Expectations Formation with Fat-Tailed Processes: Evidence and Theory

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- Large recent literature documenting **predictability** in expectation errors
  - Underreaction: lab + field, often short-term or consensus forecasts
  - Overreaction: lab + field, often longer-term individual forecasts

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- **This paper**: study expectations formation in the presence of "**fat**" **tails**
  - **Takeaway**: helps match data + parsimonious model of under & overreaction

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$\Rightarrow$ Allowing for **fat tails** is helpful for understanding belief formation!

OUTLINE

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- **Sample**: 122K observations from 2000-2023 of US and foreign firms in IBES
- **Forecasting variable**:

$$g_{it} \equiv \log \text{sales}_{it} - \log \text{sales}_{it-1 \text{ year}}$$

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  - g_{it} standardized by firm: accounts for heterogenous DGPs Wyatt-Bouchaud 03

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- **Forecasts**:

$$F_t g_{it+h} \equiv \log F_t \text{sales}_{it+h \text{ years}} - \log F_t \text{sales}_{it+(h-1) \text{ years}}$$

  - F_t = consensus analyst forecasts after year t FY-end announcement
  - $F_t g_{it+h}$ standardized using same firm-specific mean and SD as g_{it}
  - Ignores a Jensen's term, but results similar with % growth

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Coibion-Gorodnichenko Error-Revision Regressions

$$\underbrace{g_{it+1} - F_t g_{it+1}}_{\text{forecast error}} = \alpha + \beta \underbrace{\left(F_t g_{it+1} - F_{t-1} g_{it+1}\right)}_{\text{forecast revision}} + \epsilon_{it+1}$$

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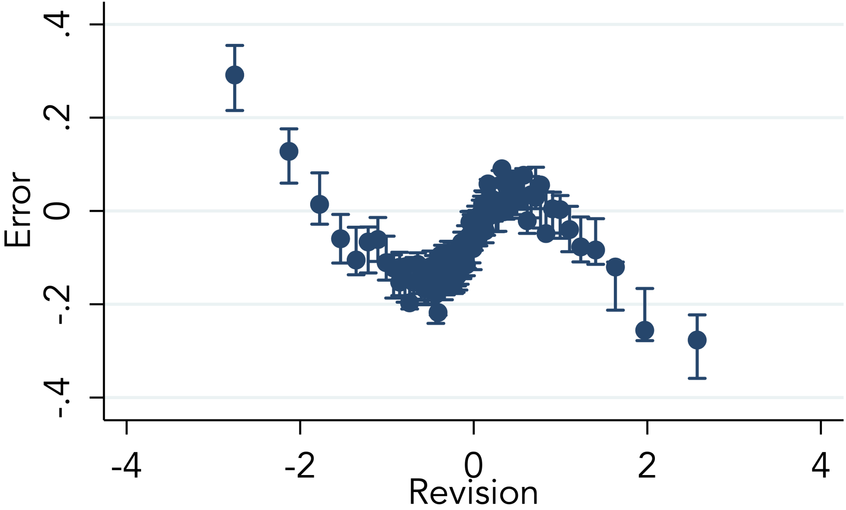
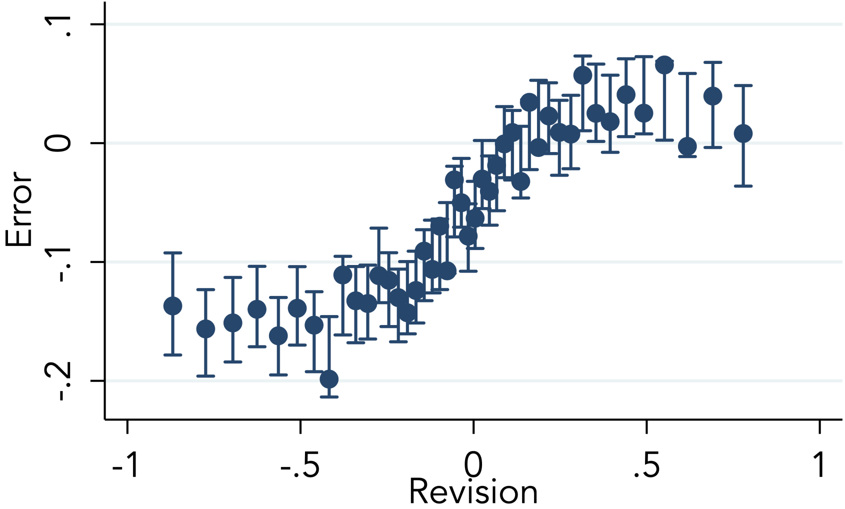
- $\beta \neq 0$ is inconsistent with rational expectations
  - Revisions are in forecasters' information set $\Rightarrow$ should not predict errors
- $\beta > 0 \Rightarrow$ revisions do not update "enough" $\Rightarrow$ **underreaction** Bouchaud et al. 19
- $\beta < 0 \Rightarrow$ revisions update "too much" $\Rightarrow$ **overreaction** Bordalo et al. 19
- Now a standard way of characterizing deviations from RE across datasets

Fact 1: Non-Linear Error-Revision Relationship

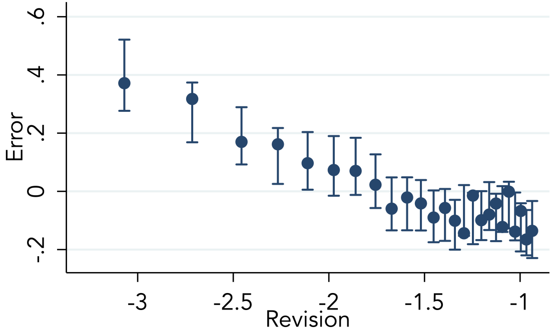


- Forecasts underreact **and** overreact within **same** variable and horizon

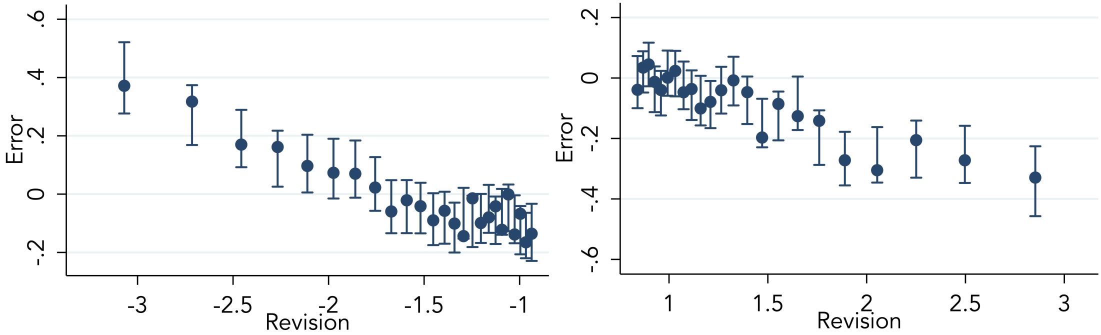
Underreaction in the Bulk of the Distribution...



- Between **10-90%** of revisions, error-revision slope is **positive** Bouchaud et al. 19



- Between **0-10%** of revisions, error-revision slope is **negative**

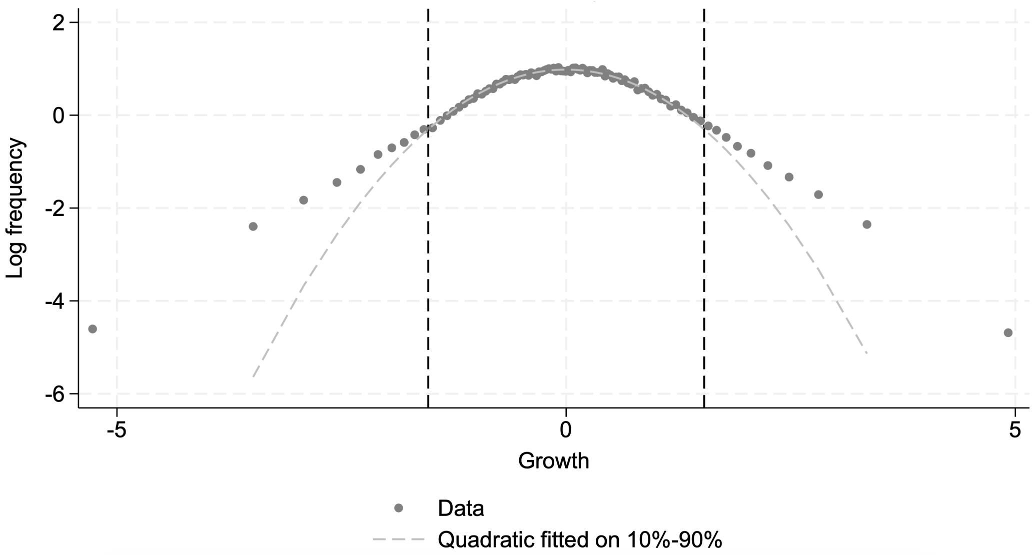


- Between **0-10%** and **90-100%** of revisions, error-revision slope is **negative**

1. Not driven by within-firm adjustment: holds with **raw growth**
2. Does not reflect omitted Jensen's term: holds with **percent growth**
3. Does not arise because of aggregate **time-varying volatility**
4. Not driven by aggregation: present in **individual** forecasts
5. Does not reflect sample: similar for both US and foreign firms

Outline

Tails of g_{it} are Fatter than Gaussian



Tail Behavior in Top Deciles is Approximately a Power Law

Power Law : $\log P(|g_{it}| > x) = -\nu \log x + \text{constant}$

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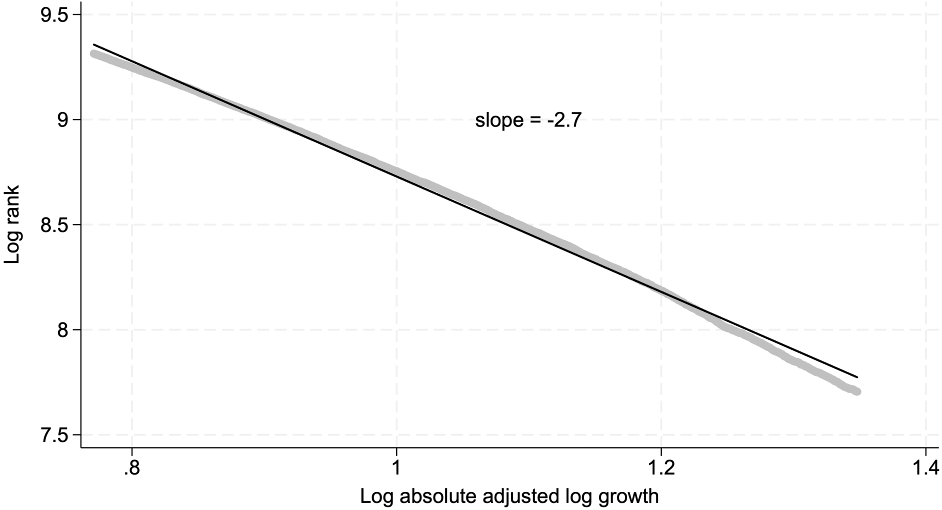
Power Law : $\log P(|g_{it}| > x) \approx -\mathbf{2.7} \log x + \text{constant}$



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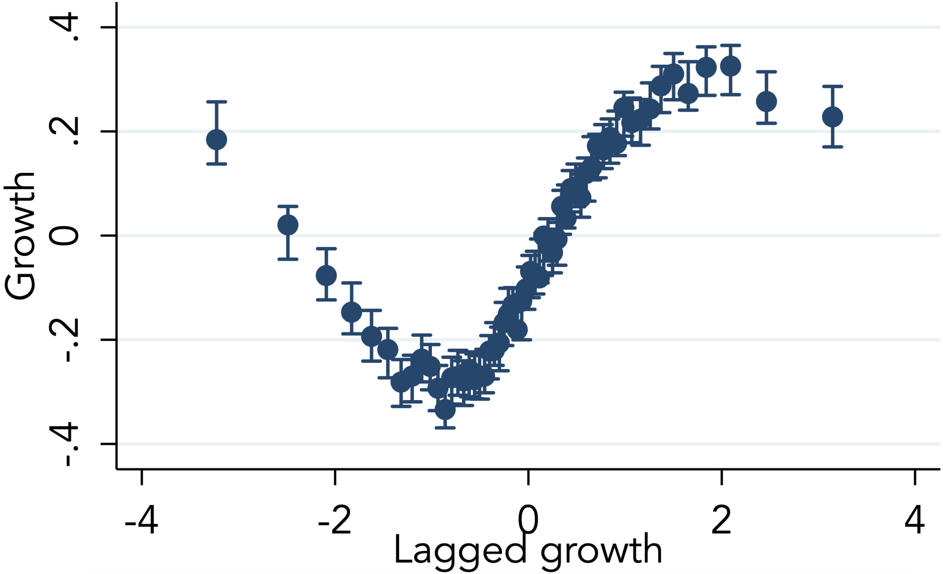
FACT 3: $\mathbf{E}(g_{it}|g_{it-1})$ IS NON-LINEAR



Data-Generating Process

- DGP for sales growth (dropping *i* subscripts):

$$
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g_{t+1} &= g^*_{t+1} + \sigma_\epsilon \epsilon_{t+1} \quad && \epsilon_t \sim f(\cdot) \\
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  - g^*_t = **unobservable** persistent latent state
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- Remarks:
  - If ϵ_t was Gaussian, rational expectation would be the Kalman filter
  - **Key**: tail parameter in u_t smaller than ϵ_t, otherwise inconsistent with Fact 3

DGP Replicates Facts 2 and 3

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  - **Intution**: moderate values of g_t likely reflect $g^*_t \Rightarrow$ likely persistent

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  - **Intuition**: extreme values of g_t likely reflect ϵ_t $\Rightarrow$ likely transitory

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- Note: forecasts overreact to weak + underreact to strong signals Augenblick et al. 24

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Model Fit: DGP

- Estimate DGP parameters using SMM by matching Facts 2 and 3
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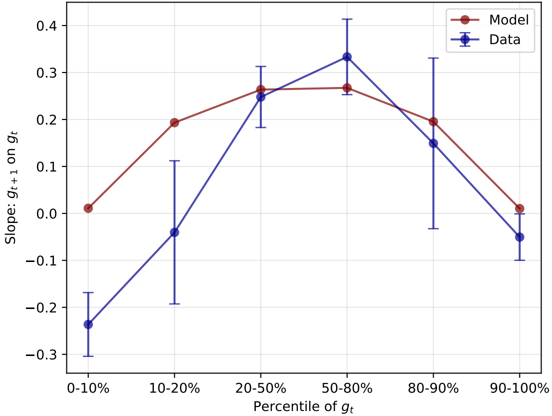
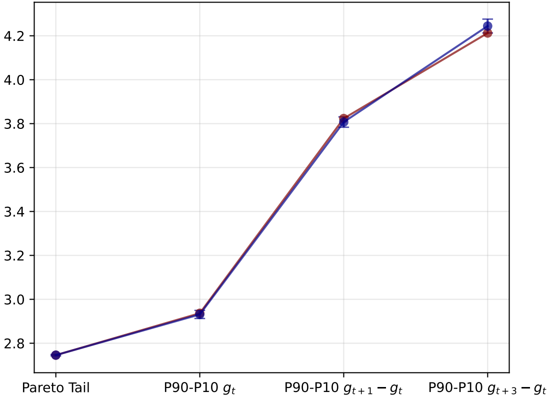
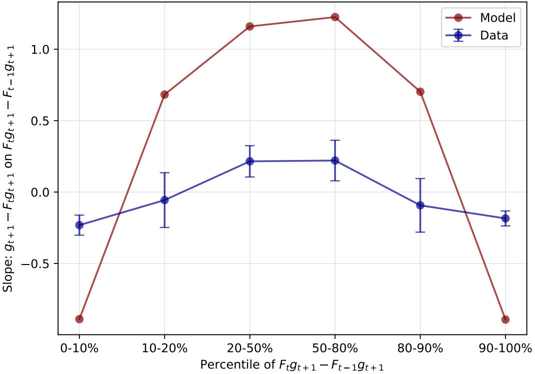
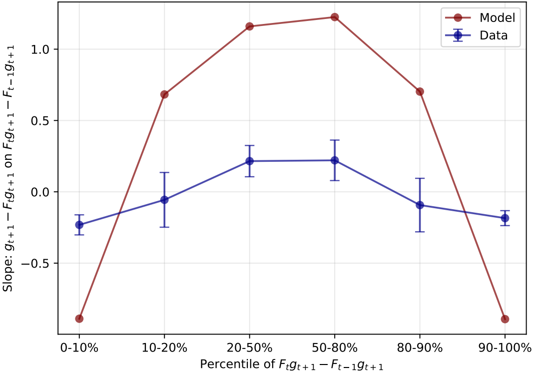
  - Assume $\epsilon \sim t$-distribution with ν degrees of freedom
- Parameter estimates: $\rho = 0.53$, $\nu = 2.53$, $\sigma_u = 0.63$, $\sigma_\epsilon = 1.33$






Model
Data
1.0
0.5
0.0
−0.5
Slope: $g_{t+1} - F_t g_{t+1}$ on $F_t g_{t+1} - F_{t-1} g_{t+1}$
0-10%
10-20%
20-50%
50-80%
80-90%
90-100%
Percentile of $F_t g_{t+1} - F_{t-1} g_{t+1}$

Model Fit: Beliefs



- Given DGP, model generates Fact 1 **qualitatively**, but not **quantitatively**

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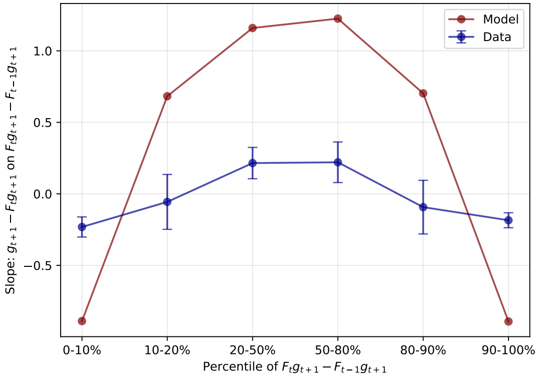

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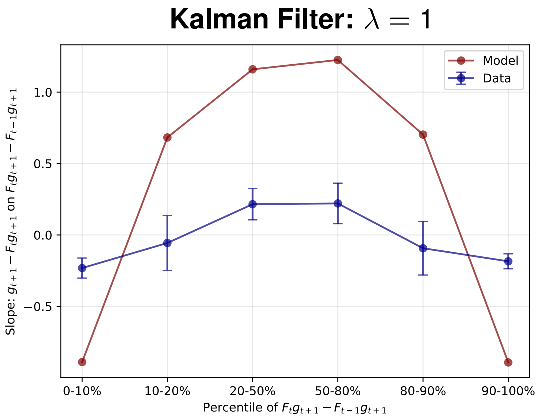

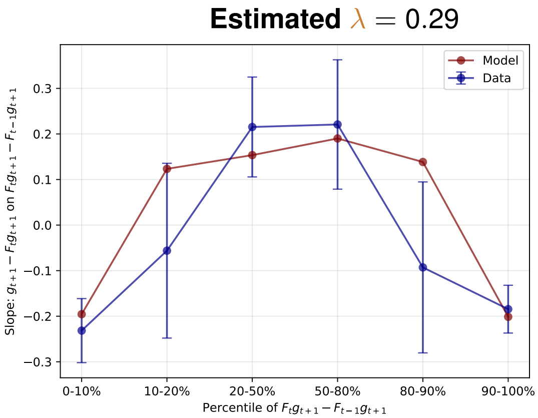

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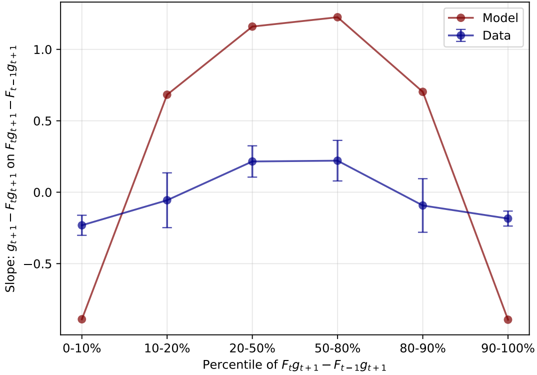
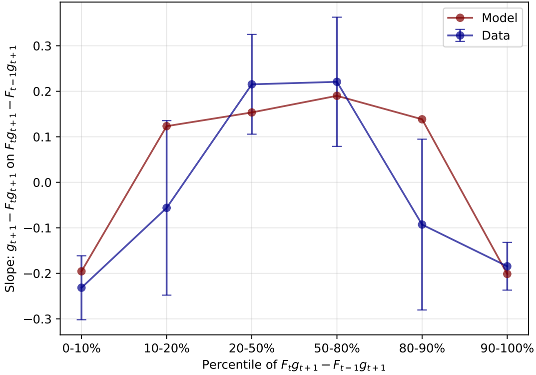
- $\lambda = 0.29 \Rightarrow$ replicate Fact 1

Kalman Filter: $\lambda = 1$



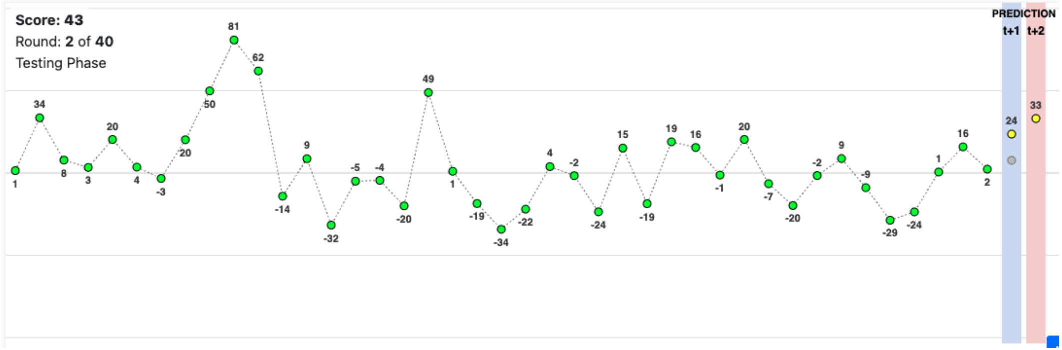
Estimated $\lambda = 0.29$



- Allow anchoring to RE: $F_t^{\lambda} g_{t+h} = \lambda F_t g_{t+h} + (1-\lambda) E_t g_{t+h}$ Fuster et al. 10, Gabaix 19
- $\lambda = 0.29 \Rightarrow$ replicate Fact 1, but accuracy loss is small: **0.1%** of MSE

Outline

- Design follows Afrouzi et al. 23: participants make one and two-period forecasts
- 201 participants make 40 forecasts ⇒ 8K observations possibly scale up?
- DGPs: 1. rescaled estimated DGP, 2. Gaussian AR1 with $\rho = 0.2$ Afrouzi et al. 23



Fat Tails Create Non-Linear Error-Revision Relationship

	Dependent Variable: Error	
	Estimated DGP	Gaussian AR1
	(1)	
Revision	-0.40***	
	(0.02)	
Revision × Bottom 20%		
Revision × Top 20%		
Revision × Top & Bottom 20%		
Constant	✓	
Clustering by Participant	✓	
Number of Observations	7839	

Fat Tails Create Non-Linear Error-Revision Relationship

	Dependent Variable: Error	
	Estimated DGP	Gaussian AR1
	(1)	(4)
Revision	-0.40***	-0.44***
	(0.02)	(0.02)
Revision $\times$ Bottom 20%		
Revision $\times$ Top 20%		
Revision $\times$ Top & Bottom 20%		
Constant	✓	✓
Clustering by Participant	✓	✓
Number of Observations	7839	5421

Fat Tails Create Non-Linear Error-Revision Relationship

	Dependent Variable: Error		
	Estimated DGP		Gaussian AR1
	(1)	(2)	(4)
Revision	-0.40***	-0.28***	-0.44***
	(0.02)	(0.06)	(0.02)
Revision × Bottom 20%		-0.27***	
		(0.09)	
Revision × Top 20%		-0.11	
		(0.08)	
Revision × Top & Bottom 20%			
Constant	✓	✓	✓
Clustering by Participant	✓	✓	✓
Number of Observations	7839	7839	5421

	Dependent Variable: Error			
	Estimated DGP			Gaussian AR1
	(1)	(2)	(3)	(4)
Revision	-0.40***	-0.28***	-0.28***	-0.44***
	(0.02)	(0.06)	(0.06)	(0.02)
Revision × Bottom 20%		-0.27***		
		(0.09)		
Revision × Top 20%		-0.11		
		(0.08)		
Revision × Top & Bottom 20%			-0.18**	
			(0.08)	
Constant	✓	✓	✓	✓
Clustering by Participant	✓	✓	✓	✓
Number of Observations	7839	7839	7839	5421

Fat Tails Create Non-Linear Error-Revision Relationship

	Dependent Variable: Error					
	Estimated DGP			Gaussian AR1		
	(1)	(2)	(3)	(4)	(5)	(6)
Revision	-0.40***	-0.28***	-0.28***	-0.44***	-0.42***	-0.42***
	(0.02)	(0.06)	(0.06)	(0.02)	(0.06)	(0.06)
Revision × Bottom 20%		-0.27***			-0.12	
		(0.09)			(0.09)	
Revision × Top 20%		-0.11			-0.07	
		(0.08)			(0.07)	
Revision × Top & Bottom 20%			-0.18**			-0.09
			(0.08)			(0.07)
Constant	✓	✓	✓	✓	✓	✓
Clustering by Participant	✓	✓	✓	✓	✓	✓
Number of Observations	7839	7839	7839	5421	5421	5421

Outline

Positive Momentum in Bulk + Mean-Reversion in Tails

- Campbell 91 + assume constant $F_t(r_{t+k})$ & earnings growth$_t = \gamma \times g_t$

$$\Rightarrow r_{t+1} = \bar{r} + \gamma \left(F_{t+1} - F_t \right) \sum_{k=0}^{\infty} c^k g_{t+1+k}$$

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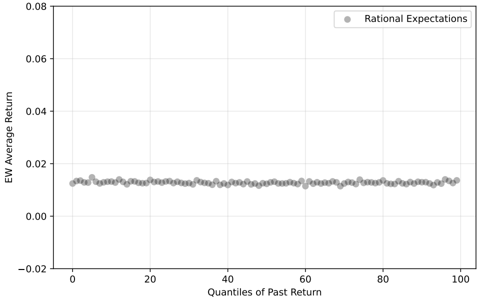
Model



Positive Momentum in Bulk + Mean-Reversion in Tails

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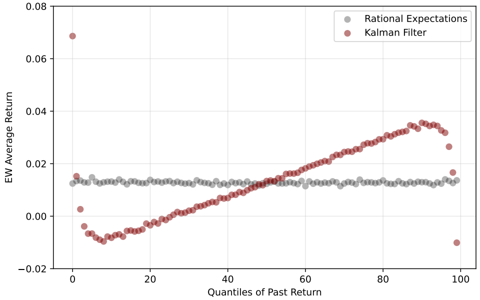
$$\Rightarrow r_{t+1} = \bar{r} + \gamma\left(F_{t+1} - F_t\right)\sum_{k=0}^{\infty} c^k g_{t+1+k}$$

Model



- Campbell 91 + assume constant $F_t(r_{t+k})$ & earnings growth$_t = \gamma \times g_t$

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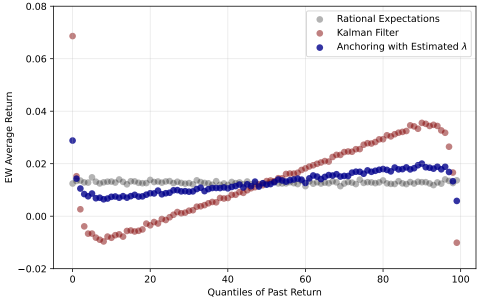
Model

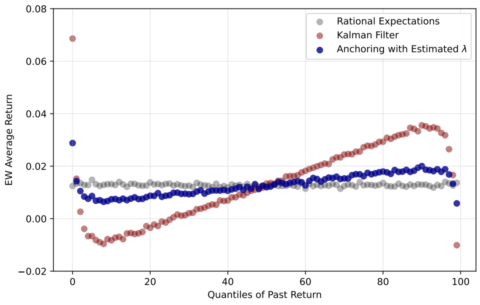


POSITIVE MOMENTUM IN BULK + MEAN-REVERSION IN TAILS

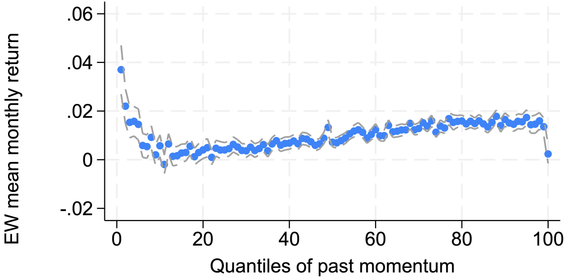
- Campbell 91 + assume constant $F_t(r_{t+k})$ & earnings growth$_t = \gamma \times g_t$

$$\Rightarrow r_{t+1} = \bar{r} + \gamma \left(F_{t+1} - F_t\right) \sum_{k=0}^{\infty} c^k g_{t+1+k}$$

Model



Data: Below Median Market Cap



1. Three Key Facts
   - Fact 1: Non-Linear Error-Revision Relationship
   - Fact 2: Fat Tails in the Distribution of Growth
   - Fact 3: Expected Growth is Non-Linear in Past Growth
2. Model of Expectations Formation
3. Additional Model Predictions
   - Quantitative Fit
   - Forecasting Experiment
   - Return Momentum
4. Conclusion

Conclusion

- Main fact: forecast errors are **non-linear** in forecast revisions
  - Underreaction in the bulk of the distribution, overreaction in the tails
- One deviation from RE can explain this: **ignoring fat tails**
  - **Intuition**: extreme realizations are less persistent than forecasters realize
  - Provides a parsimonious model of under **and** overreaction **within a DGP**
  - Also consistent with evidence from experiments and asset prices
- Broader takeaways:
  1. **Non-Gaussian models of DGP** are helpful for understanding belief formation
  2. **Combining** experiments + surveys useful for assessing important features

THANK YOU!

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