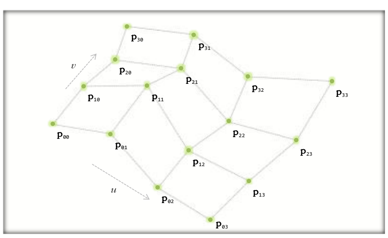
$$\mathbf{p}(u,v) = \mathbf{p}_{00} + (\mathbf{p}_{00} - \mathbf{p}_{10})v + (-\mathbf{p}_{00} + \mathbf{p}_{01})u + (\mathbf{p}_{00} - \mathbf{p}_{01} - \mathbf{p}_{10} + \mathbf{p}_{11})uv$$

$$\mathrm{p}(u,v) = \begin{bmatrix} 1 & v \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{p}_{00} & \mathrm{p}_{01} \\ \mathrm{p}_{10} & \mathrm{p}_{11} \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ u \end{bmatrix}$$

$$\mathrm{p}(u,v) = \mathrm{V}(v)^{\mathsf{T}} \mathrm{B}^{\mathsf{T}} \mathrm{GBU}(u)$$

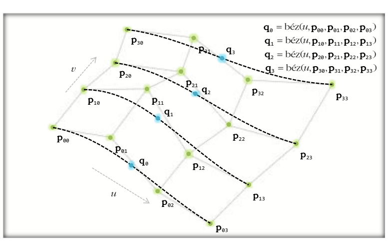
bikubična Bézierova krpa

kontrolna mreža šestnajstih točk



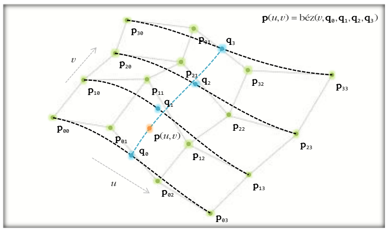
bikubična Bézierova krpa

pri korak



bikubična Bézierova krpa

drugi korak



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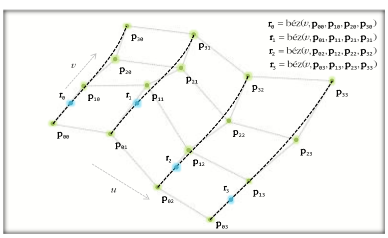
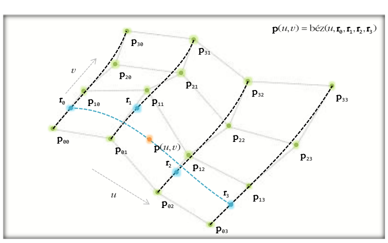
prvi korak



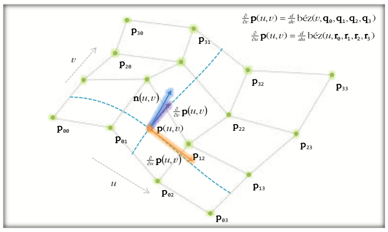
bikubična Bézierova krpa

drugi korak



bikubična Bézierova krpa

tangente



bikubična Bézierova krpa

Bernsteinovi polinomi
matrični zapis

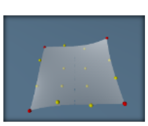
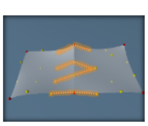
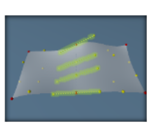
$$\mathbf{p}(u,v) = \sum_{i=0}^{3}\sum_{j=0}^{3}\mathbf{p}_{ij}B_i^3(u)B_j^3(v)$$

$$\mathbf{p}(u,v) = \mathbf{V}(v)^{\mathsf{T}}\mathbf{B}_{\mathrm{b}}{}^{\mathsf{T}}\mathbf{G}\mathbf{B}_{\mathrm{b}}\mathbf{U}(u)$$

$$\mathbf{p}(u,v) = \begin{bmatrix}1 & v & v^2 & v^3\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ -3 & 3 & 0 & 0\\ 3 & -6 & 3 & 0\\ -1 & 3 & -3 & 1\end{bmatrix}\begin{bmatrix}\mathbf{p}_{00} & \mathbf{p}_{01} & \mathbf{p}_{02} & \mathbf{p}_{03}\\ \mathbf{p}_{10} & \mathbf{p}_{11} & \mathbf{p}_{12} & \mathbf{p}_{13}\\ \mathbf{p}_{20} & \mathbf{p}_{21} & \mathbf{p}_{22} & \mathbf{p}_{23}\\ \mathbf{p}_{30} & \mathbf{p}_{31} & \mathbf{p}_{32} & \mathbf{p}_{33}\end{bmatrix}\begin{bmatrix}1 & -3 & 3 & -1\\ 0 & 3 & -6 & 3\\ 0 & 0 & 3 & -3\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\ u\\ u^2\\ u^3\end{bmatrix}$$

bikubična Bézierova krpa

risanje
sestavljanje

krivulje in ploskve

krivulje
kontrolne točke, aproksimacija, interpolacija

enačba krivulje
eksplicitna, implicitna, parametrična, utežena vsota kontrolnih točk, polinom, matrični zapis

parametrične krivulje
linearne, kvadratne, kubične

Bézierove krivulje
De Castejau konstrukcija, Bernsteinovi polinomi, risanje s prilagodljivim vzorčenjem, sestavljanje krivulj

sestavljanje krivulj
zveznost G^k, zveznost C^k

ploskve
bilinearne krpe, bikubične Bézierove krpe, sestavljanje krp

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http://www2.mat.dtu.dk/people/J.Gravesen/cagd/

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dodatna literatura

DO PRIHODNJIČ