Estimating a Quarterly Output Gap*

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1 Motivation

Although the idea of a 'potential GDP' has gained somewhat of a disputed position, it has still an important place in theoretical concepts such as the Taylor rule. In spite of its important role, it seems as there are no (openly) accessible output gap time series for a higher frequency then annually for many countries, including Germany.[1] This means, that for estimating a sub-annual Taylor rule one has to find alternatives. Without going through the process of estimating an own model for the potential output of an economy, two obvious possibilities emerge: Finding a proxy for potential output which is available with a higher frequency or to interpolate the already existing time series to a lower frequency – the ladder is done in the following. The document additionally describes the remaining steps towards a quarterly Taylor Rule for use in further in analysis.

2 Data

The analysis is carried out for Germany, for the years 1991–2024.

2.1 Time-Series

The yearly time series for the potential output is obtained from AMECO.[2] The quarterly time series for the real German GDP is obtained from DESTATIS.[3] Both of the time series are measured in 2015 Mrd. EUR, the quarterly real output data is seasonally adjusted.[4]

2.2 Interpolation

Why not directly interpolate existing output gap data, if this is the ultimative goal anyway? As the output gap is a non-stochastic function of potential GDP and real GDP, it is also possible to just interpolate the annual potential GDP time series, if one has access to a reliable lower frequency measure for the real GDP. This way, the additional data from the quarterly real GDP data is not lost, and as potential GDP is by construction a lot less prone to fluctuation than real GDP, interpolating potential GDP should also be generally a lot less costly then interpolating the output gap or real GDP. Of course, for this to work, it is crucial that the annual potential GDP estimation and the quarterly real GDP estimation follow similar guidelines. To gain an indication whether this is the case, the annual real GDP time series of AMECO (which should follow the same rules as their potential GDP time series) is compared to the sum of the quarterly real GDP data from DESTATIS, to gain an understanding, whether there could be a systematic deviation between the two time series. For this, a R^2-style measure is used, which gives the percentage of variance of the yearly AMECO time series explained by using the sum of the quarterly time series from

*The GitHub, including data and code, can be found at: https://github.com/timjasch/Quaterly-Output-Gap.

[1] There is however, quaterly data for the US available: https://fred.stlouisfed.org/series/GDPPOT.

[2] https://economy-finance.ec.europa.eu/economic-research-and-databases/economic-databases/ameco-database/download-annual-data-set-macro-economic-database-ameco_en#gross-domestic-product-income-approach-labour-costs

[3] Available at: https://www-genesis.destatis.de/genesis/online?operation=result&code=81000-0002&deep=true#abreadcrumb.

[4] The seasonal adjustment is via the 'X13 JDemetra+' directly by DESTATIS.

DESTATIS.[5] This measure returns a value of >0.9984, indicating that the two time series have compatible measures for the GDP.

Now, it comes to interpolating the yearly output data to a quarterly time series. Simple transformation methods would include to divide the yearly data by four or to use a linear trend within each year, thereby distributing the yearly growth of potential GDP equally over all four quarters. In the following, am univariate version of the Denton-Cholette-method is utilised instead. For the R-implementation, the R-package 'tempdisagg' by Christoph Sax and Peter Steiner is used.[6]

The analysis could also make use of a second, higher frequency time series as an indicator.[7] An obvious candidate for this would be to make use of the quarterly real GDP series, as real GDP and potential GDP have an obvious, large correlation over longer time horizons. However, interpolating in this fashion would lead to the quarterly potential GDP would mimic the fluctuations of the real GDP time series very closely, resulting in a time series which seems way to volatile, to be a sensible measure of potential output.

Also, my intuition leads me to think that by using the real GDP in the interpolation, the output gap estimate should carry a negative bias: As the quarterly potential GDP makes use of the real GDP time series by correcting it its direction, the gap between the quarterly real GDP and the interpolated quarterly potential GDP time series would be artificially small.

Therefore, an univariate interpolation of potential GDP from annual to quarterly data is carried out with the constraint, that the sum of the interpolated quarterly data matches the annual of AMECO for the corresponding year. The resulting time series is depicted in Figure 1 and available on GitHub.

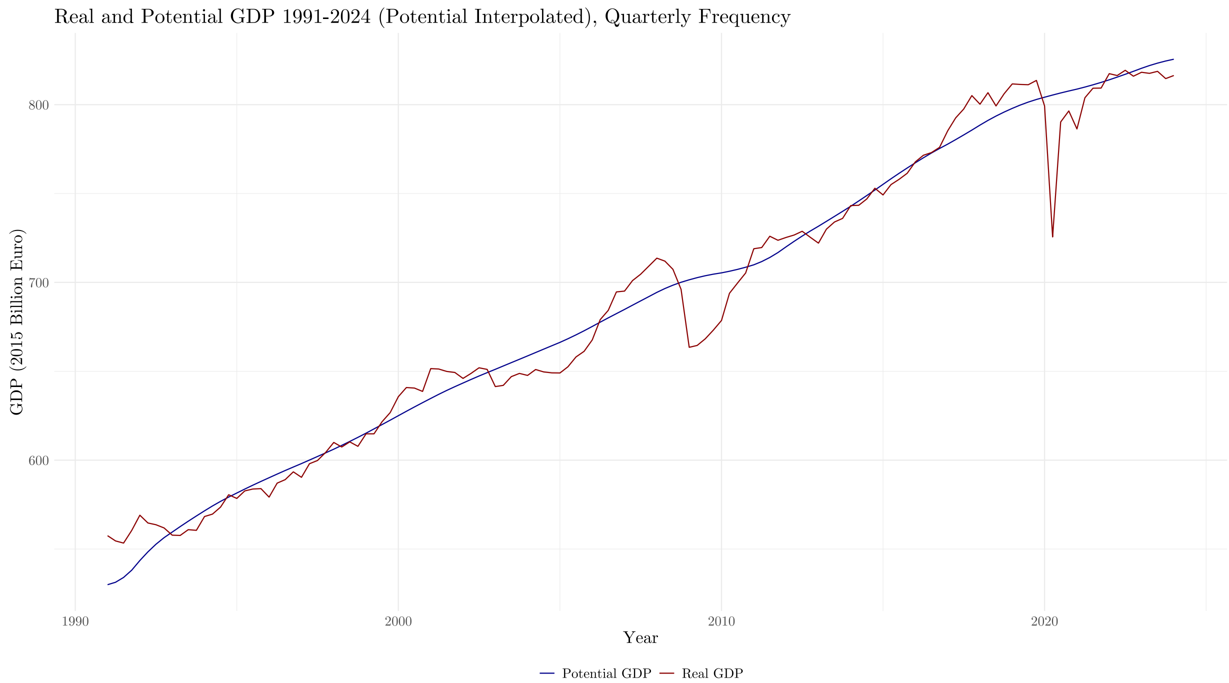

Figure 1

3 Output Gap

Now that the potential and the real output data is in a quarterly frequency, the data for the quarterly output gap can be calculated as follows:

$$Output\_Gap = (GDP_{Real} - GDP_{Potential})/GDP_{Potential}$$

[5]Where the intercept is restricted to be = 0 and the coefficient = 1.
[6]https://journal.r-project.org/archive/2013-2/sax-steiner.pdf
[7]There is a example for an analysis such as that in Sax & Steiner (2013).

The resulting time series spans from 1991Q1–2024Q1, resulting in 133 observations. The average output gap is –0.032%, which seems plausible.

A comparison of the constructed quarterly output gap to the yearly output gap is depicted in Figure 2

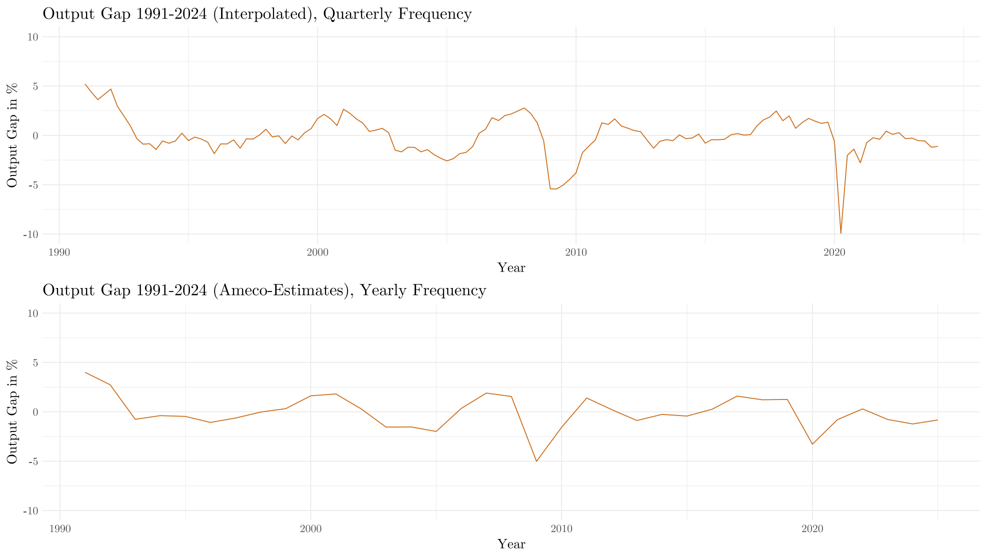

Figure 2

4 Taylor Rate

The Taylor Rule is calculated by the following equation:

$$Taylor\_Rate = 0 + 1.5 * (\pi_{yoy} - 2) + 0.5 * Output\_Gap$$

Where for the output gap, the interpolated quarterly time series from the previous section is used. Notably, the 'natural interest rate' is set equal to 0, with standard coefficients 1.5 and 0.5 for the inflation and output gap control. As the used inflation rate is measured monthly, so the frequency of the resulting Taylor Rule is.

Additionally, as an alternative measure, an rule-based interest rate solely dependent on the inflation rate is calculated, by simply taking the difference of the current year-over-year-inflation rate and the ECBs target of a 2%-inflation-level.

$$Inflation\_Overshoot = \pi_{yoy} - 2$$

Figure 3 shows the Taylor Rate implied Interest Rate, from the first quarter of 2019 until the first quarter of 2024.

Further, Figure 4 shows the movement of four interest rates: The implied Taylor Rate and the Inflation Overshoot Rate, as well as the actual set Deposit and Fixed Interest Rate by the ECB.

Finally, Figure 5 reveals the difference between the ECB set deposit rate and the implied Taylor Rate.

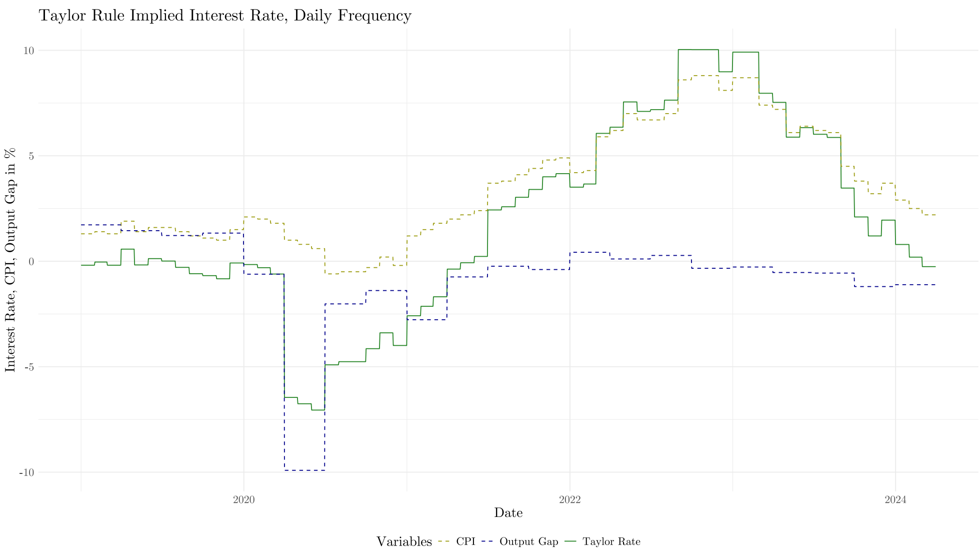

Figure 3

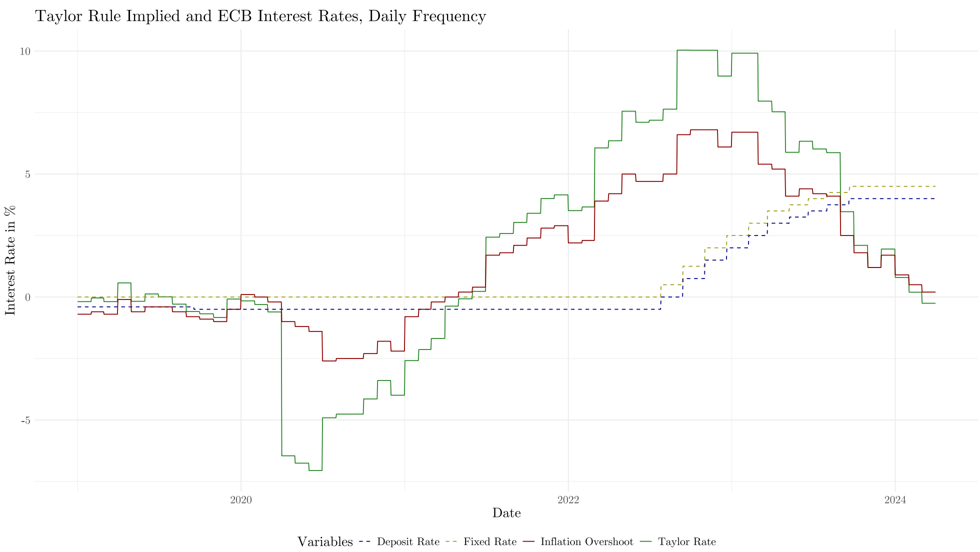

Figure 4

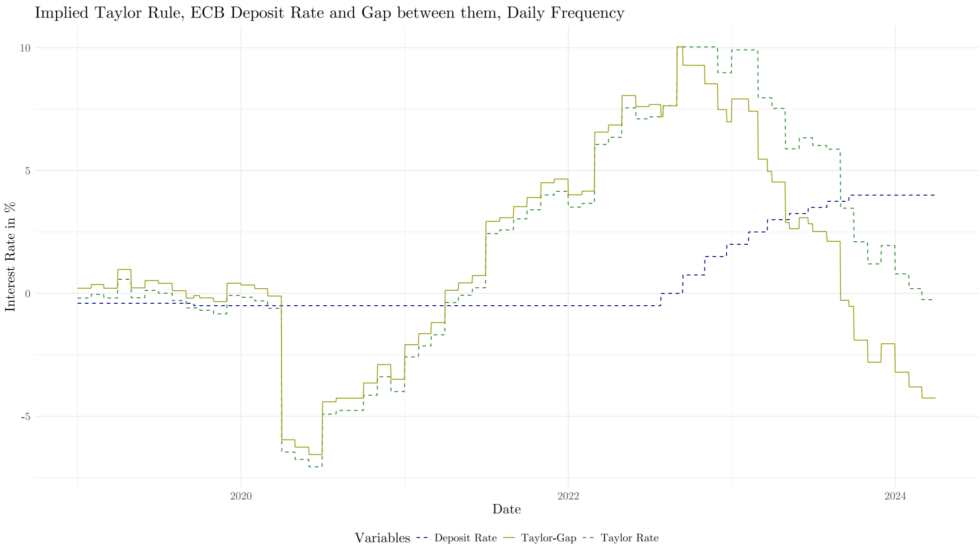
Implied Taylor Rule, ECB Deposit Rate and Gap between them, Daily Frequency
10
5
0
-5
Interest Rate in %
2020
2022
2024
Date
Variables
Deposit Rate
Taylor-Gap
Taylor Rate

Figure 5