An investigation of factors affecting inferences from and reliability of state-space age-structured assessment models

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Abstract

State-space models have been promoted as the next-generation of fisheries stock assessment and evaluations of their reliability is needed. We simulated operating models that varied fishing pressure, magnitude of observation error, and sources of process error. For each operating model, we fit a range estimating models with a range of correct and incorrect assumptions. We measured reliability of estimating models by convergence rate, accuracy of marginal AIC, estimation bias, and magnitude of retrospective patterns. All reliability measures were generally better with lower observation error, contrast in fishing pressure over time, and when median natural mortality rate is known. Magnitude of the log-likelihood gradients was not a reliable indicator of convergence. AIC can generally distinguish process error type with lower observation error and higher true process error variability. Distinguishing the stock recruit relationship with AIC required large contrast in spawning biomass and low recruitment variation, but bias in stock-recruit parameter estimation was prevalent. Retrospective patterns were not large for mis-specified models. These findings improve our understanding of when results from state space models will be reliable.

Introduction

Application of state-space models in fisheries stock assessment and management has expanded dramatically within International Council for the Exploration of the Sea (ICES), Canada, and the Northeast US (Nielsen and Berg 2014; Cadigan 2016; Pedersen and Berg 2017; Stock and Miller 2021). State-space models treat latent population characteristics as statistical time series with periodic observations that also may have error due to sampling or other other measurement properties. Traditional assessment models may use state-space approaches to account for temporal variability in population characteristics (Legault and Restrepo 1999; Methot and Wetzel 2013), but these models treat the annual parameters as penalized fixed effect parameters where the variance parameters controlling the penalties are assumed known (Thorson and Minto 2015). Modern state-space models can estimate the annually varying parameters as random effects with variance parameters estimated using maximum marginal likelihood or corresponding Bayesian approaches. These latter approaches are considered best practice and are recommended for the next generation of stock assessment models (Hoyle et al. 2022; Punt 2023).

State-space stock assessment models, with nonlinear functions of latent parameters and multiple types of observations with varying distributional assumptions, are one of the most complex examples of this analytical approach. Statistical aspects of state-space models and their application within fisheries have been studied extensively, but previous work has focused primarily on linear and Gaussian state-space models (Aeberhard et al. 2018; Auger-Méthé et al. 2021). Therefore, current understanding of the reliability of state-space models does not extend to usage for stock assessment.

As state-space models provide greater flexibility by allowing multiple processes to vary as random effects (Nielsen and Berg 2014; Aeberhard et al. 2018; Stock et al. 2021), one of the most immediate questions regards the implications of mis-specification among alternative sources of process error. Incorrect treatment of population attributes as temporally varying

(Trijoulet et al. 2020; Liljestrand et al. 2024) could lead to misidentification of stock status and biased population estimates, ultimately impacting fisheries management decisions (Legault and Palmer 2016; Szuwalski et al. 2018; Cronin-Fine and Punt 2021). Furthermore, biological, fishery, and observational processes are often confounded in catch-at-age data, which may adversely affect the ability to distinguish between true process variability and observational error (Li et al. In press; Punt et al. 2014; Stewart and Monnahan 2017; Cronin-Fine and Punt 2021; Fisch et al. 2023).

Li et al. (2024) conducted a full-factorial simulation-estimation study to assess model reliability when confounding random-effects processes (numbers-at-age, fishery selectivity, and natural mortality) were included. Their results suggest that while state-space models can generally identify sources of process error, overly complex models, even when misspecified (i.e., incorporating process error that did not exist in reality), often performed similarly to correctly specified models, with little to no bias in key management quantities. Similarly, Liljestrand et al. (2024) found little downside in assuming process error in recruitment or selectivity, even when it was absent.

Despite mounting efforts, several limitations remain. First, confounding processes that can be treated as random effects in the model have not been thoroughly examined or tested within a simulation-estimation framework. Second, previous studies relied on operating models conditioned on specific fisheries, limiting their generalizability (Li et al. In press; Liljestrand et al. 2024). In particular, the effects of observation error and underlying fishing history have not been fully isolated in simulation study designs, making it challenging to disentangle the interplay between process and observation error magnitudes, as demonstrated in Fisch et al. (2023). Third, explicitly modeling stock-recruit relationships (SRRs) as mechanistic drivers of population dynamics is promising (Fleischman et al. 2013; Pontavice et al. 2022), but reliability of inferences within integrated state-space age-structured models has not been evaluated. Evidence from other studies suggests that when both process and observation errors are unknown, estimating density dependence parameters becomes highly uncertain (Knape 2008; Polansky et al. 2009). In particular, Knape (2008) demonstrated that stronger density dependence becomes increasingly difficult to estimate in the presence of observation error. Therefore, it is crucial to assess whether density dependence mechanisms can be estimated with sufficient precision for use in fisheries management (Auger-Méthé et al. 2016). Finally, although the importance of autocorrelation in process errors is recognized, investigations of the ability to distinguish state-space assessment models with and without autocorrelation and whether such misspecification is detrimental to estimation of important population metrics are lacking (Johnson et al. 2016; Xu et al. 2019).

In the present study, we conduct a simulation study with operating models (OMs) varying by degree of observation error, source and variability of process error, and fishing history. The simulations from these OMs are fitted with estimation models (EMs) that make alternative assumptions for sources of process error, whether a SRR was estimated, and whether natural mortality is estimated. Given the confounding nature of process errors, developing diagnostic tools to detect model misspecification is of great scientific interest and could aid the next generation of stock assessments (Auger-Méthé et al. 2021). We evaluate whether OM and EM attributes affect rates of convergence and the ability of Akaike Information Criterion (AIC) to correctly determine the source of process error or the existence of a SRR. We also evaluate when retrospective patterns occur and the degree of bias in outputs of the assessment model that are important for management.

Methods

We used the Woods Hole Assessment Model (WHAM) to configure OMs and EMs in our simulation study (Stock and Miller 2021; Miller et al. 2025). WHAM is an R package freely available via a Github repository and is built on the Template Model Builder package (Kristensen et al. 2016). For this study we used version 1.0.6.9000, commit 77bbd94. WHAM has also been used to configure OMs and EMs for closed loop simulations evaluating index-based assessment methods (Legault et al. 2023) and is currently used or accepted for use in management of numerous NEUS fish stocks (e.g., NEFSC 2022a, 2022b; NEFSC 2024).

We completed a simulation study with a number of OMs that can be categorized based on where process error random effects were assumed: recruitment (R, assumed present in all models), apparent survival (denoted R+S), natural mortality (R+M), fleet selectivity (R+Sel), or index catchability (R+q). We refer to the (R+S) OMs as modeling apparent survival because on log-scale the random effects are additive to the total mortality (F+M) between numbers at age, thus they modify the survival term. However, as Stock and Miller (2021) note, these random effects can be due to events other than mortality, such as immigration, emigration, missreported catch, and other sources of misspecification. For each OM, assumptions about the magnitude of the variance of process errors and observations are required and the values we used were based on a review of the range of estimates from Northeast Unite States (NEUS) assessments using WHAM.

In total, we configured 72 OMs with alternative assumptions about the source and magnitude of process errors, magnitude of observation error in indices and age composition data, and contrast in fishing pressure over time. For each OM, we simulated 100 population time series with process errors and, for each time series, simulated observation data sets. For each data set, we fitted a number of EMs that differed in assumptions about the source of process errors, whether natural mortality (or the median for models with process error in natural mortality) was estimated, and whether a Beverton-Holt SRR was estimated within the EM. Details of each of the OMs and EMs are described below.

We did not use the log-normal bias-correction feature for process errors or observations described by (Stock and Miller 2021) for OMs and EMs to simplify interpretation of the study results (Li et al. 2025). All code we used to perform the simulation study and summarize results can be found at https://github.com/timjmiller/SSRTWG/tree/main/Project_0/code.

Operating models

Population

We intended the population demographics and observation types to represent a general NEUS groundfish stock. The population consists of 10 age classes, ages 1 to 10+, with the last being a plus group that accumulates ages 10 and older. We assume spawning occurs annually 1/4 of the way through the year. The maturity at age was a logistic curve with a_{50} = 2.89 and slope = 0.88 (Figure S1, top left).

Weight at age was generated with a von Bertalanffy growth function

$$L_a = L_\infty \left(1 - e^{-k(a-t_0)}\right)$$

where $t_0 = 0$, $L_\infty = 85$, and $k = 0.3$, and a L-W relationship such that

$$W_a = \theta_1 L_a^{\theta_2}$$

where $\theta_1 = e^{-12.1}$ and $\theta_2 = 3.2$ (Figure S1, top right).

We assumed a Beverton-Holt SRR with constant pre-recruit mortality parameters for all OMs. All biological inputs to calculations of spawning biomass per recruit (i.e., weight, maturity, and natural mortality at age) are constant in the apparent survival (R+S) selectivity (R+Sel), and survey catchability (R+q) process error OMs. Therefore, steepness and unfished recruitment are also constant over the time period for those OMs (Miller and Brooks 2021). We assumed a value of 0.2 for the natural mortality rate in OMs without process errors on natural mortality. We specified unfished recruitment equal to e^{10} and $F_{\text{MSY}} = F_{40\%} = 0.348$, which equates to a steepness of 0.69 and $a = 0.60$ and $b = 2.4 \times 10^{-5}$

for the Beverton-Holt parameterization

$$N_{1,y} = \frac{a\mathrm{SSB}_{y-1}}{1 + b\mathrm{SSB}_{y-1}}$$

(Figure S1, bottom right). For OMs with time-varying random effects for natural mortality, steepness is not constant. However, we used the same a and b parameters as other OMs, which equates to a steepness and R0 at the median of the time series process for natural mortality. Similarly, For OMs with time-varying random effects for fishery selectivity, F_{MSY} also varies temporally, so equilibrium conditions for these OMs are defined for mean selectivity parameters.

We used two fishing scenarios for OMs. In the first scenario, the stock experiences overfishing at $2.5F_{\mathrm{MSY}}$ for the first 20 years followed by fishing at F_{MSY} for the last 20 years (denoted $2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$). In the second scenario, the stock is fished at F_{MSY} for the entire time period (40 years). The magnitude of the overfishing assumptions is based on average estimates of overfishing for NEUS groundfish stocks from Wiedenmann et al. (2019) and similar to the approach in Legault et al. (2023). The second scenario represents the ideal situation where the stock is fished at an optimal level, but provides less contrast in stock sizes over time. We specified initial population abundance at age at the equilibrium distribution that corresponds to fishing at either $F = 2.5F_{\mathrm{MSY}}$ or $F = F_{\mathrm{MSY}}$. This implies that, for a deterministic model, the abundance at age would not change from the first year to the next.

Fleets

We assumed a single fleet operating year round for catch observations with logistic selectivity ($a_{50} = 5$ and slope $= 1$; Figure S1, bottom left). This selectivity was used to define F_{MSY} for the Beverton-Holt SRR parameters above. We assumed a logistic-normal distribution with no correlation on the multivariate normal scale for the corresponding annual age-composition observations.

Indices

Two time series of fishery-independent surveys measured in numbers are generated for the entire 40 year period with one occurring in the spring (0.25 of each year) and one in the fall (0.75 of each year), representing current bottom trawl surveys conducted in the NEUS. Catchability of both surveys are assumed to be 0.1. Like the fishing fleet, we assumed logistic selectivity for both indices ($a_{50} = 5$ and slope $= 1$) and a logistic-normal distribution with no correlation on the multivariate normal scale for the annual age-composition observations.

Observation Uncertainty

The standard deviation for log-aggregate catch was 0.1 for all OMs, a common assumption for NEUS stock assessments. Two levels of observation error variance (high and low) were specified for indices and all age composition observations (both indices and catch). The low uncertainty specification assumed a standard deviation of 0.1 for both series of log-aggregate index observations, and the standard deviation of the logistic-normal for age composition observations was 0.3. In the high uncertainty specification, the standard deviation for log-aggregate indices was 0.4 and that for the age composition observations was 1.5. The low standard deviation for index observations is typical for fish stocks that are consistently sampled across survey stations whereas the high value is typical for more sporadically sampled stocks. The standard deviations for the age composition observations were determined from the range of values estimated from WHAM fits to NEUS stocks that assumed the logistic-normal model. For all EMs, the standard deviation for log-aggregate observations was assumed known whereas that for the logistic-normal age composition observations was estimated.

Operating models with random effects on numbers at age

For operating models with random effects on recruitment and(or) apparent survival (R, R+S), we assumed marginal standard deviations for recruitment of $\sigma_R \in \{0.5, 1.5\}$. The marginal standard deviations for apparent survival random effects at older age classes were $\sigma_{2+} \in \{0, 0.25, 0.5\}$. The full factorial combination of these process error assumptions (2×3 levels) and scenarios for fishing history (2 levels) and observation error (2 levels) scenarios described above results in 24 different R ($\sigma_{2+} = 0$) and R+S operating models (Table S1).

Operating models with random effects on natural mortality

All R+M OMs treat natural mortality as constant across age, but with annually varying random effects. WHAM treats natural mortality as a log-transformed parameter

$$\log M_{y,a} = \mu_M + \epsilon_{M,y}$$

that is a linear combination of a mean log-natural mortality parameter that is constant across ages ($\mu_M = \log(0.2)$) and any annual random effects are marginally distributed as $\epsilon_{M,y} \sim \mathrm{N}\left(0, \sigma_M^2\right)$. The marginal standard deviations we assumed for log natural mortality random effects were $\sigma_M \in \{0.1, 0.5\}$ and the random effects were either uncorrelated or first-order autoregressive (AR1, $\rho_M \in \{0, 0.9\}$). Uncorrelated random effects were also included on recruitment with $\sigma_R = 0.5$ (hence, we denote these OMs as R+M). The full factorial combination of these process error assumptions and fishing history (2 levels) and observation error (2 levels) scenarios described above results in 16 different R+M OMs (Table S2).

Operating models with random effects on fleet selectivity

WHAM treats selectivity parameter s as a logit-transformed parameter

$$\log\left(\frac{p_{s,y} - l_s}{u_s - p_{s,y}}\right) = \mu_s + \epsilon_{s,y}$$

that is a linear combination of a mean μ_s and any annual random effects marginally distributed as $\epsilon_{s,y} \sim \mathrm{N}\left(0, \sigma_s^2\right)$, where the lower and upper bounds of the parameter (l_s and u_s) can be specified by the user. All selectivity parameters (a_{50} and slope parameters) were bounded by 0 and 10 for all OMs and EMs. The marginal standard deviations we assumed for logit scale random effects were $\sigma_s \in \{0.1, 0.5\}$ and AR1 autocorrelation parameters of $\rho_s \in \{0, 0.9\}$. Like R+M OMs, the full factorial combination of these process error assumptions (2x2 levels) and scenarios described above for fishing history (2 levels) and observation error (2 levels) results in 16 different R+Sel OMs (Table S3).

Operating models with random effects on index catchability

Like selectivity parameters, WHAM treats catchability for an index i as a logit-transformed parameter

$$\log\left(\frac{q_{i,y} - l_i}{u_i - q_{i,y}}\right) = \mu_i + \epsilon_{i,y}$$

that is a linear combination of a mean μ_i and any annual random effects marginally distributed as $\epsilon_{i,y} \sim \mathrm{N}\left(0, \sigma_i^2\right)$ where the lower and upper bounds of the catchability (l_i and u_i) can be specified by the user. We assumed bounds of 0 and 1000 for all OMs and EMs. For all OMs and EMs with process errors on catchability, the temporal variation only applies to the first index, which could be interpreted as capturing some unmeasured seasonal process that affects availability to the survey. The marginal standard deviations we assumed for logit scale random effects were $\sigma_i \in \{0.1, 0.5\}$ and AR1 autocorrelation parameters of $\rho_i \in \{0, 0.9\}$. Like R+M and R+Sel OMs, the full factorial combination of these process error assumptions and fishing history (2 levels) and observation error (2 levels) scenarios described above results in 16 different R+q OMs (Table S4).

Estimation models

For each of the data sets simulated from an OM, 20 EMs were fit. A total of 32 different EMs were fit across OMs where the subset of 20 depended on the source of process error in the OM (Table S5). The EMs have different assumptions about the source of process error (R+S, R+M, R+Sel, R+q) and whether or not 1) there is temporal autocorrelation, 2) a Beverton-Holt SRR is estimated, and 3) the natural mortality rate (μ_M, the constant or mean on log scale for R+M EMs) is estimated. For simplicity we refer to the derived estimate e^{μ_M} as the median natural mortality rate regardless of whether natural mortality random effects are estimated in the EM.

Subsets of 20 EMs in Table S5 were fit to simulate data sets from each of the OM process error categories. For R and R+S OMs, fitted EMs had matching process error assumptions as well as R+Sel, R+M, and R+q assumptions without autocorrelation. For other OM process error categories, we fit EMs with correct process error assumptions, the correct process error type but incorrect correlation assumption, and the incorrect process error types without autocorrelation. As such, EMs were configured completely correctly for the OM, or they had mis-specification in assumptions of process error autocorrelation, the type of process error, and(or) the SRR (Beverton-Holt or none).

The maturity at age, weight at age for catch and spawning stock biomass (SSB), and observation error standard deviations for aggregate catch and indices were all assumed known at the true values. However, the variance parameters for the logistic-normal distributions for age composition observations were estimated in the EMs.

Measures of reliability

Convergence

The first measure of reliability we investigated was frequency of convergence when fitting each EM to the simulated data sets. There are various ways to assess convergence of the fit (e.g., Carvalho et al. 2021; Kapur et al. 2025), but given the importance of estimates of uncertainty when using assessment models in management, we estimated probability of convergence as measured by occurrence of a positive-definite Hessian matrix at the optimized negative log-likelihood that could be inverted (i.e., providing Hessian-based standard error estimates). We also provide results in the Supplementary Materials for convergence defined by the maximum absolute gradient $< 1^{-6}$ and the maximum of the absolute gradient values for all fits of a given EM to all simulated data sets from a given OM that produced Hessian-based standard errors for all estimated fixed effects. This provides an indication of how poor the calculated gradients can be, but still presumably converged adequately enough for parameter inferences.

AIC for model selection

We investigated the reliability of AIC-based model selection for two purposes. First, we analyzed selection of each process error model structure (R, R+S, R+M, R+Sel, R+q) using marginal AIC. For a given OM simulated data set, we compared AIC for EMs with different process error assumption conditional on whether median natural mortality rate and the Beverton-Holt SRR were estimated. Second, we analyzed AIC-based selection between EMs with and without the Beverton-Holt SRR assumed. Contrast in fishing pressure and time series with recruitment at low stock size has been shown to improve estimation of SRR parameters (Magnusson and Hilborn 2007; Conn et al. 2010). Our preliminary inspections indicated generally poor performance of AIC in determining the Beverton-Holt model for a given set of OM factors (including contrast in fishing pressure), even when the EM was configured with the correct process error type. Therefore, we conditioned on the EMs having the correct process error assumption and also considered the effect of the log-standard deviation of the true log(SSB) ($\log \mathrm{SD}_{\mathrm{SSB}}$; similar to the log of the coefficient of variation for SSB) on model selection since simulations with realized SSB producing low and high recruitment would have larger variation in realized SSB.

All model selection results condition only on completion of the optimization process without failure for all of the compared EMs. We did not condition on convergence as defined above because optimization could correctly determine an inappropriate process error assumption by estimating variance parameters at the lower bound of zero. Such an optimization could indicate poor convergence but the likelihood would be equivalent to that without the misspecified random effects and the AIC would be appropriately higher because more (variance) parameters were estimated. All other measures of reliability described below (bias and Mohn's ρ) use these same criteria for inclusion of EM fits in the summarized results.

Bias

We also investigated bias in estimation of various model attributes as a measure of reliability. For a given model attribute we calculated the relative error

$$\mathrm{RE}\left(\theta_j\right) = \frac{\widehat{\theta}_j - \theta_j}{\theta_j} \tag{1}$$

from fitting a given estimating model to simulated data set j configured for a given OM where $\widehat{\theta}_j$ and θ_j are the estimated and true values for simulation j. We analyzed simulation results for estimates of terminal year estimates of SSB and recruitment, Beverton-Holt SRR parameters (a and b), and median natural mortality rate.

Mohn's ρ

Finally, we investigated presence of retrospective patterns in fitted models as a measure of reliability. We calculated Mohn's ρ for SSB, fishing mortality (averaged over all age classes), and recruitment for each EM fit to each OM simulated data set (Mohn 1999). We fit $P = 7$ peels to each simulated data set and calculated Mohn's ρ for a given attribute θ as

$$\rho\left(\theta\right) = \frac{1}{P}\sum_{p=1}^{P} \frac{\widehat{\theta}_{Y-p,Y-p} - \widehat{\theta}_{Y-p,Y}}{\widehat{\theta}_{Y-p,Y}} \tag{2}$$

where Y is last year of the full set of observations and $\widehat{\theta}_{y,y'}$ is the estimate for attribute θ in year y from a model fit using data up to year $y' \geq y$.

Summarizing results across OM and EM attributes

Because the OM and EM attributes that we investigated are numerous, we used two methods to summarize the most important factors for differences in results within a given OM process error type. The first method was fitting regression models with the response being each of the measures of reliability described above and predictor variables were defined based on OM and EM characteristics (e.g., MacKinnon et al. 1995; Wang et al. 2017; Harwell et al. 2018). For the binary indicators of convergence and AIC-based selection of a SRR, we performed logistic regressions. For indicators of AIC-based selection of EM process error type (multiple categories) we performed multinomial regressions. For other measures of reliability we fit linear regression models to transformed responses. Because relative errors (Eq. 1) and Mohn's ρ for the various parameters are bounded below at -1, we used a transformation of these values

$$y_j = \log\left[f\left(\widehat{\theta}_j, \theta_j\right) + 1\right] \tag{3}$$

where f is either the relative error (Eq. 1) or Mohn's ρ (Eq. 2) for simulation j, so that values are unbounded. For relative errors, y_j is the log-scale error. We omitted simulations where estimated attributes equal to zero (RE = -1). For all regressions we fit separate models with just individual OM and EM factors included, with all factors included, with all second order interactions, and with all third order interactions. For the multinomial regression, we used the `vglm` function from the VGAM package (Yee 2008; Yee 2015). We tabulated percent reduction in residual deviance for each of regression fits. We did not perform formal statistical analyses of effects of OM and EM attributes on results (e.g., ANOVA) because of the lack of independence of the "observations" that results from fitting multiple EMs to each simulated data set.

The second method involved fitting classification and regression trees (Breiman et al. 1984) to show how the OM and EM attributes, and their interactions, split the values for each measure of reliability (e.g., Gonzalez et al. 2018; Collier et al. 2022). We used classification trees for categorical measures (convergence and AIC) and regression trees for the other measures with continuous scales (relative error and Mohn's ρ). The response variables were the same as the regressions for the deviance reduction analyses. We used the `rpart` function in the rpart package (https://cran.r-project.org/package=rpart) to fit trees. Full trees were determined using default settings except that we increased the number of cross-validations to 100. For clarity, we pruned the full trees to show just the primary branches.

We also provide detailed results for all measures of reliability at each combination of OM and EM attributes in the Supplementary Materials. For confidence intervals of probability of convergence, we used the Clopper-Pearson exact method (Clopper and Pearson 1934; Thulin 2014). For AIC selection of process error configuration we provide estimates of the proportions of simulations where each EM type was selected. For AIC selection of the SRR (a binary indicator for each simulated data set), we fit logistic regressions and present resulting predicted probabilities of correctly selecting the SRR as a function of SSB variability ($\log \mathrm{SD}_{\mathrm{SSB}}$ described above). We estimated bias as the median of the relative errors across all simulations for a given OM and EM combination. We constructed 95% confidence intervals for the median relative bias, and Mohn's ρ using the binomial distribution approach (Thompson 1936) as in Miller and Hyun (2018) and Stock and Miller (2021).

Results

Convergence performance

For probability of convergence, EM process error assumption was the single attribute that resulted in the largest percent reduction in deviance (14-28%) for all OM process error types other than R+S OMs where the EM median natural mortality rate assumption (estimated or known) explained the most residual deviance (>11%; Table 1). However, including interactions of OM and EM factors also provided large reductions in residual deviance (35-47%), suggesting successful convergence depended on a combination OM and EM attributes.

Classification trees for each OM process error type, all had the primary branch defined using the same attribute that provided the largest reduction in deviance (Figure 1). EMs that assumed R+S process errors converged poorly for all OMs that were simulated with the alternative process error assumptions (R, R+M, R+Sel, an R+q OMs). For all trees, branches based on the OM fishing mortality history showed better convergence when the OM included a change in fishing pressure. Branches based on whether the Beverton-Holt SRR was assumed or not, showed better convergence when it was not estimated and branches based on the median natural mortality rate assumption showed better convergence when it was treated as known. For some R+M and R+Sel OMs, better convergence was also observed when there was lower observation uncertainty.

When convergence is defined by a gradient threshold, the primary factor explaining deviance reduction is the same that for Hessian-based convergence for all OM process error types, but there are some differences in deviance reduction for secondary factors (Table S6), and probability of convergence, overall, was lower (Figure S2). We found a wide range of maximum absolute values of gradients for models that had invertible Hessians (Figure S3). The largest value observed for a given EM and OM combination was typically $< 10^{-3}$, but many converged models had values greater than 1. For many OMs, EMs that assumed the correct process error type and did not estimate median natural mortality or the Beverton-Holt SRR produced the lowest gradient values.

AIC performance

Process error structure

For AIC selection of the correct process error configuration, the magnitude of observation and process error variation were the attributes that resulted in the largest percent reductions in deviance across OM process error types other than R OMs (Table 2). Both sources of variation explained large reductions in deviance for R+S (17-22%) and R+Sel (8-26%) OMs, whereas variance of process errors provided the major reductions for R+M (>9%) and R+q (>13%) OMs. Comparatively, none of the OM or EM attributes explained particularly large reductions in deviance for R OMs, but fishing history, whether a SRR was estimated, and whether median natural mortality was known or estimated provided similar and the largest reductions (approximately 5-6%). Inclusion of second and third order interactions, did not provide large reductions in deviance for any of the OM process error types.

For all OM process error types other than R OMs, the attributes defining the primary branches of classification trees matched those that provided the largest reductions in deviance (Figure 2). Across all OMs, AIC was more accurate for the process error type when process error variability was greater and when observation error was lower. For R+S OMs, there was a tendency to select R OMs when observation error was higher and apparent survival variation was lower ($\sigma_{2+} = 0.25$), but accuracy for the process error was otherwise highly accurate. Similarly, low accuracy for process error type occurred for R+Sel and R+q OMs only with high observation error and lower process error variability, but high accuracy for R+M OMs required both low observation error and high process error variance. This was also required for accurate identification of the correct correlation structure for R+M, R+q, and R+M OMs (Figure S4). No branches were estimated for classification trees fit to the R OMs, likely because accuracy was high across all simulations (0.94), although inspection of the fine-scale results shows there is some degradation in AIC selection when a SRR and median natural mortality rate are estimated for R OMs with constant fishing pressure and high observation error (Figure S4, top left).

Stock-recruit relationship

Logistic regressions for AIC selection of the Beverton-Holt SRR, showed OM fishing history and $\log \mathrm{SD}_{\mathrm{SSB}}$ provided substantial reductions in deviance for R+M (>13%), R+Sel (>26%), and R+q (>24%) OMs (Table 3). For R OMs, fishing history provided the largest reduction in deviance (>9%), whereas none of the attributes individually provided large reductions in deviance for R+S OMs (all <5%). However, inclusion of all attributes provided larger reductions in deviance than the sum of individual contributions for both R (>30%) and R+S (~19%) OMs. Further fits for R and R+S OMs including all combinations two factors showed those with $\log \mathrm{SD}_{\mathrm{SSB}}$ and recruitment variation provided essentially the same reduction in deviance as the models with all factors. For all OM process error types, inclusion of interaction terms provided relatively little reduction in residual deviance.

Attributes defining the primary branches of classification trees for AIC selection of the SRR assumption were the same as those explaining the largest reductions in deviance for the logistic regression models (Figure 3). All branches based on $\log \mathrm{SD}_{\mathrm{SSB}}$ showed better accuracy with larger variability in SSB and all branches based on fishing history showed better accuracy when there was contrast in fishing pressure. Branched based on OM observation error or recruitment variability (R and R+S OMs) showed better accuracy when they were lower. For R OMs, a combination of lower recruitment variability, contrast in fishing pressure, and higher SSB variability produced AIC accuracy over 0.8. For R+S OMs, lower recruitment variability and observation error and higher SSB variability produced AIC accuracy of 0.79. For R+M, R+Sel, and R+q OM,s accuracy of 0.87 to 0.94 was observed with just increased SSB variability.

Bias

Terminal year spawning stock biomass, fishing mortality, and recruitment

Regression models for log-scale errors in SSB that included the various OM and EM factors showed little reduction in deviance (<5%) for any of the factors across all OM process error types (Table 4). The attributes producing the largest reductions were the EM assumption for median natural mortality (known or estimated) for R, R+M, R+Sel, and R+q OMs (1-3%), EM process error type for R+S OMs (4%) and fishing history for all OM types (1-5%). Including second order interactions provided the largest reductions in residual deviance (10-26%). Including third order interactions also provided further reductions for R, R+S, and R+q OMs between 5 and 11%.

In all regression trees, branches based on fishing history and level of observation error generally showed less bias in SSB with contrast in fishing and lower observation error (Figure 4). For scenarios where there was bias, it was generally positive (over-estimation). For branches based on treatment of median natural mortality rate, bias was generally less when it was known rather than estimated. For some R+Sel and R+q OMs, less bias in SSB was shown when the EM process error configuration was correct.

Results for bias in fishing mortality and recruitment generally matched those for SSB, except that directions of bias for fishing mortality were opposite to those for SSB and recruitment. Effects of individual OM and EM factors on regression models were similarly small as measured by reduction in deviance (Tables S7 and S8). Factors defining the primary branches of regression trees were in most cases identical to those for SSB (Figures S5 and S6).

Stock-recruit parameters

Regression models for log-scale errors of estimates of both the Beverton-Holt a and b parameters showed none of the factors explained large percent reductions in deviance (Table 5). The OM fishing history provided the largest deviance reduction for most OM process error types for both parameters, but reductions were less than 5.6% except for R+Sel OMs where the reductions were 11.37% and 7.97% for the a and b parameters, respectively and for just the b parameter for R+q OMs (10%). The EM process error assumption provided similar reductions in deviance for both parameters for R OMs. Including interactions also did not produce important reductions in deviance.

For regression trees of log-scale errors in Beverton-Holt a and b parameter estimates, less bias was indicated with contrast in OM fishing pressure for all branches in trees for each OM process error type (Figures 5 and 6). For all branches based on recruitment variability in trees for R and R+S OMs, less bias in both a and b was observed with less recruitment variability. For R OMs with contrast in fishing pressure and greater recruitment variability EMs that assumed the incorrect R+M process errors produced less bias in both a and b than other process error assumptions. Across all combinations of OM and EM attributes, some bias was observed for both parameters, but there was generally less bias and(or) lower variability in estimation of the a parameter than the b parameter (Figure S7).

Median natural mortality rate

Fitted regression models for log-scale errors in median natural mortality rate showed largest percent reductions in residual deviance for R+S and R+M models (Table 6). The largest reductions for a single attribute was the EM process error assumption (>20%) and fishing history (>15%) for R+S OMs. Fishing history also provided >10% reduction for R+M OMs, but reductions for all factors in R, R+Sel, and R+q OMs were relatively low (<6%). Interactions of OM and EM factors also provided substantial further reductions for R+S and

R+M OMs (between 8 and 15% for second order interactions).

Regression trees with branches based on fishing history showed less bias in median natural mortality rate with contrast in fishing pressure and branches based on level of observation error showed less bias with more precise observations (Figure 7). For R OMs, branches based on EM process error assumption showed less bias with EMs assuming the correct R and the incorrect R+S assumption. For R+S and R+M OMs, branches based on EM process error showed only the correct EM process error assumption with less bias.

Mohn's ρ

Regression models for Mohn's ρ of SSB showed little reduction in deviance for any of the OM an EM attributes (<2%; Table 7). The lack of explanatory power is also reflected in the regression trees where median Mohn's ρ values are near zero unless a large combinations of OM and EM conditions occur (Figure 8). For example, In R+S OMs, with constant fishing pressure, high observation error, and higher apparent survival process error, EMs that assume R+M process errors have a median Mohn's $\rho = -0.068$.

Similarly, poor explanatory power of the OM and EM attributes occurred when we fit regression models for Mohn's ρ of fishing mortality and recruitment (Tables S9 and S10). Regression trees for Mohn's ρ of fishing mortality were similar to those for SSB in that median values of Mohn's ρ were close to zero for most combinations of OM and EM attributes (Figure S8). However, we observed median Mohn's ρ for recruitment greater than 0.1 at branches much closer to the base of the trees with fewer interactions of the OM and EM attributes (Figure S9). These branches with consistently large retrospective patterns were typically defined by larger OM observation error, OM constant fishing pressure, or incorrect EM process error configuration. Comparing regression model and regression tree fits, attributes defining the primary branches for all regression trees of all Mohn's ρ values (SSB, fishing mortality, and recruitment) generally matched those that explained the largest reductions in deviance.

Discussion

Assessing convergence

Analyses of model convergence across simulations can be useful for understanding the utility of alternative convergence criteria used in applications to real data for directing the practitioner to more appropriate random effects configurations. It is common during the assessment model fitting process to check that the maximum absolute gradient component is less than some threshold prior to inspecting the Hessian of the optimized likelihood for invertibility (Carvalho et al. 2021). However, there is no accepted standard for the gradient threshold (e.g., Lee et al. 2011; Hurtado-Ferro et al. 2014; Rudd and Thorson 2018) and some thresholds would exclude models that in fact have an invertible Hessian. We found the Hessian at the optimized log-likelihood can often be invertible when the maximum absolute gradient was much larger than what might be perceived to be a sensible threshold.

Li et al. (2024) found that convergence rate could be a useful diagnostic especially for separating the correct model from overly complex models. However, the criteria for convergence used in their study may also lead to limited ability to distinguish the correct model from overly simplistic models, a pattern that was also noted by Liljestrand et al. (2024) in which one process error may absorb all sources of process error when the magnitude of other process errors are low.

Often poor convergence occurs when parameter estimates are at their bounds (Carvalho et al. 2021), and this also applies to variance parameters for random effects with state-space assessment models. Even when the Hessian is invertible, parameters that are poorly informed will have extremely large variance estimates. This further inspection can lead to a more appropriate and often more parsimonious model configuration where the problematic parameters are not estimated. For example, process error variance parameters that are estimated close to 0 indicates that the random effects are estimated to have little or no variability and removing these process errors is warranted. Generally, our results suggest we can expect lower probability of convergence of state-space assessment models when estimating natural mortality or SRRs because of the difficulty distinguishing these parameters from others being estimated in assessment model in common scenarios where data quality is less than ideal. Our experiments did not aim to emulate the practitioner decision process in developing model configurations (e.g. removing a source of process error and refitting the model when process error variance parameters were estimated close to 0). Evaluating the efficacy of such a decision process when applying EMs might be important in closed loop simulations (e.g. MSE) aimed at quantifying management performance.

A factor affecting the convergence criteria, particularly for maximum likelihood estimation of models with random effects, is numerical accuracy. All optimizations performed in these simulations are of the Laplace approximation of the marginal likelihood and, therefore, gradients and Hessians are also with respect to this approximation (see TMB::sdreport in the Template Model Builder package). Functionality within the Template Model Builder package exists (i.e., TMB::checkConsistency) to check the validity of the Laplace approximation and the utility of this as a diagnostic for state-space assessment models should be explored further. Furthermore, numerical methods are used to calculate and invert the Hessian for variance estimation for models with random effects. Our results, along with the potential lack of accuracy imposed by these approximations, suggest at least investigating whether the Hessian is positive definite when the calculated absolute gradients are not terribly large (e.g, < 1).

Configuring process error

We found accuracy of marginal AIC for process error type required only low observation error for R, R+S, R+Sel, and R+q OMs. R+M OMs further required higher process error variability, but this also improved accuracy for the other OM process errors types when there was higher observation error.

Stock recruitment relationships

Variation in SSB was the most important factor for using marginal AIC to distinguish the the Beverton-Holt SRR. For R+M, R+Sel, and R+q OMs, the SRR was accurately detected when the CV of SSB over the time series was at least 40 to 50% ($\log \mathrm{SD}_{\mathrm{SSB}}$ = -0.9 to -0.7) regardless of any other OM or EM attributes. Detection of the SRR for R and R+S OMs required lower recruitment variability, but this lower level ($\sigma_R = 0.5$) was assumed for all of the other OM process error types. Our results assumed that the EM process error configuration was correct, but this may be a strong limation given the ability of AIC to distinguish the process error type in many scenarios.

Although we did not compare models with alternative SRRs (e.g., Ricker and Beverton-Holt), we do not expect AIC to perform any better distinguishing between relationships and may be more difficult than distinguishing from the null model even with larger variability in SSB. Our finding that AIC tended to choose simpler recruitment models in many cases contrasts with the noted bias in AIC for more complex models (Shibata 1976; Katz 1981; Kass and Raftery 1995), but, whereas those findings apply to the much more common comparison of models that are fit to raw and independent observations, here we are comparing state-space models which account for observation error and estimate process errors in latent variables.

Our results comport with those of de Valpine and Hastings (2002) who found AIC could not distinguish among state-space SRRs that were fit just to SSB and recruitment observations (i.e., not an assessment model). Similarly, Britten et al. (In review) found AIC could not reliably distinguish alternative environmental effects on SRR parameters. However, Miller et al. (2016) did find AIC to prefer a SRR with environmental effects when applied to data for the SNEMA yellowtail flounder stock and AIC also selected an environmental covariate on a SRR for the most recent stock assessment of Georges Bank yellowtail flounder (NEFSC 2025). Both of these yellowtail flounder stocks have large changes in stock size and the values of environmental covariates over time. Additionally, this species is well-observed by the bottom trawl survey that is used for an index in assessment models.

However, estimation of SRR parameters was only moderately reliable in ideal scenarios of low observation error and contrast in fishing for R+Sel and R+M OMs with large temporal variability in process errors. Otherwise, SRR parameter estimation was biased and(or) highly variable. We found substantial bias in estimated SRR parameters in R and R+S OMs particularly with high variability in recruitment and apparent survival process errors, suggesting that practitioners should be cautious SRR inferences when fitted assessment models have these properties. We only evaluated effects of SSB variability on accuracy of AIC in identifying the SRR, but those results suggests we might find less bias for the SRR parameters in such cases as well. Similarly, restricting results to fits that converged may also yield better accuracy of SRR parameter estimation.

Estimating assessment model quantities

As expected, bias in parameters, SSB, and other assessment output was generally improved with lower observation error. Estimation of median natural mortality was reliable in many OM scenarios with contrast in fishing pressure, consistent with Hoenig et al. (2025). However, we found poor accuracy in terminal SSB estimation when estimating median natural mortality in many OMs when there was no contrast in fishing pressure over time and higher observation error. Therefore, estimating median natural mortality should be approached with caution in state-space assessment models, particularly given its significant impact on determination of reference point and stock status (Li et al. 2024).

Negligible retrospective patterns

Incorrect EM process error assumptions did not produce strong retrospective patterns for SSB for any OMs regardless of whether median natural mortality or a SRR was estimated although some weak patterns occurred when observation error was high and there was contrast in fishing pressure. However, retrospective patterns tended to be more variable for recruitment and were sometimes large even when the EM was correct. Therefore, we recommend de-emphasis on inspection of patterns for recruitment, but further research on retrospective patterns in other assessment model parameters, management quantities such as biological reference points, and projections may be beneficial (Brooks and Legault 2016).

The general lack of retrospective patterns with mis-specified process errors is perhaps to be expected. Retrospective patterns are often induced in simulation studies by rapid changes in a quantity such as index catchability, natural mortality, or perceived catch during years toward the end of the time series (Legault 2009; Miller and Legault 2017; Huynh et al. 2022; Breivik et al. 2023). In our simulations, the process errors changing over time may have trends in particular simulations, particularly when strong autocorrelation is imposed, but the random effects have no trend on average across simulations. Szuwalski et al. (2018) and Li et al. (2024) also found relatively small retrospective patterns when the source of mis-specification was temporal variation in demography attributes. Indeed, it is common for the flexibility provided by temporal random effects to reduce retrospective patterns (Miller et al. 2018; Stock et al. 2021; Stock and Miller 2021), though it does not necessarily indicate a more accurate assessment model (Perretti et al. 2020; Li et al. 2024; Liljestrand et al. 2024). Our results together with the existing literature seem to suggest that when a strong retrospective pattern is observed in an assessment it is more likely to be due to a mis-specification of a rapid shift in some model attribute rather than whether a particular process is assumed to be randomly varying temporally.

Summarization approach

We found the use of regression models and classification and regression trees extremely useful in understanding the most important OM and EM attributes explaining variation in the measures of reliability we examined across all simulations. The classification and regression trees are generally a good tool for determining the values of the OM and EM attributes that produce better or worse measures of reliability. However, determining the combination of attributes that produce the best or worst measures of reliability can be challenging using the trees alone. For example, in the regression tree for median natural mortality rate estimates in R OMs (Figure 7), both of the first branches imply bias is low regardless of OM fishing history, but when OM fishing pressure is constant, results are much better when OM observation error is low (median RE about -6%) than when OM observation error is high (median RE about 40%). The default pruning of the trees can exclude these lower branches. However, inspection of deviance explained by various regression models shows the ~9% reduction in residual deviance by including second order interaction of all OM and EM factors (Table 6), indicating that the interaction of factors may be important, thereby complimenting the regression tree analysis. Higher order interactions of some factors could also provide reductions in deviance and, therefore, inspection of results for each combinations of OM and EM factors, as provided in the Supplementary Materials, can also be important.

Recommendations and conclusions

Our findings regarding model convergence suggests practitioners using state-space models and maximum marginal likelihood for estimation should not heavily weight the magnitude of the gradient values in determining convergence as long as the maximum absolute values is around 1 or lower. Instead, positive-definiteness of the Hessian of the minimized negative log-likelihood should be evaluated.

Unfortunately, whether the practitioner includes a Beverton-Holt SRR will often depend on biological plausibility of this particular SRR because using AIC to determine its validity required a combination of low recruitment variability, contrast in fishing pressure, large variation in SSB over time, and lower observation error, which applies to a limited number of managed stocks. Furthermore, some bias in estimation of the SRR parameters should be expected, which presumably also applies to MSY-based reference points. Because bias in terminal SSB and retrospective patterns were indifferent to whether or not the SRR was estimated, the prevalence of bias in SRR parameter estimation, and often better convergence without the SRR, we recommend a sensible default is to exclude a SRR when fitting assessment models, as also suggested by Brooks (2024).

We found marginal AIC can, in many cases, accurately distinguished models with process errors. We saw the best accuracy for models with process errors on recruitment only (R), recruitment and apparent survival (R+S), and recruitment and selectivity (R+Sel), especially with lower observation error. However, AIC could also distinguish R+M and R+q process errors when variability of those processes was greater. The R+S assumption for process errors is common in applications of WHAM in the NEUS and the SAM assessment framework (Nielsen and Berg 2014) in ICES, and we can have some confidence that practitioners are correctly arriving at this assumption over other sources of process error using marginal AIC.

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References

Aeberhard, W.H., Flemming, J.M., and Nielsen, A. 2018. Review of State-Space Models for Fisheries Science. Annual Review of Statistics and Its Application **5**(1): 215–235. doi:10.1146/annurev-statistics-031017-100427.

Auger-Méthé, M., Field, C., Albertsen, C.M., Derocher, A.E., Lewis, M.A., Jonsen, I.D., and Mills Flemming, J. 2016. State-space models' dirty little secrets: Even simple linear Gaussian models can have estimation problems. Scientific reports **6**(1): 26677. doi:10.1038/srep26677.

Auger-Méthé, M., Newman, K., Cole, D., Empacher, F., Gryba, R., King, A.A., Leos-Barajas, V., Mills Flemming, J., Nielsen, A., Petris, G., and others. 2021. A guide to state–space modeling of ecological time series. Ecological Monographs **91**(4): e01470. doi:10.1002/ecm.1470.

Breiman, L., Friedman, J.H., Olshen, R.A., and Stone, C.J. 1984. Classification and regression trees. Chapman; Hall/CRC, New York, NY USA. doi:10.1201/9781315139470.

Breivik, O.N., Aldrin, M., Fuglebakk, E., and Nielsen, A. 2023. Detecting significant retrospective patterns in state space fish stock assessment. Canadian Journal of Fisheries and Aquatic Sciences **80**(9): 1509–1518. doi:10.1139/cjfas-2022-0250.

Britten, G., Brooks, E.N., and Miller, T.J. In review. Identification and performance of environmentally-driven stock-recruitment relationships in state space assessment models. Canadian Journal of Fisheries and Aquatic Sciences.

Brooks, E.N. 2024. Pragmatic approaches to modeling recruitment in fisheries stock assessment: A perspective. Fisheries Research **270**: 106896. doi:10.1016/j.fishres.2023.106896.

Brooks, E.N., and Legault, C.M. 2016. Retrospective forecasting – evaluating performance of stock projections for New England groundfish stocks. Canadian Journal of Fisheries and Aquatic Sciences **73**(6): 935–950. doi:10.1139/cjfas-2015-0163.

Cadigan, N.G. 2016. A state-space stock assessment model for northern cod, including underreported catches and variable natural mortality rates. Canadian Journal of Fisheries and Aquatic Sciences **73**(2): 296–308. doi:10.1139/cjfas-2015-0047.

Carvalho, F., Winker, H., Courtney, D., Kapur, M., Kell, L., Cardinale, M., Schirripa, M., Kitakado, T., Yemane, D., Piner, K.R., Maunder, M.N., Taylor, I., Wetzel, C.R., Doering, K., Johnson, K.F., and Methot, R.D. 2021. A cookbook for using model diagnostics in integrated stock assessments. Fisheries Research **240**: 105959. doi:https://doi.org/10.1016/j.fishres.2021.105959.

Clopper, C.J., and Pearson, E.S. 1934. The use of confidence or fiducial limits illustrated in the case of the binomial. Biometrika **26**(4): 404–413. doi:10.1093/biomet/26.4.404.

Collier, Z.K., Zhang, H., and Soyoye, O. 2022. Alternative methods for interpreting Monte Carlo experiments. Communications in Statistics - Simulation and Computation: 1–16. doi:10.1080/03610918.2022.2082474.

Conn, P.B., Williams, E.H., and Shertzer, K.W. 2010. When can we reliably estimate the productivity of fish stocks? Canadian Journal of Fisheries and Aquatic Sciences **67**(3): 511–523. doi:10.1139/F09-194.

Cronin-Fine, L., and Punt, A.E. 2021. Modeling time-varying selectivity in size-structured assessment models. Fisheries Research **239**: 105927. Elsevier.

de Valpine, P., and Hastings, A. 2002. Fitting population models incorporating process noise and observation error. Ecological Monographs **72**(1): 57–76.

Fisch, N., Shertzer, K., Camp, E., Maunder, M., and Ahrens, R. 2023. Process and sampling variance within fisheries stock assessment models: Estimability, likelihood choice, and the consequences of incorrect specification. ICES Journal of Marine Science **80**(8): 2125–2149. doi:10.1093/icesjms/fsad138.

Fleischman, S.J., Catalano, M.J., Clark, R.A., and Bernard, D.R. 2013. An age-structured state-space stock–recruit model for Pacific salmon (*Oncorhynchus spp.*). Canadian Journal of Fisheries and Aquatic Sciences **70**(3): 401–414. doi:10.1139/cjfas-2012-0112.

Gonzalez, O., O'Rourke, H.P., Wurpts, I.C., and Grimm, K.J. 2018. Analyzing Monte Carlo simulation studies with classification and regression trees. Structural Equation Modeling: A Multidisciplinary Journal **25**(3): 403–413. doi:10.1080/10705511.2017.1369353.

Harwell, M., Kohli, N., and Peralta-Torres, Y. 2018. A survey of reporting practices of computer simulation studies in statistical research. The American Statistician **72**(4): 321–327. doi:10.1080/00031305.2017.1342692.

Hoenig, J.M., Hearn, W.S., Leigh, G.M., and Latour, R.J. 2025. Principles for estimating natural mortality rate. Fisheries Research **281**: 107195. doi:10.1016/j.fishres.2024.107195.

Hoyle, S.D., Maunder, M.N., Punt, A.E., Mace, P.M., Devine, J.A., and A'mar, Z.T. 2022. Preface: Developing the next generation of stock assessment software. Fisheries Research **246**: 106176. doi:10.1016/j.fishres.2021.106176.

Hurtado-Ferro, F., Szuwalski, C.S., Valero, J.L., Anderson, S.C., Cunningham, C.J., Johnson, K.F., Licandeo, R., McGilliard, C.R., Monnahan, C.C., Muradian, M.L., Ono, K., Vert-Pre, K.A., Whitten, A.R., and Punt, A.E. 2014. Looking in the rear-view mirror: Bias and retrospective patterns in integrated, age-structured stock assessment models. ICES Journal of Marine Science **72**(1): 99–110. doi:10.1093/icesjms/fsu198.

Huynh, Q.C., Legault, C.M., Hordyk, A.R., and Carruthers, T.R. 2022. A closed-loop simulation framework and indicator approach for evaluating impacts of retrospective patterns in stock assessments. ICES Journal of Marine Science **79**(7): 2003–2016. doi:10.1093/icesjms/fsac066.

Johnson, K.F., Councill, E., Thorson, J.T., Brooks, E., Methot, R.D., and Punt, A.E. 2016. Can autocorrelated recruitment be estimated using integrated assessment models and how does it affect population forecasts? Fisheries Research **183**: 222–232. doi:10.1016/j.fishres.2016.06.004.

Kapur, M.S., Ducharme-Barth, N., Oshima, M., and Carvalho, F. 2025. Good practices, trade-offs, and precautions for model diagnostics in integrated stock assessments. Fisheries Research **281**: 107206. doi:10.1016/j.fishres.2024.107206.

Kass, R.E., and Raftery, A.E. 1995. Bayes factors. Journal of the American Statistical

Association **90**(430): 773–795. doi:10.1080/01621459.1995.10476572.

Katz, R.W. 1981. On some criteria for estimating the order of a Markov chain. Technometrics **23**(3): 243–249. doi:10.1080/00401706.1981.10486293.

Knape, J. 2008. Estimability of density dependence in models of time series data. Ecology **89**(11): 2994–3000. doi:10.1890/08-0071.1.

Kristensen, K., Nielsen, A., Berg, C.W., Skaug, H., and Bell, B.M. 2016. TMB: Automatic differentiation and Laplace approximation. Journal of Statistical Software **70**(5): 1–21. doi:10.18637/jss.v070.i05.

Lee, H.-H., Maunder, M.N., Piner, K.R., and Methot, R.D. 2011. Estimating natural mortality within a fisheries stock assessment model: An evaluation using simulation analysis based on twelve stock assessments. Fisheries Research **109**(1): 89–94. doi:10.1016/j.fishres.2011.01.021.

Legault, C.M. 2009. Report of the retrospective working group, 14-16 january 2008. US Department of Commerce Northeast Fisheries Science Center Reference Document 09-01. US Department of Commerce Northeast Fisheries Science Center. Woods Hole, MA.

Legault, C.M., and Palmer, M.C. 2016. In what direction should the fishing mortality target change when natural mortality increases within an assessment? Canadian Journal of Fisheries and Aquatic Sciences **73**(3): 349–357. doi:10.1139/cjfas-2015-0232.

Legault, C.M., and Restrepo, V.R. 1999. A flexible forward age-structured assessment program. Col. Vol. Sci. Pap. ICCAT **49**(2): 246–253.

Legault, C.M., Wiedenmann, J., Deroba, J.J., Fay, G., Miller, T.J., Brooks, E.N., Bell, R.J., Langan, J.A., Cournane, J.M., Jones, A.W., and Muffley, B. 2023. Data-rich but model-resistant: An evaluation of data-limited methods to manage fisheries with failed age-based stock assessments. Canadian Journal of Fisheries and Aquatic Sciences **80**(1): 27–42. doi:10.1139/cjfas-2022-0045.

Li, C., Deroba, J.J., Berger, A.M., Goethel, D.R., Langseth, B.J., Schueller, A.M., and

Miller, T.J. In press. Random effects on numbers-at-age transitions implictly account for movement dynamics and improve performance within a state-space stock assessment. Canadian Journal of Fisheries and Aquatic Sciences.

Li, C., Deroba, J.J., Miller, T.J., Legault, C.M., and Perretti, C. 2025. Guidance on bias-correction of log-normal random effects and observations in state-space assessment models. Canadian Journal of Fisheries and Aquatic Sciences. doi:10.1139/cjfas-2025-0093.

Li, C., Deroba, J.J., Miller, T.J., Legault, C.M., and Perretti, C.T. 2024. An evaluation of common stock assessment diagnostic tools for choosing among state-space models with multiple random effects processes. Fisheries Research **273**: 106968. doi:10.1016/j.fishres.2024.106968.

Liljestrand, E.M., Bence, J.R., and Deroba, J.J. 2024. The effect of process variability and data quality on performance of a state-space stock assessment model. Fisheries Research **275**: 107023. doi:10.1016/j.fishres.2024.107023.

MacKinnon, D.P., Warsi, G., and Dwyer, J.H. 1995. A simulation study of mediated effect measures. Multivariate Behavioral Research **30**(1): 41–62. doi:10.1207/s15327906mbr3001_3.

Magnusson, A., and Hilborn, R. 2007. What makes fisheries data informative? Fish and Fisheries **8**(4): 337–358. doi:10.1111/j.1467-2979.2007.00258.x.

Methot, R.D., and Wetzel, C.R. 2013. Stock synthesis: A biological and statistical framework for fish stock assessment and fishery management. Fisheries Research **142**: 86–99. doi:10.1016/j.fishres.2012.10.012.

Miller, T.J., and Brooks, E.N. 2021. Steepness is a slippery slope. Fish and Fisheries **22**(3): 634–645. doi:10.1111/faf.12534.

Miller, T.J., Curti, K.L., and Hansell, A.C. 2025. Space for WHAM: A multi-region, multi-stock generalization of the woods hole assessment model with an application to black sea bass. Canadian Journal of Fisheries and Aquatic Sciences **82**: 1–26. doi:10.1139/cjfas-2025-0097.

Miller, T.J., Hare, J.A., and Alade, L. 2016. A state-space approach to incorporating environmental effects on recruitment in an age-structured assessment model with an application to Southern New England yellowtail flounder. Canadian Journal of Fisheries and Aquatic Sciences **73**(8): 1261–1270. doi:10.1139/cjfas-2015-0339.

Miller, T.J., and Hyun, S.-Y. 2018. Evaluating evidence for alternative natural mortality and process error assumptions using a state-space, age-structured assessment model. Canadian Journal of Fisheries and Aquatic Sciences **75**(5): 691–703. doi:10.1139/cjfas-2017-0035.

Miller, T.J., and Legault, C.M. 2017. Statistical behavior of retrospective patterns and their effects on estimation of stock and harvest status. Fisheries Research **186**: 109–120. doi:10.1016/j.fishres.2016.08.002.

Miller, T.J., O'Brien, L., and Fratantoni, P.S. 2018. Temporal and environmental variation in growth and maturity and effects on management reference points of Georges Bank Atlantic cod. Canadian Journal of Fisheries and Aquatic Sciences **75**(12): 2159–2171. doi:10.1139/cjfas-2017-0124.

Mohn, R. 1999. The retrospective problem in sequential population analysis: An investigation using cod fishery and simulated data. ICES Journal of Marine Science **56**(4): 473–488. doi:10.1006/jmsc.1999.0481.

NEFSC. 2022a. Final report of the haddock research track assessment working group. Available at https://s3.us-east-1.amazonaws.com/nefmc.org/14b_EGB_Research_Track_Haddock_WG_F

NEFSC. 2022b. Report of the American plaice research track working group. Available at https://s3.us-east-1.amazonaws.com/nefmc.org/2_American-Plaice-WG-Report.pdf.

NEFSC. 2024. Butterfish research track assessment report. US Dept Commer Northeast Fish Sci Cent Ref Doc. 24-03; 191 p.

NEFSC. 2025. Yellowttail flounder research track working group report. Available at https://d23h0vhsm26o6d.cloudfront.net/10c.-Yellowtail-Flounder-RT-WG-Report.pdf.

Nielsen, A., and Berg, C.W. 2014. Estimation of time-varying selectivity in stock assessments using state-space models. Fisheries Research **158**: 96–101. doi:10.1016/j.fishres.2014.01.014.

Pedersen, M.W., and Berg, C.W. 2017. A stochastic surplus production model in continuous time. Fish and Fisheries **18**(2): 226–243. doi:10.1111/faf.12174.

Perretti, C.T., Deroba, J.J., and Legault, C.M. 2020. Simulation testing methods for estimating misreported catch in a state-space stock assessment model. ICES Journal of Marine Science **77**(3): 911–920. doi:10.1093/icesjms/fsaa034.

Polansky, L., De Valpine, P., Lloyd-Smith, J.O., and Getz, W.M. 2009. Likelihood ridges and multimodality in population growth rate models. Ecology **90**(8): 2313–2320. doi:10.1890/08-1461.1.

Pontavice, H. du, Miller, T.J., Stock, B.C., Chen, Z., and Saba, V.S. 2022. Ocean model-based covariates improve a marine fish stock assessment when observations are limited. ICES Journal of Marine Science **79**(4): 1259–1273. doi:10.1093/icesjms/fsac050.

Punt, A.E. 2023. Those who fail to learn from history are condemned to repeat it: A perspective on current stock assessment good practices and the consequences of not following them. Fisheries Research **261**: 106642. doi:10.1016/j.fishres.2023.106642.

Punt, A.E., Hurtado-Ferro, F., and Whitten, A.R. 2014. Model selection for selectivity in fisheries stock assessments. Fisheries Research **158**: 124–134. doi:10.1016/j.fishres.2013.06.003.

Rudd, M.B., and Thorson, J.T. 2018. Accounting for variable recruitment and fishing mortality in length-based stock assessments for data-limited fisheries. Canadian Journal of Fisheries and Aquatic Sciences **75**(7): 1019–1035. doi:10.1139/cjfas-2017-0143.

Shibata, R. 1976. Selection of the order of an autoregressive model by Akaike's information criterion. Biometrika **63**(1): 117–126. doi:10.1093/biomet/63.1.117.

Stewart, I.J., and Monnahan, C.C. 2017. Implications of process error in selectivity for approaches to weighting compositional data in fisheries stock assessments. Fisheries Research **192**: 126–134. doi:10.1016/j.fishres.2016.06.018.

Stock, B.C., and Miller, T.J. 2021. The Woods Hole Assessment Model (WHAM): A general state-space assessment framework that incorporates time- and age-varying processes via random effects and links to environmental covariates. Fisheries Research **240**: 105967.

doi:10.1016/j.fishres.2021.105967.

Stock, B.C., Xu, H., Miller, T.J., Thorson, J.T., and Nye, J.A. 2021. Implementing two-dimensional autocorrelation in either survival or natural mortality improves a state-space assessment model for Southern New England-Mid Atlantic yellowtail flounder. Fisheries Research **237**: 105873. doi:10.1016/j.fishres.2021.105873.

Szuwalski, C.S., Ianelli, J.N., and Punt, A.E. 2018. Reducing retrospective patterns in stock assessment and impacts on management performance. ICES Journal of Marine Science **75**(2): 596–609. doi:10.1093/icesjms/fsx159.

Thompson, W.R. 1936. On confidence ranges for the median and other expectation distributions for populations of unknown distribution form. Annals of Mathematical Statistics **7**(3): 122–128. doi:10.1214/aoms/1177732502.

Thorson, J.T., and Minto, C. 2015. Mixed effects: A unifying framework for statistical modelling in fisheries biology. ICES Journal of Marine Science **72**(5): 1245–1256. doi:10.1093/icesjms/fsu213.

Thulin, M. 2014. The cost of using exact confidence intervals for a binomial proportion. Electronic Journal of Statistics **8**(1): 817–840. doi:10.1214/14-EJS909.

Trijoulet, V., Fay, G., and Miller, T.J. 2020. Performance of a state-space multispecies model: What are the consequences of ignoring predation and process errors in stock assessments? Journal of Applied Ecology **57**(1): 121–135. doi:10.1111/1365-2664.13515.

Wang, S., Cadigan, N.G., and Benoît, H.P. 2017. Inference about regression parameters using highly stratified survey count data with over-dispersion and repeated measurements. Journal of Applied Statistics **44**(6): 1013–1030. doi:10.1080/02664763.2016.1191622.

Wiedenmann, J., Free, C.M., and Jensen, O.P. 2019. Evaluating the performance of data-limited methods for setting catch targets through application to data-rich stocks: A case study using northeast U.S. Fish stocks. Fisheries Research **209**(1): 129–142. doi:10.1016/j.fishres.2018.09.018.

Xu, H., Thorson, J.T., Methot, R.D., and Taylor, I.G. 2019. A new semi-parametric method for autocorrelated age- and time-varying selectivity in age-structured assessment models. Canadian Journal of Fisheries and Aquatic Sciences **76**(2): 268–285. doi:10.1139/cjfas-2017-0446.

Yee, T.W. 2008. The VGAM package. R News **8**(2): 28–39. Available from https://journal.r-project.org/articles/RN-2008-014/.

Yee, T.W. 2015. Vector generalized linear and additive models: With an implementation in R. Springer, New York, NY USA. doi:10.1007/978-1-4939-2818-7.

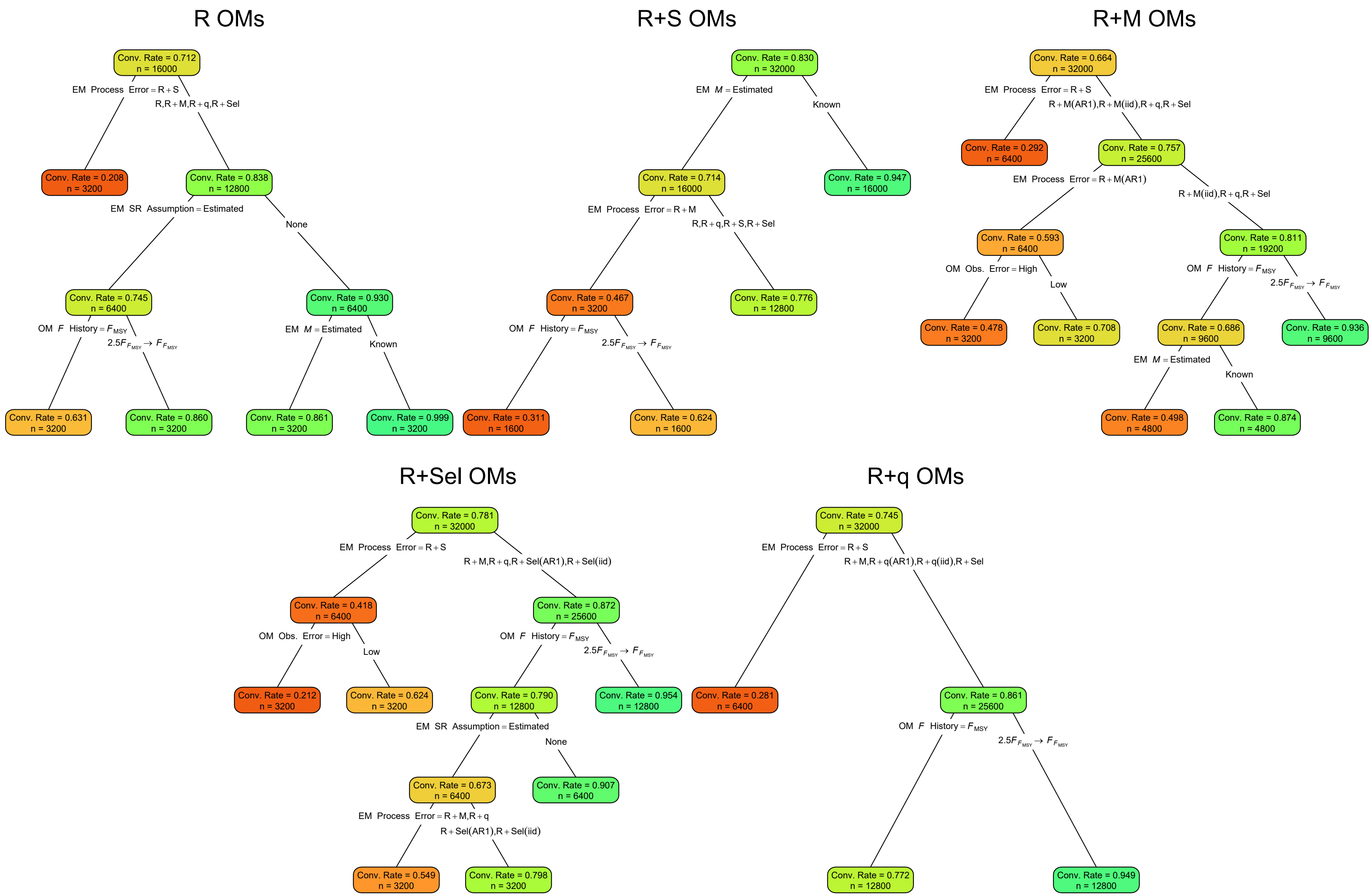

Fig. 1. Classification trees indicating primary factors determining convergence as defined by providing Hessian-based standard errors for R, R+S, R+M, R+Sel and R+q OMs. Lower or higher convergence rates are indicated by more red or green polygons, respectively

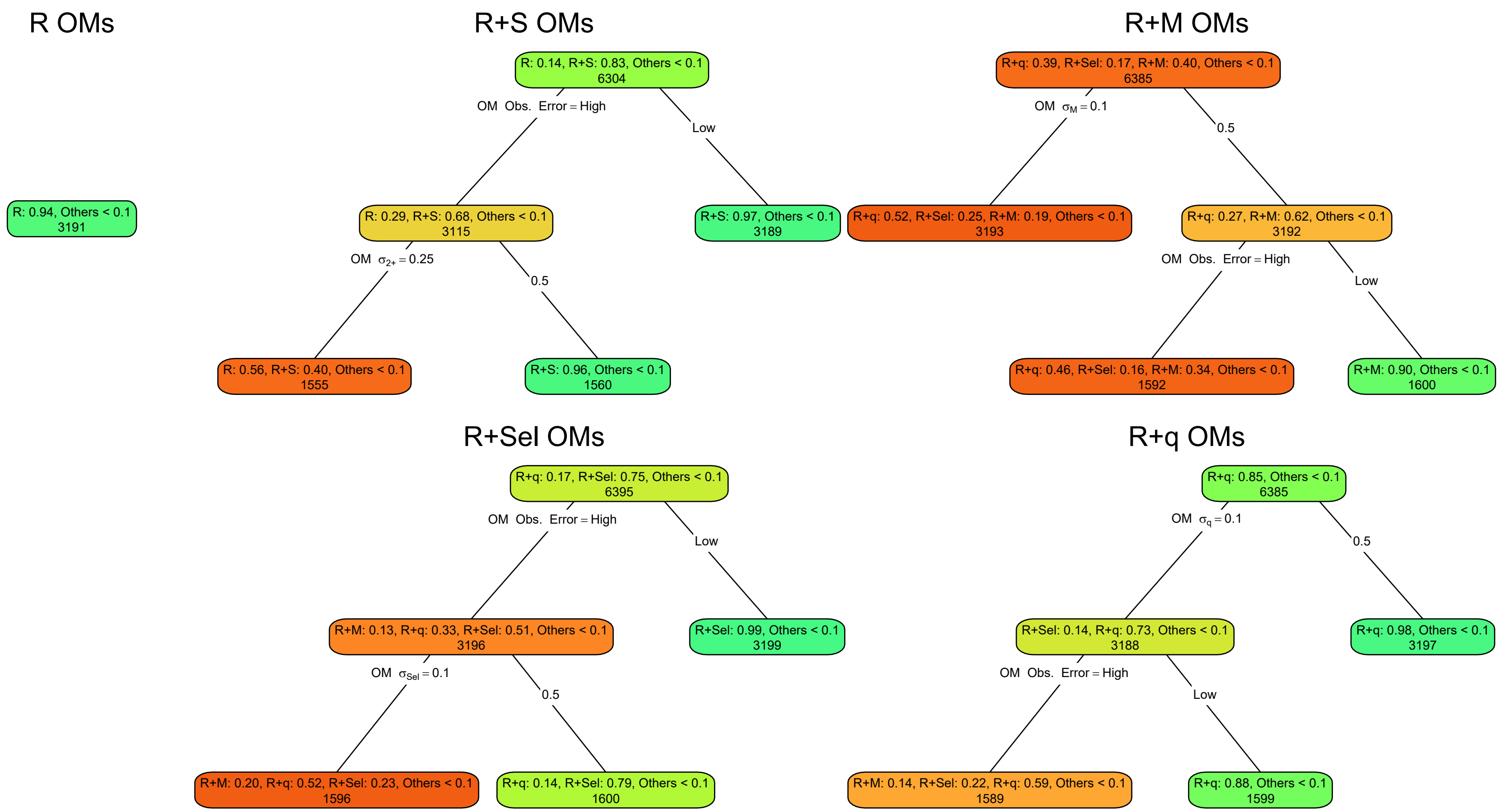

Fig. 2. Classification trees indicating primary factors determining which EM process error assumption provides the lowest AIC for R+S, R+M, R+Sel and R+q OMs. Each node shows the proportion of EM process error models with lowest AIC (top) and number of observations (bottom) for the corresponding subset. Lower or higher accuracy of the process error assumption are indicated by more red or green polygons, respectively.

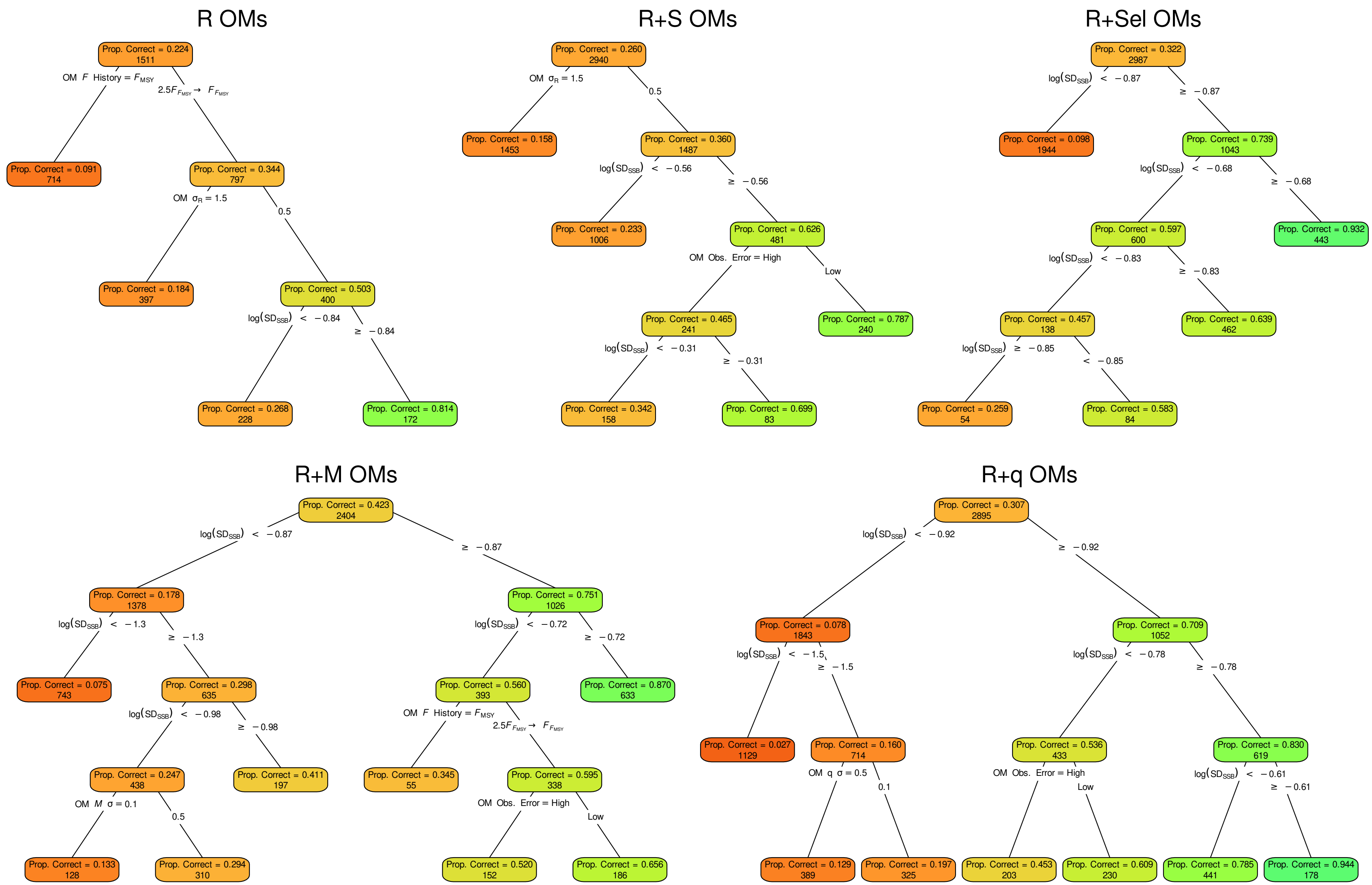

Fig. 3. Classification trees indicating primary factors determining which EM SRR assumption (none or Beverton-Holt) provides the lowest AIC for R, R+S, R+M, R+Sel and R+q OMs. Each node shows the proportion of EMs that assume the SRR with lowest AIC (top) and number of observations (bottom) for the corresponding subset. Lower or higher accuracy of the process error assumption are indicated by more red or green polygons, respectively.

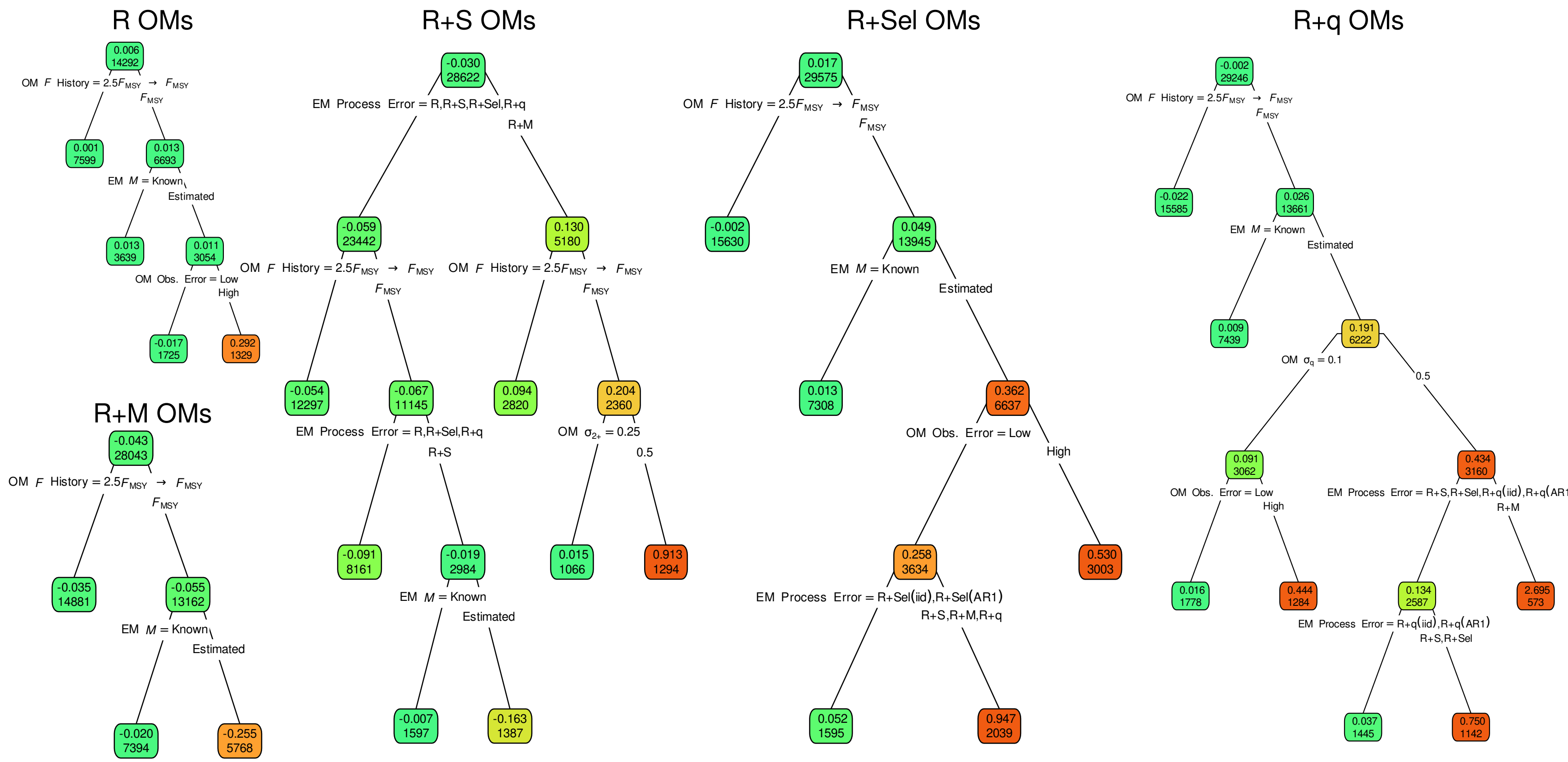

Fig. 4. Regression trees indicating primary factors determining reductions in sums of squares of errors in estimation measured by Eq. 3 for terminal year SSB for R+S, R+M, R+Sel and R+q OMs. Each node shows the median error (top) and number of observations (bottom) for the corresponding subset. Median errors closer to or further from zero are indicated by more green or red polygons, respectively.

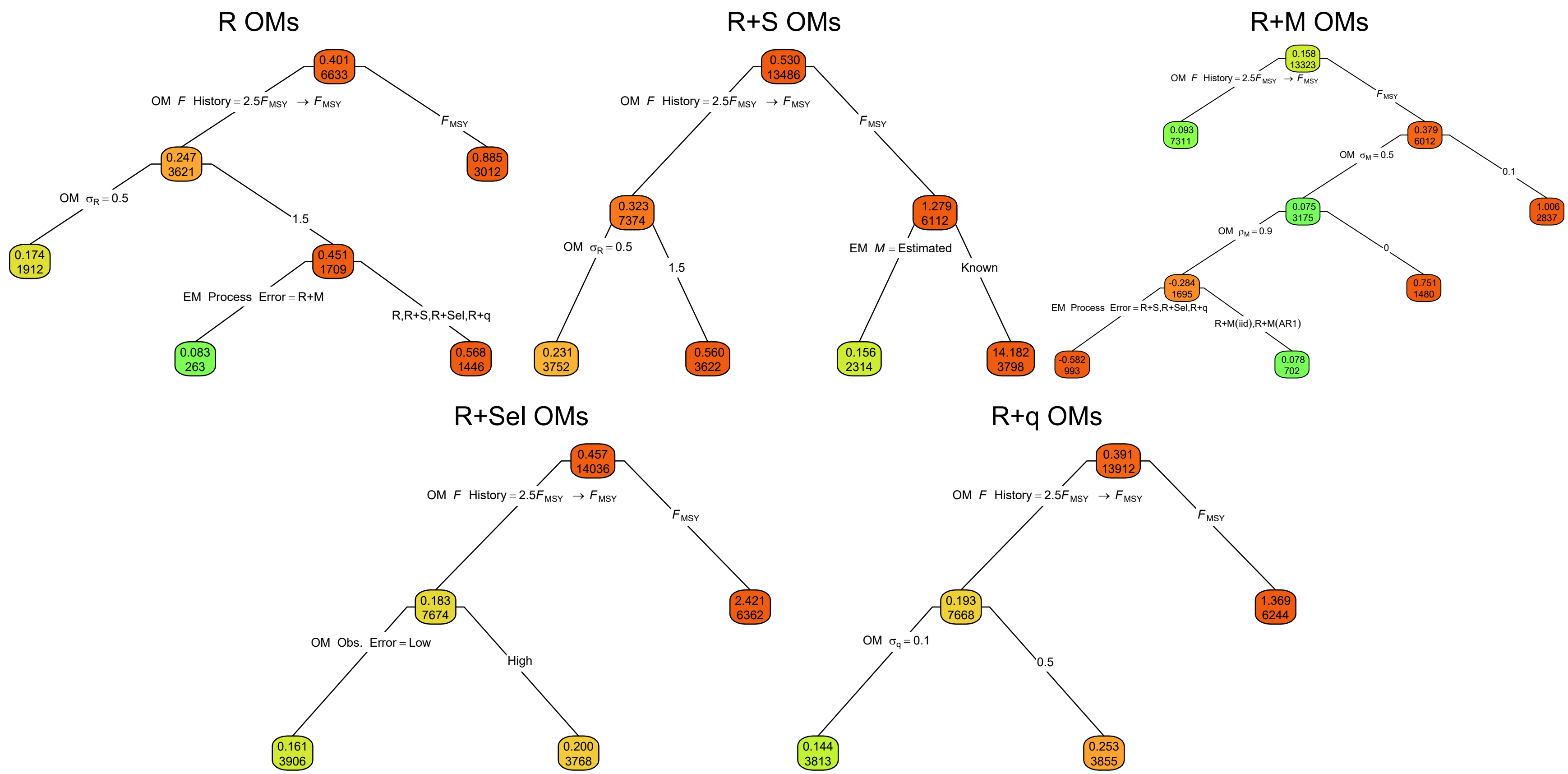

Fig. 5. Regression trees indicating primary factors determining reductions in sums of squares of errors in estimation measured by Eq. 3 for the Beverton-Holt SRR parameter a for R+S, R+M, R+Sel and R+q OMs. Each node shows the median error (top) and number of observations (bottom) for the corresponding subset. Lower or higher median absolute errors of the process error assumption are indicated by more green or red polygons, respectively.

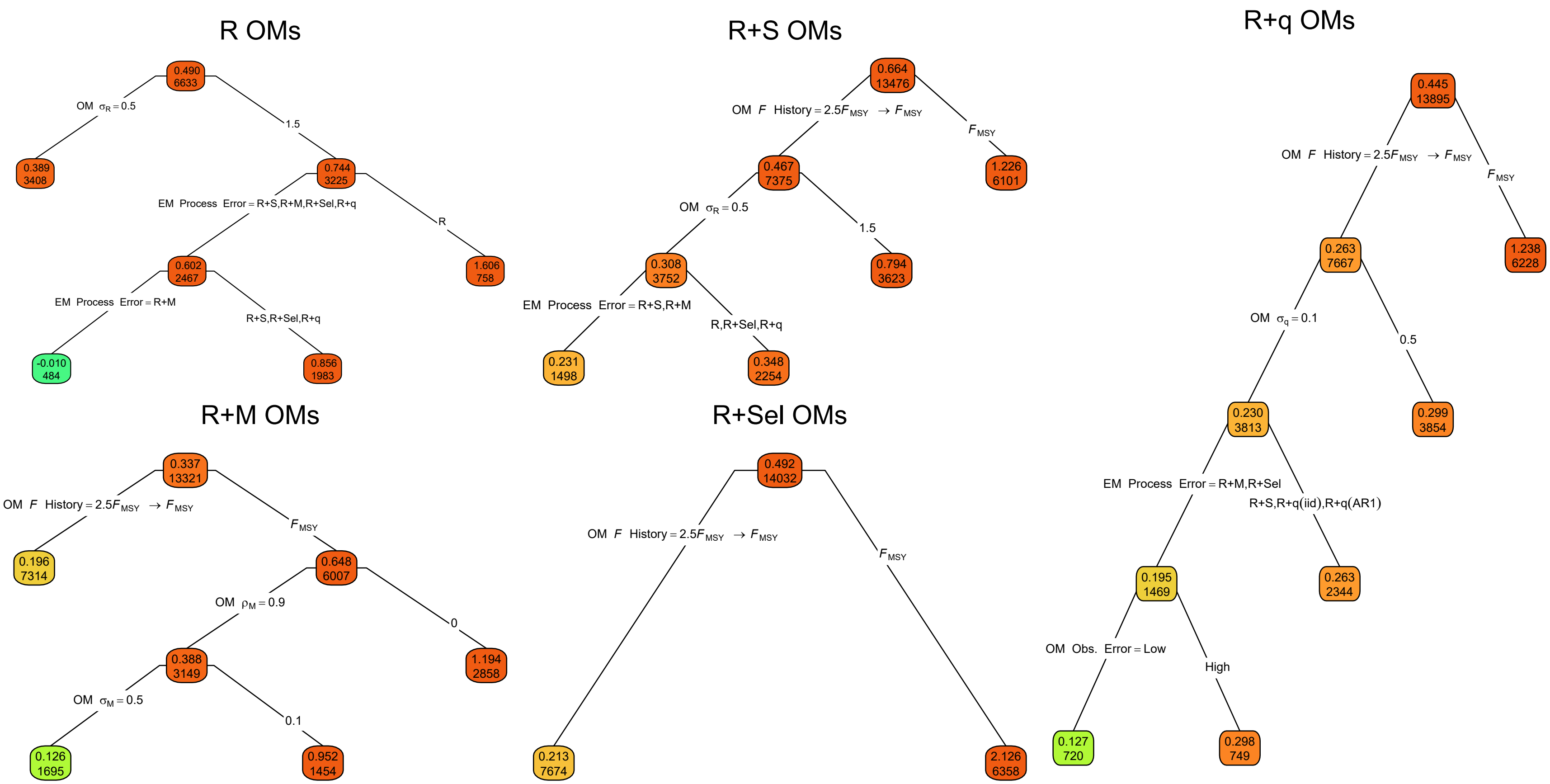

Fig. 6. Regression trees indicating primary factors determining reductions in sums of squares of errors in estimation measured by Eq. 3 for the Beverton-Holt SRR parameter b for R+S, R+M, R+Sel and R+q OMs. Each node shows the median error (top) and number of observations (bottom) for the corresponding subset. Lower or higher median absolute errors of the process error assumption are indicated by more green or red polygons, respectively.

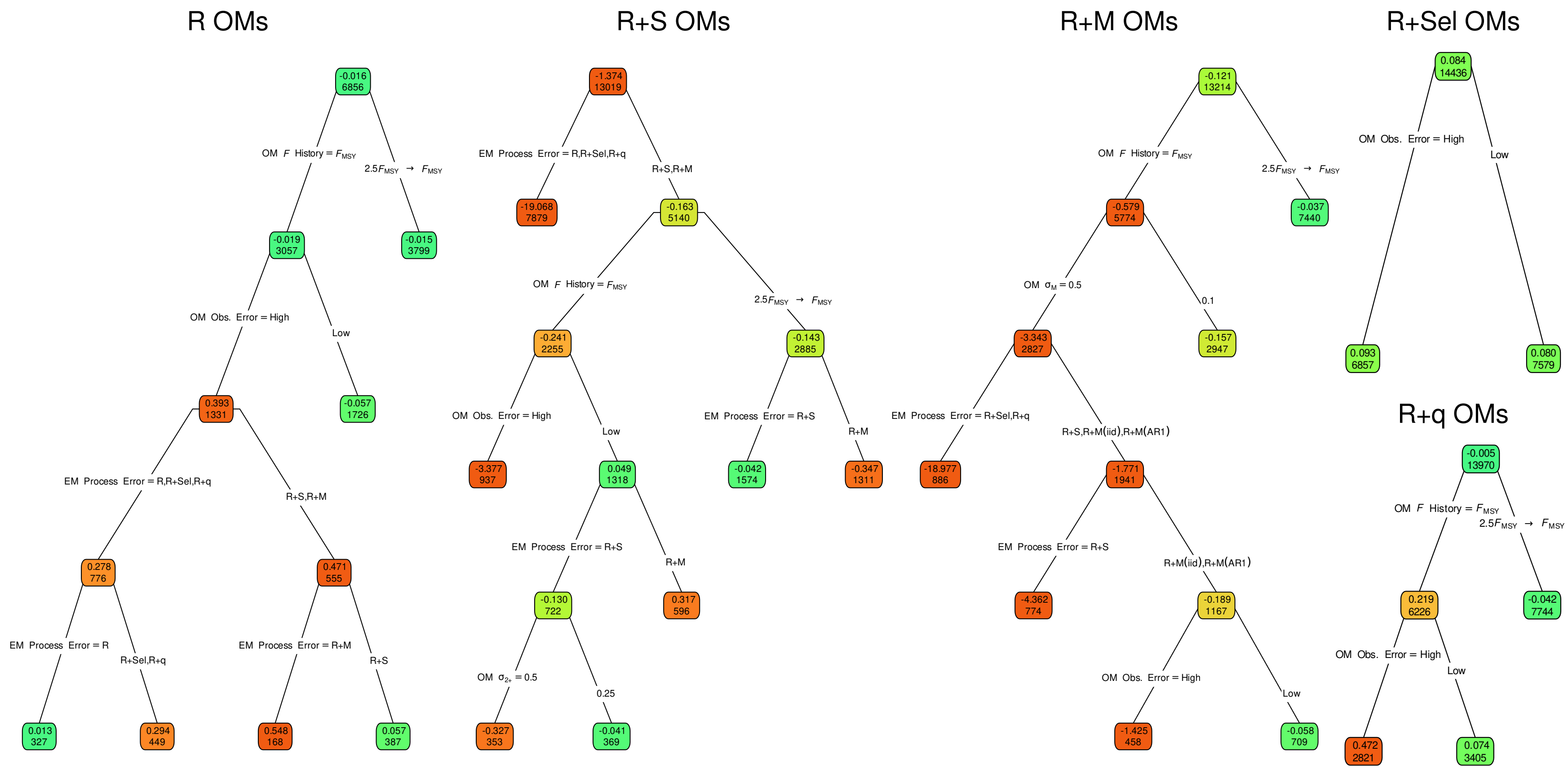

Fig. 7. Regression trees indicating primary factors determining reductions in sums of squares of errors in estimation measured by Eq. 3 for the median natural mortality rate for R+S, R+M, R+Sel and R+q OMs. Each node shows the median error (top) and number of observations (bottom) for the corresponding subset. Lower or higher median absolute errors of the process error assumption are indicated by more green or red polygons, respectively.

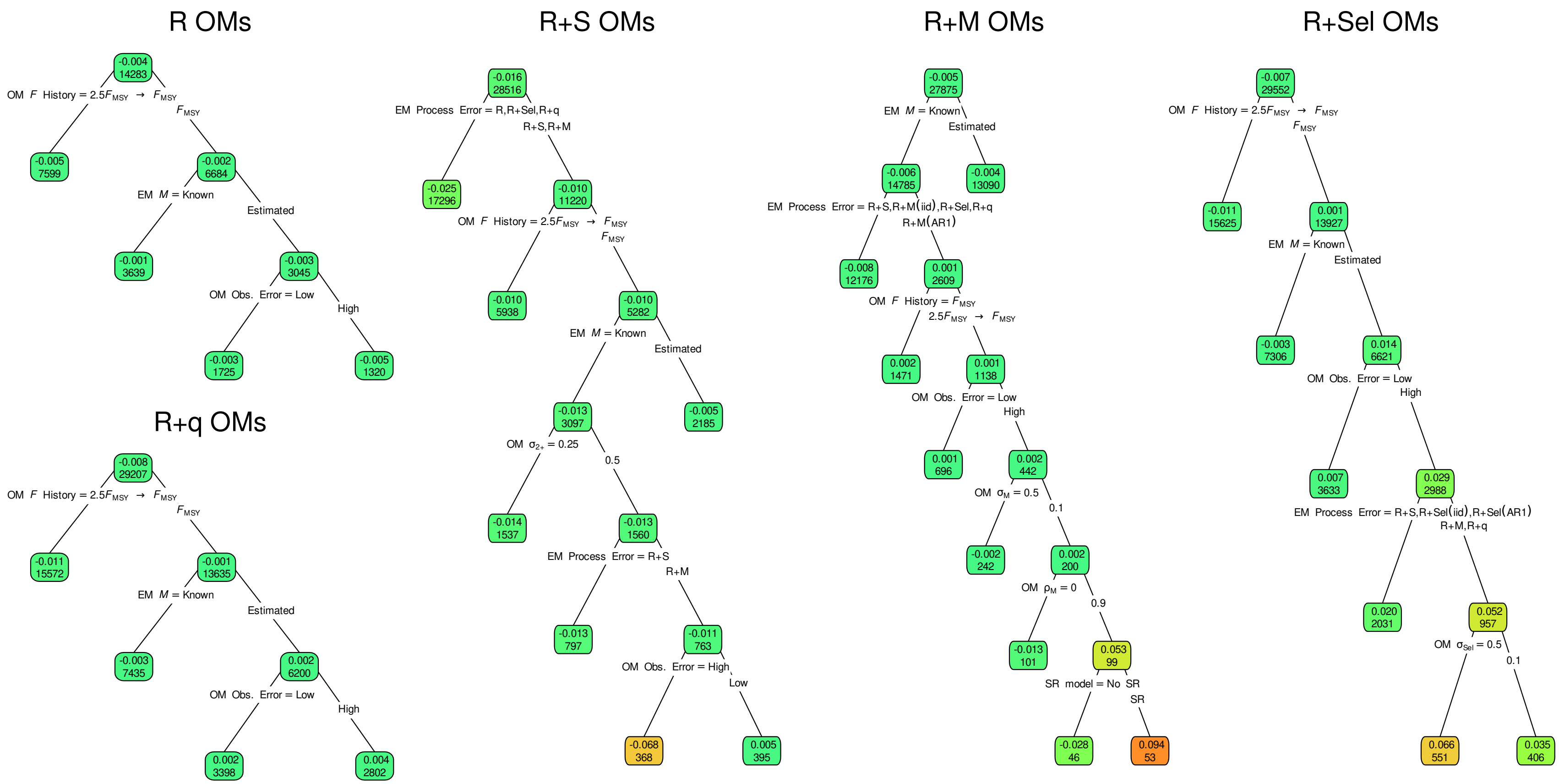

Fig. 8. Regression trees indicating primary factors determining reductions in sums of squares of errors in transformed Mohn's ρ (Eq. 3) for SSB for R+S, R+M, R+Sel and R+q OMs. Each node shows the median Mohn's ρ (top) and number of observations (bottom) for the corresponding subset. Median Mohn's ρ closer to or further from zero are indicated by more green or red polygons, respectively.

Table 1. For each OM process error type (columns), percent reduction in deviance for logistic regression models fit to indicators of convergence (providing Hessian-based standard errors) with each OM and EM factor (rows) included individually, combined, and with second and third order interactions.

Factor	R	R+S	R+M	R+Sel	R+q
EM Process Error	27.95	4.58	14.68	17.24	24.66
EM M Assumption	1.07	11.43	2.45	0.56	1.46
EM SR Assumption	2.88	3.30	1.24	2.47	1.59
OM Obs. Error	0.75	4.64	2.06	4.54	1.60
OM F History	2.32	3.37	1.63	3.30	2.59
OM σ_R	0.10	0.02	–	–	–
OM σ_{2+}	–	0.40	–	–	–
OM σ_M	–	–	0.22	–	–
OM ρ_M	–	–	0.17	–	–
OM σ_{Sel}	–	–	–	1.81	–
OM ρ_{Sel}	–	–	–	0.02	–
OM σ_q	–	–	–	–	0.34
OM ρ_q	–	–	–	–	<0.01
All factors	39.54	31.46	24.85	34.83	36.31
+ All Two Way	45.03	39.89	35.20	42.81	43.70
+ All Three Way	47.02	44.57	37.88	45.51	46.87

Table 2. For each OM process error type (columns), percent reduction in deviance for multinomial logistic regression models fit to indicators of EM process error assumption with lowest AIC with each OM and EM factor (rows) included individually, combined, and with second and third order interactions.

Factor	R	R+S	R+M	R+Sel	R+q
EM M Assumption	5.52	1.05	0.52	0.61	1.32
EM SR Assumption	5.60	0.75	1.13	0.93	1.95
OM Obs. Error	2.96	22.46	3.42	25.67	5.03
OM F History	5.77	0.62	0.94	0.91	2.05
OM σ_R	0.10	0.66	–	–	–
OM σ_{2+}	–	16.86	–	–	–
OM σ_M	–	–	9.06	–	–
OM ρ_M	–	–	0.38	–	–
OM σ_{Sel}	–	–	–	7.59	–
OM ρ_{Sel}	–	–	–	0.60	–
OM σ_q	–	–	–	–	13.50
OM ρ_q	–	–	–	–	0.75
All factors	20.98	46.12	16.58	40.83	25.99
+ All Two Way	22.02	48.94	21.63	44.08	30.17
+ All Three Way	22.05	49.98	22.36	44.54	31.38

Table 3. For each OM process error type (columns), percent reduction in deviance for logistic regression models fit to indicators of EM SRR assumption (none or Beverton-Holt) with lowest AIC with each OM and EM factor (rows) included individually, combined, and with second and third order interactions.

Factor	R	R+S	R+M	R+Sel	R+q
EM M Assumption	0.04	0.21	0.18	0.02	0.01
OM Obs. Error	<0.01	0.65	0.14	0.04	0.02
OM F History	9.17	3.79	13.08	26.56	24.60
OM σ_R	3.54	4.74	–	–	–
OM σ_{2+}	–	0.14	–	–	–
OM σ_M	–	–	1.14	–	–
OM ρ_M	–	–	0.05	–	–
OM σ_{Sel}	–	–	–	0.02	–
OM ρ_{Sel}	–	–	–	0.17	–
OM σ_q	–	–	–	–	0.36
OM ρ_q	–	–	–	–	0.02
$\log(\mathrm{SD}_{\mathrm{SSB}})$	4.11	1.59	33.39	41.36	39.23
All factors	31.52	18.99	34.23	43.77	42.31
+ All Two Way	34.79	22.24	35.99	45.84	44.04
+ All Three Way	35.41	23.09	37.57	46.39	44.63

Table 4. For each OM process error type (columns), percent reduction in deviance for linear regression models fit to errors in estimation measured by Eq. 3 for the terminal year SSB with each OM and EM factor (rows) included individually, combined, and with second and third order interactions.

Factor	R	R+S	R+M	R+Sel	R+q
EM M Assumption	2.28	1.15	1.04	2.92	3.26
EM SR assumption	0.10	0.06	0.08	0.06	0.08
EM Process Error	0.43	4.28	0.40	0.11	1.05
OM Obs. Error	1.63	0.07	0.78	0.32	<0.01
OM F History	2.62	3.15	1.28	3.22	4.72
OM σ_R	0.03	0.01	–	–	–
OM σ_{2+}	–	0.93	–	–	–
OM σ_M	–	–	0.18	–	–
OM ρ_M	–	–	0.01	–	–
OM σ_{Sel}	–	–	–	0.16	–
OM ρ_{Sel}	–	–	–	0.04	–
OM σ_q	–	–	–	–	1.02
OM ρ_q	–	–	–	–	0.06
All factors	7.59	9.86	3.93	7.04	10.64
+ All Two Way	17.99	25.56	10.06	13.44	22.43
+ All Three Way	23.39	36.74	13.76	16.55	31.11

Table 5. For each OM process error type (columns), percent reduction in deviance for linear regression models fit to errors in estimation measured by Eq. 3 for the Beverton-Holt SRR parameters with each OM and EM factor (rows) included individually, combined, and with second and third order interactions.

Factor	**Beverton-Holt a**					**Beverton-Holt b**				
	R	R+S	R+M	R+Sel	R+q	R	R+S	R+M	R+Sel	R+q
EM M Assumption	0.02	1.05	0.02	0.11	0.02	0.05	1.06	0.03	0.01	0.40
EM Process Error	2.74	0.18	0.20	1.25	1.90	2.29	1.21	0.12	1.40	3.06
OM Obs. Error	0.16	<0.01	0.01	0.04	<0.01	<0.01	0.01	0.05	0.01	0.01
OM F History	3.15	3.34	5.60	11.37	10.00	1.16	1.17	2.01	7.97	3.87
OM σ_R	2.31	0.74	–	–	–	1.67	0.52	–	–	–
OM σ_{2+}	–	0.29	–	–	–	–	0.01	–	–	–
OM σ_M	–	–	0.30	–	–	–	–	0.13	–	–
OM ρ_M	–	–	0.51	–	–	–	–	0.22	–	–
OM σ_{Sel}	–	–	–	0.13	–	–	–	–	0.05	–
OM ρ_{Sel}	–	–	–	0.07	–	–	–	–	0.04	–
OM σ_q	–	–	–	–	0.04	–	–	–	–	0.10
OM ρ_q	–	–	–	–	<0.01	–	–	–	–	<0.01
All factors	8.07	5.15	6.73	12.64	11.79	4.91	3.75	2.55	9.12	7.22
+ All Two Way	9.96	7.37	9.76	13.59	13.65	7.55	7.15	4.32	10.08	12.16
+ All Three Way	11.22	8.15	11.13	14.48	14.87	9.78	9.02	5.26	11.08	14.73

Table 6. For each OM process error type (columns), percent reduction in deviance for linear regression models fit to errors in estimation measured by Eq. 3 for the median natural mortality rate parameter with each OM and EM factor (rows) included individually, combined, and with second and third order interactions.

Factor	R	R+S	R+M	R+Sel	R+q
EM SR assumption	0.21	0.38	0.11	0.26	0.43
EM Process Error	1.98	20.36	3.16	0.94	1.31
OM Obs. Error	4.74	0.79	0.40	2.23	1.88
OM F History	5.07	15.11	10.65	0.24	2.38
OM σ_R	<0.01	0.01	–	–	–
OM σ_{2+}	–	5.04	–	–	–
OM σ_M	–	–	5.32	–	–
OM ρ_M	–	–	0.85	–	–
OM σ_{Sel}	–	–	–	1.30	–
OM ρ_{Sel}	–	–	–	0.37	–
OM σ_q	–	–	–	–	0.46
OM ρ_q	–	–	–	–	0.06
All factors	12.64	40.10	21.29	5.54	6.52
+ All Two Way	21.17	48.12	36.19	9.87	11.71
+ All Three Way	23.03	50.38	42.82	11.58	14.64

Table 7. For each OM process error type (columns), percent reduction in deviance for linear regression models fit to transformed Mohn's ρ values for each simulation (Eq. 3) for SSB with each OM and EM factor (rows) included individually, combined, and with second and third order interactions.

Factor	R	R+S	R+M	R+Sel	R+q
EM M Assumption	0.79	0.18	0.15	0.95	1.24
EM SR assumption	<0.01	0.01	<0.01	<0.01	<0.01
EM Process Error	<0.01	0.22	0.14	0.08	0.04
OM Obs. Error	0.12	0.03	0.05	0.18	0.21
OM F History	0.84	0.14	0.07	1.08	1.56
OM σ_R	0.01	0.01	–	–	–
OM σ_{2+}	–	0.02	–	–	–
OM σ_M	–	–	0.01	–	–
OM ρ_M	–	–	<0.01	–	–
OM σ_{Sel}	–	–	–	0.01	–
OM ρ_{Sel}	–	–	–	0.02	–
OM σ_q	–	–	–	–	0.01
OM ρ_q	–	–	–	–	0.01
All factors	1.89	0.63	0.43	2.43	3.29
+ All Two Way	3.63	1.10	0.91	4.75	6.22
+ All Three Way	4.27	1.65	1.50	5.73	7.53

Supplementary Materials

Referenced Figures

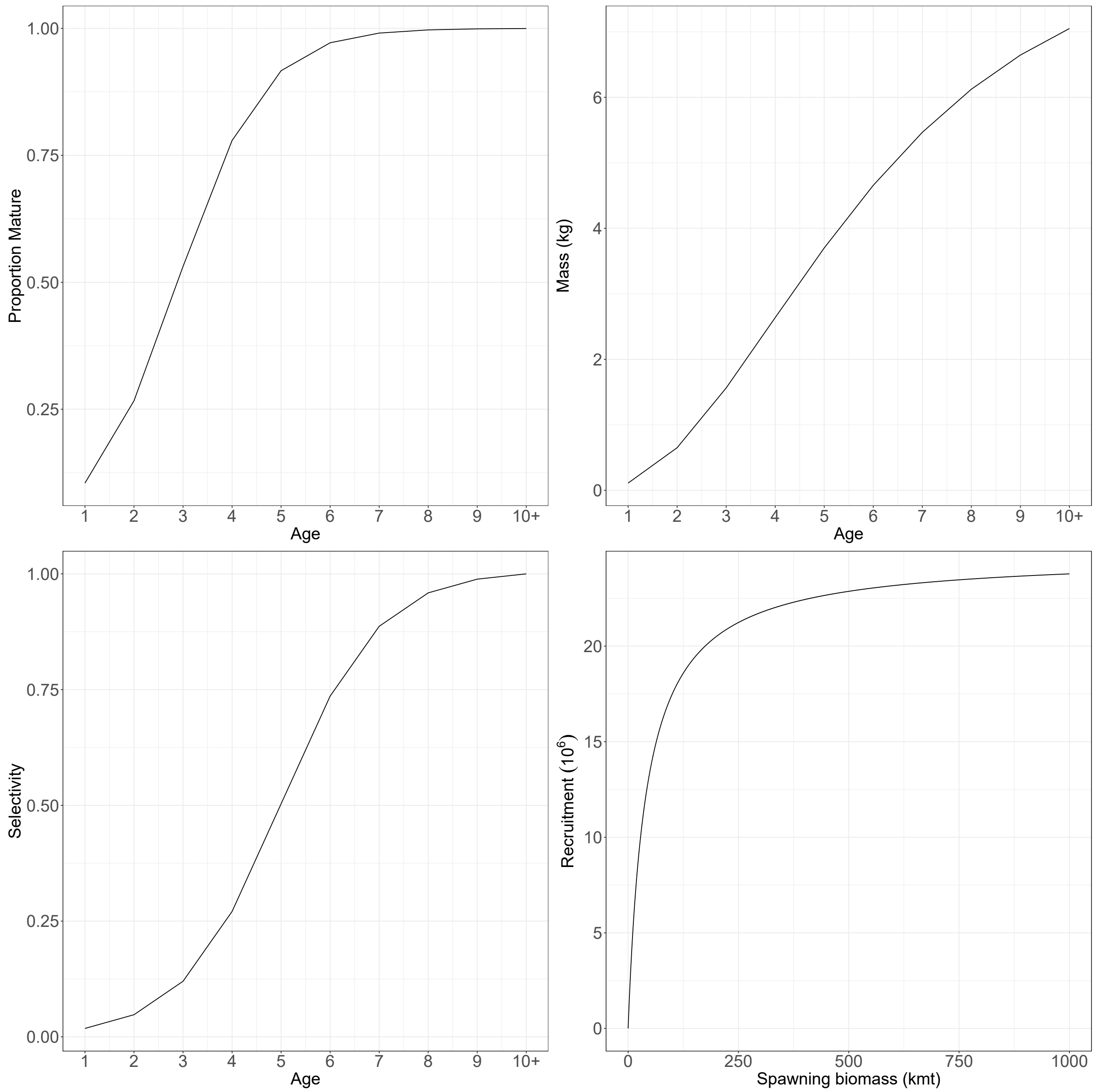

Fig. S1. The proportion mature at age, weight at age, fleet and index selectivity at age, and Beverton-Holt SRR assumed for the population in all OMs. For OMs with random effects on fleet selectivity, this represents the selectivity at the mean of the random effects.

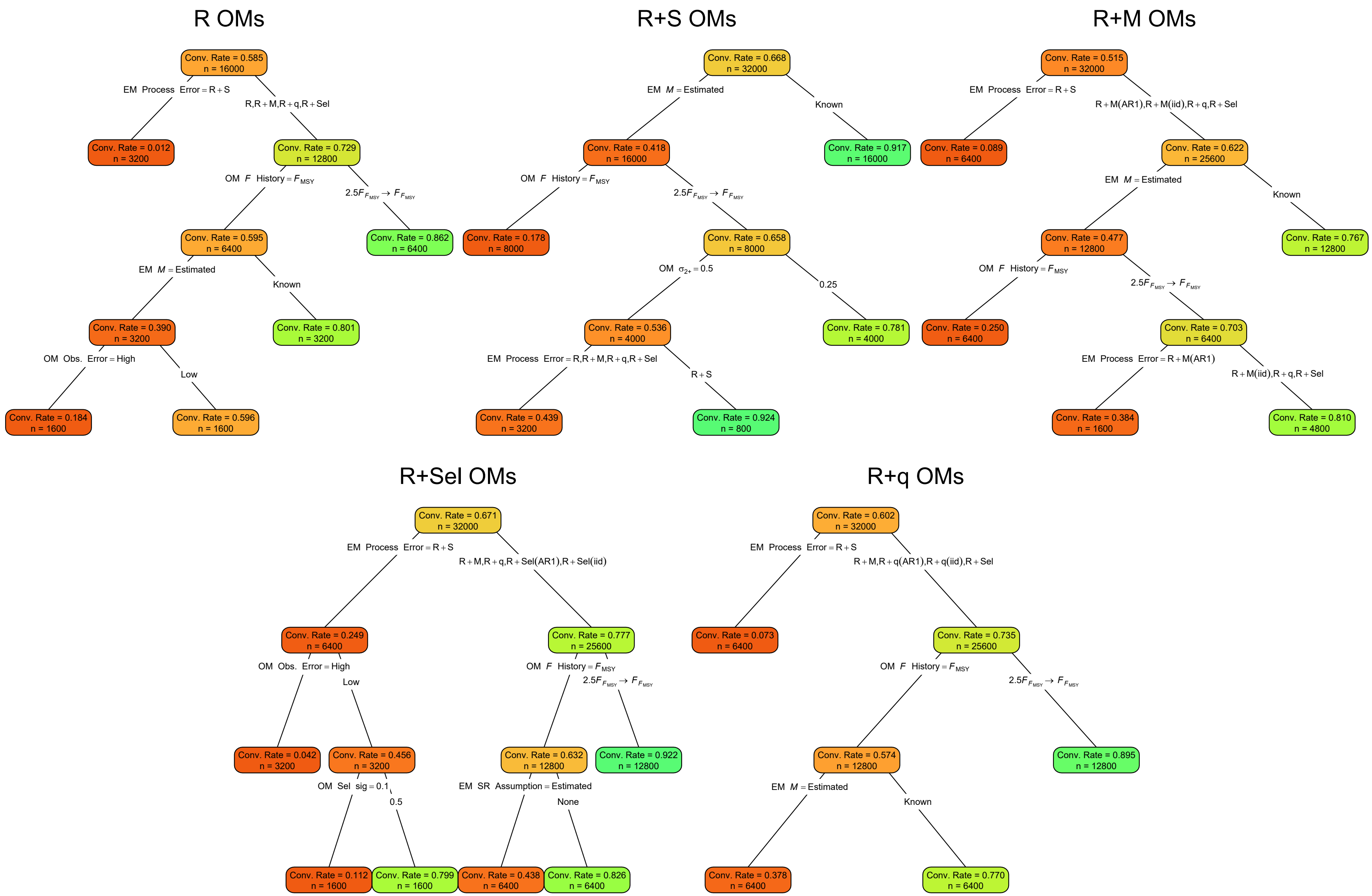

Fig. S2. Classification trees indicating primary factors determining convergence as defined by a maximum absolute gradient $< 10^{-6}$ for R, R+S, R+M, R+Sel and R+q OMs. Lower or higher convergence rates are indicated by more red or green polygons, respectively

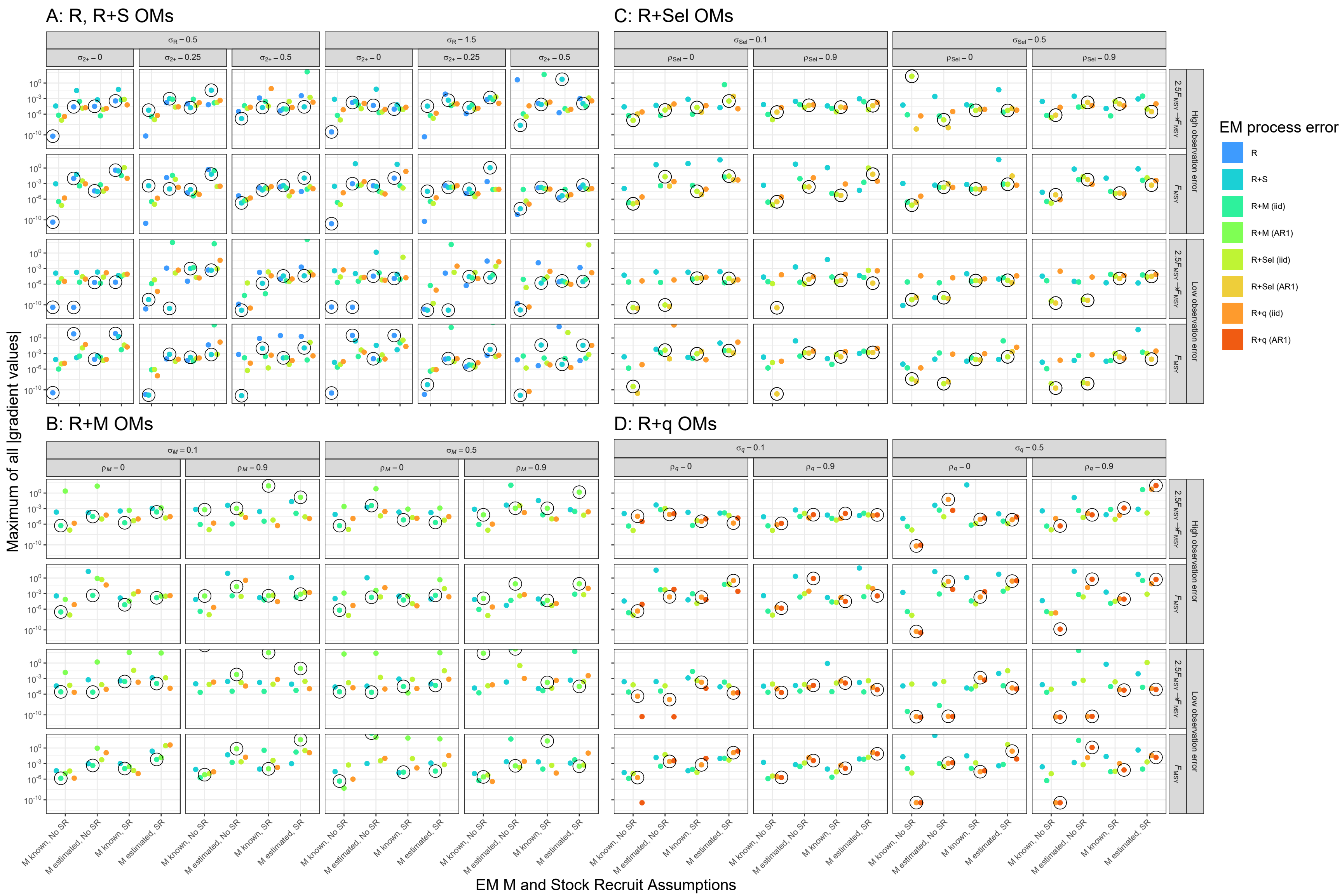

Fig. S3. The maximum of the absolute values of all gradient values for all fits that provided Hessian-based standard errors across all simuated data sets of a given OM configuration (A: R and R+S, B: R+M, C: R+Sel, or D: R+q). Results are conditional on EM fits with alternative process error type (colored points and lines), median natural mortality (estimated or known) and recruitment assumptions (Beverton-Holt SRR or not). Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

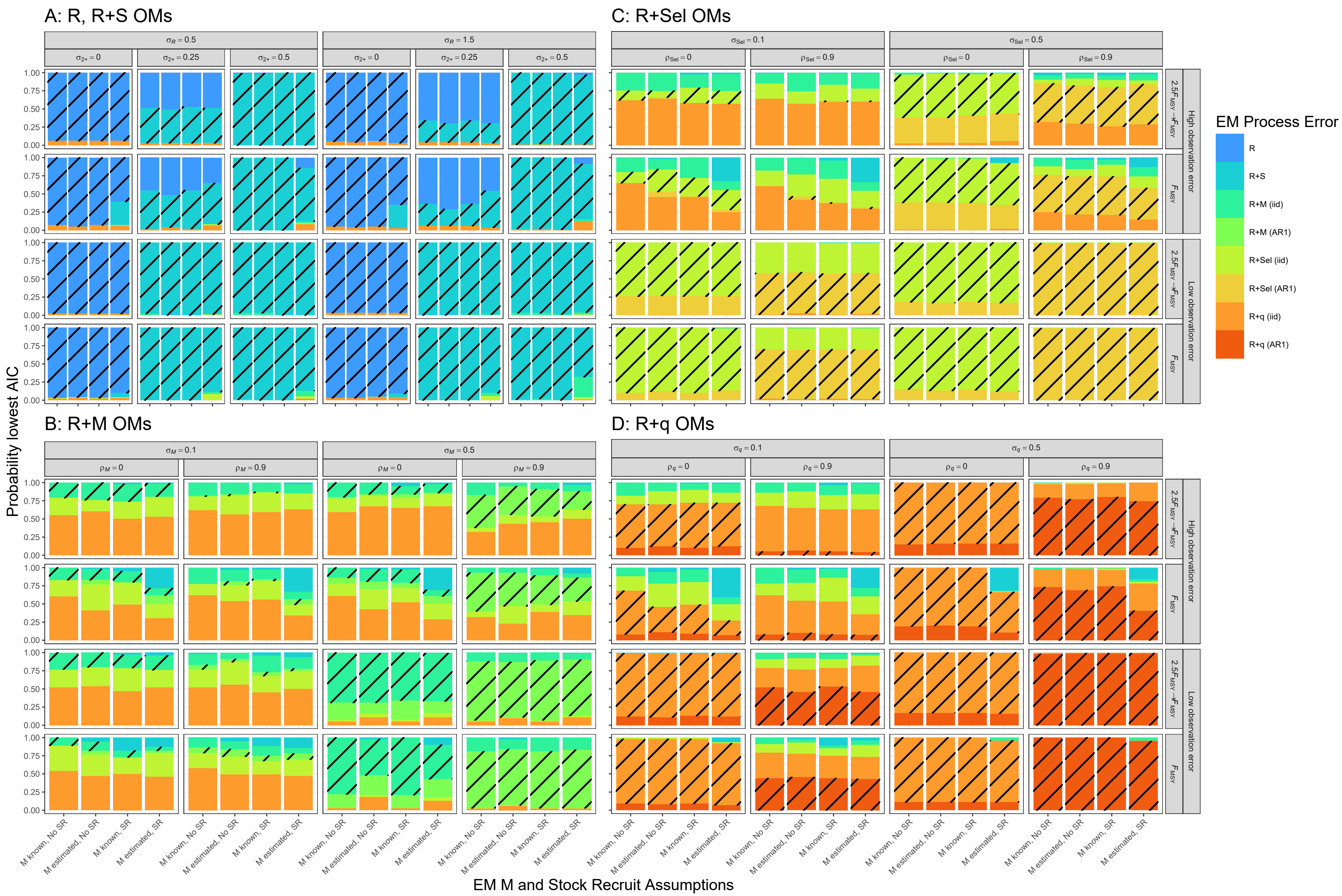

Fig. S4. Estimated probability of lowest AIC for EMs assuming alternative process error structures (colored bars) conditional on alternative assumptions for median natural mortality (estimated or known) and Beverton-Holt SRR (estimated or not; along x-axis) when fitted to OMs that have R and R+S (A), R+Sel (B), R+M (C), or R+q (D) process error structures. Striped bars indicate results where the EM process error structure matches that of the OM.

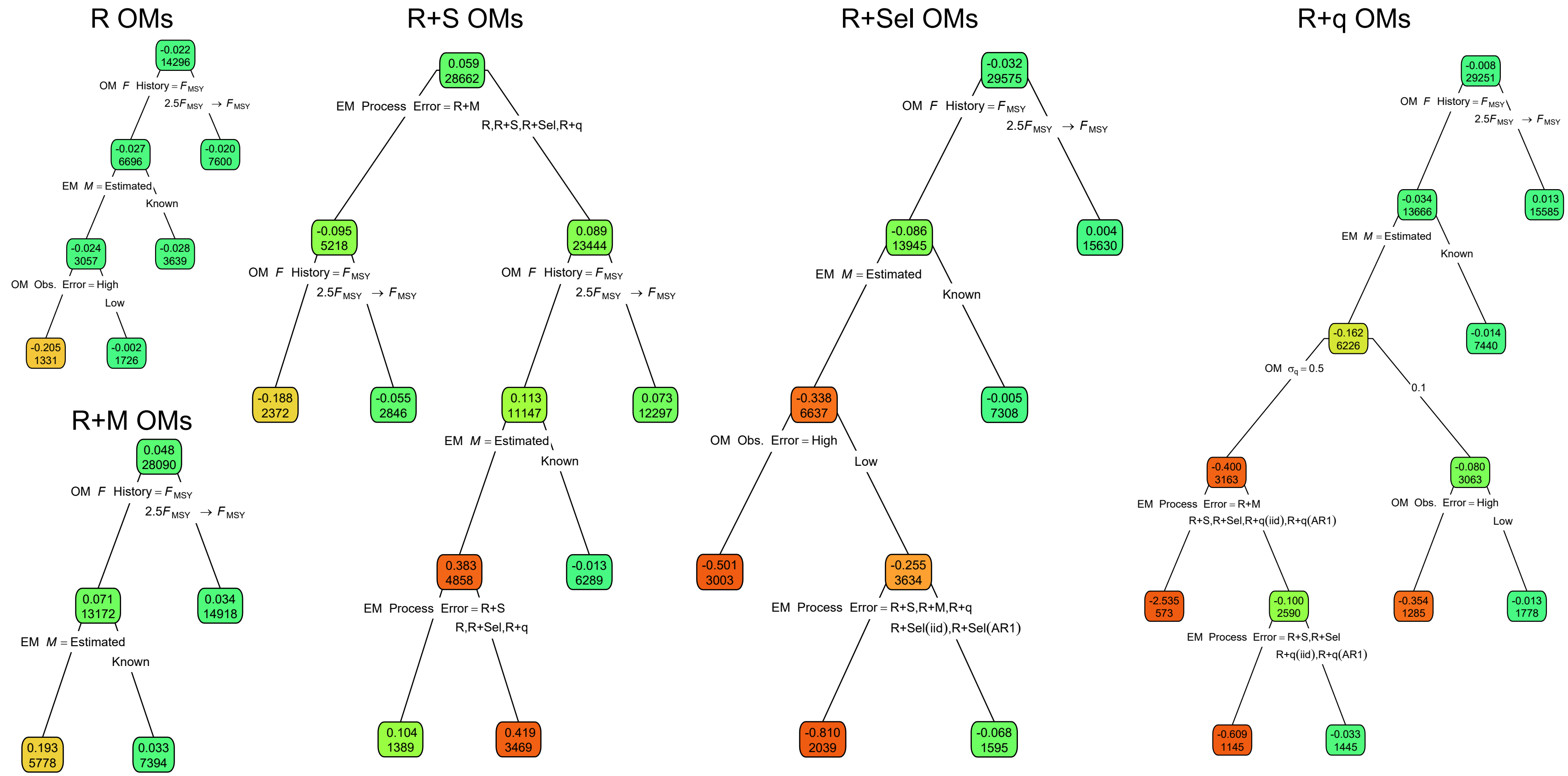

Fig. S5. Regression trees indicating primary factors determining reductions in sums of squares of errors in estimation measured by Eq. 3 for terminal year fully-selected fishing mortality for R+S, R+M, R+Sel and R+q OMs. Each node shows the median error (top) and number of observations (bottom) for the corresponding subset. Median errors closer to or further from zero are indicated by more green or red polygons, respectively.

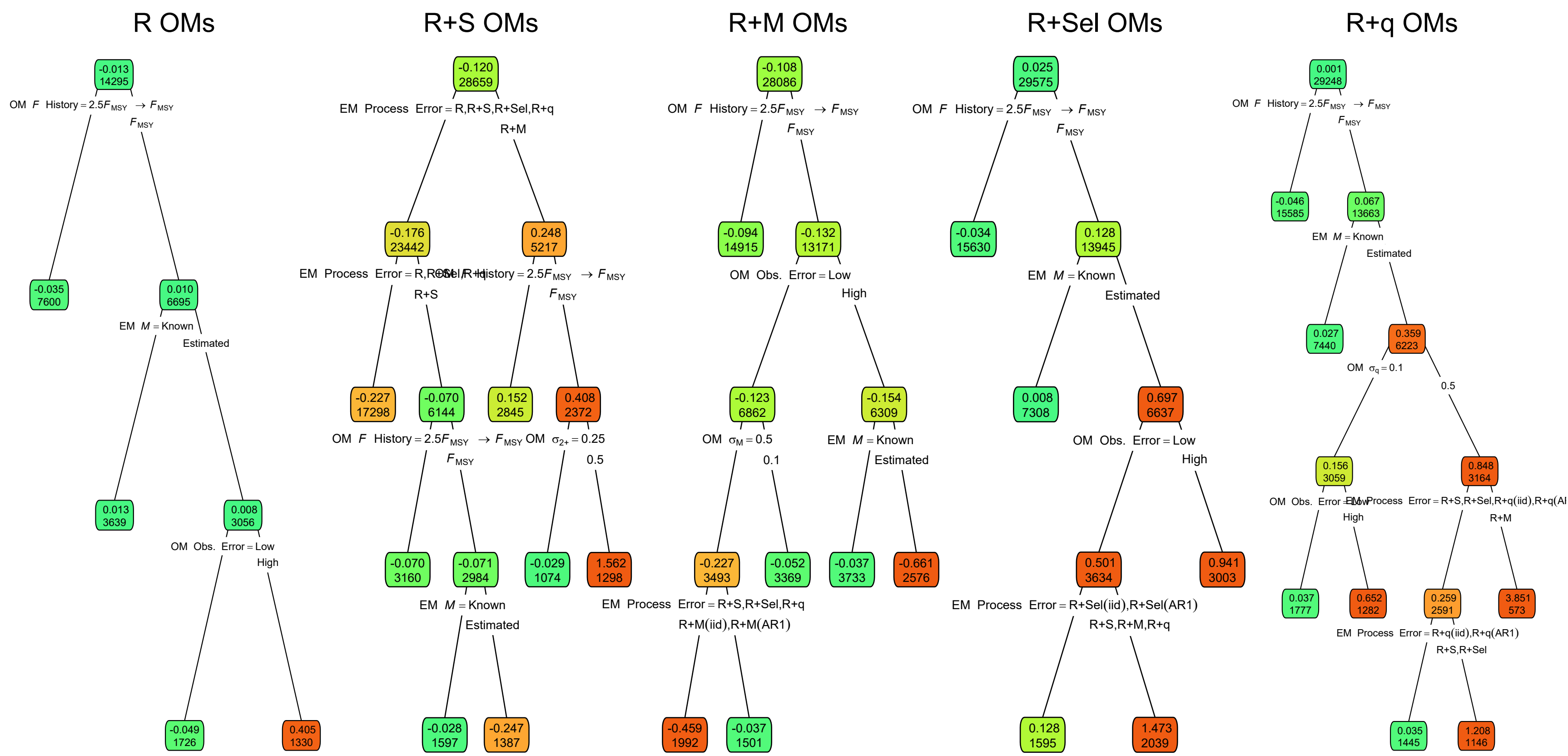

Fig. S6. Regression trees indicating primary factors determining reductions in sums of squares of errors in estimation measured by Eq. 3 for terminal year recruitment for R+S, R+M, R+Sel and R+q OMs. Each node shows the median error (top) and number of observations (bottom) for the corresponding subset. Median errors closer to or further from zero are indicated by more green or red polygons, respectively.

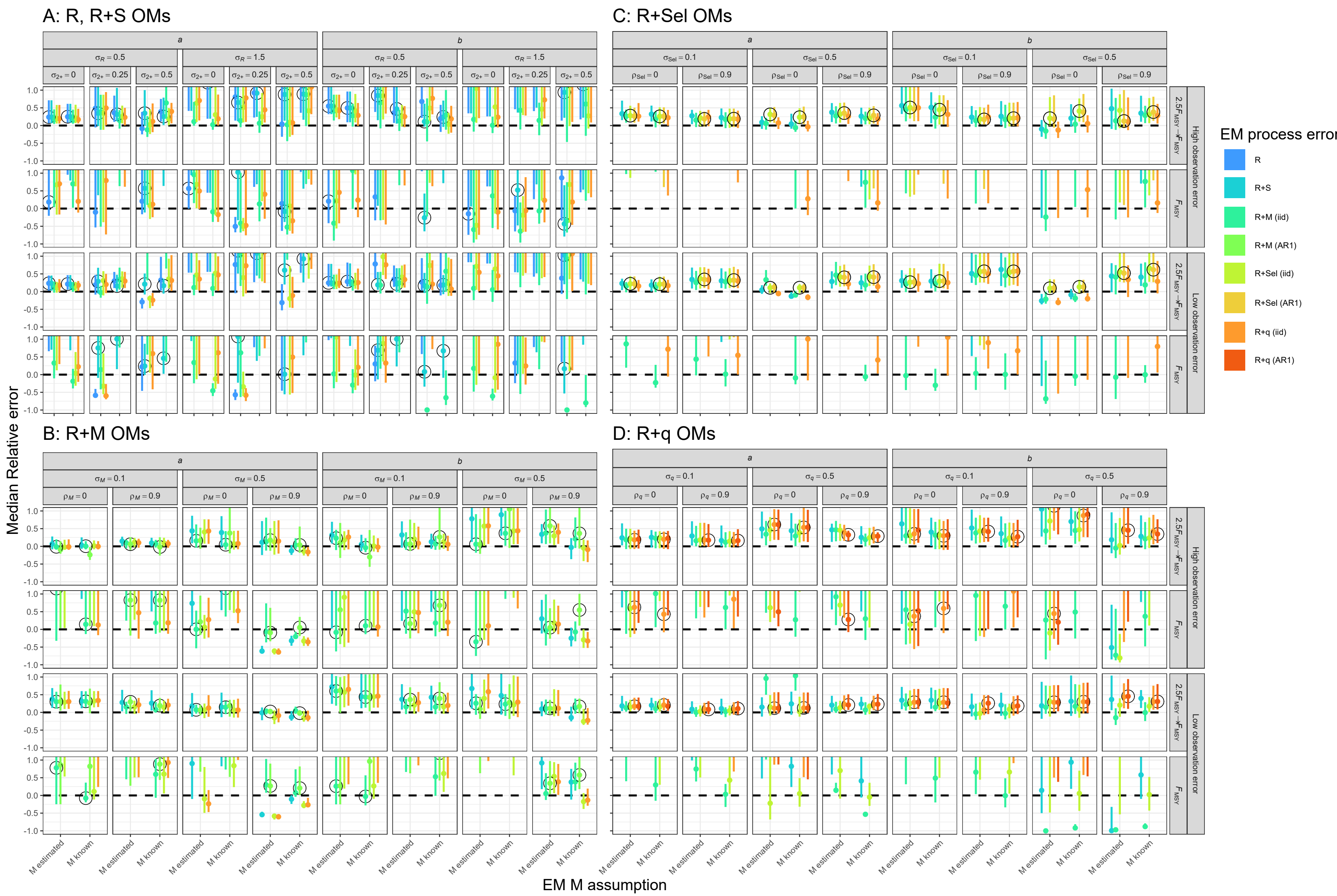

Fig. S7. Median relative error of Beverton-Holt SRR parameters (a and b) for EMs fitted to data sets simulated with alternative process error structures: R and R+S (A), R+Sel (B), R+M (C), or R+q (D). Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

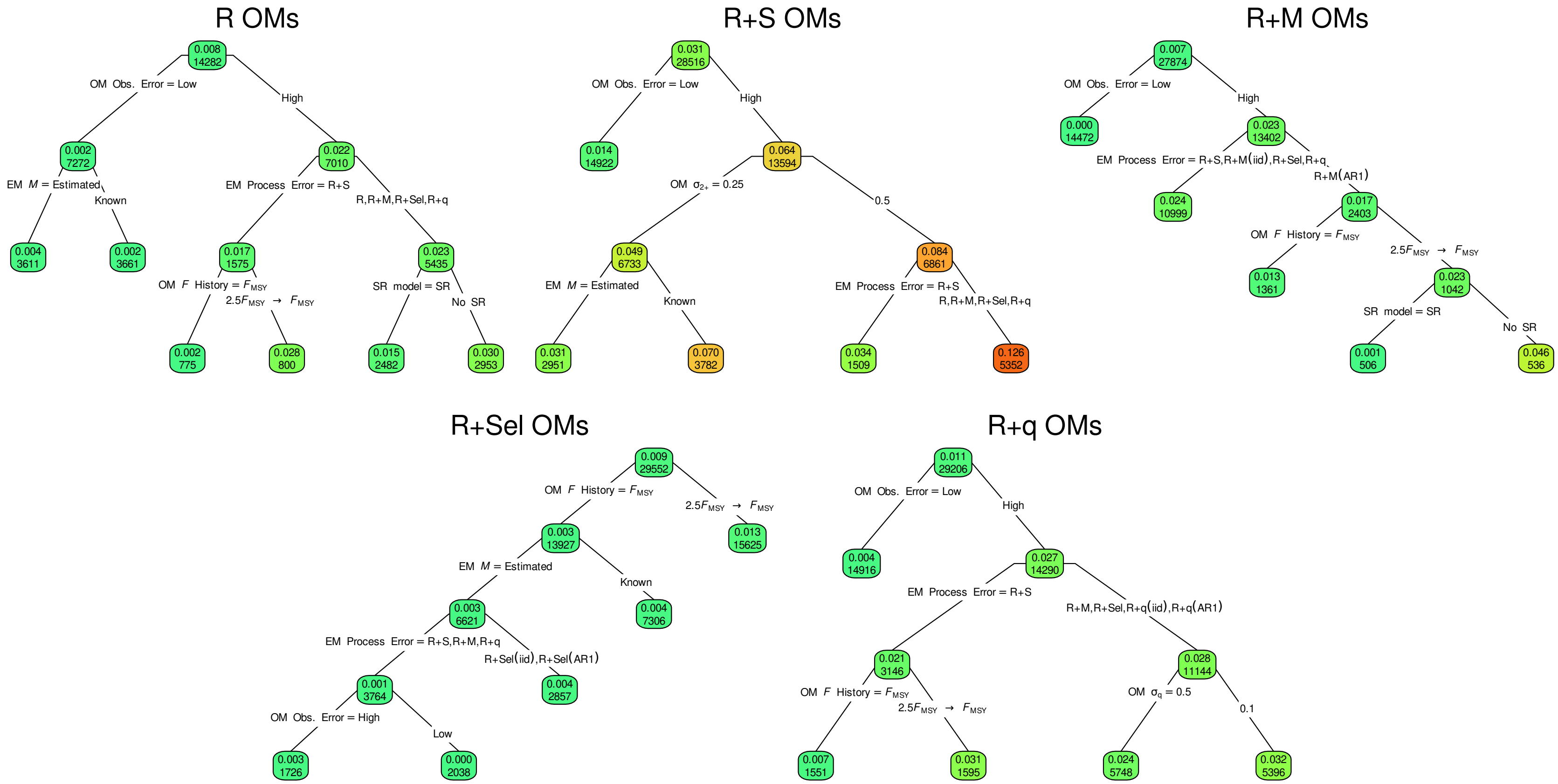

Fig. S8. Regression trees indicating primary factors determining reductions in sums of squares of errors in transformed Mohn's ρ (Eq. 3) for fishing mortality averaged over all age classes for R+S, R+M, R+Sel and R+q OMs. Each node shows the median Mohn's ρ (top) and number of observations (bottom) for the corresponding subset. Median Mohn's ρ closer to or further from zero are indicated by more green or red polygons, respectively.

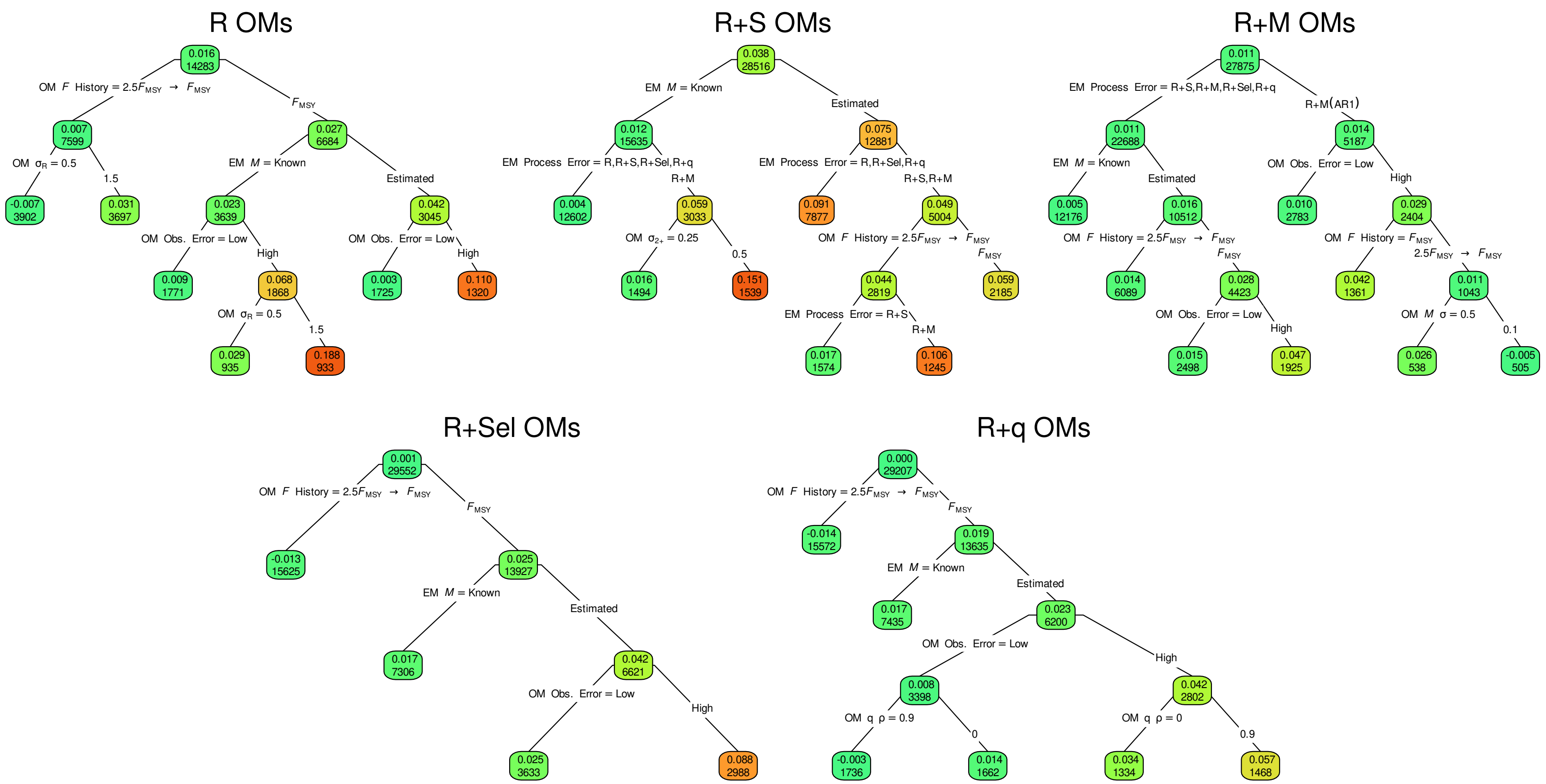

Fig. S9. Regression trees indicating primary factors determining reductions in sums of squares of errors in transformed Mohn's ρ (Eq. 3) for recruitment for R+S, R+M, R+Sel and R+q OMs. Each node shows the median Mohn's ρ (top) and number of observations (bottom) for the corresponding subset. Median Mohn's ρ closer to or further from zero are indicated by more green or red polygons, respectively.

Referenced Tables

Table S1. Distinguishing characteristics of the OMs with random effects on recruitment and apparent survival (R, R+S). When observation uncertainty is low, standard deviations for log-normal distributed indices and logistic normal distributed age composition observations are 0.1 and 0.3, respectively, and when it is high, standard deviations are 0.4 and 1.5, respectively. Fishing mortality either changes from $2.5F_{\mathrm{MSY}}$ to F_{MSY} after year 20 (of 40) or is constant at F_{MSY} over all years.

Model	σ_R	σ_{2+}	Fishing History	Observation Uncertainty
1	0.5		$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	Low
2	1.5		$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	Low
3	0.5	0.25	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	Low
4	1.5	0.25	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	Low
5	0.5	0.50	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	Low
6	1.5	0.50	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	Low
7	0.5		F_{MSY}	Low
8	1.5		F_{MSY}	Low
9	0.5	0.25	F_{MSY}	Low
10	1.5	0.25	F_{MSY}	Low
11	0.5	0.50	F_{MSY}	Low
12	1.5	0.50	F_{MSY}	Low
13	0.5		$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	High
14	1.5		$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	High
15	0.5	0.25	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	High
16	1.5	0.25	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	High
17	0.5	0.50	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	High
18	1.5	0.50	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	High
19	0.5		F_{MSY}	High
20	1.5		F_{MSY}	High
21	0.5	0.25	F_{MSY}	High
22	1.5	0.25	F_{MSY}	High
23	0.5	0.50	F_{MSY}	High
24	1.5	0.50	F_{MSY}	High

Table S2. Distinguishing characteristics of the OMs with random effects on recruitment and natural mortality (R+M). When observation uncertainty is low, standard deviations for log-normal distributed indices and logistic normal distributed age composition observations are 0.1 and 0.3, respectively, and when it is high, standard deviations are 0.4 and 1.5, respectively. Fishing mortality either changes from $2.5F_{\mathrm{MSY}}$ to F_{MSY} after year 20 (of 40) or is constant at F_{MSY} over all years. For AR1 process errors, σ_M is defined for the marginal distribution of the processes.

Model	σ_R	σ_M	ρ_M	Fishing History	Observation Uncertainty
1	0.5	0.1	0.0	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	Low
2	0.5	0.5	0.0	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	Low
3	0.5	0.1	0.9	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	Low
4	0.5	0.5	0.9	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	Low
5	0.5	0.1	0.0	F_{MSY}	Low
6	0.5	0.5	0.0	F_{MSY}	Low
7	0.5	0.1	0.9	F_{MSY}	Low
8	0.5	0.5	0.9	F_{MSY}	Low
9	0.5	0.1	0.0	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	High
10	0.5	0.5	0.0	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	High
11	0.5	0.1	0.9	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	High
12	0.5	0.5	0.9	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	High
13	0.5	0.1	0.0	F_{MSY}	High
14	0.5	0.5	0.0	F_{MSY}	High
15	0.5	0.1	0.9	F_{MSY}	High
16	0.5	0.5	0.9	F_{MSY}	High

Table S3. Distinguishing characteristics of the OMs with random effects on recruitment and selectivity (R+Sel). When observation uncertainty is low, standard deviations for log-normal distributed indices and logistic normal distributed age composition observations are 0.1 and 0.3, respectively, and when it is high, standard deviations are 0.4 and 1.5, respectively. Fishing mortality either changes from $2.5F_{\mathrm{MSY}}$ to F_{MSY} after year 20 (of 40) or is constant at F_{MSY} over all years. For AR1 process errors, σ_{Sel} is defined for the marginal distribution of the processes.

Model	σ_R	σ_{Sel}	ρ_{Sel}	Fishing History	Observation Uncertainty
1	0.5	0.1	0.0	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	Low
2	0.5	0.5	0.0	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	Low
3	0.5	0.1	0.9	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	Low
4	0.5	0.5	0.9	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	Low
5	0.5	0.1	0.0	F_{MSY}	Low
6	0.5	0.5	0.0	F_{MSY}	Low
7	0.5	0.1	0.9	F_{MSY}	Low
8	0.5	0.5	0.9	F_{MSY}	Low
9	0.5	0.1	0.0	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	High
10	0.5	0.5	0.0	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	High
11	0.5	0.1	0.9	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	High
12	0.5	0.5	0.9	$2.5F_{\mathrm{MSY}} \rightarrow F_{\mathrm{MSY}}$	High
13	0.5	0.1	0.0	F_{MSY}	High
14	0.5	0.5	0.0	F_{MSY}	High
15	0.5	0.1	0.9	F_{MSY}	High
16	0.5	0.5	0.9	F_{MSY}	High

Table S4. Distinguishing characteristics of the OMs with random effects on recruitment and catchability (R+q). When observation uncertainty is low, standard deviations for log-normal distributed indices and logistic normal distributed age composition observations are 0.1 and 0.3, respectively, and when it is high, standard deviations are 0.4 and 1.5, respectively. Fishing mortality either changes from $2.5F_{\mathrm{MSY}}$ to F_{MSY} after year 20 (of 40) or is constant at F_{MSY} over all years. For AR1 process errors, σ_q is defined for the marginal distribution of the processes.

Model	σ_R	σ_q	ρ_q	Fishing History	Observation Uncertainty
1	0.5	0.1	0.0	$2.5F_{\mathrm{MSY}} \to F_{\mathrm{MSY}}$	Low
2	0.5	0.5	0.0	$2.5F_{\mathrm{MSY}} \to F_{\mathrm{MSY}}$	Low
3	0.5	0.1	0.9	$2.5F_{\mathrm{MSY}} \to F_{\mathrm{MSY}}$	Low
4	0.5	0.5	0.9	$2.5F_{\mathrm{MSY}} \to F_{\mathrm{MSY}}$	Low
5	0.5	0.1	0.0	F_{MSY}	Low
6	0.5	0.5	0.0	F_{MSY}	Low
7	0.5	0.1	0.9	F_{MSY}	Low
8	0.5	0.5	0.9	F_{MSY}	Low
9	0.5	0.1	0.0	$2.5F_{\mathrm{MSY}} \to F_{\mathrm{MSY}}$	High
10	0.5	0.5	0.0	$2.5F_{\mathrm{MSY}} \to F_{\mathrm{MSY}}$	High
11	0.5	0.1	0.9	$2.5F_{\mathrm{MSY}} \to F_{\mathrm{MSY}}$	High
12	0.5	0.5	0.9	$2.5F_{\mathrm{MSY}} \to F_{\mathrm{MSY}}$	High
13	0.5	0.1	0.0	F_{MSY}	High
14	0.5	0.5	0.0	F_{MSY}	High
15	0.5	0.1	0.9	F_{MSY}	High
16	0.5	0.5	0.9	F_{MSY}	High

Table S5. Distinguishing characteristics of the EMs and indication (+) of which OM process error categories (R, R+S, R+M, R+Sel, R+q) each EM configuration was fit.

Model	Recruitment model	Median M	Process error	R,R+S OMs	R+M OMs	R+Sel OMs	R+q OMs
1	Mean recruitment	0.2	R ($\sigma_{2+} = 0$)	+			
2	Beverton-Holt	0.2	R ($\sigma_{2+} = 0$)	+			
3	Mean recruitment	Estimated	R ($\sigma_{2+} = 0$)	+			
4	Beverton-Holt	Estimated	R ($\sigma_{2+} = 0$)	+			
5	Mean recruitment	0.2	R+S (σ_{2+} estimated)	+	+	+	+
6	Beverton-Holt	0.2	R+S (σ_{2+} estimated)	+	+	+	+
7	Mean recruitment	Estimated	R+S (σ_{2+} estimated)	+	+	+	+
8	Beverton-Holt	Estimated	R+S (σ_{2+} estimated)	+	+	+	+
9	Mean recruitment	0.2	R+M ($\rho_M = 0$)	+	+	+	+
10	Beverton-Holt	0.2	R+M ($\rho_M = 0$)	+	+	+	+
11	Mean recruitment	Estimated	R+M ($\rho_M = 0$)	+	+	+	+
12	Beverton-Holt	Estimated	R+M ($\rho_M = 0$)	+	+	+	+
13	Mean recruitment	0.2	R+Sel ($\rho_{\text{Sel}} = 0$)	+	+	+	+
14	Beverton-Holt	0.2	R+Sel ($\rho_{\text{Sel}} = 0$)	+	+	+	+
15	Mean recruitment	Estimated	R+Sel ($\rho_{\text{Sel}} = 0$)	+	+	+	+
16	Beverton-Holt	Estimated	R+Sel ($\rho_{\text{Sel}} = 0$)	+	+	+	+
17	Mean recruitment	0.2	R+q ($\rho_q = 0$)	+	+	+	+
18	Beverton-Holt	0.2	R+q ($\rho_q = 0$)	+	+	+	+
19	Mean recruitment	Estimated	R+q ($\rho_q = 0$)	+	+	+	+
20	Beverton-Holt	Estimated	R+q ($\rho_q = 0$)	+	+	+	+
21	Mean recruitment	0.2	R+M (ρ_M estimated)		+		
22	Beverton-Holt	0.2	R+M (ρ_M estimated)		+		
23	Mean recruitment	Estimated	R+M (ρ_M estimated)		+		
24	Beverton-Holt	Estimated	R+M (ρ_M estimated)		+		
25	Mean recruitment	0.2	R+Sel (ρ_{Sel} estimated)			+	
26	Beverton-Holt	0.2	R+Sel (ρ_{Sel} estimated)			+	
27	Mean recruitment	Estimated	R+Sel (ρ_{Sel} estimated)			+	
28	Beverton-Holt	Estimated	R+Sel (ρ_{Sel} estimated)			+	
29	Mean recruitment	0.2	R+q (ρ_q estimated)				+
30	Beverton-Holt	0.2	R+q (ρ_q estimated)				+
31	Mean recruitment	Estimated	R+q (ρ_q estimated)				+
32	Beverton-Holt	Estimated	R+q (ρ_q estimated)				+

Table S6. For each OM process error type (columns), percent reduction in deviance for logistic regression models fit to indicators of convergence (maximum absolute gradient < 10^{-6}) with each OM and EM factor (rows) included individually, combined, and with second and third order interactions.

Factor	R	R+S	R+M	R+Sel	R+q
EM Process Error	30.40	0.45	17.57	16.04	24.03
EM M Assumption	2.38	24.11	4.42	1.02	2.66
EM SR Assumption	1.80	0.32	0.96	3.38	2.13
OM Obs. Error	0.12	0.77	0.33	1.76	0.28
OM F History	3.51	6.33	2.36	5.86	5.30
OM σ_R	<0.01	<0.01	–	–	–
OM σ_{2+}	–	<0.01	–	–	–
OM σ_M	–	–	0.39	–	–
OM ρ_M	–	–	0.09	–	–
OM σ_{Sel}	–	–	–	1.08	–
OM ρ_{Sel}	–	–	–	0.01	–
OM σ_q	–	–	–	–	0.06
OM ρ_q	–	–	–	–	<0.01
All factors	43.69	35.72	29.33	34.57	40.69
+ All Two Way	50.53	42.99	43.91	45.93	48.62
+ All Three Way	52.30	48.41	46.81	47.71	50.40

Table S7. For each OM process error type (columns), percent reduction in deviance for linear regression models fit to errors in estimation measured by Eq. 3 for the terminal year fully-selected fishing mortality with each OM and EM factor (rows) included individually, combined, and with second and third order interactions.

Factor	R	R+S	R+M	R+Sel	R+q
EM M Assumption	2.26	1.33	1.26	2.93	3.26
EM SR assumption	0.11	0.07	0.08	0.07	0.09
EM Process Error	0.46	4.18	0.38	0.13	1.02
OM Obs. Error	1.61	0.06	0.86	0.41	<0.01
OM F History	2.49	3.23	1.42	3.22	4.55
OM σ_R	0.02	0.02	–	–	–
OM σ_{2+}	–	0.87	–	–	–
OM σ_M	–	–	0.16	–	–
OM ρ_M	–	–	0.01	–	–
OM σ_{Sel}	–	–	–	0.24	–
OM ρ_{Sel}	–	–	–	0.05	–
OM σ_q	–	–	–	–	1.03
OM ρ_q	–	–	–	–	0.05
All factors	7.42	9.96	4.37	7.26	10.43
+ All Two Way	17.63	25.76	10.94	13.88	22.07
+ All Three Way	22.97	37.03	14.74	17.32	30.74

Table S8. For each OM process error type (columns), percent reduction in deviance for linear regression models fit to errors in estimation measured by Eq. 3 for the terminal year recruitment with each OM and EM factor (rows) included individually, combined, and with second and third order interactions.

Factor	R	R+S	R+M	R+Sel	R+q
EM M Assumption	1.96	0.40	0.69	3.52	3.03
EM SR assumption	0.06	0.02	0.05	0.02	0.05
EM Process Error	0.39	4.74	0.41	0.12	1.16
OM Obs. Error	1.47	0.08	0.64	0.18	<0.01
OM F History	2.54	2.66	1.11	4.18	5.06
OM σ_R	0.03	0.01	–	–	–
OM σ_{2+}	–	1.05	–	–	–
OM σ_M	–	–	0.36	–	–
OM ρ_M	–	–	0.02	–	–
OM σ_{Sel}	–	–	–	0.23	–
OM ρ_{Sel}	–	–	–	0.06	–
OM σ_q	–	–	–	–	1.09
OM ρ_q	–	–	–	–	0.06
All factors	6.90	9.01	3.43	8.58	10.90
+ All Two Way	16.48	24.64	9.73	15.76	22.75
+ All Three Way	21.46	35.60	13.56	19.07	31.15

Further Detailed Results

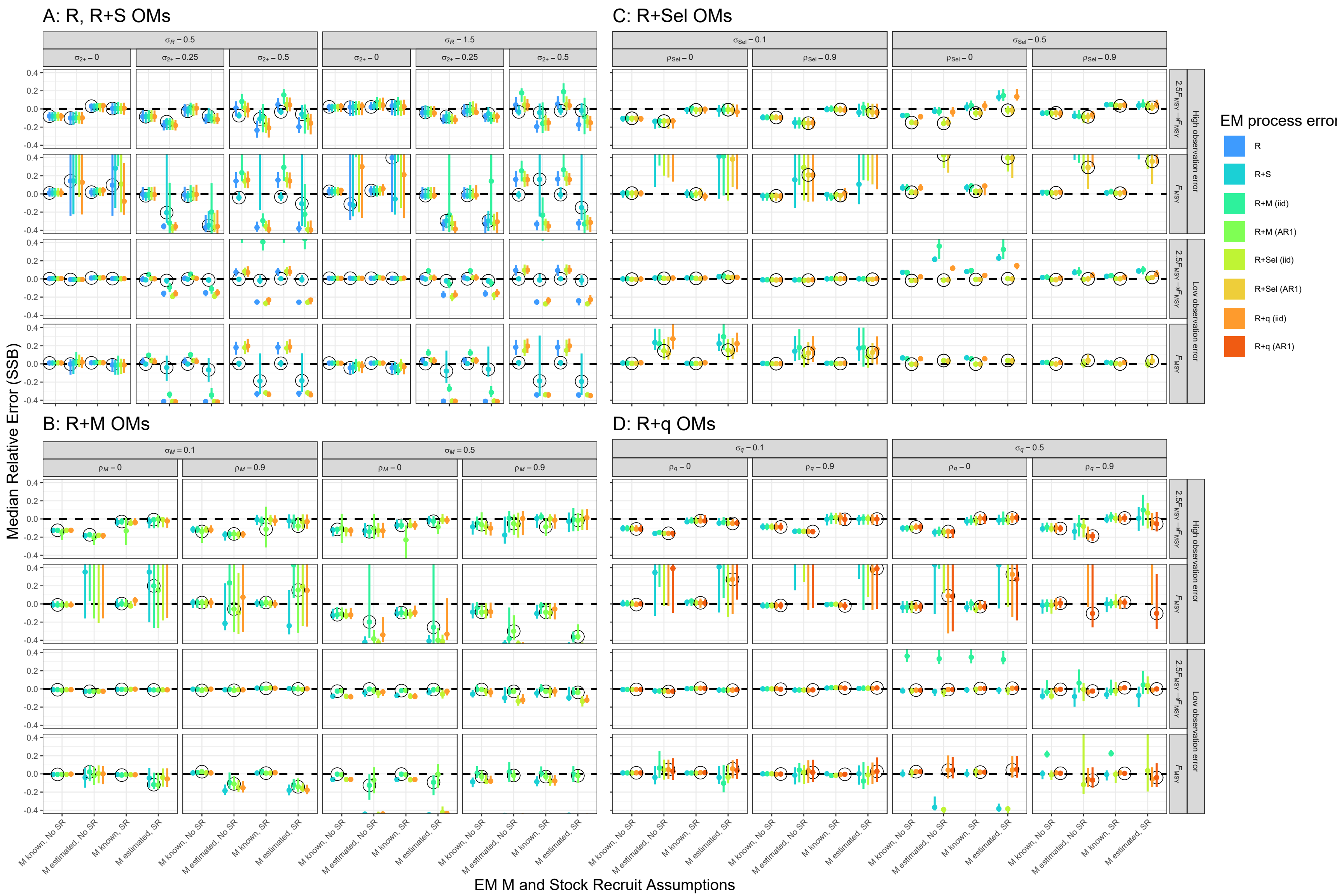

Fig. S10. Median relative error of terminal year SSB for EMs fitted to data sets simulated with alternative process error structures: R and R+S (A), R+Sel (B), R+M (C), or R+q (D). Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

Fig. S11. Median relative error of terminal year fully-selected fishing mortality for EMs fitted to data sets simulated with alternative process error structures: R and R+S (A), R+Sel (B), R+M (C), or R+q (D). Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

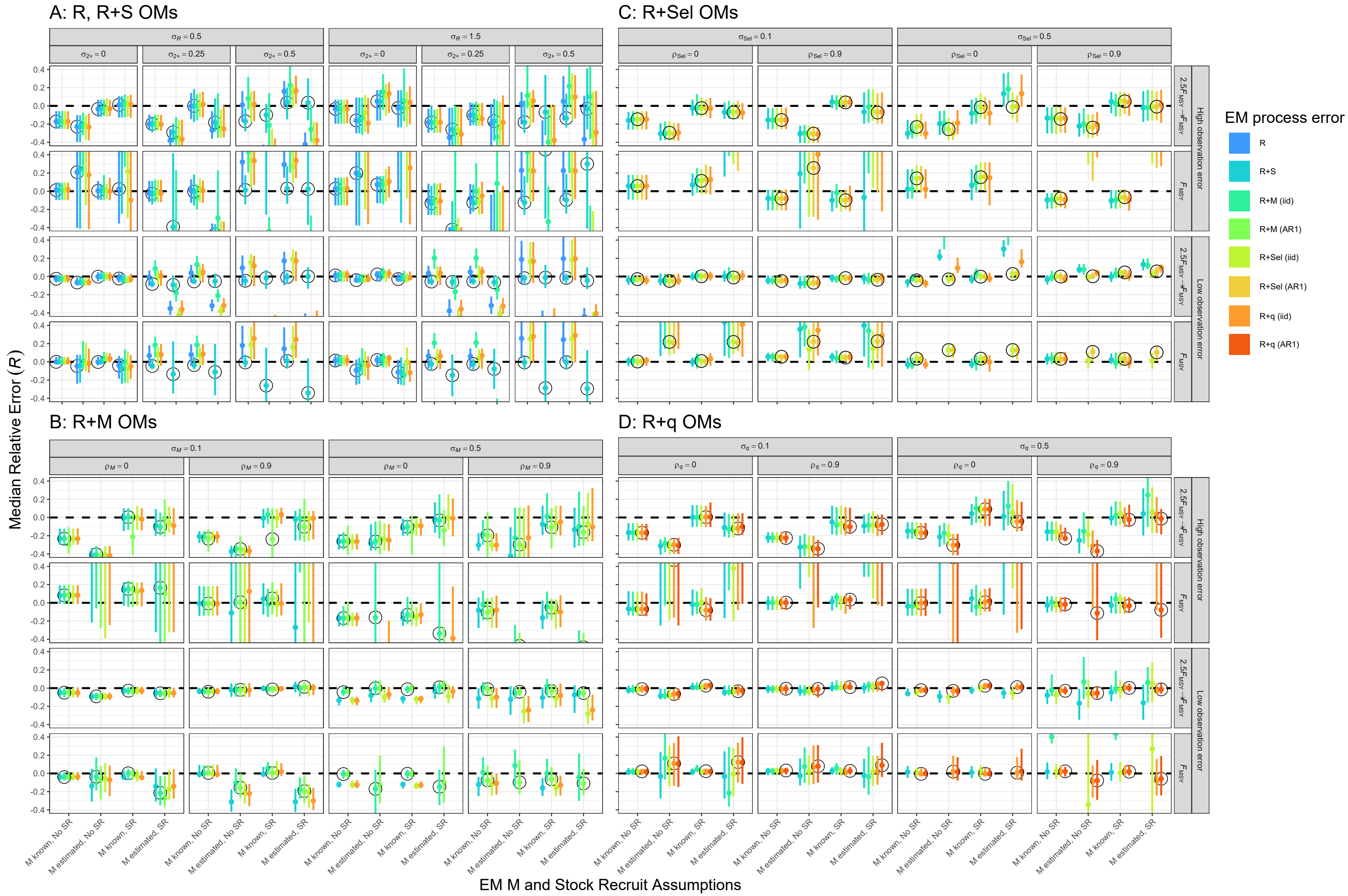

Fig. S12. Median relative error of terminal year recruitment for EMs fitted to data sets simulated with alternative process error structures: R and R+S (A), R+Sel (B), R+M (C), or R+q (D). Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

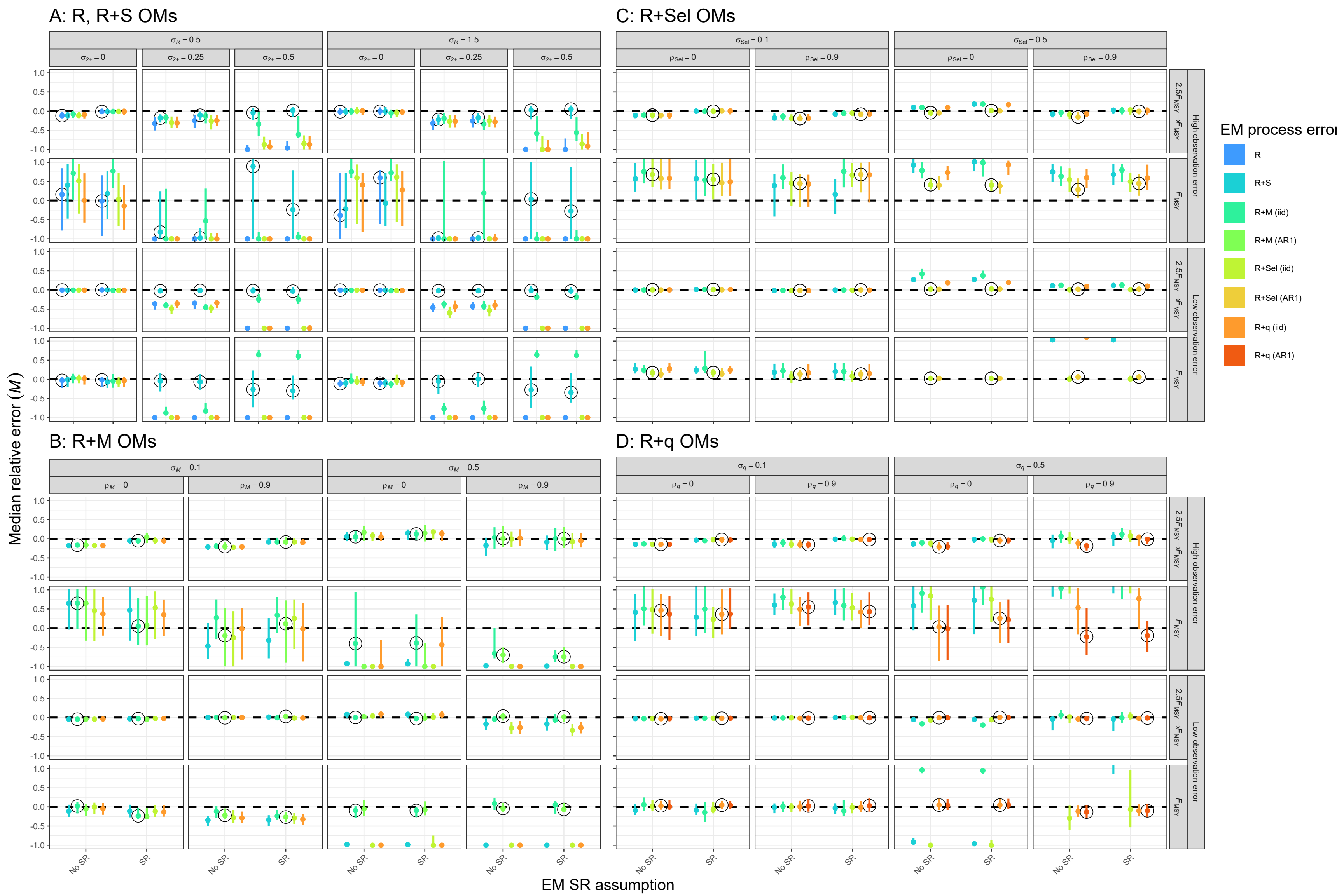

Fig. S13. Median relative error of median natural mortality for EMs fitted to data sets simulated with alternative process error structures: R and R+S (A), R+Sel (B), R+M (C), or R+q (D). Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

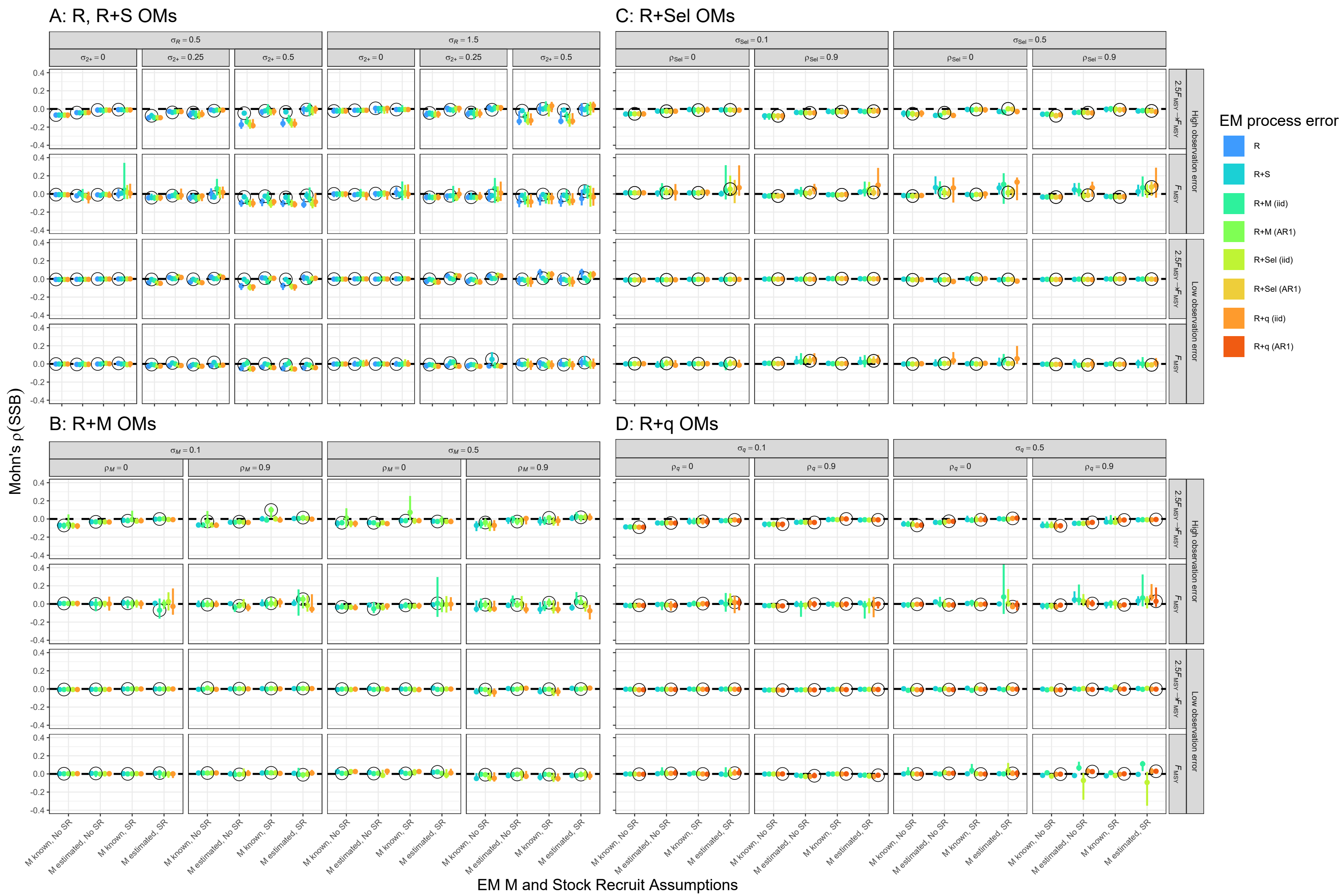

Fig. S14. Median Mohn's ρ for SSB for EMs fitted to data sets simulated with alternative process error structures: R and R+S (A), R+Sel (B), R+M (C), or R+q (D). Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

Fig. S15. Median Mohn's ρ of fishing mortality averaged over all age classes for EMs fitted to data sets simulated with alternative process error structures: R and R+S (A), R+Sel (B), R+M (C), or R+q (D). Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

Fig. S16. Median Mohn's ρ of recruitment for EMs fitted to data sets simulated with alternative process error structures: R and R+S (A), R+Sel (B), R+M (C), or R+q (D). Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

Table S9. For each OM process error type (columns), percent reduction in deviance for linear regression models fit to transformed Mohn's ρ values for each simulation (Eq. 3) for fishing mortality averaged over all age classes with each OM and EM factor (rows) included individually, combined, and with second and third order interactions.

Factor	R	R+S	R+M	R+Sel	R+q
EM M Assumption	0.06	0.09	0.01	0.12	0.01
EM SR assumption	0.01	<0.01	0.01	0.02	0.01
EM Process Error	0.03	0.07	0.02	0.06	0.03
OM Obs. Error	0.16	0.10	0.05	0.02	0.07
OM F History	0.07	0.02	0.03	0.24	0.03
OM σ_R	<0.01	0.01	–	–	–
OM σ_{2+}	–	0.09	–	–	–
OM σ_M	–	–	<0.01	–	–
OM ρ_M	–	–	<0.01	–	–
OM σ_{Sel}	–	–	–	0.01	–
OM ρ_{Sel}	–	–	–	<0.01	–
OM σ_q	–	–	–	–	<0.01
OM ρ_q	–	–	–	–	0.01
All factors	0.32	0.38	0.12	0.48	0.15
+ All Two Way	0.65	0.67	0.30	0.95	0.43
+ All Three Way	1.18	1.11	0.63	1.34	0.90

Table S10. For each OM process error type (columns), percent reduction in deviance for linear regression models fit to transformed Mohn's ρ values for each simulation (Eq. 3) for recruitment with each OM and EM factor (rows) included individually, combined, and with second and third order interactions.

Factor	R	R+S	R+M	R+Sel	R+q
EM M Assumption	0.86	0.56	0.16	1.00	1.27
EM SR assumption	<0.01	0.02	0.01	0.01	0.01
EM Process Error	0.01	0.59	0.18	0.07	0.04
OM Obs. Error	0.34	0.01	0.08	0.24	0.27
OM F History	0.91	0.22	0.06	1.20	1.67
OM σ_R	<0.01	0.14	–	–	–
OM σ_{2+}	–	0.11	–	–	–
OM σ_M	–	–	0.01	–	–
OM ρ_M	–	–	<0.01	–	–
OM σ_{Sel}	–	–	–	0.01	–
OM ρ_{Sel}	–	–	–	0.01	–
OM σ_q	–	–	–	–	0.01
OM ρ_q	–	–	–	–	0.01
All factors	2.28	1.74	0.51	2.66	3.51
+ All Two Way	4.20	2.74	1.08	5.08	6.51
+ All Three Way	4.83	3.79	1.79	6.03	7.82

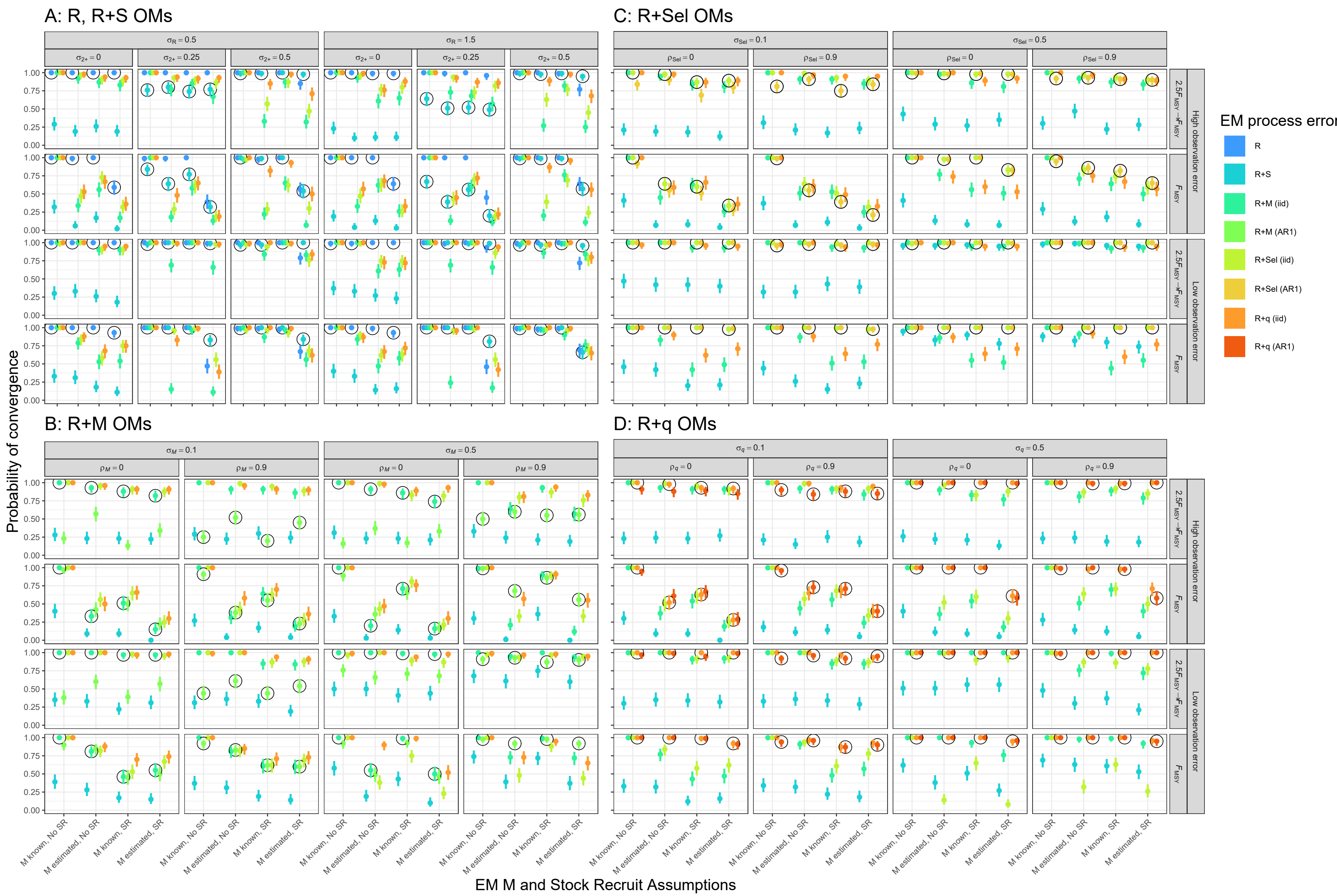

Fig. S17. Probability of EMs providing Hessian-based standard errors with alternative process error (colored points and lines), and median natural mortality (estimated or known) and Beverton-Holt SRR (estimated or not; along x-axis) assumptions when fitted to OMs that have R and R+S (A), R+Sel (B), R+M (C), or R+q (D) process error structures. Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

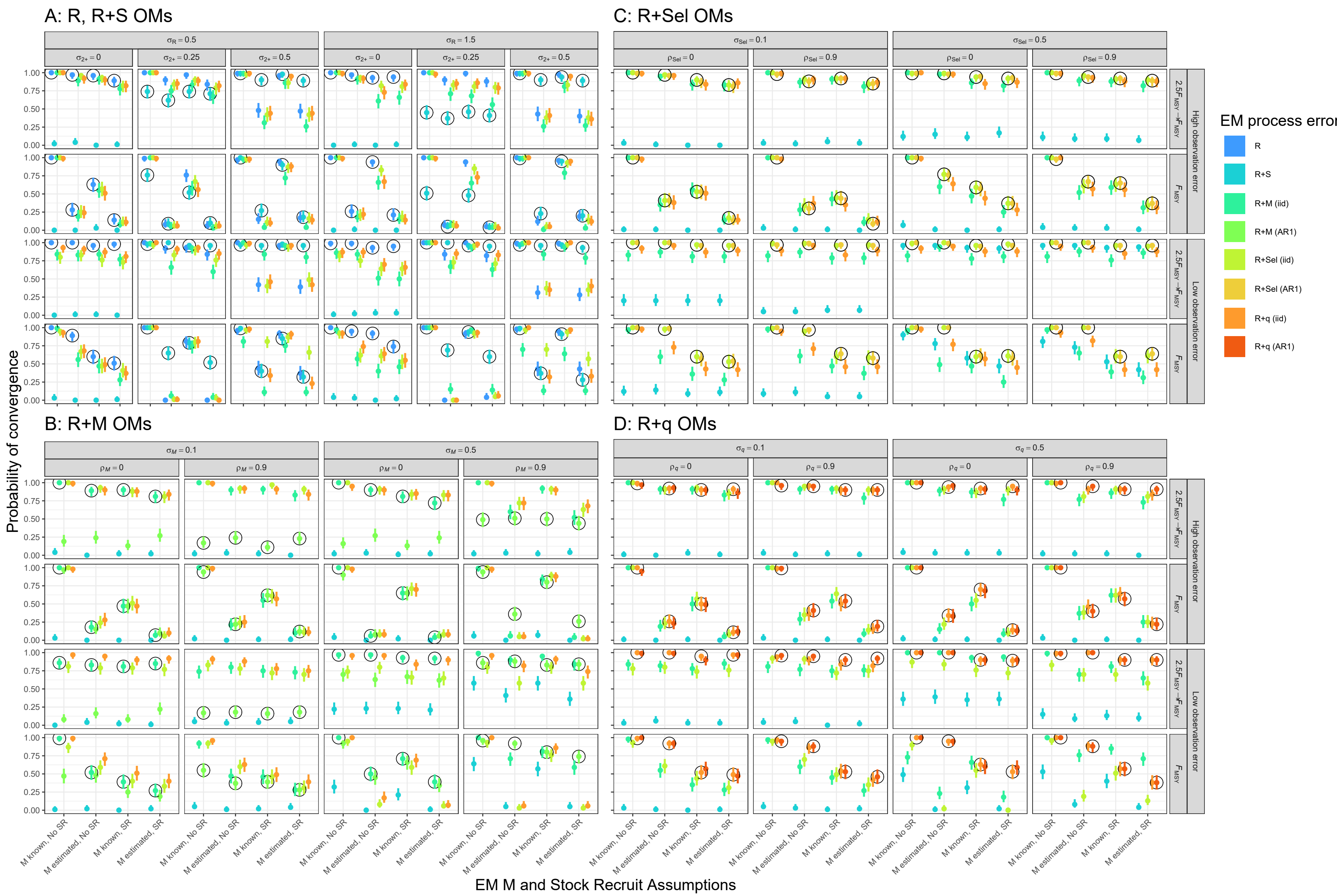

Fig. S18. Probability of EMs providing maximum absolute values of gradients less than 10^{-6} with alternative process error (colored points and lines), and median natural mortality (estimated or known) and Beverton-Holt SRR (estimated or not; along x-axis) assumptions when fitted to OMs that have R and R+S (A), R+Sel (B), R+M (C), or R+q (D) process error structures. Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

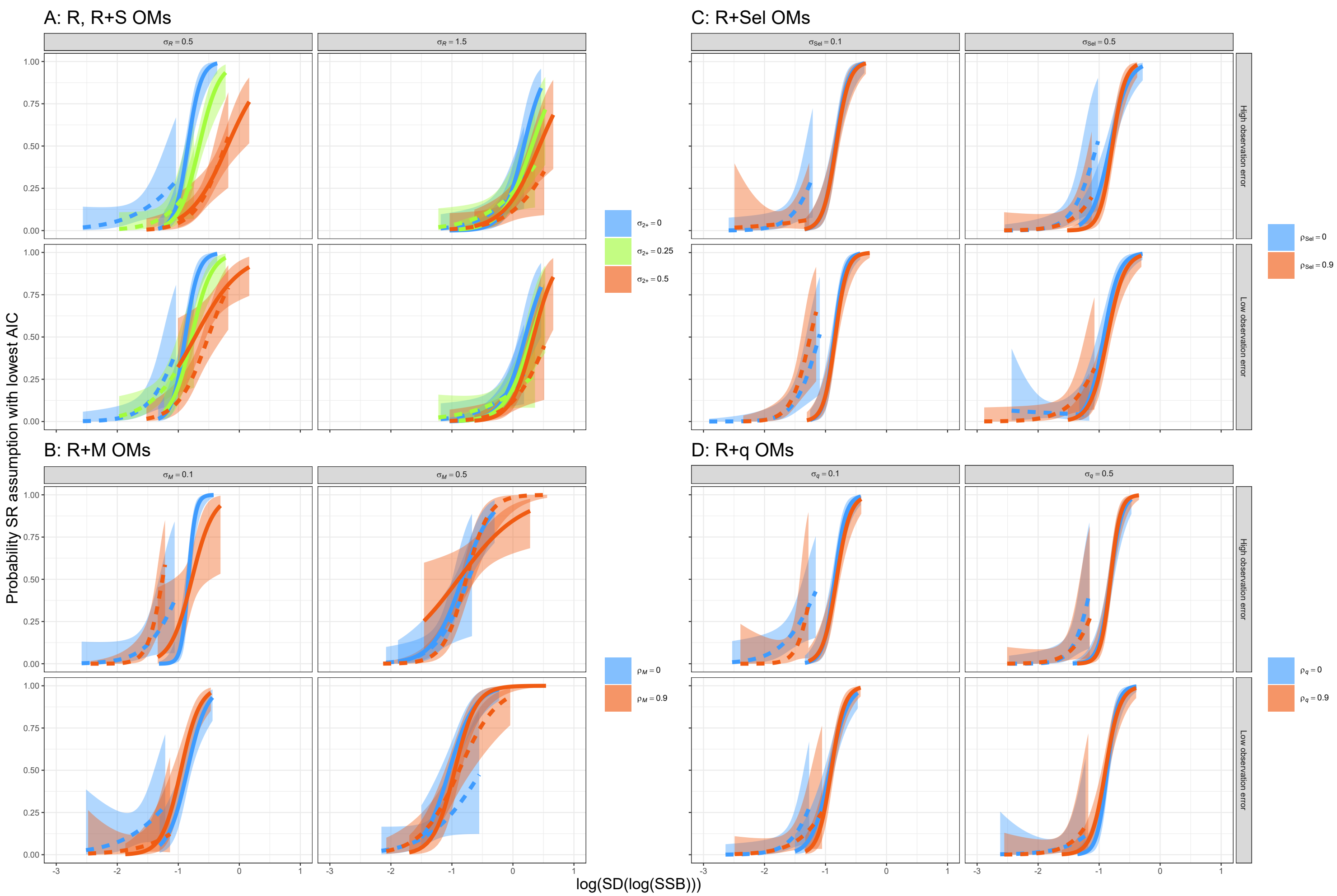

Fig. S19. Probability of lowest AIC from logistic regression on the log-standard deviation of the true log(SSB) in each simulation for EM with Beverton-Holt SRRs, rather than the otherwise equivalent EM without the SRR. Results are conditional on median M is known in the EM and alternative assumptions EMs having the correct process error structure: R and R+S (A), R+Sel (B), R+M (C), or R+q (D), and median M is assumed known in the EM. Solid and dashed lines are for OMs with and without temporal contrast in fishing pressure, respectively, and polygons represent 95% confidence intervals. Range of results indicates the range of log-standard deviation of log(SSB) for simulations of the particular OM.

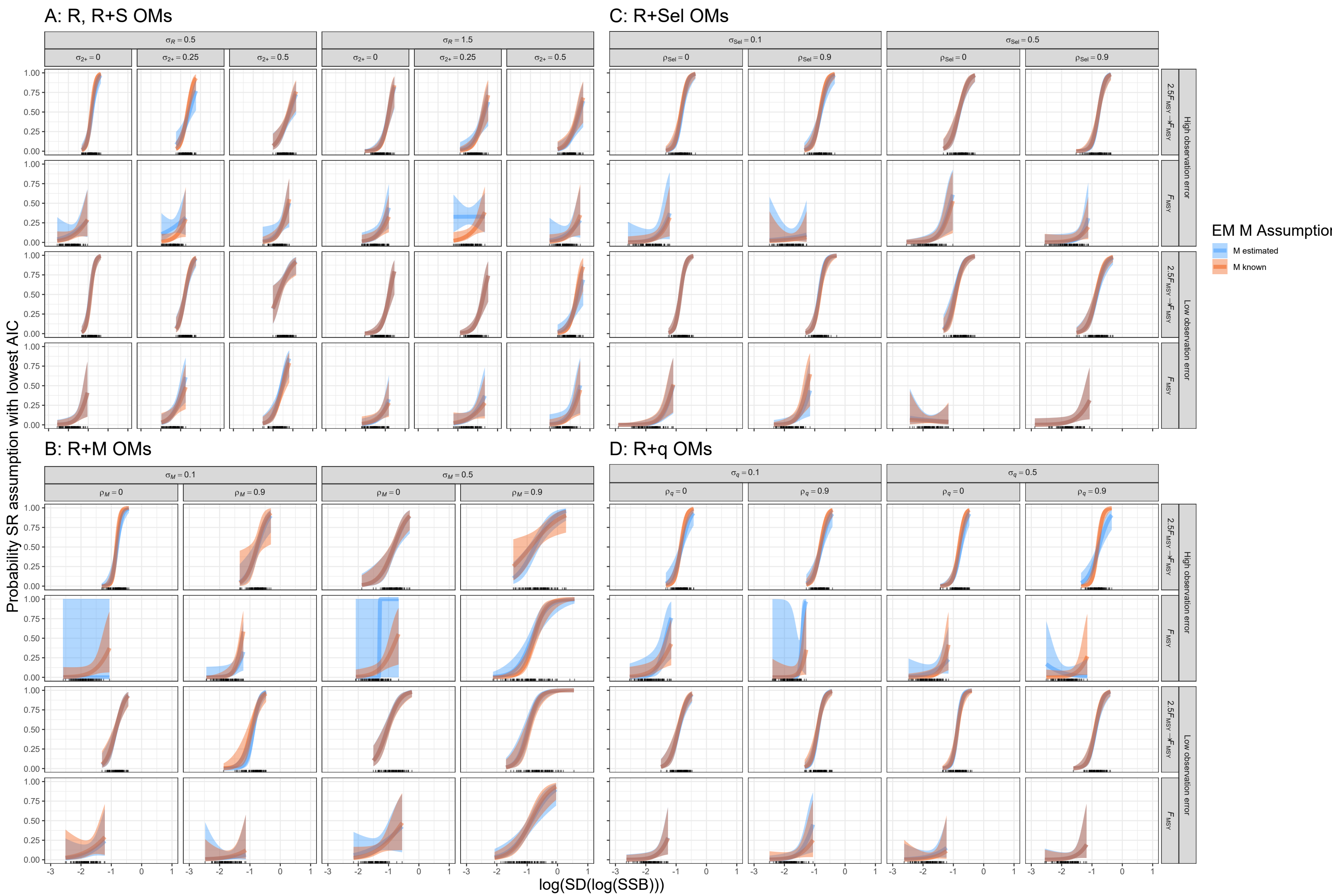

Fig. S20. Estimated probability of lowest AIC from logistic regression on the log-standard deviation of the true log(SSB) in each simulation for EM with Beverton-Holt SRRs, rather than the otherwise equivalent EM without the SRR. Results are conditional on alternative assumptions for median natural mortality (estimated or known) and on EMs having the correct process error structure: R and R+S (A), R+Sel (B), R+M (C), or R+q (D). Rug along x-axis denotes $SD(\log(SSB))$ values for each simulation and polygons represent 95% confidence intervals.