2D AR1 random effects on NAA vs. M (+CPI)

## Tables

Converged = converged with positive definite Hessian. Model in each table with **lowest retro (not AIC)** is highlighted in grey and plotted in the figures.

#### Table 1. NAA only (models where only NAA are random effects)

Model m1, NAA\_sigma = rec + NAA\_cor = iid, estimates recruitment as independent random effects with . This is closest to a SCAA model and referred to as the “base” model in figures. NAA\_sigma = rec + NAA\_cor = ar1\_y estimates recruitment as correlated random effects (AR1 by year). NAA\_sigma = rec+1 is the ‘full state-space’ model with all numbers-at-age as random effects, one for age-1 and one for all other ages. Models m3-m6 are the four in Haikun’s draft.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Model |  |  |  |  | -log | AIC |  | Mohn’s | Mohn’s | Mohn’s |
| Base | 1.67 (0.18) | — | — | — | -919.603 | -1685.2 | 252.6 | 6.4180 | 1.0190 | -0.4265 |
| NAA-1 | 0.66 (0.08) | — | 0.92 (0.05) | — | -957.164 | -1758.3 | 179.5 | 4.5580 | 0.9014 | -0.3848 |
| NAA-2 | 1.22 (0.15) | 0.61 (0.05) | — | — | -1024.455 | -1892.9 | 44.9 | 0.8120 | 0.1702 | -0.1010 |
| NAA-3 | 0.93 (0.13) | 0.54 (0.05) | — | 0.50 (0.09) | -1036.632 | -1915.3 | 22.5 | 0.4032 | 0.0384 | 0.0204 |
| NAA-4 | 0.78 (0.11) | 0.48 (0.05) | 0.61 (0.08) | — | -1045.568 | -1933.1 | 4.7 | 0.6720 | 0.0983 | -0.0440 |
| NAA-5 | 0.73 (0.11) | 0.47 (0.05) | 0.53 (0.09) | 0.33 (0.12) | -1048.883 | -1937.8 | 0.0 | 0.5133 | 0.0583 | 0.0016 |

#### Table 2. M only (models where only M is a random effect)

NAA as in m1 above (base model SCAA, NOT “full state-space”). Just estimating , without any random effects on NAA or M, reduces AIC and a lot (compare m1 to m6).

## Warning in rm(M\_re, est\_M, conv, runtime, pdHess): object 'est\_M' not found

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Model |  |  |  |  | -log | AIC |  | Mohn’s | Mohn’s | Mohn’s |
| Base | — | — | — | — | -919.603 | -1685.2 | 233.3 | 6.4180 | 1.0190 | -0.4265 |
| M-1 | — | 1.21 (0.10) | — | — | -1031.676 | -1907.4 | 11.1 | 0.1242 | 0.1827 | -0.1085 |
| M-2 | — | 1.15 (0.43) | — | 0.26 (0.48) | -968.364 | -1778.7 | 139.8 | 2.8848 | 0.1163 | -0.1026 |
| M-3 | — | 0.17 (0.08) | 0.98 (0.02) | — | -981.554 | -1805.1 | 113.4 | 1.5005 | -0.1372 | 0.3245 |
| M-4 | — | 0.79 (0.14) | 0.63 (0.16) | 0.40 (0.16) | -1039.268 | -1918.5 | 0.0 | -0.1035 | 0.0687 | -0.0047 |

#### Table 3. NAA + M.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Model | Random effects NAA | Random effects M | -log |  | AIC | Mohn’s | Mohn’s | Mohn’s |
| NAA-M-1 | IID | IID | -1047.749 | 30.2 | -1937.5 | 0.2127 | 0.1212 | -0.0548 |
| NAA-M-2 | IID | 2D AR1 | -1046.981 | 35.7 | -1932 | -0.137 | 0.027 | 0.0425 |
| NAA-M-3 | 2D AR1 | IID | -1064.853 | 0.0 | -1967.7 | 0.4516 | 0.0532 | 0.01 |
| NAA-M-4 | 2D AR1 | 2D AR1 |  |  |  |  |  |  |

#### Table 4. NAA + CPI.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Model | Random effects on NAA | NLL |  | AIC |  |  |  |
| NAA-CPI-1 | IID | -978.419 | 23.7 | -1790.8 | 0.6474 | 0.1634 | -0.0950 |
| NAA-CPI-2 | ar1\_a | -986.085 | 10.3 | -1804.2 | 0.3926 | 0.0626 | -0.0029 |
| NAA-CPI-3 | ar1\_y | -989.044 | 4.4 | -1810.1 | 0.5592 | 0.1188 | -0.0567 |
| NAA-CPI-4 | 2D AR1 | -992.244 | 0.0 | -1814.5 | 0.4479 | 0.0781 | -0.0189 |

#### Table 5. Best models with CPI fit so can compare AIC, F=0 in projections so can compare SSB predictions.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Model | NAA cor | NAA dev | M\_re | runtime | NLL |  | AIC |  |  |  | 2019 | 2020 | 2021 |
| Base | IID | Recruit | — | 1.23 | -854.771 | 281.5 | -1549.5 | 6.4179 | 1.0190 | -0.4265 | 312 (194, 500) | 1151 (117, 11282) | 2324 (294, 18377) |
| NAA-2 | IID | All NAA | — | 0.24 | -959.623 | 73.8 | -1757.2 | 0.8120 | 0.1702 | -0.1010 | 482 (201, 1153) | 1556 (233, 10384) | 2793 (466, 16722) |
| NAA-5 | 2D AR1 | All NAA | — | 0.69 | -984.051 | 28.9 | -1802.1 | 0.5134 | 0.0583 | 0.0016 | 298 (119, 745) | 602 (119, 3042) | 1133 (154, 8342) |
| M-1 | IID | Recruit | IID | 7.77 | -966.844 | 59.3 | -1771.7 | 0.1242 | 0.1827 | -0.1085 | 250 (71, 884) | 1972 (231, 16873) | 4512 (764, 26638) |
| M-4 | IID | Recruit | 2D AR1 | 7.78 | -974.436 | 48.1 | -1782.9 | -0.1074 | 0.0582 | 0.0024 | 203 (79, 518) | 704 (45, 11091) | 1782 (127, 24955) |
| NAA-M-2 | IID | All NAA | 2D AR1 | 0.99 | -982.149 | 34.7 | -1796.3 | -0.1352 | 0.0275 | 0.0464 | 185 (67, 510) | 383 (28, 5163) | 793 (39, 16020) |
| NAA-M-3 | 2D AR1 | All NAA | IID | 1.21 | -999.513 | 0.0 | -1831.0 | 0.4191 | 0.0045 | 0.0715 | 321 (131, 782) | 586 (132, 2602) | 1010 (137, 7447) |