CS 135 Spring 2019: Problem Set 2.

Problem 1. (20 points) For each of the following, use ***only the tree method*** to either show that the argument is valid, or else find a counterexample.

a. (5 points)

$$\begin{array}{r} A \Rightarrow B \\ \neg C \Rightarrow A \\ \hline \therefore \neg(B \Rightarrow C) \end{array}$$

b. (5 points)

$$\begin{array}{r} U \Rightarrow W \\ A \Rightarrow W \\ S \Rightarrow U \\ A \Rightarrow S \\ \hline \therefore \neg U \Rightarrow \neg W \end{array}$$

c. (10 points)
Hypothesis 1: If Superman were able and willing to prevent evil, he would do so.
Hypothesis 2: If Superman were unable to prevent evil, he would be impotent.
Hypothesis 3: If Superman were unwilling to prevent evil, he would be malevolent.
Hypothesis 4: Superman does not prevent evil.
Hypothesis 5: If Superman exists, he is neither malevolent nor impotent.
Conclusion: Therefore, Superman does not exist.

Problem 2. (15 points) Let $Loves(x, y, d)$ be the predicate "*x loves y on day d.*" Thus, for example, $\exists x \, \forall d \; Loves(x, Juliet, d)$ means that there is someone who loves Juliet every day.

a. (1 point) What are the domains of variables x and d in the example above?

Express each of the following statements as a quantified predicate.

b. (2 points) Every day Juliet is loved by someone.
c. (3 points) Iago never loves himself.

Now, let $Future(d_1, d_2)$ denote "day d_1 $comes\ before$ day d_2." Also let $EQ(x, y)$ denote "$x\ and\ y\ are\ the\ same\ person$" and $EQ(d_1, d_2)$ denote "$day\ d_1 is\ the\ same\ as\ d_2$." Use these in addition to $Loves(x, y, d)$ to express the following statements.

d. (3 points) There is a person who, on each day, loves someone other than himself.
e. (3 points) Everyone who someday loves a person who loves everyone everyday loves that person ever after.
f. (3 points) No one loves anyone on days when they are not loved by anyone else.
g. (Extra Credit 5 points) Anyone who loves one person one day, and on a later day loves someone else but not the person he loved the first day is not loved after the later day by the person he loved earlier.