Tim Baer

TARC 2017 Flight Simulator Documentation

The motion of the inputted rockets is modeled in 2D space (horizontal and vertical). The equations of motion are defined over the interval $[0,\ \textit{rocket touches ground})$.

The positive x-axis is defined as the ray with endpoint at the launch pad base extending parallel to ground along the intended horizontal direction of flight and the positive y-axis as the perpendicular ray extending upward.

Variables

Initial mass $\leq 650\ (g)$

Rocket drag coefficient: Long elliptical nose cone typically has the least amount of drag compared to other common shapes.

Parachute drag coefficient: Typically near 1.75.

Propellant burn rate: Most model rocket motors use black powder (gunpowder). Its burn rate varies significantly with its grain size and shape.

Wind Speed: Assumes rocket is flown into the wind, and that wind acts horizontally on the rocket. Additionally, wind speed is assumed to constant throughout the flight.

Note: All variables are pre-assigned 'reasonable' values for a 2017 TARC rocket. User can override these values.

Transitional Time Points

0, liftoff begins
$\textit{Root of } l - p[t]$, rocket leaves launch pad
r^{-1}, propellant burn out
$r^{-1} + b$, parachute deploys
$\textit{Root of } py[t]$, rocket lands

$0 < \textit{Root of } l - py[t] \leq r^{-1} \leq r^{-1} + b$

Additional transitional time points exist when terminal velocity is reached. More precisely, whenever $\textit{drag} = \textit{gravity}$ over a decreasing interval.

Relevant Equations

$$\vec{F} = m\vec{a} = m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = m\frac{\mathrm{d}^2\vec{p}}{\mathrm{d}t^2} = thrust + drag + weight$$

$$thrust = \begin{cases} \dot{m}u_e + A_e(p_e - p_0),\ r^{-1} \geq t \\ 0,\ t > r^{-1} \end{cases},\ where\ p_e = p_0\ is\ assumed$$

$$drag = \begin{cases} -\frac{1}{2}pC_{d1}A_1v[t]^2,\ 0 \leq t \leq r^{-1} + b \\ \frac{1}{2}pC_{d1}A_1v[t]^2 + \frac{1}{2}pC_{d2}A_2v[t]^2,\ t > r^{-1} + b \end{cases},\ where\ p\ is\ assumed\ to\ be\ constant\ (1.225\ kg/m^3)$$

$$weight = \begin{cases} 0,\ 0 \leq t \leq Root\ of\ l - p[t] \\ mg\sin\theta,\ Root\ of\ l - p[t] \leq t \leq r^{-1} + b \\ mg,\ t > r^{-1} + b \end{cases},\ where\ g\ is\ assumed\ to\ be\ constant\ (9.80665\ m/s^2)$$

$$\theta = \begin{cases} initial\ angle,\ 0 \leq t \leq\ Root\ of\ l - p[t] \\ \tan^{-1}\frac{vy[t]}{vx[t]},\ Root\ of\ l - p[t] < t \leq r^{-1} + b \\ 90,\ t > r^{-1} + b \end{cases}$$

Jump Discontinuities

Red lines connect the points at the time of the discontinuity on all graphs.

Margin of Error for Methods

NDSolve: $AccuracyGoal = PrecisionGoal \approx 7.98$

FindRoot: $AccuracyGoal = PrecisionGoal \approx 7.98$

FindMaximum: $AccuracyGoal = PrecisionGoal \approx 7.98$

References

Benson, Tom. Index of Rocket Slides. NASA, Web. 24 July 2016.

16.Unified: Thermodynamics and Propulsion
Prof. Z. S. Spakovszky

Mathematica Source Code

```
Off[InterpolatingFunction::dmval]
Style["Mathematica Version",Bold]
"ReleaseID" /. ("Kernel" /. SystemInformation["Small"])



(* If you input data, evaluate cells for updated results *)
Subscript[m,in]=0.65;
\[CapitalTheta]=75;
rc=0.75;
Subscript[a,1]=1;
pc=1.75;
d=0.5;
Subscript[m,p]=0.1;
Subscript[v,p]=0.1;
r=0.2;
Subscript[a,2]=1;
l=2;
q="C6-3";
k=1;
wind= -2;

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Style["Steps",Bold]

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"1. Select output options"

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"2. Evaluation\[LongRightArrow][Evaluate All Cells]"

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"3. Edit variable values"

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"4. Select visible code and Evaluation\[LongRightArrow] [Evaluate Cells]"
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Style["Graph Selection",Bold]
Panel[{"TARC 2017 Scoring: " Checkbox[Dynamic[g0]]}[[1]]]
Panel[{"Position Graphs: " Checkbox[Dynamic[g1]]}[[1]]]
Panel[{"Velocity Graphs: " Checkbox[Dynamic[g2]]}[[1]]]
Panel[{"Acceleration Graphs: " Checkbox[Dynamic[g3]]}[[1]]]
Panel[{"\[CapitalSigma] Forces Graphs: " Checkbox[Dynamic[g4]]}[[1]]]
Panel[{"Angle Graph: " Checkbox[Dynamic[g5]]}[[1]]]


Grid[Prepend[{{"motor code",InputField[Dynamic[q],String]},{"\[NumberSign] of
motors",InputField[Dynamic[k]]},{"initial mass
(kg)",InputField[Dynamic[Subscript[m,in]]]},{"launch angle
(degrees)",InputField[Dynamic[\[CapitalTheta]]]},{"rocket drag
coefficient",InputField[Dynamic[rc]]},{"rocket drag reference area
(m^2)",InputField[Dynamic[Subscript[a,1]]]},{"parachute drag
coefficient",InputField[Dynamic[pc]]},{"parachute diameter
(m)",InputField[Dynamic[d]]},{"propellant mass
(kg)",InputField[Dynamic[Subscript[m,p]]]},{"propellant volume
(m^3)",InputField[Dynamic[Subscript[v,p]]]},{"propellant burn rate
(m/s)",InputField[Dynamic[r]]},{"area fluid propellant passes through
(m^2)",InputField[Dynamic[Subscript[a,2]]]},{"launch rod length
(m)",InputField[Dynamic[l]]},{"wind speed", InputField[Dynamic[wind]]}},{"Variable","Editable
Value (as decimal)"}],Alignment->Left,Spacings->{2, 1},Frame->All]

b=ToExpression[StringTake[q,{StringPosition[q,"-"][[1]][[1]]+1,StringLength[q]}]];
If[ToExpression[StringTake[q,{1}]]===A,i=k*1.88,If[ToExpression[StringTake[q,{1}]]===B,i=k*3.7
55,If[ToExpression[StringTake[q,{1}]]===C,i=k*7.505,If[ToExpression[StringTake[q,{1}]]===D,i=
k*15.05,If[ToExpression[StringTake[q,{1}]]===E,i=k*30.005,If[ToExpression[StringTake[q,{1}]]==
=F,i=k*60.005,If[ToExpression[StringTake[q,{1}]]===G,i=k*120.005]]]]]]];


gravity=9.80665;
\[Epsilon]=1*10^(-10)  (* sufficiently small *);
n=1*10^3        (* sufficiently large*);
\[Theta]=If[\[CapitalTheta]==90,\[Pi]/2,ArcTan[(vya[t])/(vxa[t])]];
mv= Max[Subscript[m,in]-r*t*Subscript[m,p],Subscript[m,in]-Subscript[m,p]]     (* moving mass*);
thrustvc=
Piecewise[{{(Subscript[m,p]/Subscript[v,p])*Subscript[a,2]*(i/(Subscript[m,in]-r*t*Subscript[m,p])),
0<=t<=r^(-1) },{0,t>r^(-1)}}] (* thrust variable constants *);
dragvc=Piecewise[{{0.6125*rc*Subscript[a,1], 0<=t<=r^(-1)+b},{0.6125*rc*Subscript[a,1]+
1.93*pc*(d/2)^2,t>r^(-1)+b}}]    (* drag variable constants *);
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s3=NDSolve[{mv*vxd'[t]==-wind+thrustvc*Cos[\[CapitalTheta]*\[Pi]/180]-(vxd[t]/Abs[vxd[t]])*dragvc*Cos[\[CapitalTheta]*\[Pi]/180]*vxd[t]^2,mv*vyd'[t]==thrustvc*Sin[\[CapitalTheta]*\[Pi]/180]-(vyd[t]/Abs[vyd[t]])*dragvc*Sin[\[CapitalTheta]*\[Pi]/180]*vyd[t]^2,vxd[0]==\[Epsilon]*Cos[\[CapitalTheta]*\[Pi]/180],vyd[0]==\[Epsilon]*Sin[\[CapitalTheta]*\[Pi]/180]},{vxd',vyd',vxd,vyd},{t,0,r^(-1)}]     (* liftoff, assumes rocket leaves launch pad before fuel burnout  *);
x=t/.FindRoot[l-Sqrt[Evaluate[Integrate[vxd[t]/.s3,{Hold[t],0,t}]]^2+Evaluate[Integrate[vyd[t]/.s3,{Hold[t],0,t}]]^2],{t,\[Epsilon],0,n}];


s1=NDSolve[{mv*vxa'[t]==-wind+thrustvc*Cos[\[Theta]]*vxa[t]-(vxa[t]/Abs[vxa[t]])*dragvc*Cos[\[Theta]]*vxa[t]^2,mv*vya'[t]==thrustvc*Sin[\[Theta]]*vya[t]-(vxa[t]/Abs[vxa[t]])dragvc*Sin[\[Theta]]*vya[t]^2-gravity,vxa[x]==vxd[x]/.s3,vya[x]==vyd[x]/.s3},{vxa',vya',vxa,vya},{t,x,r^(-1)+b}]     (* Tsiolkovsky rocket equation written as an ODE, describes rocket flight before parachute ejection *);


terminalvelocity=-Sqrt[(dragvc)/(mv*gravity)];


w=Piecewise[{{terminalvelocity/.{t->r^(-1)+b},(vya[r^(-1)+b]/.s1)[[1]]<terminalvelocity/.{t->r^(-1)+b}},{(vya[r^(-1)+b]/.s1)[[1]],(vya[r^(-1)+b]/.s1)[[1]]>=terminalvelocity/.{t->r^(-1)+b}}}]    (*vertical acceleration initial value*);
s2=NDSolve[{mv*vyb'[t]==-wind-(vyb[t]/Abs[vyb[t]])*dragvc*vyb[t]^2-gravity,vyb[r^(-1)+b]==w},{vyb',vyb},{t,r^(-1)+b,n}]     (* object falling toward Earth with parachute drag, describes rocket flight after parachute ejection *);


vx[t_]:= Piecewise[{{(vxd[t]/.s3)[[1]],0<=t<=x},{(vxa[t]/.s1)[[1]],x<t<=r^(-1)+b},{0,t>r^(-1)+b}}]     (* Horizontal  Velocity *);
vy[t_]:=Piecewise[{{(vyd[t]/.s3)[[1]],0<=t<=x},{Max[(vya[t]/.s1),terminalvelocity],x<t<=r^(-1)+b},{Max[(vyb[t]/.s2),terminalvelocity],r^(-1)+b<t}}]     (* Vertical Velocity*);
vn[t_]:=Sqrt[vx[t]^2+vy[t]^2];


ax[t_]:=vx'[t];
ay[t_]:=vy'[t];
an[t_]:=Sqrt[ax[t]^2+ay[t]^2]
(*\[LeftDoubleBracketingBar]Acceleration\[RightDoubleBracketingBar] *);
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```
fx[t_]:= mv*ax[t];
fy[t_]:= mv*ay[t];
fn[t_]:= Sqrt[fx[t]^2+fy[t]^2];


sol=NDSolve[{py'[t]==vy[t],py[0]==\[Epsilon]*Sin[\[CapitalTheta]]},py,{t,0,10}];
sol10=NDSolve[{px'[t]==vx[t],px[0]==\[Epsilon]*Cos[\[CapitalTheta]]},px,{t,0,10}];
pn[t_]:=Sqrt[(px[t]/.sol10)^2+(py[t]/.sol)^2];



s4=FindRoot[py[t]/.sol,{t,10*n,r^(-1),100*n}]     (* assumes rocket hits ground after thrusting
stage *);
td=Round[t/.s4,0.01];
s5=Piecewise[{{4*Abs[41-t],t<41},{0,41<=t<=43},{4*Abs[t-43],t>43}}];
am=Round[3.28084*FindMaximum[py[t]/.sol,{t,r^(-1),0,t/.s4}][[1]]]     (* in feet *);


If[g0,"Flight duration: ",Null]
If[g0,td,Null]
If[g0,"Maximum altitude: ",Null]
If[g0,am,Null]
If[g0,"TARC 2017 SCORE: ",Null]
If[g0,(Abs[775-am]+s5/.{t->td}),Null]

If[g0,ParametricPlot3D[{t,px[t]/.sol10,py[t]/.sol},{t,0,t/.s4},PlotRange->All,
Exclusions->"Discontinuities",ExclusionsStyle->Red, PlotLabel->"Parametric Plot (t, horizontal
position, vertical position)"],Null]
If[g0,Grid[Prepend[Table[{t,(px[t]/.sol10)[[1]],(py[t]/.sol)[[1]]},{t,0,t/.s4}],{"time","horizontal
position","vertical position"}],Alignment->Left,Spacings->{2, 1},Frame->All],Null]


If[g1,Plot[px[t]/.sol10,{t,0,t/.s4},PlotRange->All,
Exclusions->"Discontinuities",ExclusionsStyle->Red, PlotLabel->"Horizontal Position (m)"],Null]
If[g1,Plot[py[t]/.sol,{t,0,t/.s4},PlotRange->All,
Exclusions->"Discontinuities",ExclusionsStyle->Red, PlotLabel->"Vertical Position (m)"],Null]
If[g1,Plot[pn[t],{t,0,t/.s4},PlotRange->All, Exclusions->"Discontinuities",ExclusionsStyle->Red,
PlotLabel->" \[LeftDoubleBracketingBar]Position\[RightDoubleBracketingBar] (m)"],Null]


If[g2,Plot[vx[t],{t,0,t/.s4},PlotRange->All, Exclusions->"Discontinuities",ExclusionsStyle->Red,
PlotLabel->"Horizontal Velocity (m/s)"],Null]
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If[g2,Plot[vy[t],{t,0,t/.s4},PlotRange->All,
Exclusions->"Discontinuities",ExclusionsStyle->Red,PlotLabel->"Vertical Velocity (m/s)"],Null]
If[g2,Plot[vn[t],{t,0,t/.s4},PlotRange->All,
Exclusions->"Discontinuities",ExclusionsStyle->Red,PlotLabel->"\[LeftDoubleBracketingBar]Velo
city\[RightDoubleBracketingBar] (m/s)"],Null]


If[g3,Plot[ax[t],{t,0,t/.s4},Exclusions->"Discontinuities",ExclusionsStyle->Red,PlotLabel->"Horizo
ntal Acceleration (m/s^2)"],Null]
If[g3,Plot[ay[t],{t,0,t/.s4},Exclusions->"Discontinuities",ExclusionsStyle->Red,PlotLabel->"Vertical
Acceleration (m/s^2)"],Null]
If[g3,Plot[an[t],{t,0,t/.s4},Exclusions->"Discontinuities",ExclusionsStyle->Red,PlotLabel->"\[LeftD
oubleBracketingBar]Acceleration\[RightDoubleBracketingBar] (m/s^2)"],Null]


If[g4,Plot[fx[t],{t,0,t/.s4},Exclusions->"Discontinuities",ExclusionsStyle->Red,PlotLabel->"\[Capita
lSigma] Horizontal Forces (N)"],Null]
If[g4,Plot[fy[t],{t,0,t/.s4},Exclusions->"Discontinuities",ExclusionsStyle->Red,PlotLabel->"\[Capita
lSigma] Vertical Forces (N)"],Null]
If[g4,Plot[fn[t],{t,0,t/.s4},Exclusions->"Discontinuities",ExclusionsStyle->Red,PlotLabel->"\[LeftDo
ubleBracketingBar]\[CapitalSigma] Forces\[RightDoubleBracketingBar] (N)"],Null]


If[g5,Plot[Piecewise[{{\[CapitalTheta],0<=t<x},{ArcTan[(vy[t])/(vx[t])]*180/\[Pi],x<t<=r^(-1)+b}}],{t,0
,r^(-1)+b},PlotRange->All,
Exclusions->"Discontinuities",ExclusionsStyle->Red,PlotLabel->"Angle (degrees)" ],Null]
```