2D AR1 random effects on NAA vs. M (+CPI)

## Tables

Converged = converged with positive definite Hessian. Model in each table with **lowest retro (not AIC)** is highlighted in grey and plotted in the figures.

#### Table 1. NAA only (models where only NAA are random effects)

Model m1, NAA\_sigma = rec + NAA\_cor = iid, estimates recruitment as independent random effects with . This is closest to a SCAA model and referred to as the “base” model in figures. NAA\_sigma = rec + NAA\_cor = ar1\_y estimates recruitment as correlated random effects (AR1 by year). NAA\_sigma = rec+1 is the ‘full state-space’ model with all numbers-at-age as random effects, one for age-1 and one for all other ages. Models m3-m6 are the four in Haikun’s draft.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Model |  |  |  |  | -log | AIC |  | Mohn’s | Mohn’s | Mohn’s |
| Base | 1.67 (0.18) | — | — | — | -919.603 | -1685.2 | 252.6 | 6.4180 | 1.0166 | -0.4265 |
| NAA-1 | 0.66 (0.08) | — | 0.92 (0.05) | — | -957.164 | -1758.3 | 179.5 | 4.5580 | 0.8969 | -0.3848 |
| NAA-2 | 1.22 (0.15) | 0.61 (0.05) | — | — | -1024.455 | -1892.9 | 44.9 | 0.8120 | 0.1656 | -0.1010 |
| NAA-3 | 0.93 (0.13) | 0.54 (0.05) | — | 0.50 (0.09) | -1036.632 | -1915.3 | 22.5 | 0.4032 | 0.0351 | 0.0204 |
| NAA-4 | 0.78 (0.11) | 0.48 (0.05) | 0.61 (0.08) | — | -1045.568 | -1933.1 | 4.7 | 0.6720 | 0.0952 | -0.0440 |
| NAA-5 | 0.73 (0.11) | 0.47 (0.05) | 0.53 (0.09) | 0.33 (0.12) | -1048.883 | -1937.8 | 0.0 | 0.5133 | 0.0552 | 0.0016 |

#### Table 2. M only (models where only M is a random effect)

NAA as in m1 above (base model SCAA, NOT “full state-space”). Just estimating , without any random effects on NAA or M, reduces AIC and a lot (compare m1 to m6).

## Warning in rm(M\_re, est\_M, conv, runtime, pdHess): object 'est\_M' not found

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Model |  |  |  |  | -log | AIC |  | Mohn’s | Mohn’s | Mohn’s |
| Base | — | — | — | — | -919.603 | -1685.2 | 233.3 | 6.4180 | 1.0166 | -0.4265 |
| M-1 | — | 1.21 (0.10) | — | — | -1031.676 | -1907.4 | 11.1 | 0.1242 | 0.1802 | -0.1085 |
| M-2 | — | 1.15 (0.43) | — | 0.26 (0.48) | -968.364 | -1778.7 | 139.8 | 2.8848 | 0.1083 | -0.1026 |
| M-3 | — | 0.17 (0.08) | 0.98 (0.02) | — | -981.554 | -1805.1 | 113.4 | 1.5005 | -0.1409 | 0.3245 |
| M-4 | — | 0.79 (0.14) | 0.63 (0.16) | 0.40 (0.16) | -1039.268 | -1918.5 | 0.0 | -0.1035 | 0.0664 | -0.0047 |

#### Table 3. NAA + M.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Model | Random effects NAA | Random effects M | -log |  | AIC | Mohn’s | Mohn’s | Mohn’s |
| NAA-M-1 | IID | IID | -1047.749 | 30.2 | -1937.5 | 0.2127 | 0.1176 | -0.0548 |
| NAA-M-2 | IID | 2D AR1 | -1046.981 | 35.7 | -1932 | -0.137 | 0.0248 | 0.0425 |
| NAA-M-3 | 2D AR1 | IID | -1064.853 | 0.0 | -1967.7 | 0.4516 | 0.0505 | 0.01 |
| NAA-M-4 | 2D AR1 | 2D AR1 |  |  |  |  |  |  |

#### Table 5. Best models, compare SSB predictions.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Model | NAA cor | NAA dev | M\_re | runtime | NLL |  | AIC |  |  |  | 2019 | 2020 | 2021 |
| Base | IID | Recruit | — | 1.15 | -919.603 | 282.5 | -1685.2 | 6.4180 | 1.0166 | -0.4265 | 288 (182, 456) | 1029 (113, 9386) | 2141 (271, 16939) |
| NAA-2 | IID | All NAA | — | 0.23 | -1024.455 | 74.8 | -1892.9 | 0.8120 | 0.1656 | -0.1010 | 439 (186, 1037) | 1389 (219, 8817) | 2561 (426, 15386) |
| NAA-5 | 2D AR1 | All NAA | — | 0.70 | -1048.883 | 29.9 | -1937.8 | 0.5133 | 0.0552 | 0.0016 | 273 (111, 674) | 545 (111, 2671) | 1031 (142, 7504) |
| M-1 | IID | Recruit | IID | 7.43 | -1031.676 | 60.3 | -1907.4 | 0.1242 | 0.1802 | -0.1085 | 233 (67, 804) | 1733 (209, 14377) | 4136 (692, 24720) |
| M-4 | IID | Recruit | 2D AR1 | 7.55 | -1039.268 | 49.2 | -1918.5 | -0.1035 | 0.0664 | -0.0047 | 191 (77, 476) | 628 (43, 9255) | 1611 (115, 22625) |
| NAA-M-2 | IID | All NAA | 2D AR1 | 0.96 | -1046.981 | 35.7 | -1932.0 | -0.1370 | 0.0248 | 0.0425 | 174 (65, 470) | 346 (28, 4305) | 715 (36, 14166) |
| NAA-M-3 | 2D AR1 | All NAA | IID | 1.23 | -1064.853 | 0.0 | -1967.7 | 0.4516 | 0.0505 | 0.0100 | 294 (113, 766) | 615 (130, 2917) | 1168 (152, 8958) |