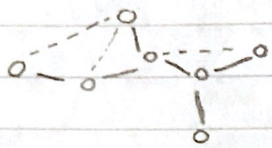
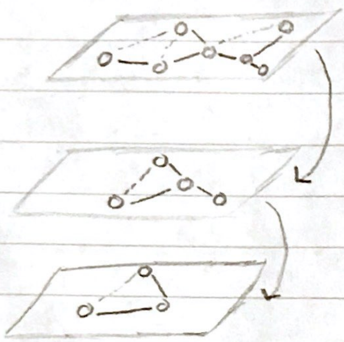
ultra-sparsifier : G' with $n + O(n)$ edges is <u>ultra-sparsifier</u> if it is a good approximation of G



(take spanning tree & add edges)

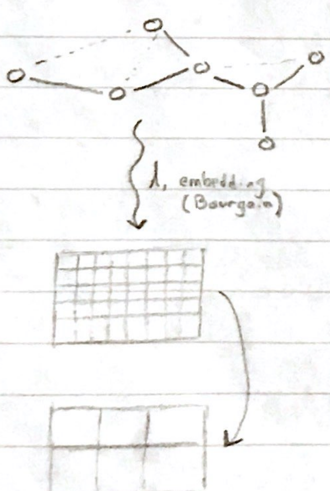
to "fix" eigenvalues

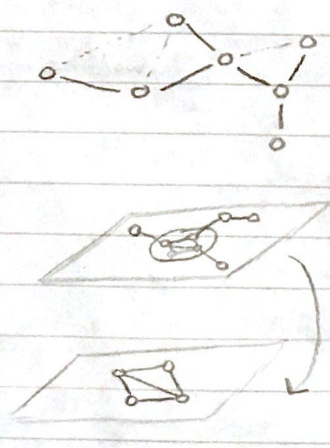


Cholesky factorization
(to eliminate vertices of deg 1 & 2)

problem: can one design an algorithm for solving linear equations in Laplacian in time $O(m \log \varepsilon^{-1})$.

Sherman : min-cost flow





j-trees

$$\tilde{\kappa}_{X \to Y}(A) = \min \{ \|A\| \|G\| : G : Y \to X,\ A \circ G \circ A = A \}$$

Let F be an $(\alpha, \frac{\beta}{\tilde{\kappa}})$ algorithm then F^* is an $(\frac{\alpha}{1-\beta}, \frac{\beta^t}{\tilde{\kappa}})$ algorithm.

(a final $\circ$ with a $(M, 0)$ solver reduces β-error).

Andoni : where does low diameter decompositions/metric tree embeddings appear?