

<sup>1</sup> Factors affecting inferences on natural mortality and  
<sup>2</sup> associated environmental effects in state-space  
<sup>3</sup> age-structured assessment models

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19

20 **Abstract**

21 Treatment of natural mortality is a major consideration in assessment models and state-space  
22 approaches allow estimation of temporal variation in this mortality rate as well as effects of  
23 specific covariates. However, there has been no investigation of the reliability of inferences  
24 made regarding natural mortality, associated covariate effects, and important assessment  
25 output from state-space assessment models. We conducted a large-scale simulation study  
26 that considers models fit to data simulated from operating models with alternative assump-  
27 tions defined by several factors, but we focus on scenarios where there is temporal contrast  
28 in fishing pressure and lower uncertainty in population observations (age composition and  
29 indices of abundance). We fit estimating models to simulated observations with alternative  
30 assumptions on inclusion of environmental effects, estimation of the median natural mortal-  
31 ity rate, and the source of temporal variability in the population demography. Our results  
32 suggest that estimation of environmental effects on natural mortality is possible and reliable  
33 even when the population process error source was misspecified in some scenarios with lower  
34 uncertainty in covariate observations, and higher covariate temporal variability.

35 **keywords:** state-space assessment models, time-varying natural mortality, bias, AIC

## <sup>36</sup> Introduction

<sup>37</sup> State-space population models are now used widely for fisheries stock assessment in Europe,  
<sup>38</sup> the United States, and Canada (Nielsen and Berg, 2014; Cadigan, 2016; Pedersen and Berg,  
<sup>39</sup> 2017; Stock and Miller, 2021). Because application of these methods are considered best  
<sup>40</sup> practice and recommended for the next generation of stock assessment models (Hoyle et al.,  
<sup>41</sup> 2022; Punt, 2023), it is expected their use will only grow globally. An appeal of state-  
<sup>42</sup> space models lies in their separation of sources of biological and measurement variability by  
<sup>43</sup> treating latent population characteristics as statistical time series with periodic observations  
<sup>44</sup> measured with error. Through advances in computational capacity, we can use sophisticated  
<sup>45</sup> numerical approaches to estimate model parameters as mixed effects (Thorson and Minto,  
<sup>46</sup> 2015; Kristensen et al., 2016).

<sup>47</sup> State-space stock assessment models, with non-linear functions of latent processes and nu-  
<sup>48</sup> merous observation types with different probability distribution assumptions represent one  
<sup>49</sup> of most complex classes of state-space models. The literature on the effects of various factors  
<sup>50</sup> on reliability of inferences from state-space assessment models is growing (Li et al., 2024;  
<sup>51</sup> Miller et al., In reviewa). The importance of contrast in population size and fishing mortality  
<sup>52</sup> ( $F$ ) and quality of data used to fit assessment models including the state-space variety is  
<sup>53</sup> known (Magnusson and Hilborn, 2007; Miller et al., In reviewa). Furthermore, estimation  
<sup>54</sup> of natural mortality ( $M$ ), and even temporal variability in  $M$  is possible in many scenarios  
<sup>55</sup> (Lee et al., 2011; Cadigan, 2016; Miller and Hyun, 2018; Miller et al., In reviewa).

<sup>56</sup> The effects of temporal variation in recruitment via undefined or explicit environmental  
<sup>57</sup> factors have been extensively investigated in both traditional assessment models and state  
<sup>58</sup> space models (Myers, 1998; Haltuch and Punt, 2011; Johnson et al., 2016; Miller et al., 2016).  
<sup>59</sup> Reliability of estimating environmental and spawning biomass effects on recruitment in state-  
<sup>60</sup> space assessment models requires a combination of strong effects, good age composition data  
<sup>61</sup> quality, contrast in the environmental covariate and lower recruitment variability (Britten

62 et al., In review; Miller et al., In reviewa).

63 A critical aspect of fisheries assessment models and their use in management is short-term  
64 projections that are used to determine catch advice. While understanding drivers of re-  
65 cruitment is important particularly for subsequent effects on reference points, recruitment  
66 in short-term projections typically has little impact on the exploitable biomass in the first  
67 few projection years. However, assumptions for  $M$  have immediate and larger effects on  
68 projected biomass because they affect the abundances at older age classes at the end of the  
69 data time series that constitute spawning biomass and catch (Brodziak et al., 2008; Stock  
70 et al., 2021).

71 Because of the effects of  $M$  on both biological reference points and short term projections,  
72 better understanding sources of variation in  $M$  would provide more accurate estimation of  
73 abundance and productivity and therefore improved management. Temporal variation in  $M$   
74 is less studied than recruitment, but its importance for explaining variability in observations  
75 has been demonstrated in state-space assessment models for Atlantic cod and yellowtail  
76 flounder (Cadigan, 2016; Stock et al., 2021). Deriso et al. (2008) also demonstrated the  
77 importance of several factors affecting  $M$  for Pacific herring.

78 Assessment models could include temporal variation in many aspects of population dynamics  
79 or how observations are related to the population. For example the Woods Hole Assessment  
80 Model (WHAM) can include process errors treated as random effects for transition in cohorts  
81 over time (hereafter referred to as apparent survival), catchability for indices of abundance,  
82 selectivity of fishing fleets or indices, movement between regions, or in  $M$  (Stock and Miller,  
83 2021; Miller et al., In reviewb). However, misspecified temporal population process errors  
84 could lead to biased population and stock status estimation, and, therefore, poor fisheries  
85 management decisions (Legault and Palmer, 2016; Szuwalski et al., 2018). Studies of the  
86 reliability of inferences regarding the presence of temporal variability in  $M$  are limited. Miller  
87 et al. (In reviewa) found AIC could accurately distinguish process errors in apparent survival,

88 but not for those specifically due to  $M$  except when uncertainty in population observations  
89 (indices, catch and age composition) was low and there was greater temporal variation in  $M$ .  
90 In their simulation studies looking at models with multiple sources of process error, Li et al.  
91 (2024) found including more sources of process error than existed in the operating model  
92 was a better model-building approach than excluding them a priori.  
93 Here we conduct a simulation study with operating models (OMs) varying by degree of ob-  
94 servation error uncertainty, sources of process error, fishing history, temporal variation in  
95 environmental covariates, and magnitude of the effect of the covariate on  $M$ . The simu-  
96 lated observations from these OMs are fitted with estimating models that make alternative  
97 assumptions for sources of process error, and whether median  $M$  and covariate effects are  
98 estimated. We evaluate the effects of these factors on convergence of fitted models, whether  
99 Akaike's information criterion (AIC) can determine the correct source of process error and  
100 correct assumption about covariate effects on  $M$ , and the accuracy for estimators of relevant  
101 parameters and stock size and harvest rates derived from the assessment model.

## 102 Methods

103 Our analyses used the Woods Hole Assessment Model (WHAM) to construct both OMs and  
104 EMs (Miller and Stock, 2020; Stock and Miller, 2021; Miller et al., In reviewb). The WHAM  
105 package has been used extensively to configure OMs and EMs for several other simulation  
106 studies (Stock et al., 2021; Legault et al., 2023; Li et al., 2024; Britten et al., In review; Li  
107 et al., In reviewa) and is used to assess many commercially important stocks in the Northeast  
108 U.S. (e.g., NEFSC, 2022a,b, 2024). We used version 1.0.6.9000, commit 77bbd94 to generate  
109 all results.

110 We completed a simulation study with 288 operating models. The factors defining the con-  
111 figuration of each operating model, described in detail in subsequent sections, include source  
112 of population process error (3 levels), index and catch observation uncertainty (2 levels),

113 environmental covariate uncertainty (2 levels), temporal variation in the latent environmen-  
114 tal covariate (4 levels), and fishing history (2 levels). We simulated 100 data sets for each  
115 operating model that included simulations of process errors.

116 For each simulated data set we fit a set of 12 estimating models (EMs). The factors that  
117 distinguish the estimating models, also described in detail below, include source of population  
118 process error type (3 levels) whether (median)  $M$  was estimated or assumed known (2 levels),  
119 and whether the effect of the environmental covariate on  $M$  was estimated or not (2 levels).

120 The sources of population process error that were used in the OMs or assumed in the EMs  
121 were on recruitment only (R), recruitment and apparent survival (R+S), or recruitment  
122 and  $M$  (R+M). We did not use the log-normal bias-correction feature for process errors or  
123 observations described by Stock and Miller (2021) for OMs and EMs (Li et al., In reviewb).

124 Simulations were all carried out on the University of Massachusetts Green High-Performance  
125 Computing Cluster. Code for completing the simulations and summarizing results can be  
126 found at [https://github.com/timjmiller/SSRTWG/ecov\\_study/mortality](https://github.com/timjmiller/SSRTWG/ecov_study/mortality).

## 127 Operating models

### 128 Environmental covariate

129 In the WHAM model, environmental covariates are assumed to be described as state-space  
130 processes with annual observations of the true latent covariate (Miller et al., 2016; Stock  
131 and Miller, 2021). In our simulations, the latent covariate is assumed to be a stationary first  
132 order autoregressive (AR1) process

$$X_y | X_{y-1} \sim N \left( \mu_E (1 - \rho_E) + \rho_E X_{y-1}, (1 - \rho_E^2) \sigma_E^2 \right)$$

133 with marginal mean  $\mu_E = 0$  and variance  $\sigma_E^2$ . The four configurations of the latent environ-  
134 mental covariate in the operating models assume one of two values for the marginal standard

<sup>135</sup> deviation  $\sigma_E \in \{0.1, 0.5\}$  and for the autocorrelation parameter  $\rho_E \in \{0, 0.5\}$ .

<sup>136</sup> The observations of the latent environmental covariate are assumed to be unbiased and  
<sup>137</sup> Gaussian

$$x_y | X_y \sim N(X_y, \sigma_e^2)$$

<sup>138</sup> The standard deviation of the environmental observations in the operating models is one of  
<sup>139</sup> two values  $\sigma_e \in \{0.1, 0.5\}$ . Figure S2 provides example simulations of the latent covariate  
<sup>140</sup> and observations under the alternative configurations.

## <sup>141</sup> Population

<sup>142</sup> Many of the characteristics of the population biology and structure including the age classes  
<sup>143</sup> (10 age classes (ages 1 to 10+)), time span (40 years), maturity (Figure S1, top left), growth  
<sup>144</sup> (Figure S1, top right), time of spawning (1/4 of the year), and recruitment (Figure S1,  
<sup>145</sup> bottom right) are identical to Miller et al. (In reviewa). The maturity at age is a logistic  
<sup>146</sup> function with age at 50% maturity ( $a_{50} = 2.89$ ) and slope = 0.88 and weight at age is  
<sup>147</sup> derived from a von Beralanffy growth function where  $t_0 = 0$ ,  $L_\infty = 85$ , and  $k = 0.3$ , and a  
<sup>148</sup> length-weight relationship

$$W_a = \theta_1 L_a^{\theta_2}$$

<sup>149</sup> where  $\theta_1 = e^{-12.1}$  and  $\theta_2 = 3.2$ .

<sup>150</sup> The general model for  $M$  in year  $y$  is a log-linear function of both covariate effects  $X_y$  and  
<sup>151</sup> process errors  $\varepsilon_{M,y}$  and a parameter  $\beta_M$  that defines median  $M$

$$\log M_y = \beta_M + \beta_E X_y + \varepsilon_{M,y}$$

<sup>152</sup> where the process errors are modeled as random effects that may, in general, be autocorre-

153 lated normal random variables

$$\varepsilon_{M,y} | \varepsilon_{M,y-1} \sim N(\varepsilon_{M,y-1}, (1 - \rho_M^2) \sigma_M^2)$$

154 (Stock and Miller, 2021), but we assume  $\rho_M = 0$  in our R+M OMs. We assume the median  
155  $M$  rate  $e^{\beta_M} = 0.2$  is constant across ages. For R and R+S OMs and EMs,  $\varepsilon_{M,y} = 0$ . For  
156 all R+M OMs, we assume the same standard deviation  $\sigma_M = 0.3$ , which is estimated in  
157 the R+M EMs. The covariate effect is one of 3 alternative values in the operating models,  
158  $\beta_E \in \{0, 0.25, 0.5\}$ . The parameters defining the simulated covariate time series, size of the  
159 covariate effect, and any  $M$  random effects result in a range of different levels of variation  
160 in annual values (Figure S3).

161 We assumed expected recruitment each year is from a Beverton-Holt stock-recruit relation-  
162 ship (SRR)

$$R_y = \frac{aSSB_{y-1}}{1 + bSSB_{y-1}}.$$

163 All biological inputs to calculations of spawning biomass per recruit (i.e., weight, maturity,  
164 and  $M$  at age) are constant in the R and R+S OMs without covariate effects on  $M$ . There-  
165 fore, steepness and equilibrium unfished recruitment are also constant over the time period  
166 for those OMs (Miller and Brooks, 2021). As in Miller et al. (In reviewa), our assumed bio-  
167 logical inputs and selectivity (defined below) with constant  $M$  result in equilibrium  $F$  that  
168 reduces spawning biomass per recruit to 40% of the unfished level is  $F_{40\%} = 0.348$ . With  
169 an assumed unfished recruitment of  $R_0 = e^{10}$ , setting  $F_{MSY} = F_{40\%}$  results in a steepness of  
170 0.69 and  $a = 0.60$  and  $b = 2.4 \times 10^{-5}$ . For R+M OMs and all OMs with covariate effects on  
171  $M$ , steepness is not constant, but we used the same  $a$  and  $b$  parameters as other operating  
172 models which equates to a steepness and  $R_0$  at the median of the time series models for  $M$   
173 and the covariate.

174 We also used the same two fishing scenarios as Miller et al. (In reviewa) for OMs. In the first  
175 scenario, the stock experiences overfishing at  $2.5F_{MSY}$  for the first 20 years followed by fishing

176 at  $F_{\text{MSY}}$  for the last 20 years (denoted  $2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$ ). In the second scenario, the stock  
177 is fished at  $F_{\text{MSY}}$  for the entire time period (40 years). The magnitude of the overfishing  
178 assumptions is intended to reflect estimates of overfishing for Northeast US groundfish stocks  
179 from Wiedenmann et al. (2019).

180 We configured all R, R+S, and R+M OMs with uncorrelated random effects on recruitment  
181 with standard deviation on log(recruitment)  $\sigma_R = 0.5$ . This same assumption was used by  
182 Miller et al. (In reviewa) for R+M OMs and other OMs with fishery selectivity and index  
183 catchability process errors. For R+S OMs, apparent survival process errors were uncorrelated  
184 with  $\sigma_{2+} = 0.3$ .

## 185 Catch and index observations

186 We define the generation of observations of aggregate (total combined across ages) catch and  
187 indices, and corresponding age composition identical to Miller et al. (In reviewa). There is a  
188 single fleet operating year round for catch observations with logistic selectivity for the fleet  
189 with  $a_{50} = 5$  and slope = 1 (Figure S1, bottom left). Observations are generated for all 40  
190 years of the model. There are two index time series intended to represent fishery-independent  
191 surveys occurring in the spring (0.25 way through the year) and the fall (0.75 way through  
192 the year). Catchability of both surveys are assumed to be 0.1. We assumed catch and index  
193 age composition observations are generated from a logistic-normal distribution where errors  
194 on the multivariate normal scale are independent. The standard deviation parameter is also  
195 constant across ages.

196 Standard deviation for log-aggregate catch was 0.1. There were two levels of observation error  
197 variance for aggregate indices and age composition for both indices and fleet catch. A low  
198 uncertainty specification assumed standard deviation of both series of log-aggregate index  
199 observations was 0.1 and the standard deviation of the logistic-normal for age composition  
200 observations was 0.3. In the high uncertainty specification the standard deviation for log-

201 aggregate indices was 0.4 and that for the age composition observations was 1.5. For all  
202 estimating models, the standard deviation for log-aggregate observations was assumed known  
203 at the true value whereas that for the logistic-normal age composition observations was  
204 estimated.

205 **Estimating models**

206 Estimating models were fit to each of 100 simulated data sets from each operating model.  
207 There were three factors defining the configuration of each estimating model: 1) whether  $\beta_M$   
208 was estimated or assumed known, 2) whether an environmental effect  $\beta_E$  was estimated or  
209 not, and 3) whether the process errors were assumed on recruitment only (R), recruitment  
210 and survival (R+S), or recruitment and  $M$  (R+M).

211 The configuration of the process errors in the estimating models generally matched the  
212 corresponding options in the operating models. For example, uncorrelated R+S was assumed  
213 for both the estimating and operating model. However, R+M EMs did not assume  $M$  random  
214 effects were uncorrelated ( $\rho_M$  was estimated). The environmental covariate observations  
215 were included in all estimation models, whether effects on  $M$  were estimated or not, to  
216 ensure comparability of AIC. All fixed effects parameters for selectivity, catchability, fully-  
217 selected  $F$ , mean recruitment, initial abundance at age, and variances for logistic-normal  
218 age composition distributions were estimated. Any process error variance parameters for  
219 recruitment, survival, and  $M$  were also estimated. The observation error variance of the  
220 environmental observations and aggregate catch and indices were all assumed known at the  
221 true values.

222 **Performance measures**

223 **EM convergence**

224 We measured the frequency of convergence when fitting each EM to the simulated data  
225 sets for each OM. There are various ways to assess convergence of the fit (e.g., Carvalho  
226 et al., 2021), but we defined successful convergence as the Hessian of the marginal log-  
227 likelihood being invertible and providing variance estimates for the fixed effects parameters as  
228 recommended by Miller et al. (In reviewa). We calculated 95% confidence intervals (CIs) for  
229 probability of convergence using the Clopper-Pearson exact method (Clopper and Pearson,  
230 1934; Thulin, 2014).

231 **AIC for model selection**

232 We measured the frequency of correct model selection using marginal AIC. For a given  
233 operating model the set of models that were considered all made the same assumptions on  
234 whether or not to estimate  $\beta_M$  or it is assumed at the true value. For model  $m$ , the marginal  
235 AIC is a function of the marginal log-likelihood maximized with respect to the fixed effects  
236 in the model  $\boldsymbol{\theta}$  and the number of fixed effects  $n(\boldsymbol{\theta})$  estimated,

$$\text{AIC}_m = -2 [\text{argmax}_{\boldsymbol{\theta}} \log L_m(\boldsymbol{\theta}) - n(\boldsymbol{\theta})].$$

237 All model fits that successfully completed the optimization were used for this set of analyses.  
238 We used all of these fits because some lack of convergence would be expected for the correct  
239 behavior of more complicated models that include process errors that did not exist in the  
240 operating model. For example R+M EMs fit to R OMs would be expected to estimate no  
241 variance in the  $M$  random effects and the estimated variance parameter going to zero would  
242 cause poor convergence.

243 **Parameter estimation**

244 All results here use OM simulations with fits that satisfied the convergence criterion described  
245 above. We used this conditioning to reflect how practitioners would proceed in analyses of  
246 model fits with real assessment data. That is, practitioners would ensure models converged  
247 such that Hessian-based standard errors were available for all model parameter estimates.

248 We focused on statistical behavior of estimators of the covariate effect on  $M$  ( $\hat{\beta}_E$ ), the esti-  
249 mator of the median  $M$  parameter ( $\hat{\beta}_M$ ), and estimators of terminal year  $M$  ( $\hat{M}$ ), spawning  
250 stock biomass ( $\widehat{\text{SSB}}$ ), and fully-selected  $F$  ( $\hat{F}$ ). In preliminary analyses we examined results  
251 for the estimators of all the annual values for  $M$ , SSB and  $F$  over the whole time series,  
252 but we found no appreciable differences in patterns across the various factors defining the  
253 OMs and EMs. Furthermore, results for terminal year  $F$  were generally inversely related to  
254 those for spawning stock biomass, and are provided in the Supplementary Materials and not  
255 discussed further.

256 We calculated median errors (ME) of  $\hat{\beta}_E$  and  $\hat{\beta}_M$ , and the Hessian-based standard error  
257 estimators of these parameters ( $\widehat{SE}(\hat{\beta}_E)$  and  $\widehat{SE}(\hat{\beta}_M)$ ). We calculated the median relative  
258 errors (MRE) of terminal year  $\hat{M}$ ,  $\widehat{\text{SSB}}$ , and  $\hat{F}$ . We also calculated the root mean square  
259 error (RMSE) and estimated probability of coverage of constructed 95% CIs for  $\hat{\beta}_E$  and  $\hat{\beta}_M$   
260 for EMs that estimated these parameters. We constructed the CIs for probabilities of CI  
261 coverage using the same methods as those for probabilities of convergence.

262 The true values for terminal year SSB and  $F$  vary among simulations. For the  $i$ th simulated  
263 data the relative error for terminal year value  $\theta_i$  provided from the fitted estimation model  
264 is

$$\text{RE}_i(\theta) = \frac{\hat{\theta}_i - \theta_i}{\theta_i}$$

265 We calculated RMSE as

$$\text{RMSE}(\hat{\theta}) = \sqrt{\frac{1}{n} \sum_{i=1}^n (\hat{\theta}_i - \theta_i)^2}$$

266 For ME and MRE of estimators, we constructed 95% CIs using the binomial distribution  
267 approach as in Stock and Miller (2021) and Miller et al. (In reviewa). Because of the inverse  
268 relationship of the CI and the null hypothesis significance test, when the CI contains zero  
269 we do not have evidence of bias from the simulation study.

## 270 Results

271 Miller et al. (In reviewa) found inferences are most reliable for distinguishing process error  
272 sources and stock-recruit relationships in scenarios with lower observation error for indices  
273 and age composition data and with temporal contrast in fishing pressure. Our expectation is  
274 that inferences for covariate effects on  $M$  would require data to be at least that informative  
275 and inspection of the comprehensive set of results across all OM scenarios generally confirms this  
276 (See Supplemental Materials). Therefore, we restrict our attention to these OM scenarios  
277 with more informative data (temporal contrast in fishing and lower population observation  
278 error).

## 279 EM Convergence

280 In OM scenarios with contrast in fishing pressure and lower uncertainty in population observations  
281 (catch, indices and age composition), R EMs generally converged with high frequency except  
282 when EMs estimated covariate effects on  $M$  and the covariate uncertainty ( $\sigma_e$ ) was high  
283 (Figure 1). R+S EMs generally converged with low frequency when the OM process error  
284 did not match the EM configuration. R+M EMs generally did not converge with high  
285 frequency except for R+S OM scenarios with low covariate uncertainty and where the EMs assumed  
286 median  $M$  as known. There was generally little effect of the treatment of median  $M$  (known  
287 or estimated) on convergence for any of the other EMs. The largest impact of the size of the  
288 true covariate effect ( $\beta_E$ ) on convergence was observed for R and R+M EMs that estimated

289 covariate effects in R and R+M OMs with high uncertainty in covariate observations ( $\sigma_e$ ) and  
290 high temporal variability in the latent covariate ( $\sigma_E$ ). There was an increase in convergence  
291 frequency with increased temporal variability in the covariate for many EMs in OMs with  
292 large covariate uncertainty.

## 293 AIC performance

294 We only present results for EMs where the median  $M$  rate parameter was estimated because  
295 results differed little whether this parameter was estimated or assumed known (see Figures  
296 S7 to S9). When there was temporal contrast in fishing pressure and lower uncertainty in  
297 population observations, the only OM scenarios where AIC was consistently accurate for  
298 both covariate effect and process error configurations were R OMs with high variability in  
299 the covariate and low uncertainty in observations of the covariate (Figure 2). R+M OMs  
300 with those same covariate attributes were only accurate for the covariate effect (low Type I  
301 and II errors) and the incorrect process error almost always chosen was R rather than R+S.  
302 We also observed high accuracy for both covariate effect and process error configurations for  
303 R+S OMs where either no effect of the covariate or the strongest effect of the covariate was  
304 simulated. All R and R+S OMs were accurate for process error configuration and across  
305 all OMs, AIC accurately selects models without covariate effects when there are none (low  
306 Type I error). See Figures S7 to S9 for full results.

## 307 Covariate effect estimation

308 When there was temporal contrast in fishing pressure, lower uncertainty in population ob-  
309 servations, low uncertainty in environmental observations, and larger temporal contrast in  
310 the simulated true environmental covariate, we observed little or no bias in estimation of  
311 covariate effects ( $\hat{\beta}_E$ ) across all EM and OM process error assumptions, both EM configu-  
312 rations of median  $M$ , and all true covariate effect sizes (Figure 3, rows 3 and 4). The only

313 exception to this was when R EMs with median  $M$  estimated were fit to R+S OMs with  
314 those configurations. Decreasing trends in bias of  $\hat{\beta}_E$  for EMs fit to many configurations of  
315 R and R+M OMs indicate that  $\hat{\beta}_E$  values were closer to zero on average even when the true  
316 effect increased. For R+S OMs, EMs with the matching process error configuration showed  
317 little evidence of bias in estimating  $\beta_E$  across a range of covariate and covariate observation  
318 configurations. R+S and R+M EMs generally performed similarly when fitted to either R+S  
319 or R+M OMs except R+S OMs with high covariate uncertainty and low covariate variability  
320 (Figure 3, rows 5 and 6 of middle columns).

321 Bias of Hessian-based standard error estimation for  $\hat{\beta}_E$  was also close to zero in the same  
322 OM scenarios where bias in estimation of  $\beta_E$  itself was close to zero (Figure S13, rows 3 and  
323 4). However, in these high information OMs, bias of standard error estimation was observed  
324 for several EMs that did not have the matching R+S and R+M process error configuration.  
325 Estimation of standard errors was essentially unreliable for most EMs when OMs had higher  
326 uncertainty in covariate observations (Figure S13, rows 5 to 8).

327 Despite the reliable estimation of covariate effects and standard errors with low uncertainty  
328 in environmental observations, and larger temporal contrast in the simulated true environ-  
329 mental covariate, we found poor CI coverage for many EMs fitted to R+S and R+M OMs  
330 (Figure 4, rows 3 and 4). For example, R+M EMs fit to R+M OMs showed little bias in  
331 estimation of the covariate effect and standard errors, but CI coverage was negatively biased  
332 (rows 3 and 4 in right two columns of Figures 3, 4, and S13). However, there appears to be  
333 a positive correlation of the covariate and standard error estimates such that estimates less  
334 than the true value had negatively biased standard error estimates which would make CIs  
335 too small (Figure S24). CI coverage was most reliable for R+S OMs when the EMs had the  
336 matching process error configuration (Figure 4, middle 2 columns).

<sup>337</sup> **Median natural mortality rate estimation**

<sup>338</sup> We observed negligible bias in estimating the median  $M$  parameter ( $\beta_M$ ) over a much wider  
<sup>339</sup> set of operating models than that for estimating the covariate effect on  $M$ . When there is  
<sup>340</sup> temporal contrast in fishing pressure, lower uncertainty in population observations, all EMs  
<sup>341</sup> fit to R and R+M OMs showed little evidence of bias for  $\hat{\beta}_M$  (Figure 5, left and right sets of  
<sup>342</sup> columns). For R+S OMs with those conditions, only EMs with the matching process error  
<sup>343</sup> assumption showed little evidence of bias.

<sup>344</sup> We also found little or no bias of Hessian-based standard error estimation for  $\beta_M$  in many  
<sup>345</sup> of the same OM scenarios where bias of  $\hat{\beta}_M$  was close to zero except for some EMs with the  
<sup>346</sup> process error source mis-specified or where the OM simulated the strongest covariate effect  
<sup>347</sup> (Figure S28). Like  $\hat{\beta}_M$ , reliability of standard error estimation in R+S OMs required the  
<sup>348</sup> EMs to have the matching process error assumption.

<sup>349</sup> we found CI coverage to be unreliable for many EM-OM combinations even for OM and  
<sup>350</sup> EM combinations where bias in  $\hat{\beta}_M$  and its standard error estimators was negligible, much  
<sup>351</sup> like that for  $\hat{\beta}_E$  (Figure 6). In the same EM OM combination we investigated above for  $\hat{\beta}_E$ ,  
<sup>352</sup> we observed an opposite negative correlation of  $\hat{\beta}_M$  and its standard error estimates (Figure  
<sup>353</sup> S35), which would result in CIs being too narrow when  $\hat{\beta}_M$  values are larger than average.  
<sup>354</sup> The exceptions to the general poor CI coverage were for all EMs fit to several R OMs with  
<sup>355</sup> low covariate uncertainty and covariate variability (Figure 6, first two columns) and for R+S  
<sup>356</sup> EMs in many R+S OMs (Figure 6, middle two columns) .

<sup>357</sup> **Terminal year natural morality rate**

<sup>358</sup> The EMs with either R or R+S process error configurations with median  $M$  assumed known  
<sup>359</sup> and covariate effects not estimated will have terminal year  $M$  correctly specified when fitted  
<sup>360</sup> to either R or R+S OMs with no covariate effect because  $M$  is constant in both the EM and  
<sup>361</sup> OM at the same value. We exclude those OM-EM combinations from our results here.

362 In many of the OMs with temporal contrast in fishing pressure and lower uncertainty in  
363 population observations, there was little difference in bias for terminal year  $M$  among EMs  
364 whether the median  $M$  parameter ( $\beta_M$ ) was estimated (Figure 7). However, when there were  
365 differences, bias in terminal year  $M$  was closer to zero when the EM assumed  $\beta_M$  known, as  
366 would be expected. For R OMs, all estimating models exhibited little evidence of bias except  
367 when OMs simulated the largest covariate effect ( $\beta_E = 0.5$ ). However, bias estimates were at  
368 worst around 10% (Figure 7, left columns). For R+S OMs, the R+S EMs generally exhibited  
369 the least evidence for bias and R+M EMs generally estimated terminal  $M$  lower than the  
370 true value, particularly when observation uncertainty in the covariate was lower. For R+M  
371 OMs, many of the EM configurations provided little bias in terminal  $\widehat{M}$ , but the biases were  
372 generally further from zero than the results for the R OMs (Figure 7, right columns).

373 Accuracy (lower RMSE) of terminal year  $\widehat{M}$  was similar for all of the EMs when fitted to  
374 R+M OMs (Figure S43). For R+S OMs, accuracy was generally better for R+S EM than  
375 other process error assumptions, particularly when  $\beta_E$  was also estimated. Accuracy for  
376 R+S EMs was also generally as good as other EM process error assumptions even when the  
377 OM process error type was different. As would be expected, the accuracy of terminal year  
378  $\widehat{M}$  where median  $M$  was assumed known was better than when it was estimated, when there  
379 were differences.

## 380 Terminal year spawning stock biomass

381 In OMs with temporal contrast in fishing pressure and lower uncertainty in population  
382 observations, we found little bias in estimates of terminal year SSB for all EM configurations  
383 when fitted to R or R+M OMs (Figure 8). We also found little or no evidence of bias for  
384 R+S OMs when the EMs assumed the correct process error configuration. We found the  
385 worst bias in terminal year SSB estimation for R EMs with  $\beta_M$  estimated when fitted to  
386 R+S OMs, and R+M EMs also had more bias than R+S EMs in these OMs. Like the bias

387 results, accuracy, as measured by RMSE, was similar for many of EMs when fit to R and  
388 R+M OMs. Accuracy was generally worse (higher RMSE) when EMs were fit to R+S OMs,  
389 and the worst accuracy occurred when the EM process error assumption did not match that  
390 of the OM (Figure S50).

## 391 Discussion

392 Our simulation study demonstrated that estimation of environmental effects on  $M$  is possible  
393 and reliable in certain scenarios even when the process error was misspecified (e.g., R and  
394 R+S EMs fit to R+M OM). In many of these same OMs, frequency of convergence of fitted  
395 models did not appear to suffer when covariate effects on  $M$  were estimated even when there  
396 was no effect simulated. However, these scenarios are information rich in that there was  
397 contrast in population size (due to changes in fishing pressure) and the covariate affecting  
398 the population, and there was low uncertainty in population and covariate observations.  
399 Previous research has shown that estimation of a constant  $M$  parameter requires contrast  
400 in time series and informative data (Lee et al., 2011), so it is no surprise that estimation  
401 of these effects also requires relatively good information via more precise observations and  
402 higher contrast in the covariate time series.

403 Even though estimation of covariate effects was unbiased in many scenarios, AIC could only  
404 reliably detect covariate effects for R and R+M OMs with contrast in covariates and low  
405 covariate uncertainty. In those scenarios where the covariate effect could be detected, CI  
406 coverage for the covariate effect was often biased even when there was little or no bias in  
407 the estimators of the effect and its standard error. Cadigan et al. (2024) found CI coverage  
408 to be biased for SSB and  $F$  estimation in a state-space model in some scenarios, but they  
409 attributed the poor coverage to bias in Hessian-based standard error estimation, and their  
410 simulations held any process error random effects constant. The coverage bias we observed,  
411 at least for some OM-EM combinations, may be related to correlation of the estimators

<sup>412</sup> of the covariate effect and the corresponding standard error and therefore consideration of  
<sup>413</sup> other methods of calculating CIs may be warranted (e.g., those based on profile likelihood  
<sup>414</sup> and/or Monte Carlo sampling of the log-likelihood surface).

<sup>415</sup> Miller et al. (In reviewa) investigated R+M OMs with two levels of  $M$  process error variability  
<sup>416</sup> ( $\sigma_M \in \{0.1, 0.5\}$ ) and only found AIC able to accurately distinguish R+M process errors with  
<sup>417</sup> the higher level of process error variability ( $\sigma_M = 0.5$ ). We assumed  $\sigma_M = 0.3$ , intermediate  
<sup>418</sup> to the values investigated by Miller et al. (In reviewa), and found process error inferences  
<sup>419</sup> unreliable for the source of process error, indicating that the level of variability required for  
<sup>420</sup> detecting  $M$  process errors must be greater than  $\sigma_M = 0.3$ , but may still be less than 0.5. In  
<sup>421</sup> our results and those by Miller et al. (In reviewa), AIC typically chose R EMs which would  
<sup>422</sup> indicate the fitted R+M EMs would estimate no variability in the  $M$  process errors. Future  
<sup>423</sup> studies like ours where R+M OMs are simulated with greater variability in  $M$  process errors  
<sup>424</sup> would better inform reliability of covariate effect inferences under such scenarios.

<sup>425</sup> Deriso et al. (2008) attempted to estimate process errors as well as covariate effects with  
<sup>426</sup>  $M$  for Pacific herring, but similarly found no variability in  $M$ , suggesting there was too  
<sup>427</sup> much uncertainty in the available observations relative to the true temporal variability in  
<sup>428</sup>  $M$ . Given that we found covariate effect inferences using R EMs was reliable in R+M OMs  
<sup>429</sup> with apparently little variability in  $M$  process errors, the findings of Deriso et al. (2008) on  
<sup>430</sup> covariate effects for Pacific herring would presumably also be robust to true low variability in  
<sup>431</sup>  $M$ . However, they did not account for uncertainty in covariate observations, some of which  
<sup>432</sup> would probably have substantial uncertainty (e.g., competition and predation covariates).  
<sup>433</sup> We found higher covariate observation uncertainty to cause true covariate effects to be less  
<sup>434</sup> detectable using AIC, but we did not investigate the implications for incorrectly assuming  
<sup>435</sup> no covariate uncertainty for covariate inferences.

<sup>436</sup> Any bias or poor accuracy for annual SSB estimation was primarily a function of whether or  
<sup>437</sup> not the median  $M$  parameter was estimated or known and the types of process errors, rather

438 than the treatment of the covariate effects on  $M$ . For example, we found estimation of SSB  
439 was better when the EM had the process error correctly specified for R+S OMs. Fortunately,  
440 our results and those by Miller et al. (In reviewa) demonstrate that marginal AIC seems to  
441 be a good tool for determining whether this source of process error should be included in the  
442 model. However, the reliability of the estimation of SSB does break down in the less ideal  
443 scenarios when there is higher population observation error, and lack of contrast in fishing  
444 pressure (e.g., Figures S47 to S49).

445 The R+S and R+M EMs both include process error for the survival of cohorts and would  
446 be expected to perform similarly, and they did in our simulations when the OM and EM  
447 matched. However, we found that the biases using R+M EMs for R+S OMs were generally  
448 worse than using R+S EMs for R+M OMs. Additionally, there are implications for biological  
449 reference points for R+M EMs (e.g., Legault and Palmer, 2016) that are not present with  
450 R+S EMs. So, we recommend following the suggestion from Li et al. (2024) to prefer R+S  
451 EMs over R+M OMs unless strong biological evidence is present to support a particular R+M  
452 OM. Such support could be found through both biological understanding (e.g., Trijoulet  
453 et al., 2020) as well as statistical properties such as a large delta AIC for R+M compared  
454 to R+S associated with greater temporal variability in natural mortality (Miller et al., In  
455 reviewa).

456 The ability to accurately infer covariate effects on  $M$  in some realistic situations indicates that  
457 such investigations may be fruitful. Ability to make inferences could improve further when  
458 WHAM is extended to incorporate tagging data (Miller et al., In reviewb). Tagging data can  
459 greatly inform natural mortality estimation (Pollock et al., 1991; Hampton, 2000), and this  
460 impact on  $M$  estimation should also apply to estimation of covariate effects or unexplained  
461 temporal variation in  $M$ . Given our findings and planned future WHAM development, we  
462 expect investigations of and accounting for covariate effects on  $M$  to become more common  
463 within the fisheries stock assessment process. At the same time, it will be equally important  
464 to conduct research that will improve our understanding of how best to measure the depletion

<sup>465</sup> of stocks and determine catch advice for these stocks with covariate effects on  $M$ .

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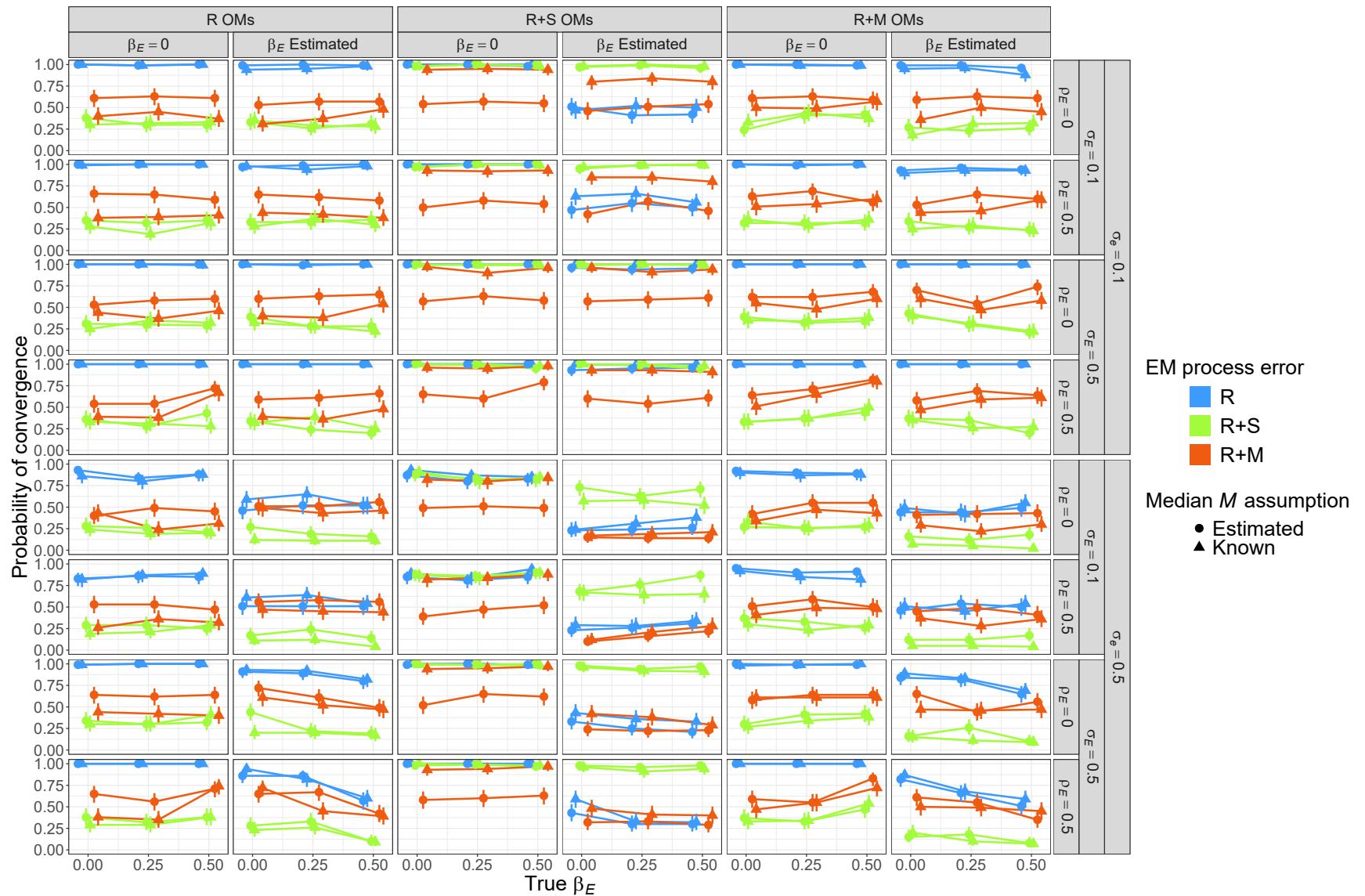


Fig. 1. Estimated probability of fits providing Hessian-based standard errors for EMs with alternative process error assumptions, treatment of median natural mortality ( $\beta_M$  known or estimated), and treatment of covariate effect ( $\beta_E = 0$  or estimated). The OMs have R (left) and R+S (middle), or R+M (right) process error structures, alternative configurations of covariate time series structure and levels of observation uncertainty (rows), and three levels of true covariate effect on median natural mortality (x axis). All OMs had low observation error for fish population observations and temporal contrast in fishing pressure. Vertical lines represent 95% confidence intervals.

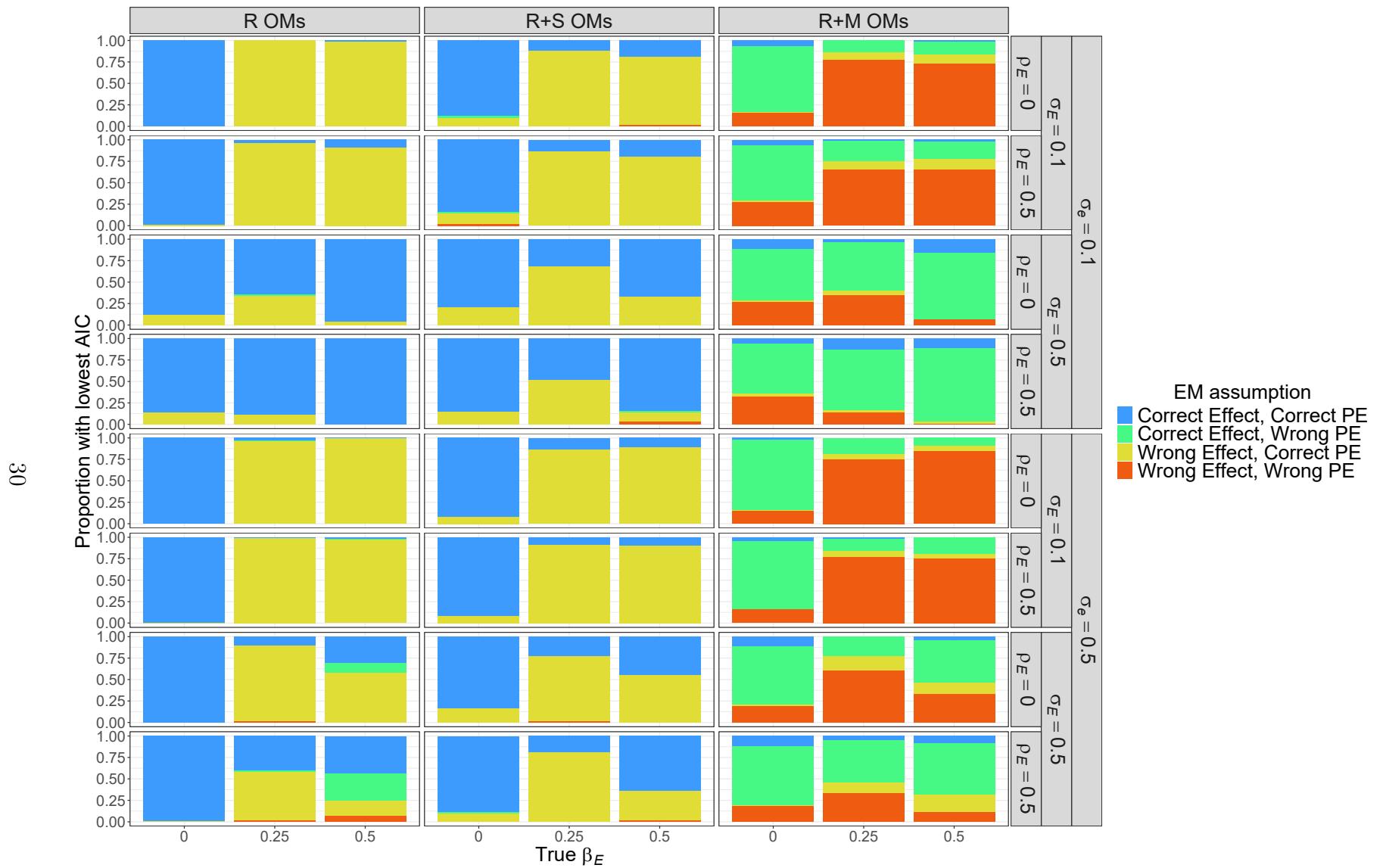


Fig. 2. For each OM, the proportion of simulated data sets where the EM type (treatment of environmental covariate effect and assumed process error type) had the lowest AIC. All OMs had low observation error for fish population observations and temporal contrast in fishing pressure. All EMs estimated median natural mortality rate.

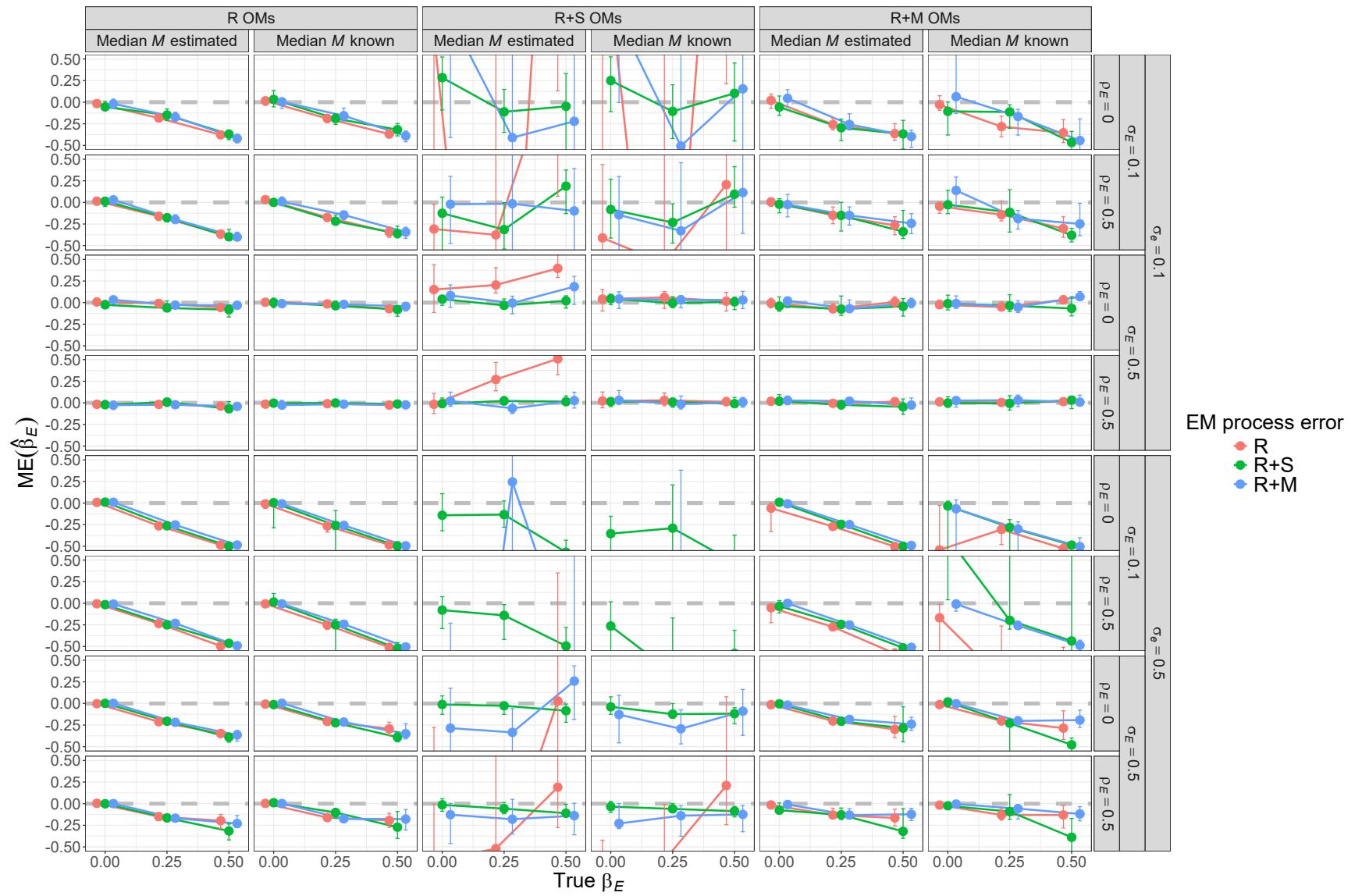


Fig. 3. Median error (ME) of estimates of environmental effect on natural mortality  $\beta_E$  from fitting EMs with alternative process error assumptions and treatment of median natural mortality ( $\beta_M$  known or estimated). All OMs had low observation error and contrast in fishing mortality. Vertical lines represent 95% confidence intervals.

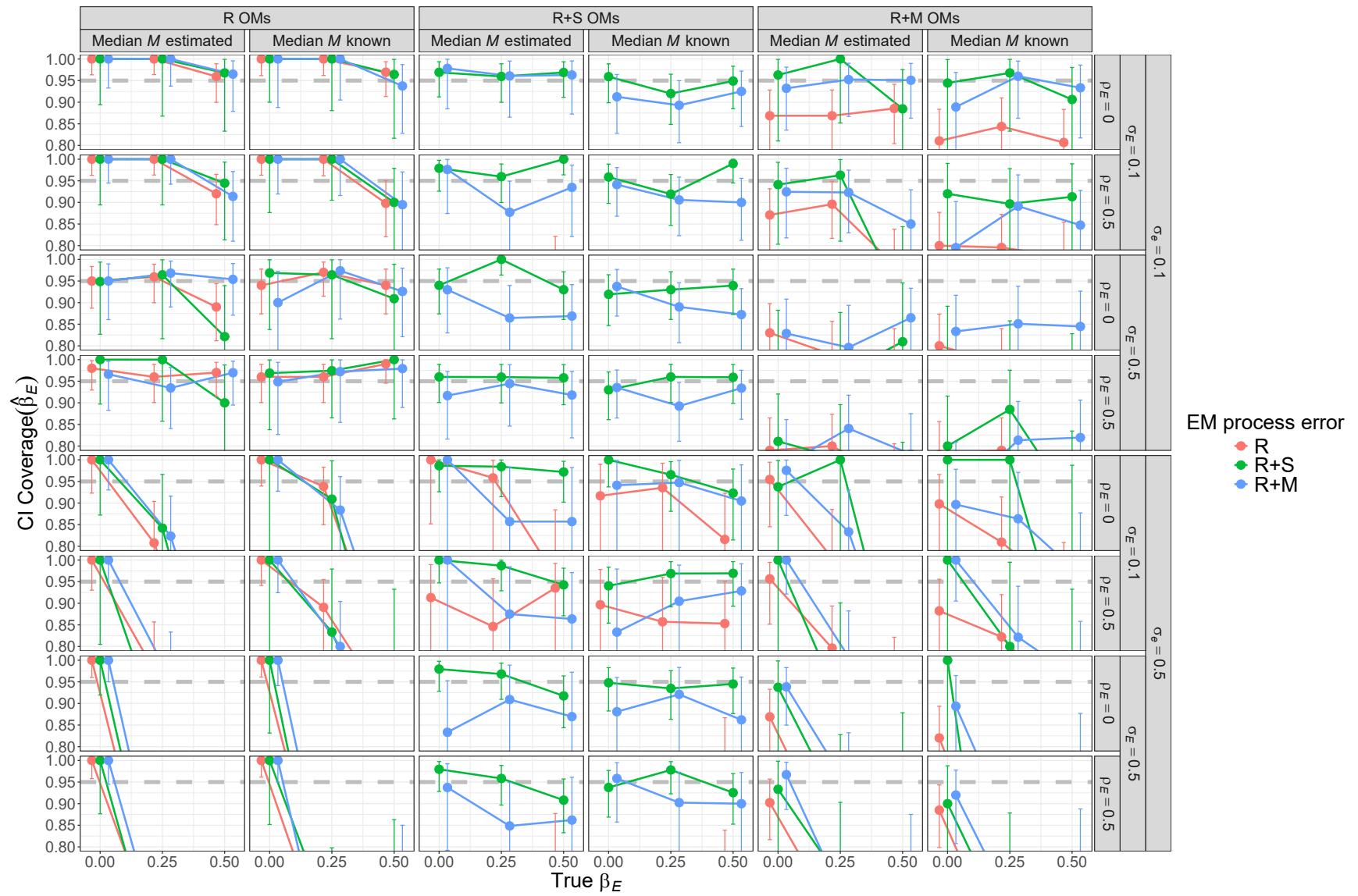


Fig. 4. Probability of 95% confidence interval for  $\beta_E$  containing the true value for EMs with alternative process error assumptions and treatment of median natural mortality ( $\beta_M$  known or estimated). All OMs had low observation error and contrast in fishing mortality. Vertical lines represent 95% confidence intervals.

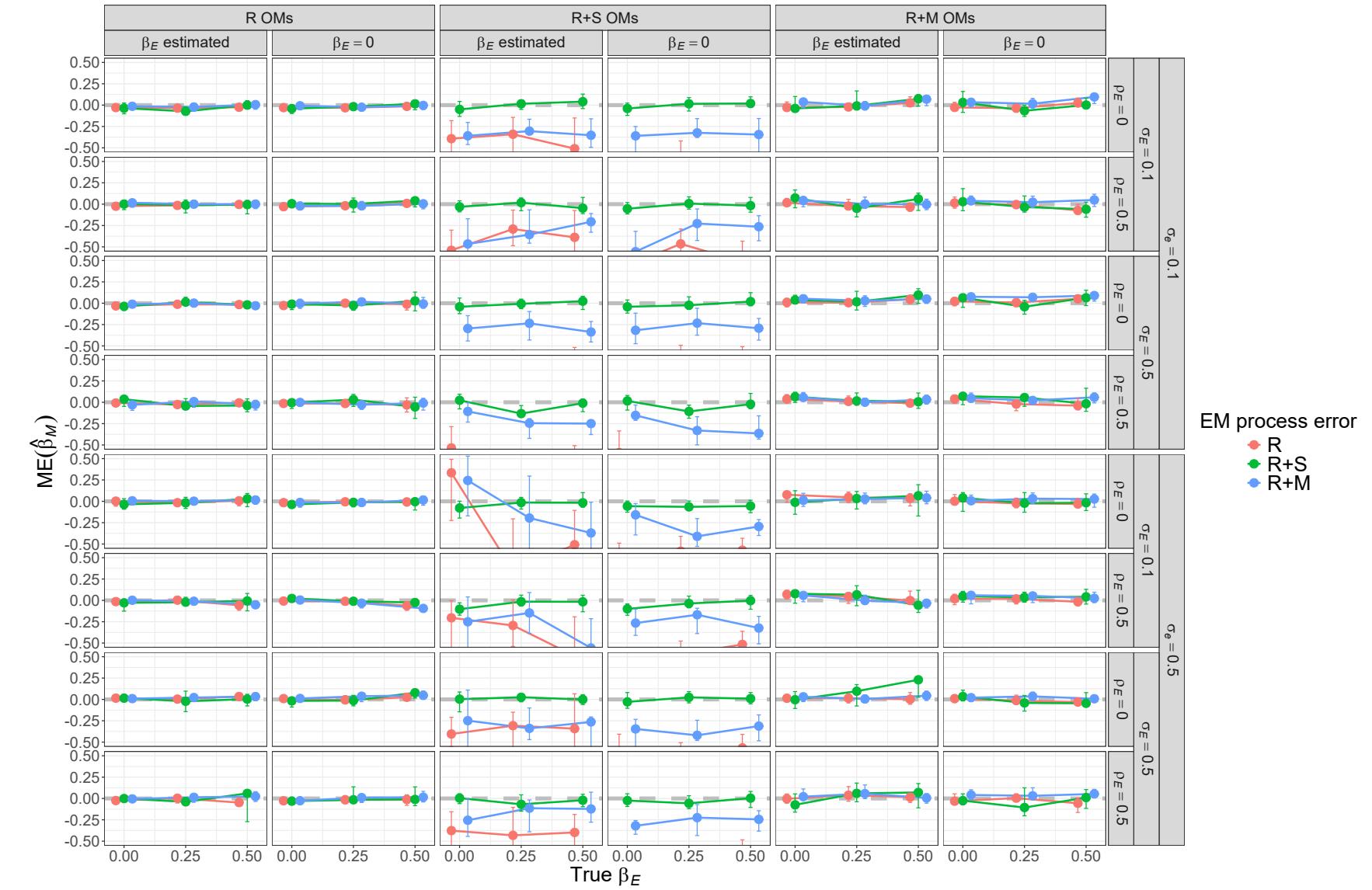


Fig. 5. Median error (ME) of estimates of  $\beta_M$  from fitting EMs with alternative process error assumptions and treatment of covariate effect ( $\beta_E = 0$  or estimated). All OMs had low observation error and contrast in fishing mortality. Vertical lines represent 95% confidence intervals.

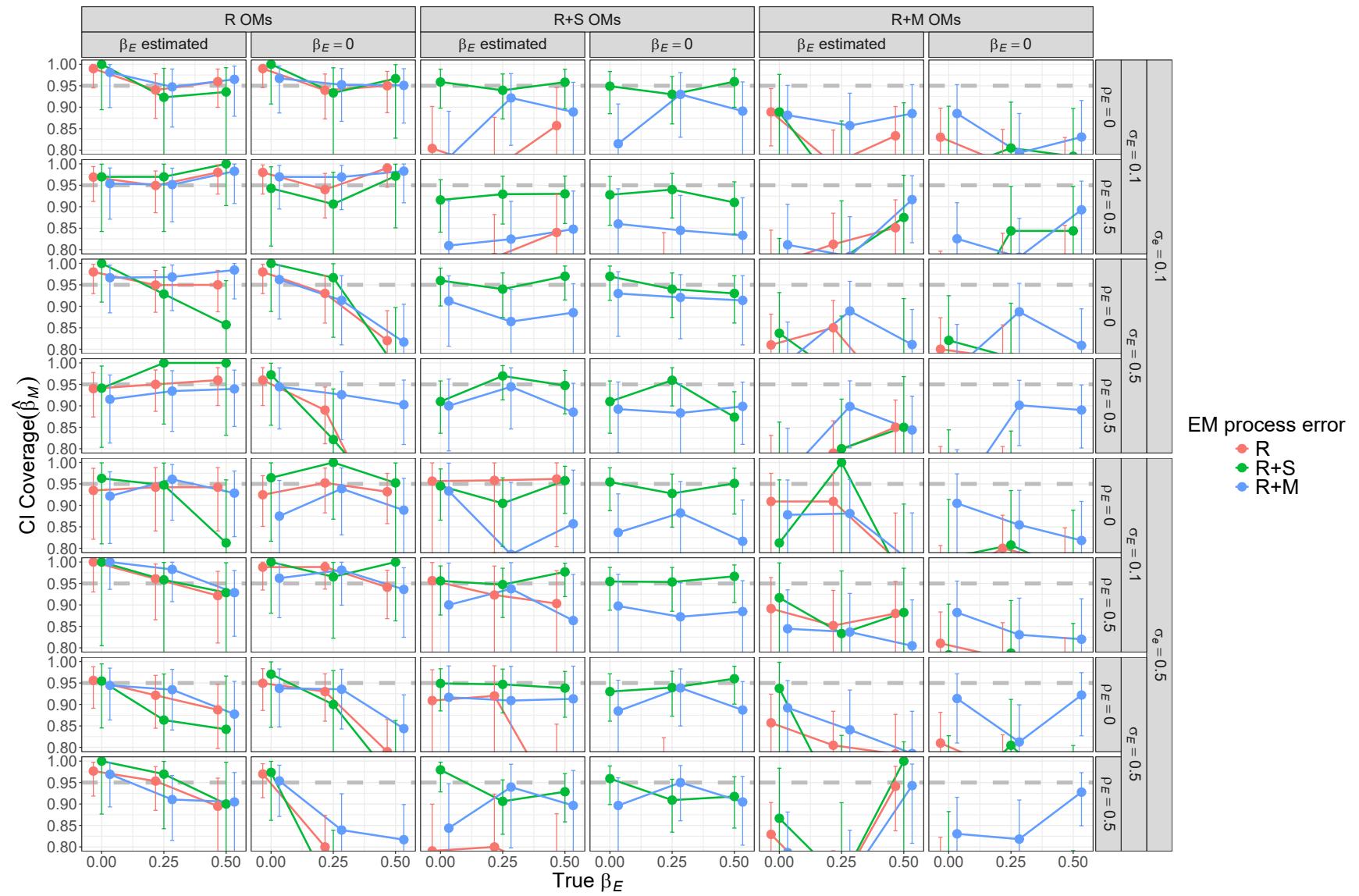


Fig. 6. Probability of 95% confidence interval for  $\beta_M$  containing the true value for EMs with alternative process error assumptions and treatment of covariate effect ( $\beta_E = 0$  or estimated). All OMs had low observation error and contrast in fishing mortality. Vertical lines represent 95% confidence intervals.

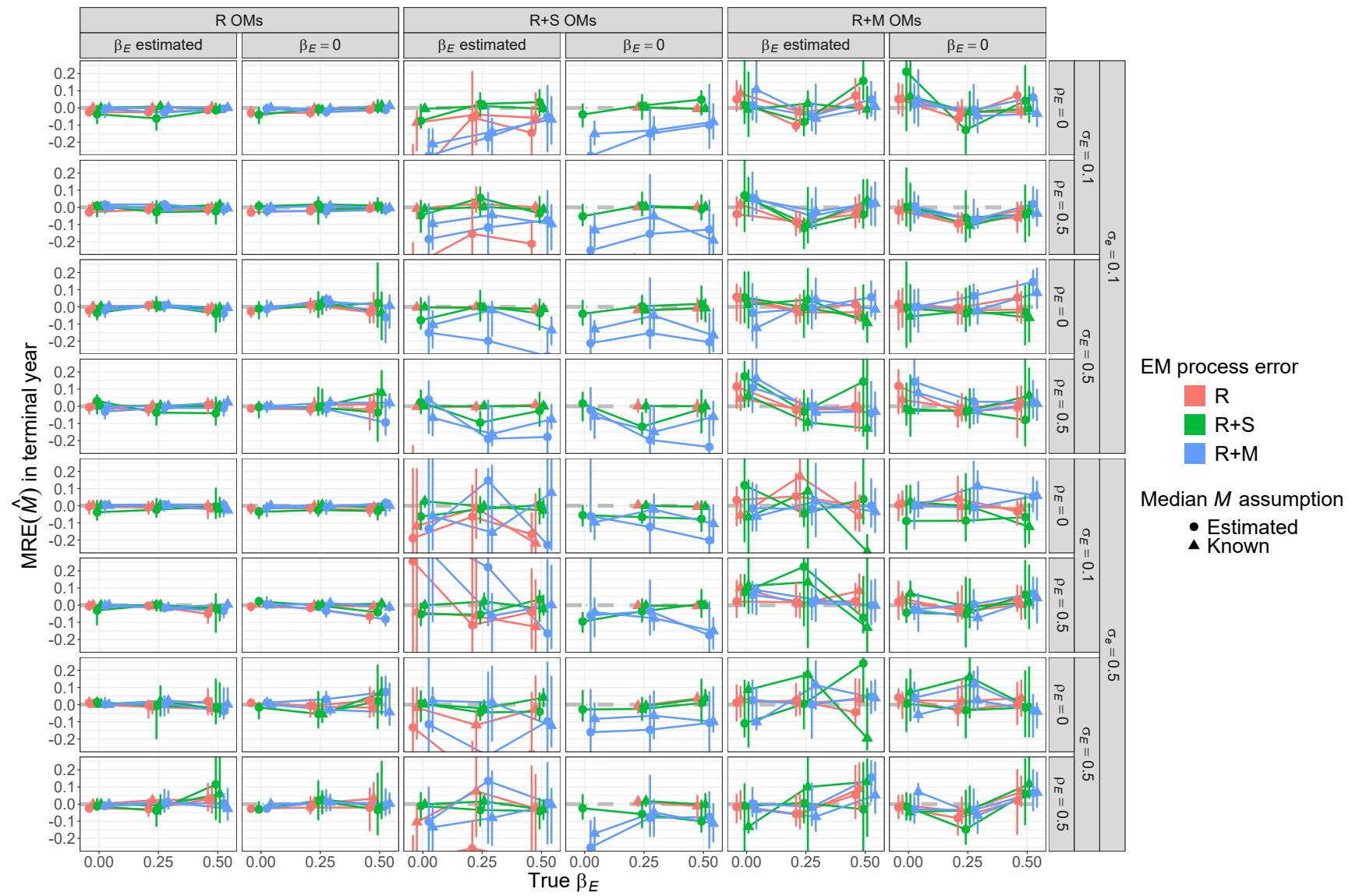


Fig. 7. Median relative error (MRE) of estimates of natural mortality rate in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known). All OMs had low observation error and contrast in fishing mortality.

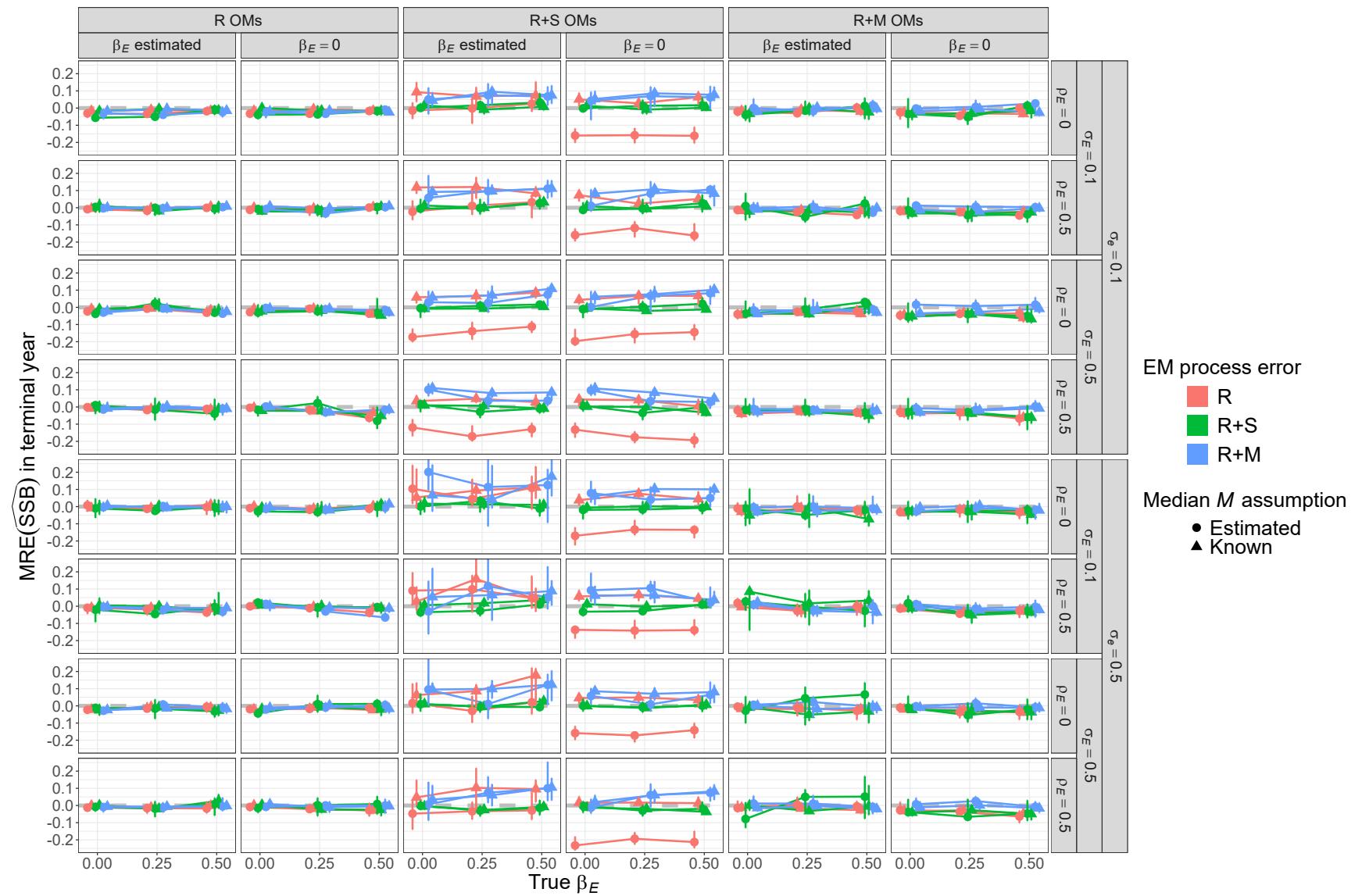


Fig. 8. Median relative error (MRE) of estimates of spawning stock biomass (SSB) in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known). All OMs had low observation error and contrast in fishing mortality.

<sup>594</sup> Supplemental Materials

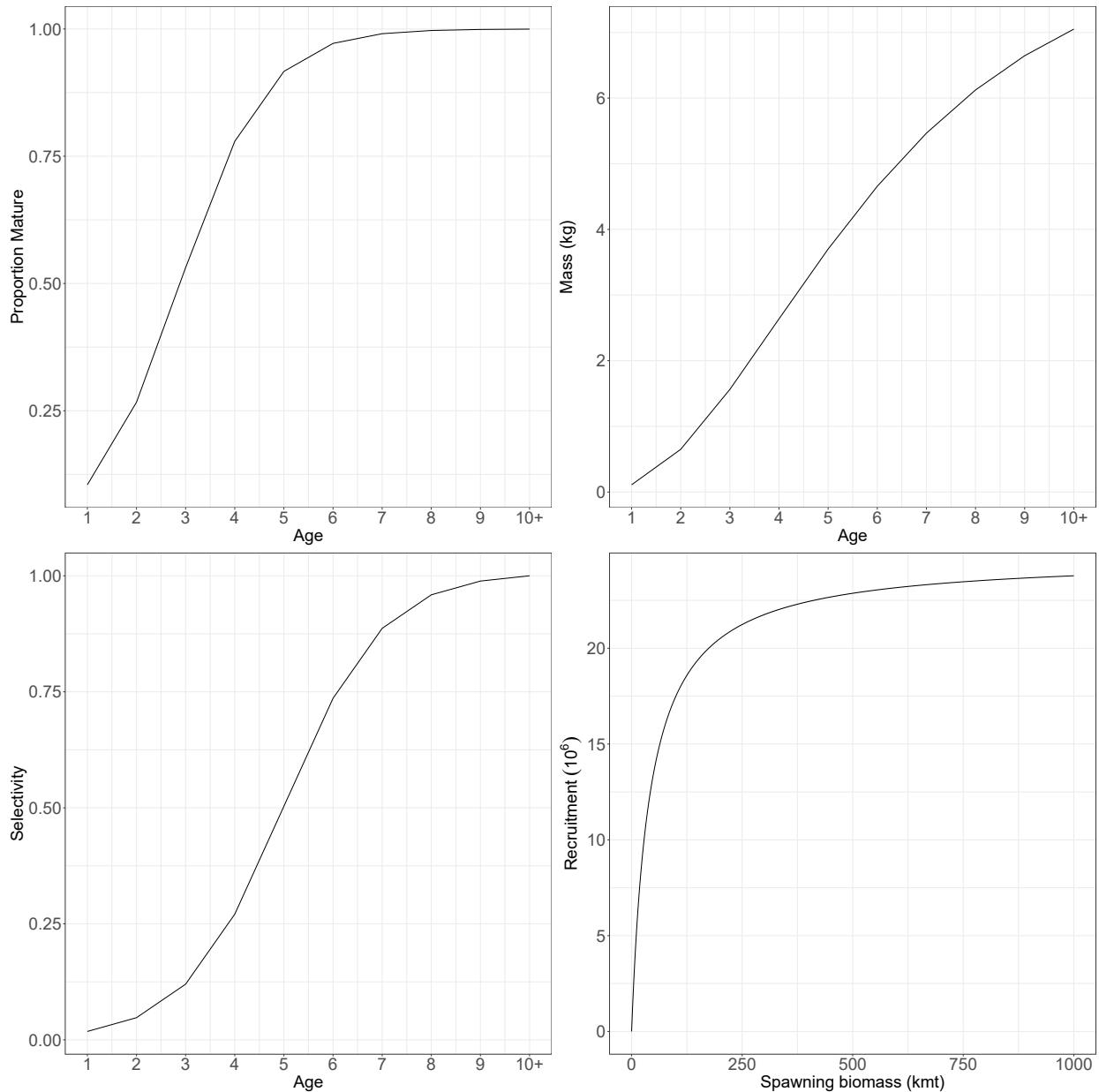


Fig. S1. The proportion mature at age, weight at age, fleet and index selectivity at age, and Beverton-Holt stock-recruit relationship assumed for the population in all operating models. For operating models with random effects on fleet selectivity, this represents the selectivity at the mean of the random effects.

Fig. S2. Example simulations of environmental covariate latent processes and observations with different levels of observation error, and different assumptions about variability of the latent process.

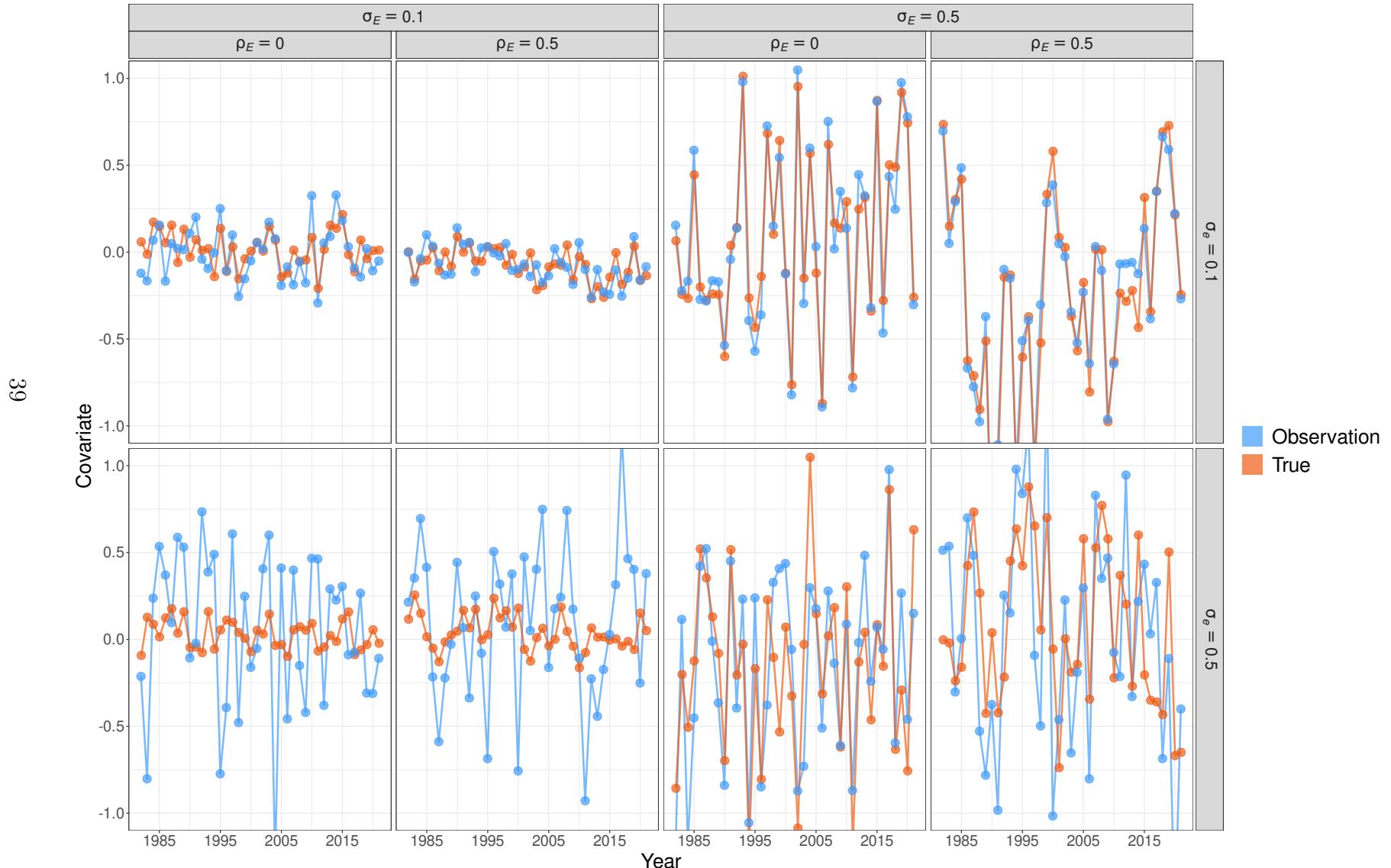
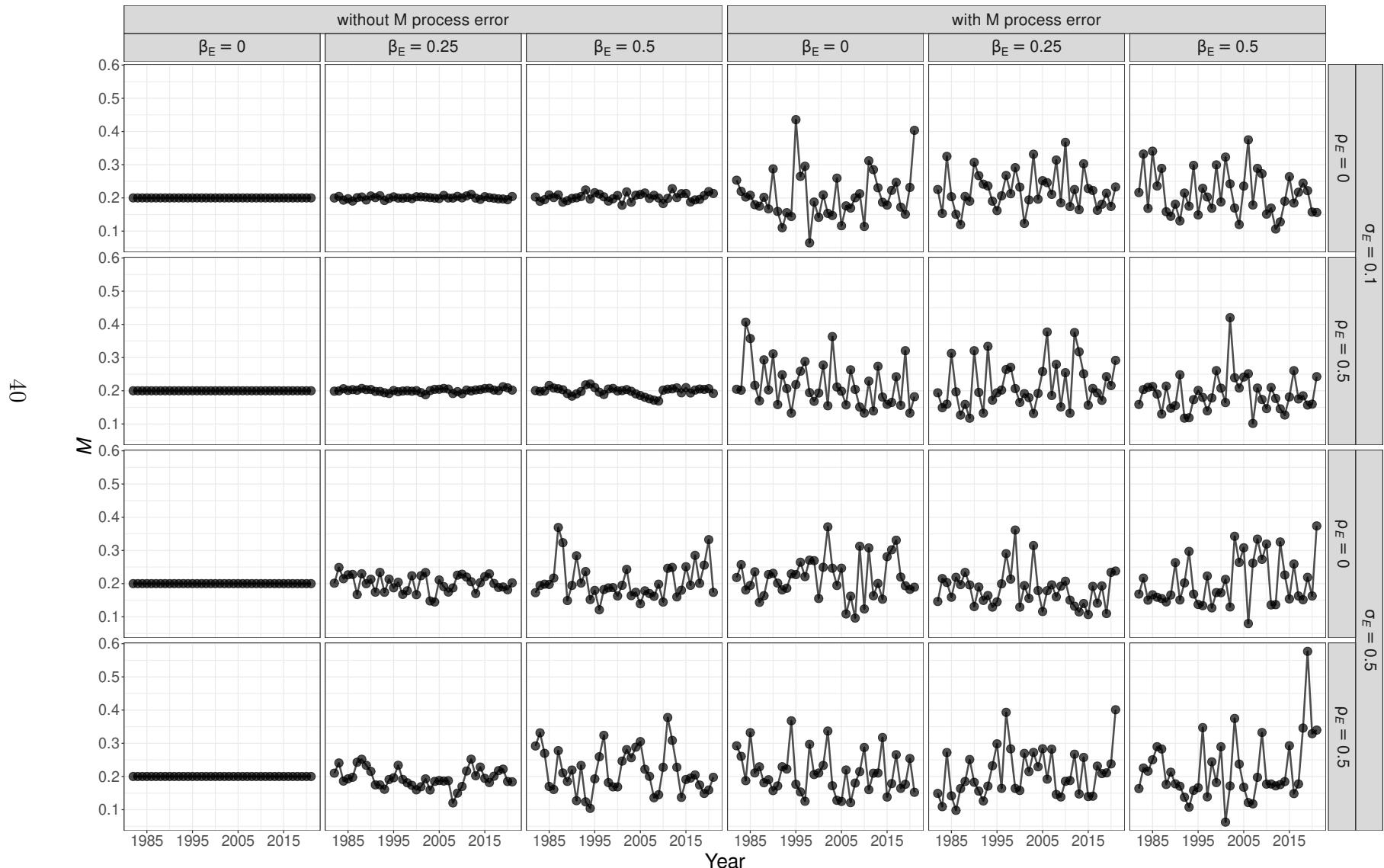


Fig. S3. Example simulations of annual natural mortality rates that may be a function of a temporally varying environmental covariate and autoregressive random effects.



595 **Convergence results**

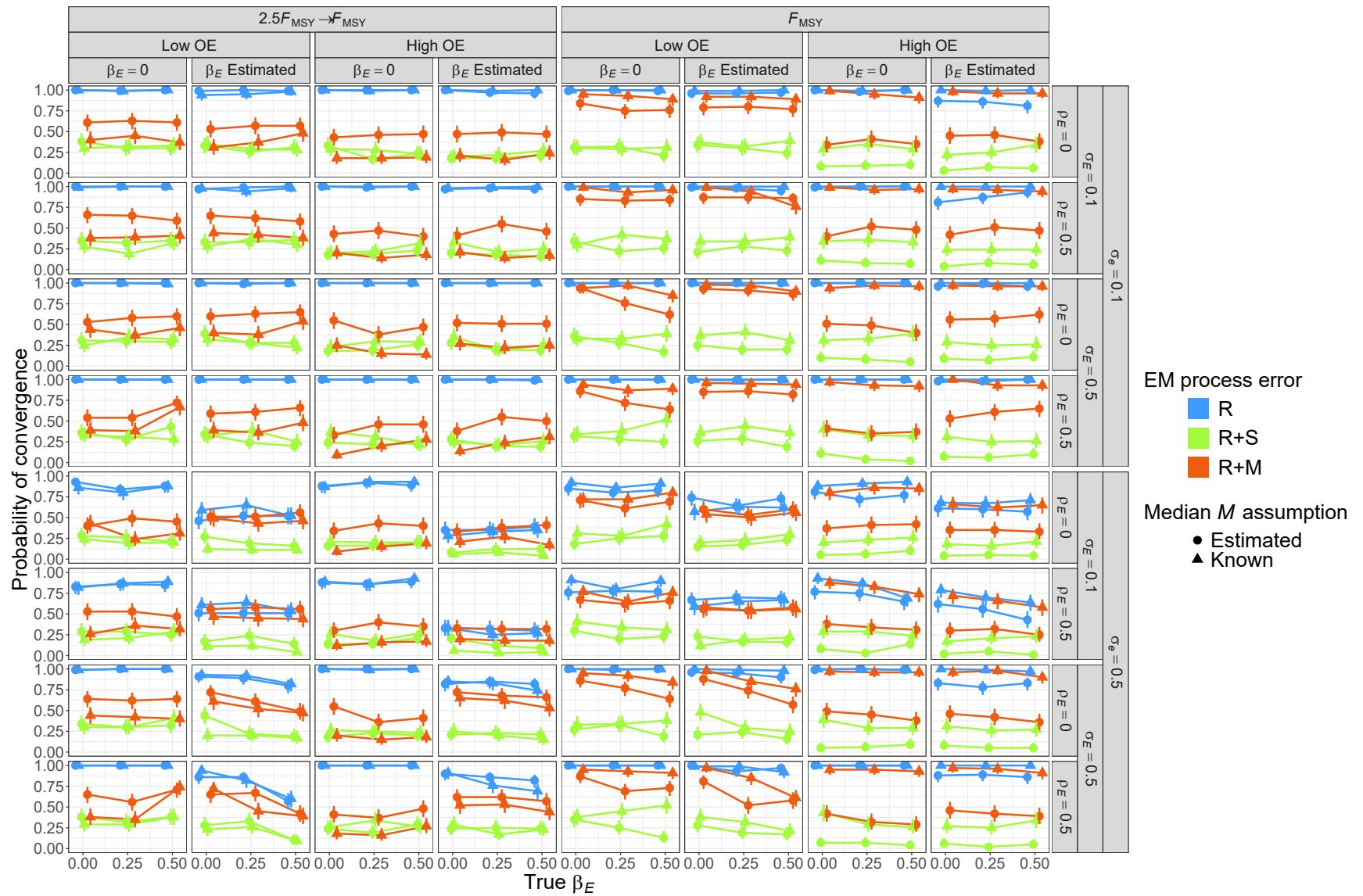


Fig. S4. Estimated probability of fits providing Hessian-based standard errors for EMs assuming alternative process error, that estimate or assume known median natural mortality, and that estimate or assume no covariate effect on median natural mortality when fitted to R OMs and three levels of true covariate effect on median natural mortality (x axis). Vertical lines represent 95% confidence intervals.

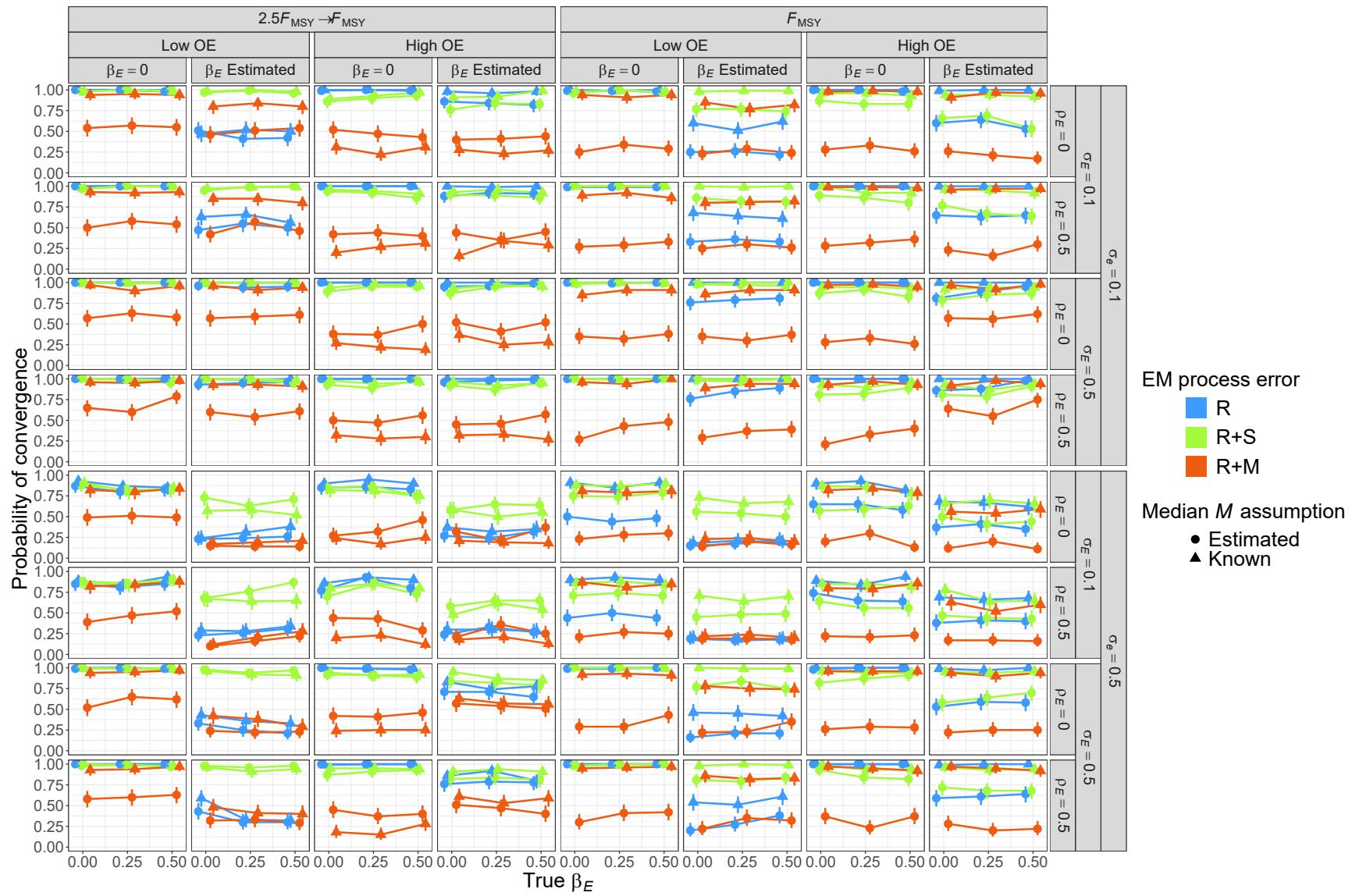


Fig. S5. Estimated probability of fits providing Hessian-based standard errors for EMs assuming alternative process error, that estimate or assume known median natural mortality, and that estimate or assume no covariate effect on median natural mortality when fitted to R+S OMs and three levels of true covariate effect on median natural mortality (x axis). Vertical lines represent 95% confidence intervals.

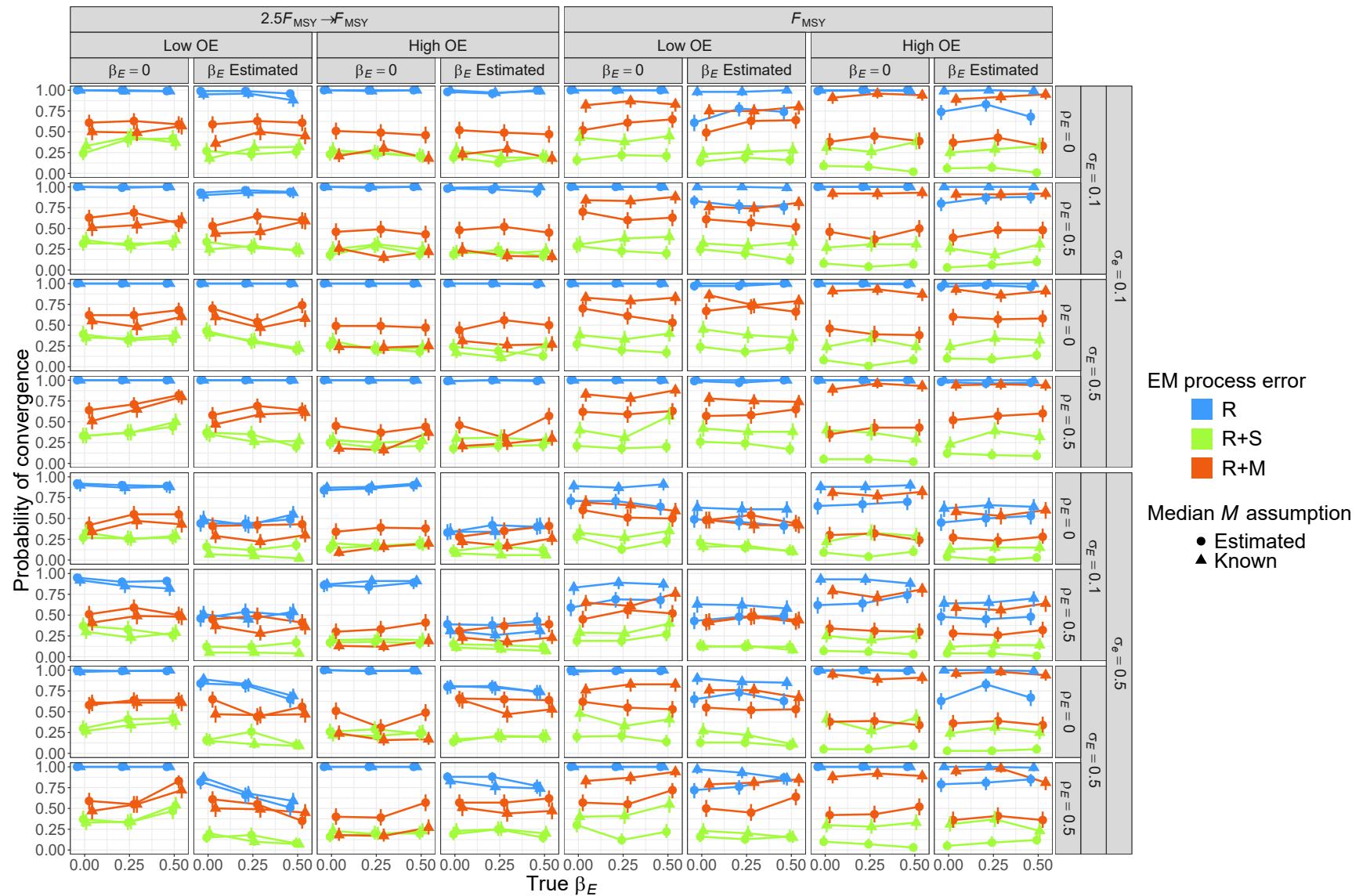


Fig. S6. Estimated probability of fits providing Hessian-based standard errors for EMs assuming alternative process error, that estimate or assume known median natural mortality, and that estimate or assume no covariate effect on median natural mortality when fitted to R+M OMs and three levels of true covariate effect on median natural mortality (x axis). Vertical lines represent 95% confidence intervals.

596 **AIC results**

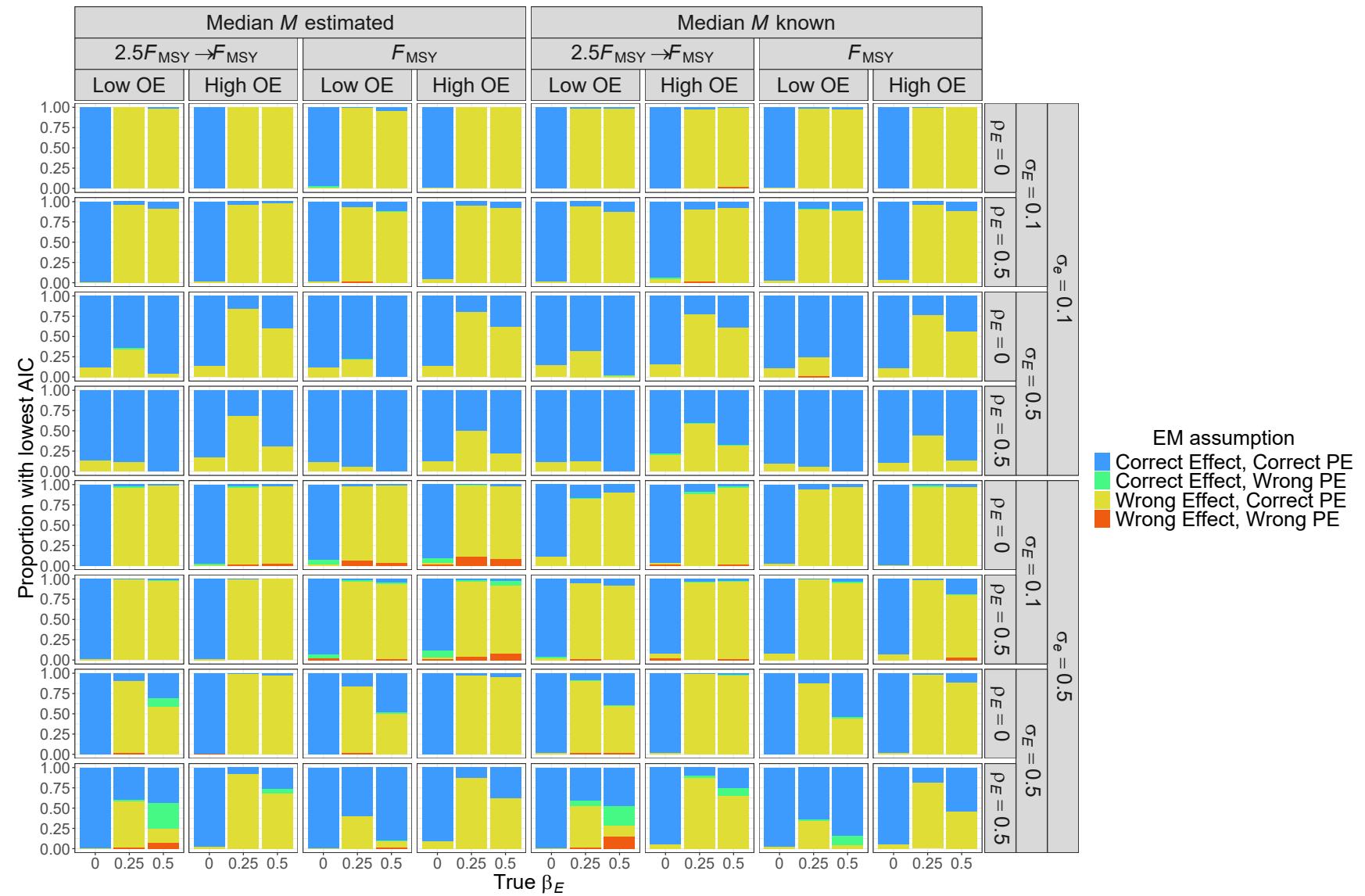


Fig. S7. Proportion of simulated data sets for R OMs where the EM type (treatment of environmental covariate and assumed process error type) had the lowest AIC.

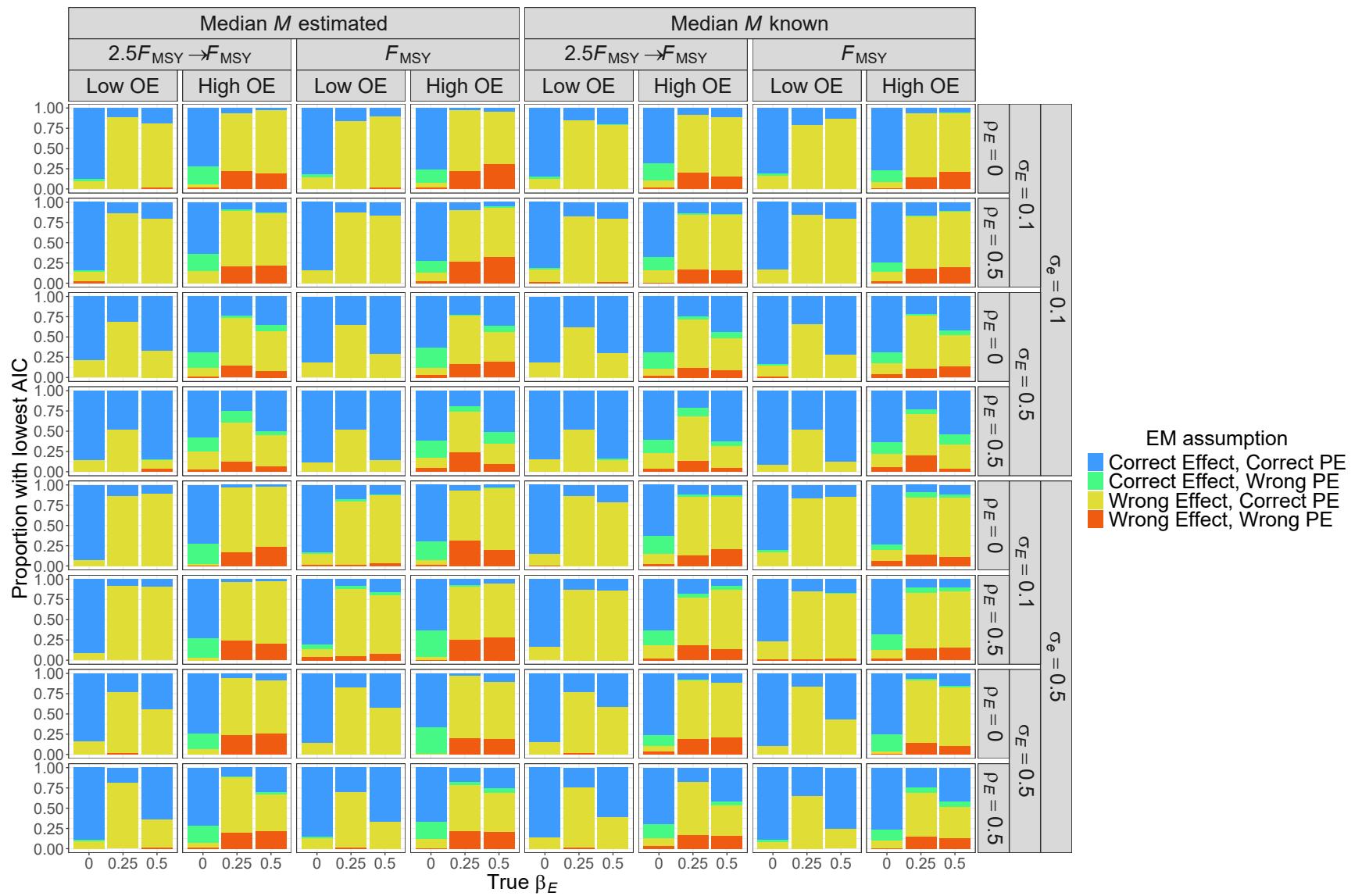


Fig. S8. Proportion of simulated data sets for R+S OMs where the EM type (treatment of environmental covariate and assumed process error type) had the lowest AIC.

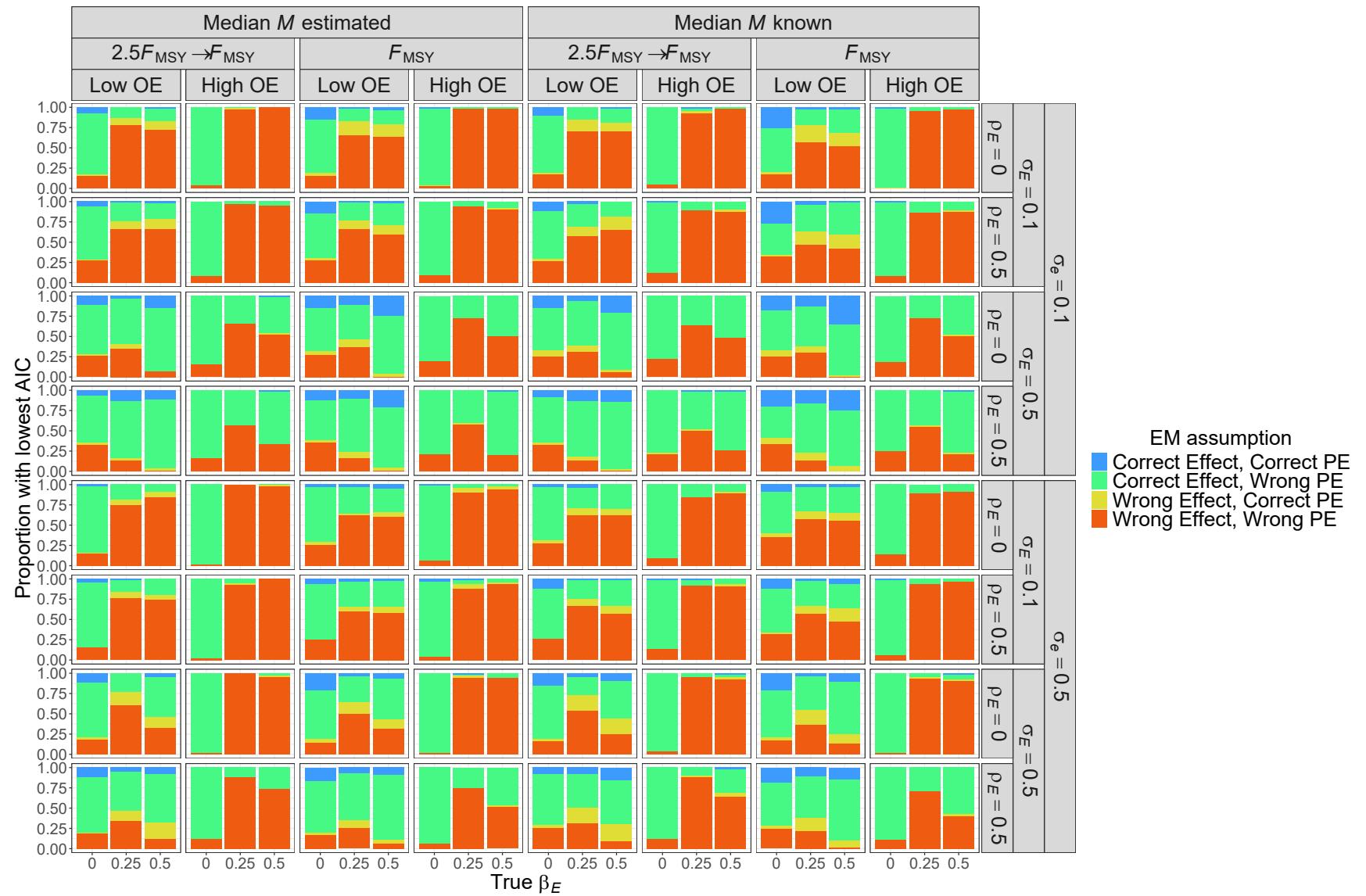


Fig. S9. Proportion of simulated data sets for R+M OMs where the EM type (treatment of environmental covariate and assumed process error type) had the lowest AIC.

597 Covariate effect bias

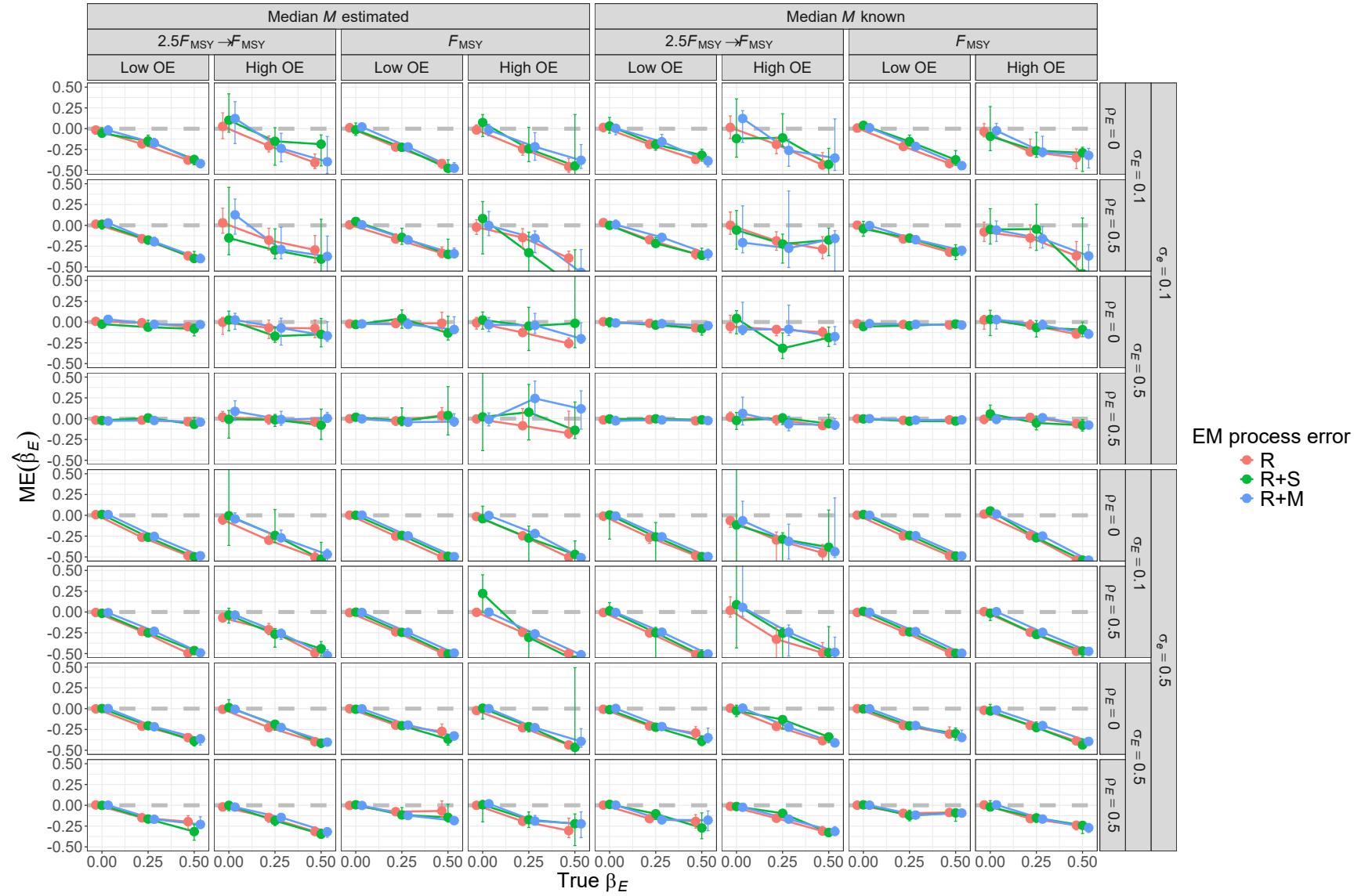


Fig. S10. For R OMs, median error (ME) of estimates of environmental effect on natural mortality  $\beta_E$  from fitting EMs with alternative process error assumptions and treatment of median natural mortality ( $e_M^\beta$  known or estimated). Vertical lines represent 95% confidence intervals.

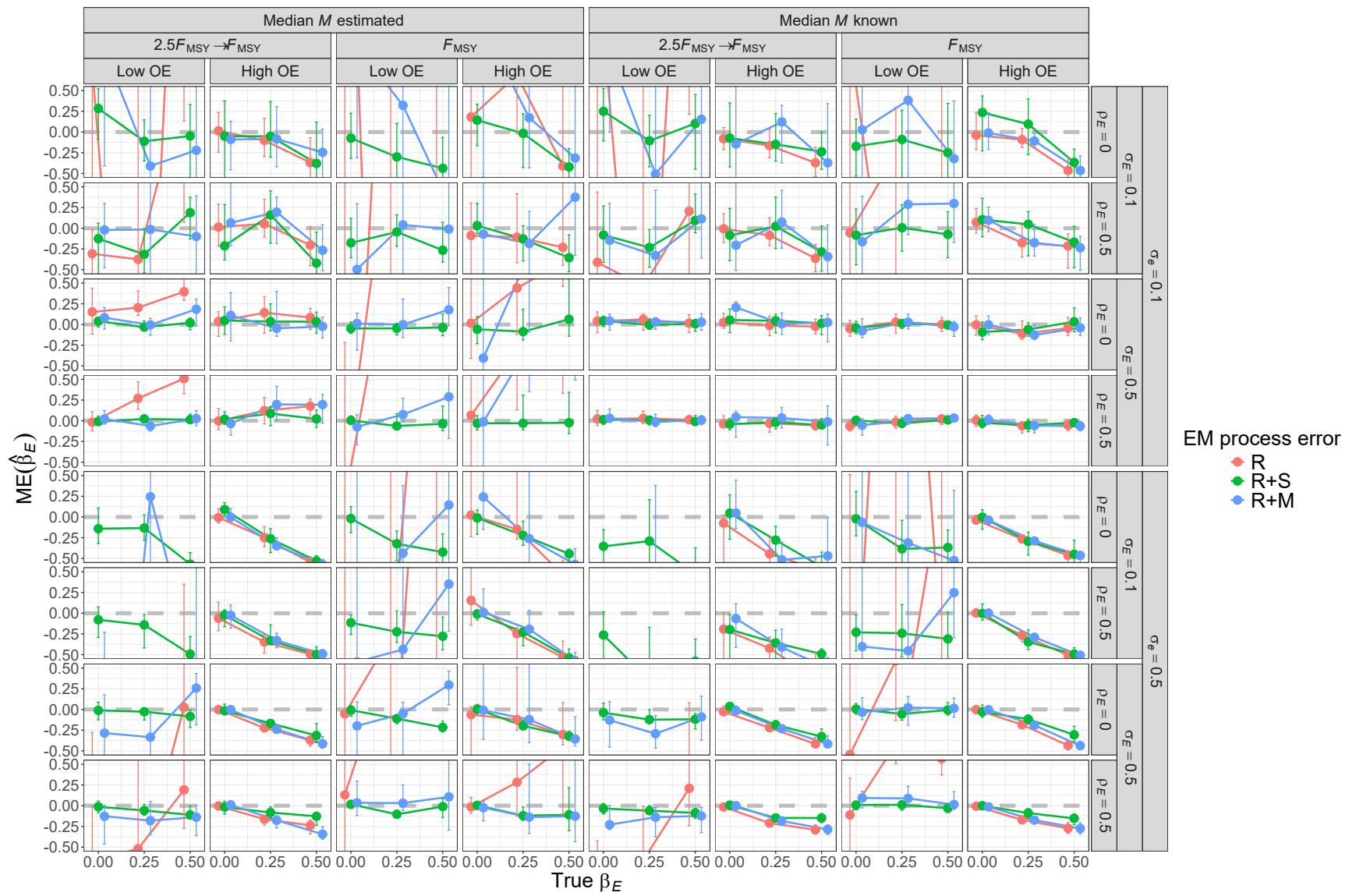


Fig. S11. For R+S OMs, median error (ME) of estimates of environmental effect on natural mortality  $\beta_E$  from fitting EMs with alternative process error assumptions and treatment of median natural mortality ( $e_M^\beta$  known or estimated). Vertical lines represent 95% confidence intervals.

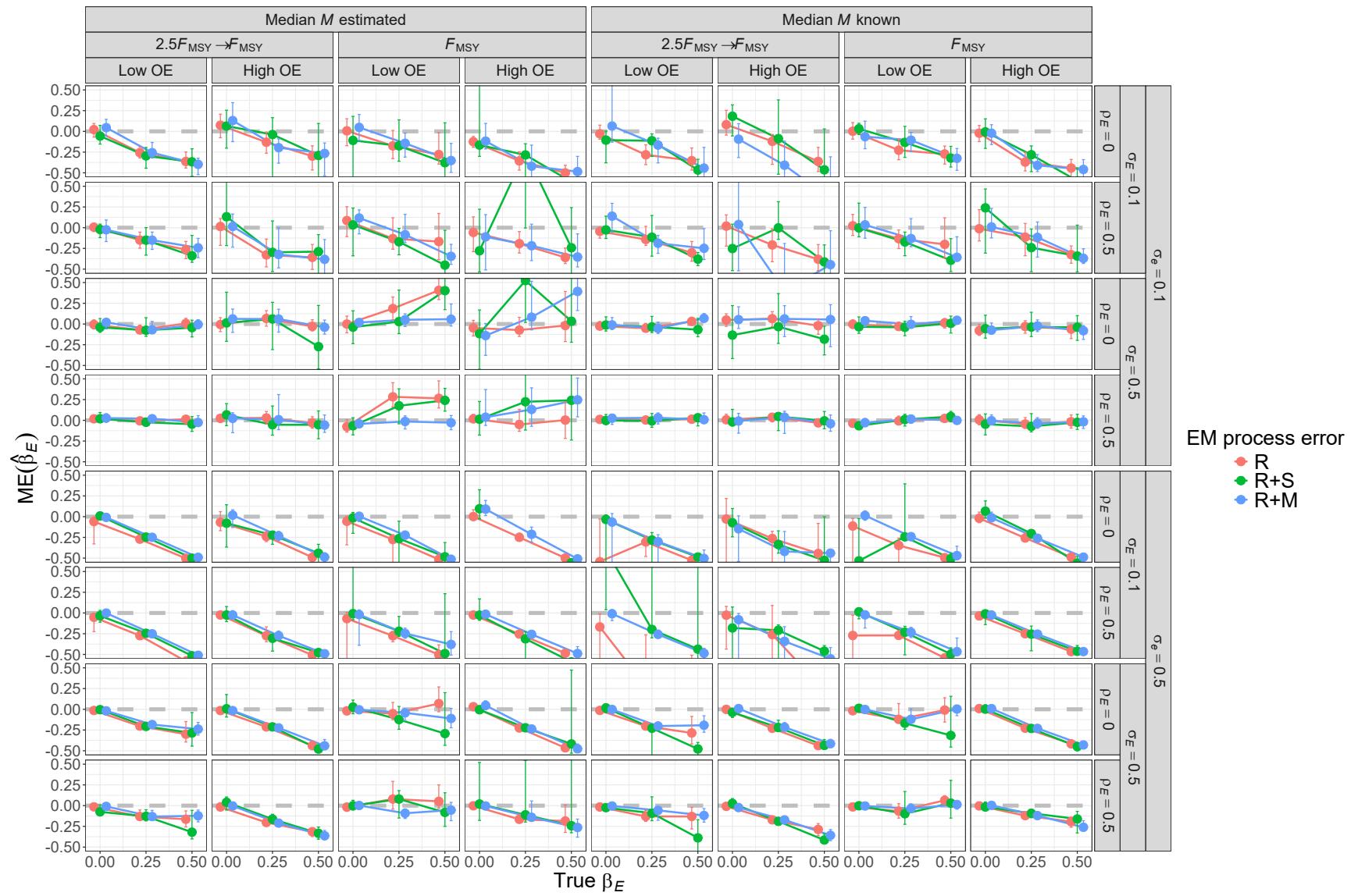


Fig. S12. For R+M OMs, median error (ME) of estimates of environmental effect on natural mortality  $\beta_E$  from fitting EMs with alternative process error assumptions and treatment of median natural mortality ( $e_M^\beta$  known or estimated). Vertical lines represent 95% confidence intervals.

<sup>598</sup> Covariate effect standard error estimation bias

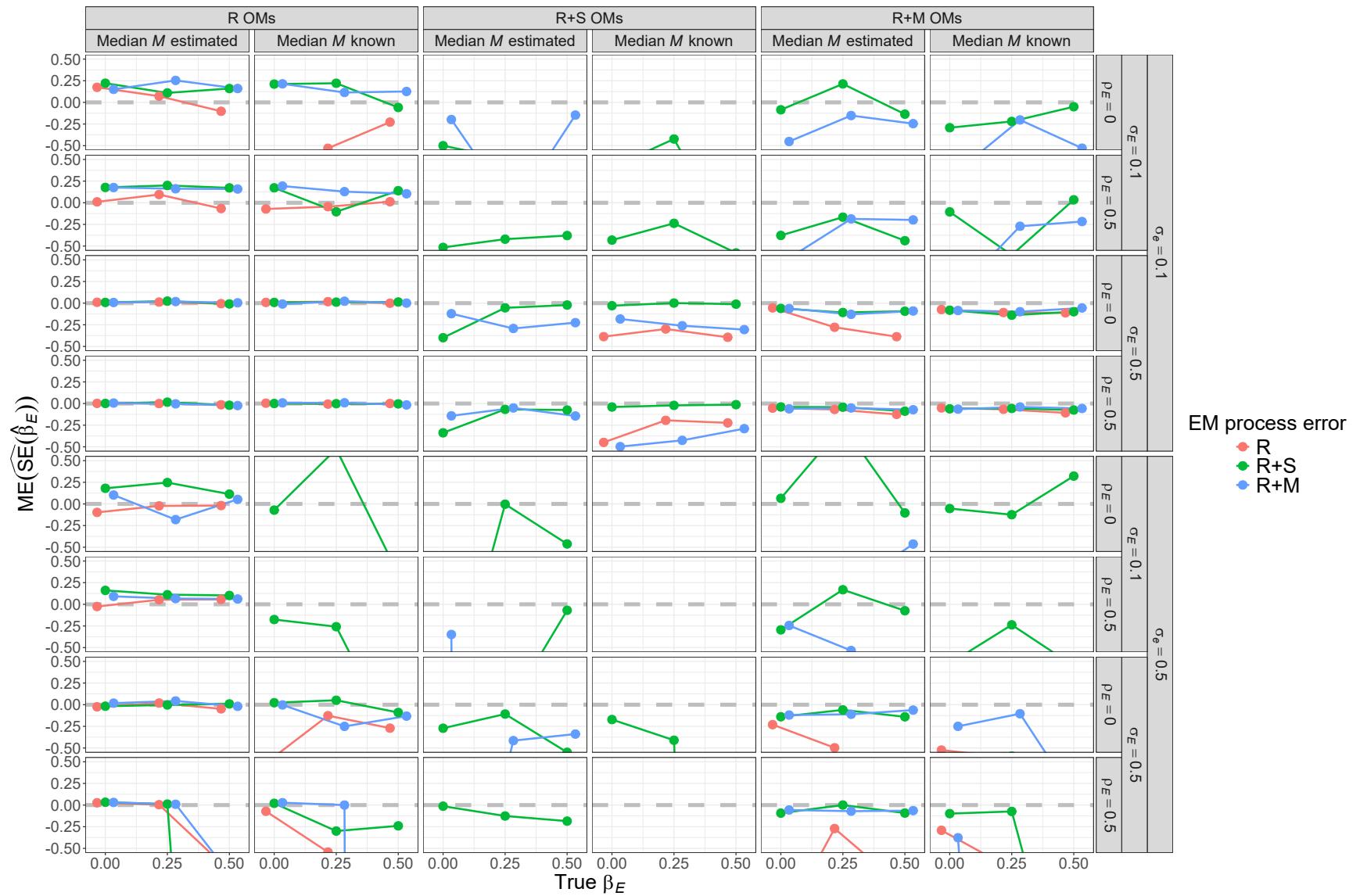


Fig. S13. Median error (ME) of Hessian-based estimates of standard error for covariate effect on natural mortality  $\beta_E$  from fitting EMs with alternative process error assumptions and treatment of median natural mortality ( $e_M^\beta$  known or estimated). All OMs had low observation error and contrast in fishing mortality. True standard error is defined as the mean of the standard error estimates across converged fits to simulated data sets for a given OM scenario.

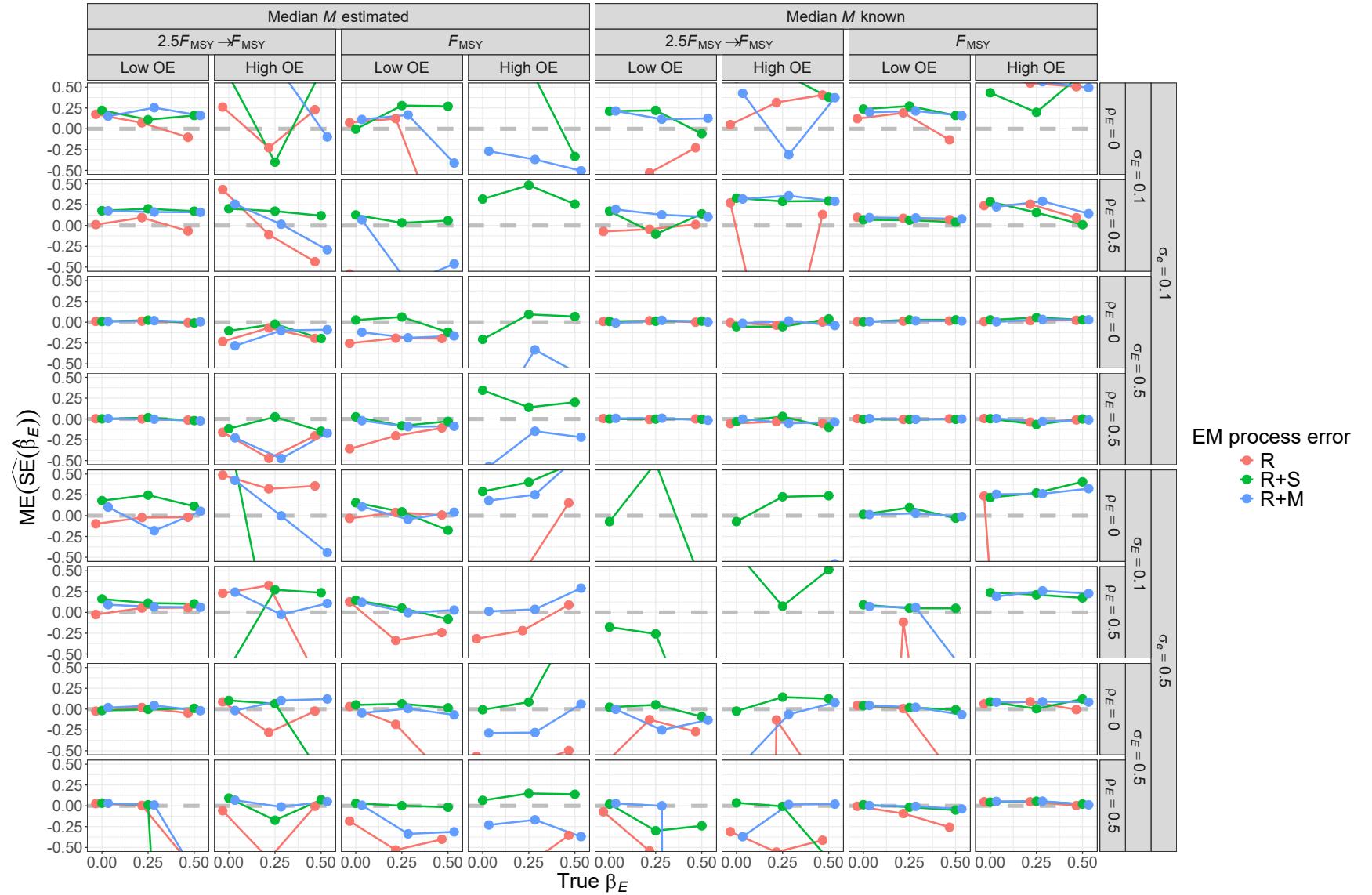


Fig. S14. For R OMs, median error (ME) of Hessian-based estimates of standard error for covariate effect on natural mortality  $\beta_E$  from fitting EMs with alternative process error assumptions and treatment of median natural mortality ( $e_M^\beta$  known or estimated). True standard error is defined as the mean of the standard error estimates across converged fits to simulated data sets for a given OM scenario.

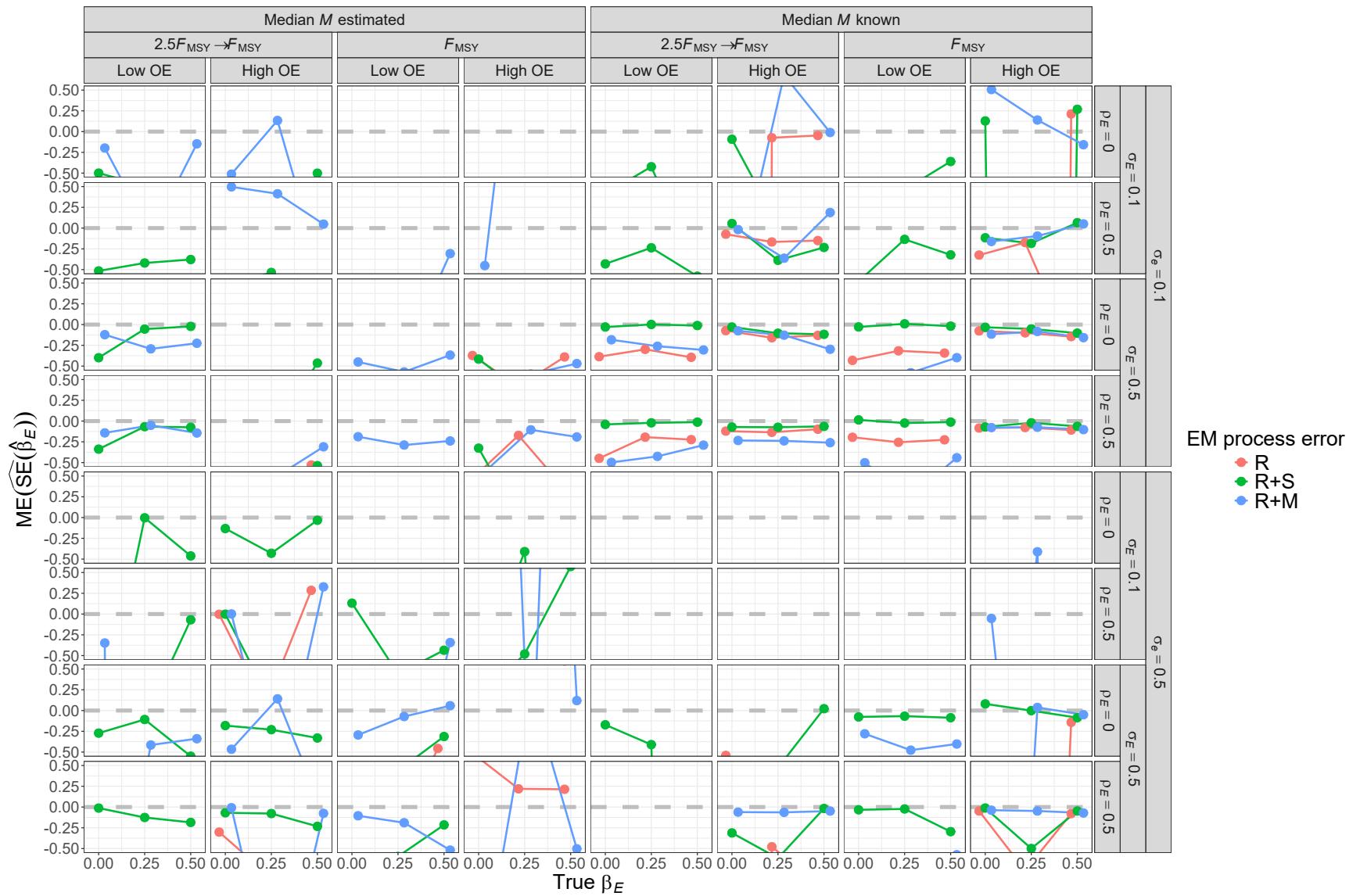


Fig. S15. For R+S OMs, median error (ME) of Hessian-based estimates of standard error for covariate effect on natural mortality  $\beta_E$  from fitting EMs with alternative process error assumptions and treatment of median natural mortality ( $e_M^\beta$  known or estimated). True standard error is defined as the mean of the standard error estimates across converged fits to simulated data sets for a given OM scenario.

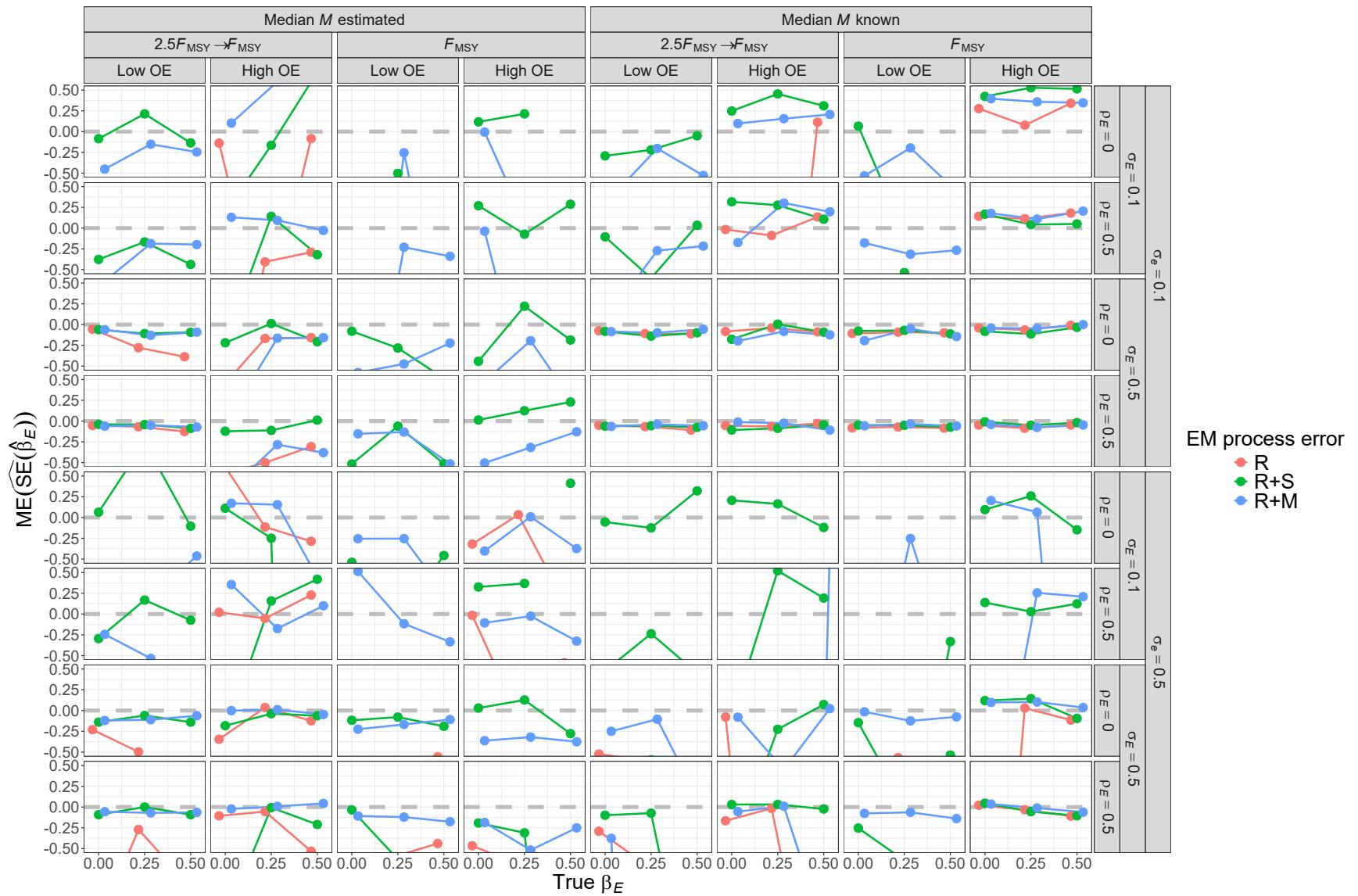


Fig. S16. For R+M OMs, median error (ME) of Hessian-based estimates of standard error for covariate effect on natural mortality  $\beta_E$  from fitting EMs with alternative process error assumptions and treatment of median natural mortality ( $e_M^\beta$  known or estimated). True standard error is defined as the mean of the standard error estimates across converged fits to simulated data sets for a given OM scenario.

<sup>599</sup> Covariate effect confidence interval coverage

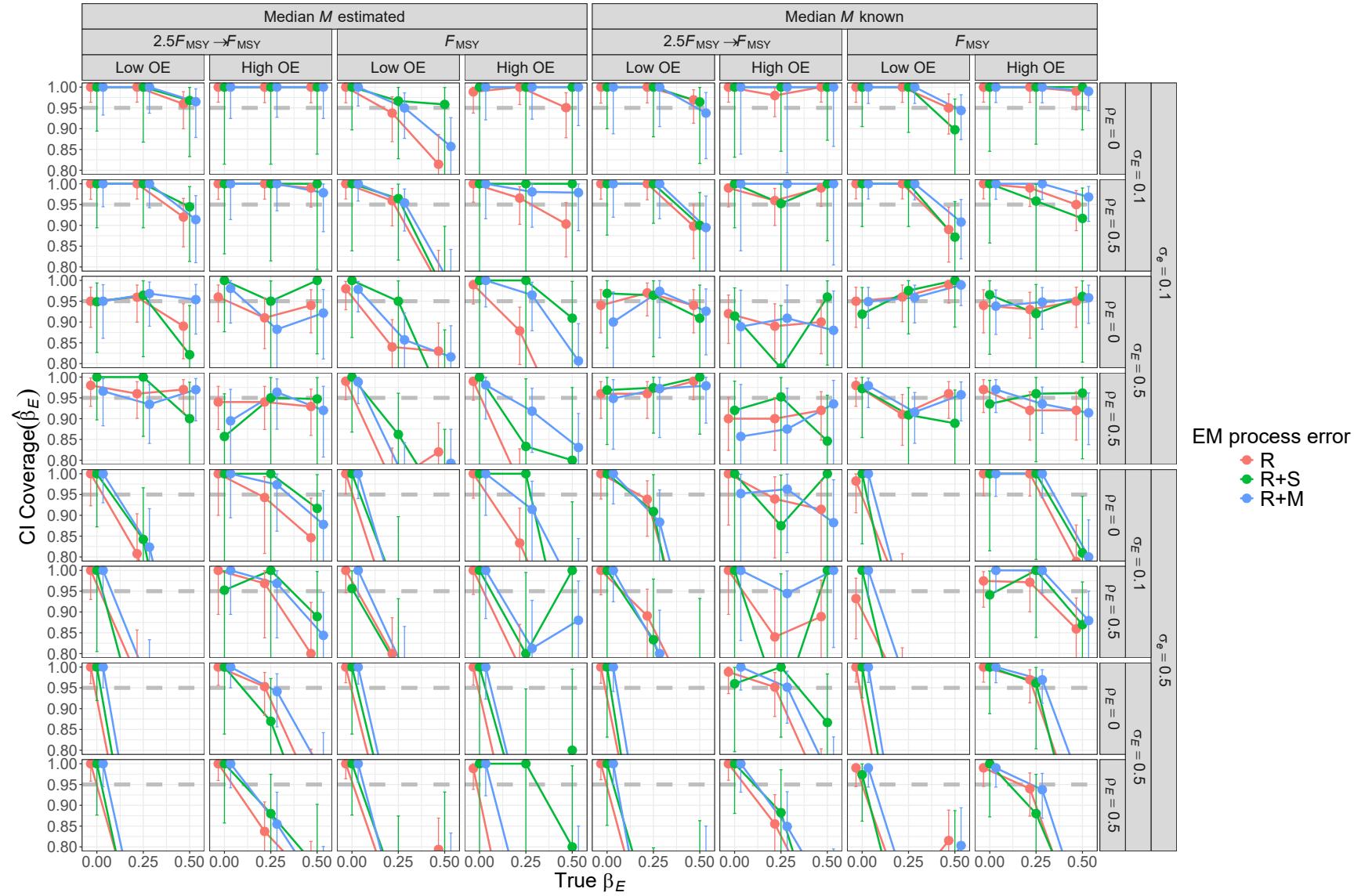


Fig. S17. For R OMs, probability of 95% confidence interval for  $\beta_E$  containing the true value for EMs with alternative process error assumptions and treatment of median natural mortality ( $e_M^\beta$  known or estimated). Vertical lines represent 95% confidence intervals.

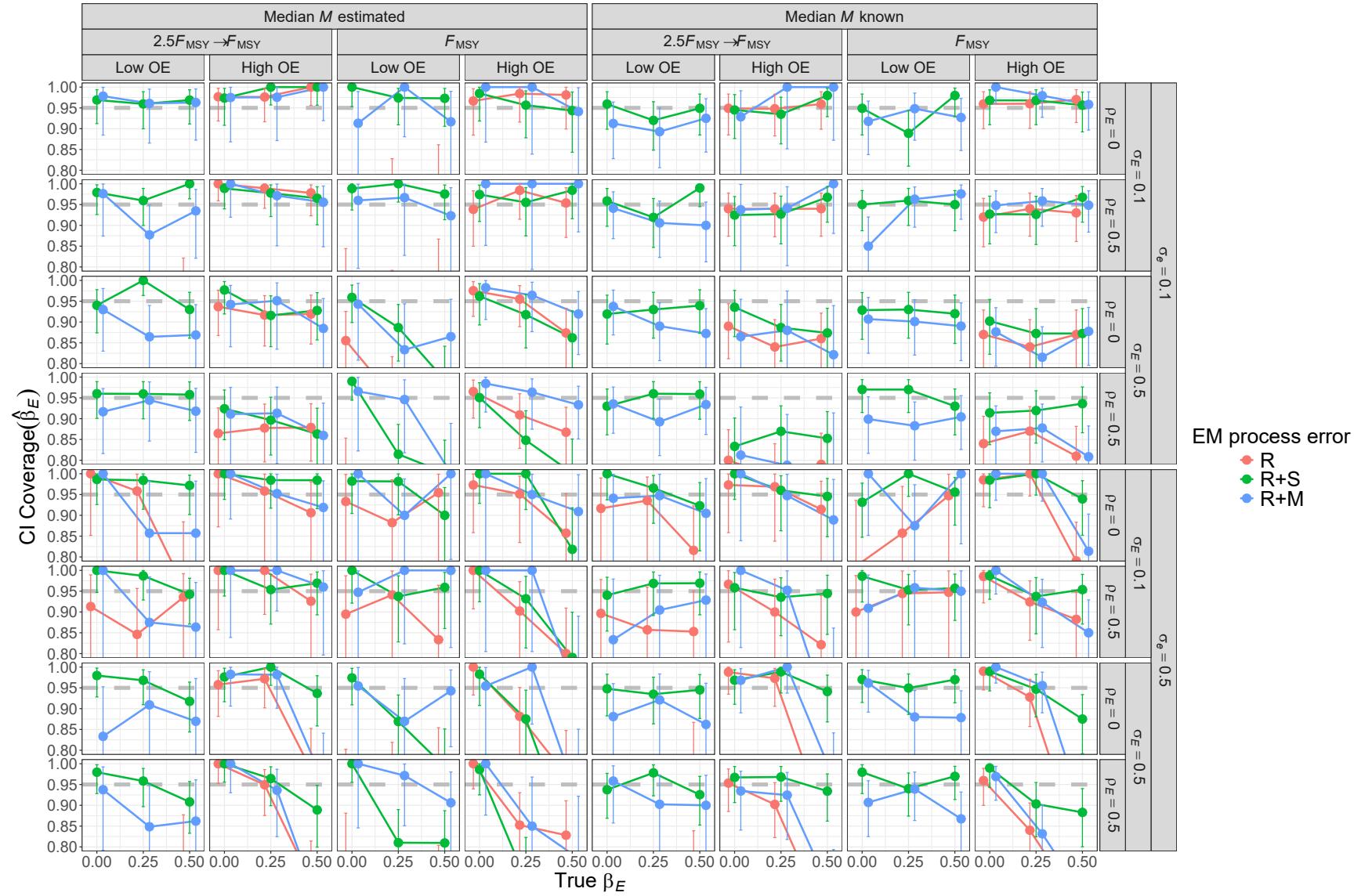


Fig. S18. For R+S OMs, probability of 95% confidence interval for  $\beta_E$  containing the true value for EMs with alternative process error assumptions and treatment of median natural mortality ( $e_M^\beta$  known or estimated). Vertical lines represent 95% confidence intervals.

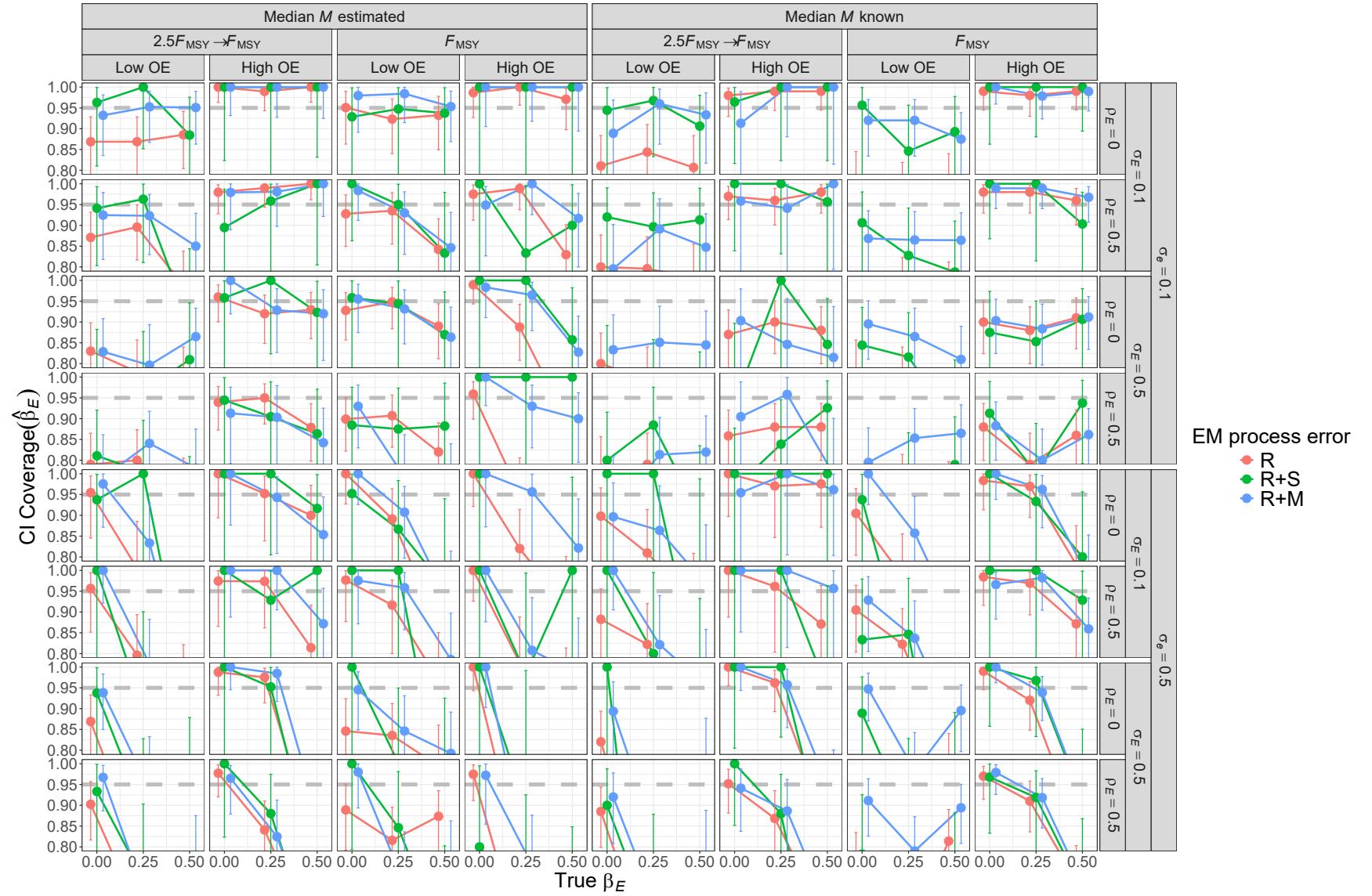


Fig. S19. For R+M OMs, probability of 95% confidence interval for  $\beta_E$  containing the true value for EMs with alternative process error assumptions and treatment of median natural mortality ( $e_M^\beta$  known or estimated). Vertical lines represent 95% confidence intervals.

<sub>600</sub> Covariate effect RMSE

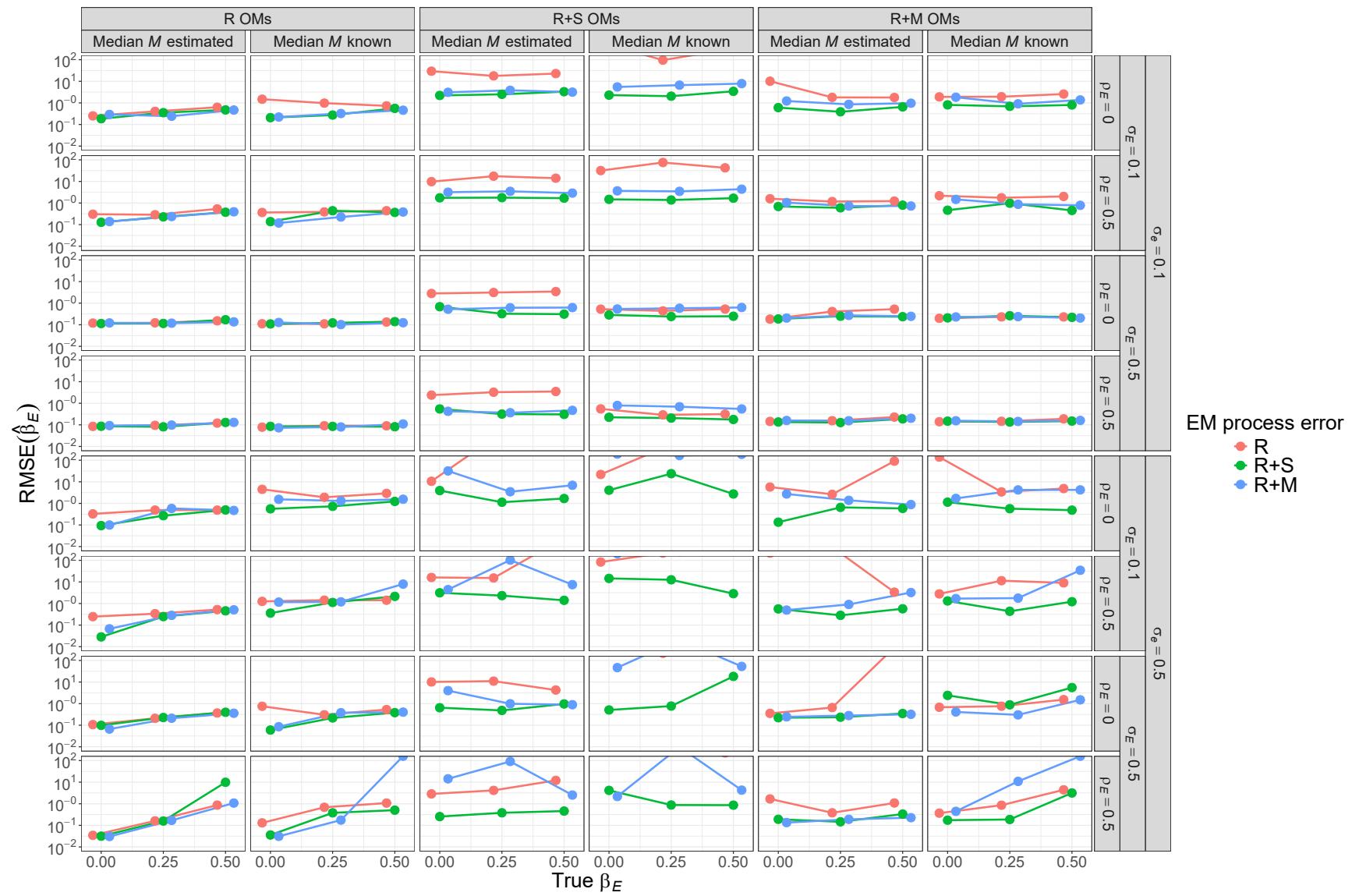


Fig. S20. Root mean square error (RMSE) of estimates of covariate effect on natural mortality  $\beta_E$  from fitting EMs with alternative process error assumptions and treatment of median natural mortality ( $e_M^\beta$  known or estimated). All OM had low observation error and contrast in fishing mortality.

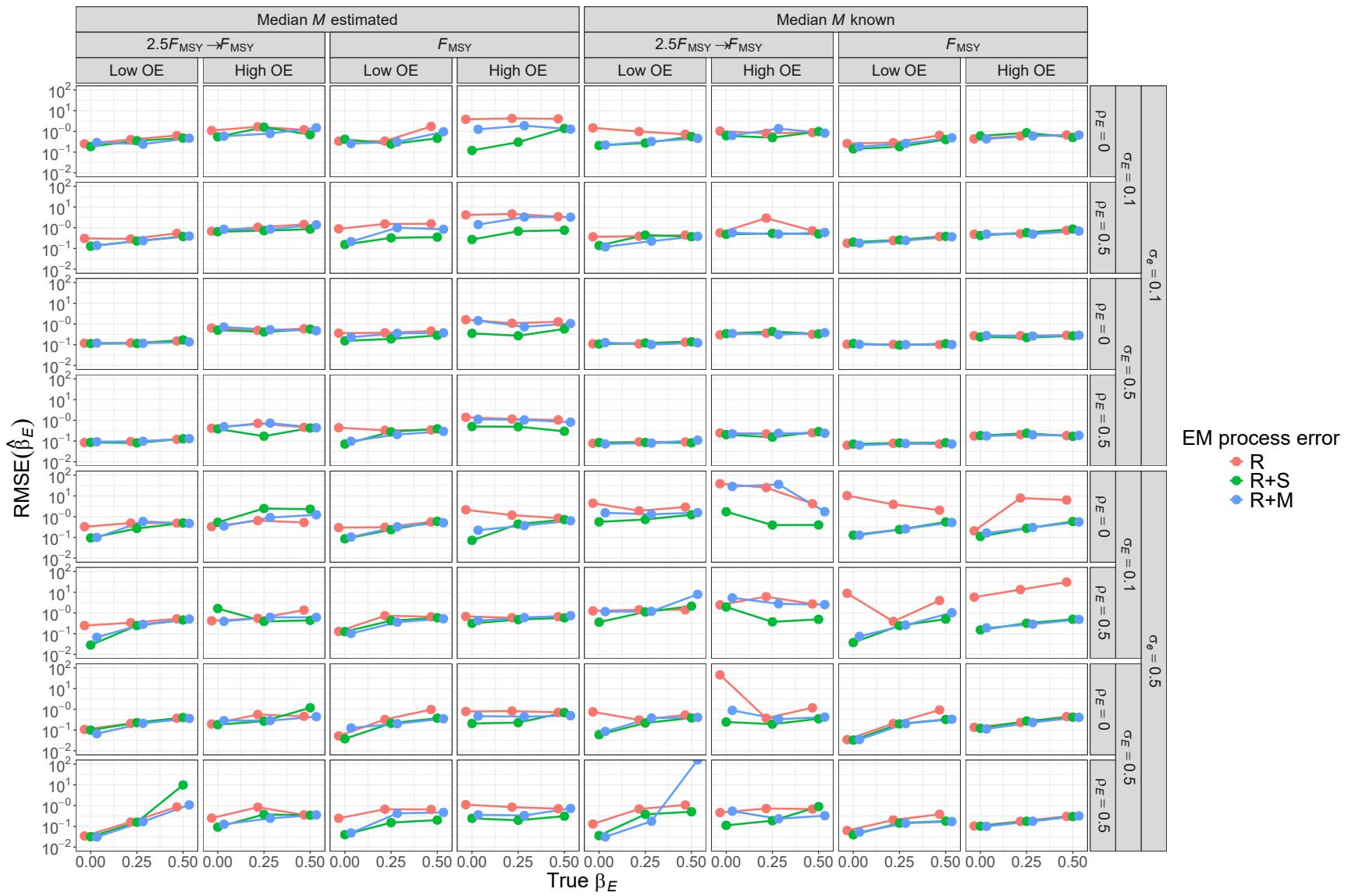


Fig. S21. For R OMs, root mean square error (RMSE) of estimates of covariate effect on natural mortality  $\beta_E$  from fitting EMs with alternative process error assumptions and treatment of median natural mortality ( $e_M^\beta$  known or estimated).

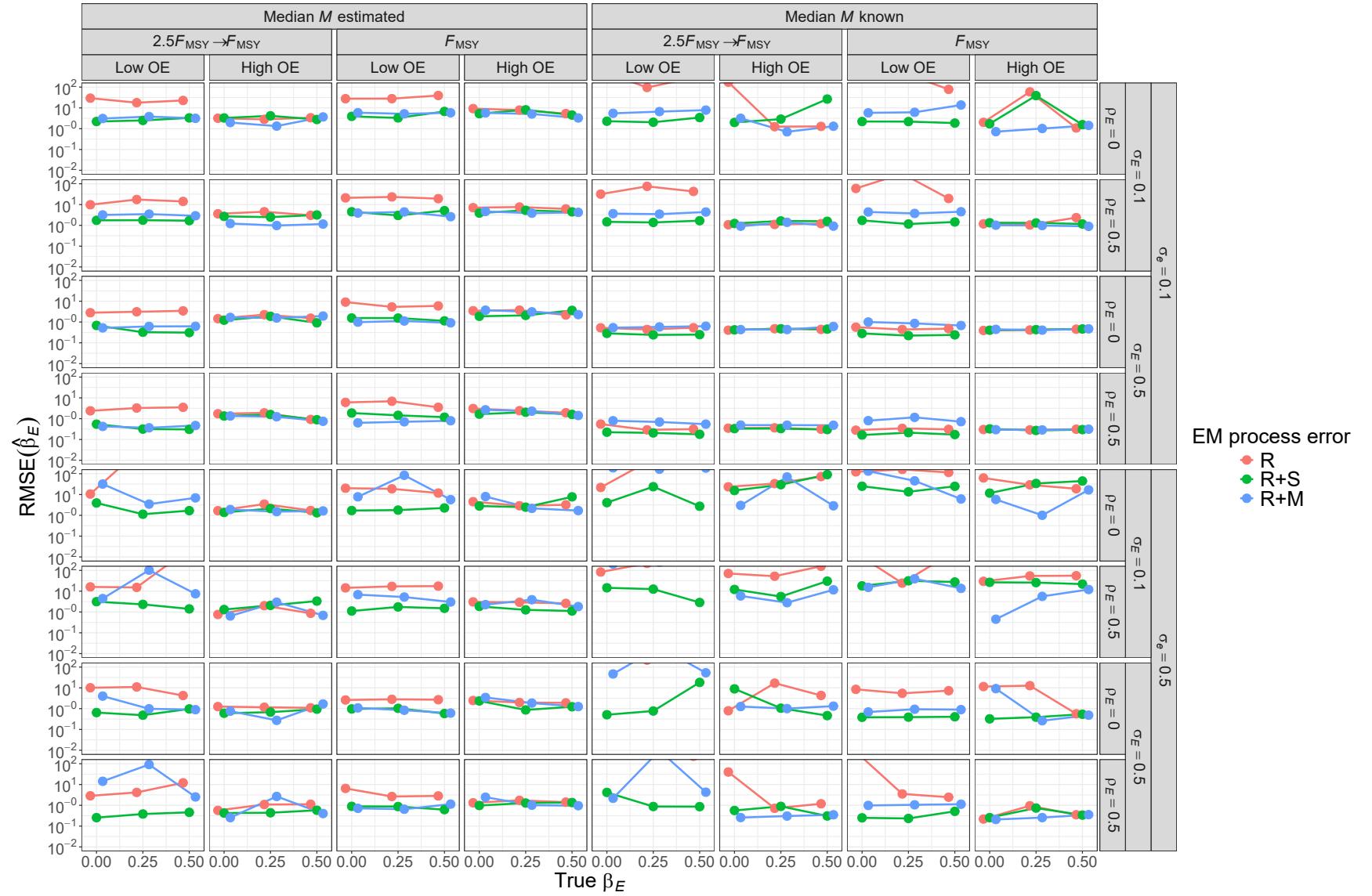


Fig. S22. For R+S OMs, root mean square error (RMSE) of estimates of covariate effect on natural mortality  $\beta_E$  from fitting EMs with alternative process error assumptions and treatment of median natural mortality ( $e_M^\beta$  known or estimated).

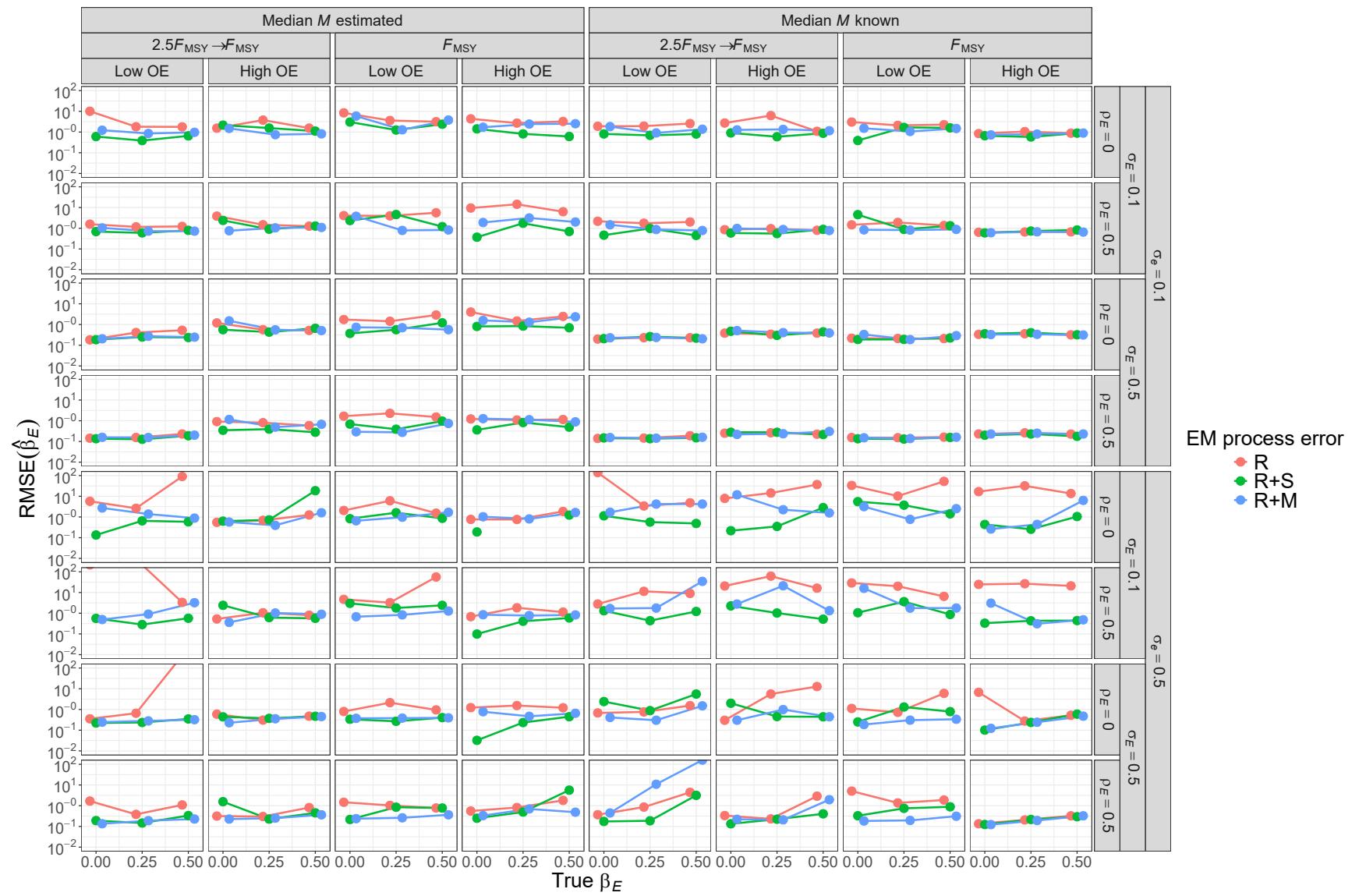


Fig. S23. For R+M OMs, root mean square error (RMSE) of estimates of covariate effect on natural mortality  $\beta_E$  from fitting EMs with alternative process error assumptions and treatment of median natural mortality ( $e_M^\beta$  known or estimated).

601 Covariate effect estimate and standard error example

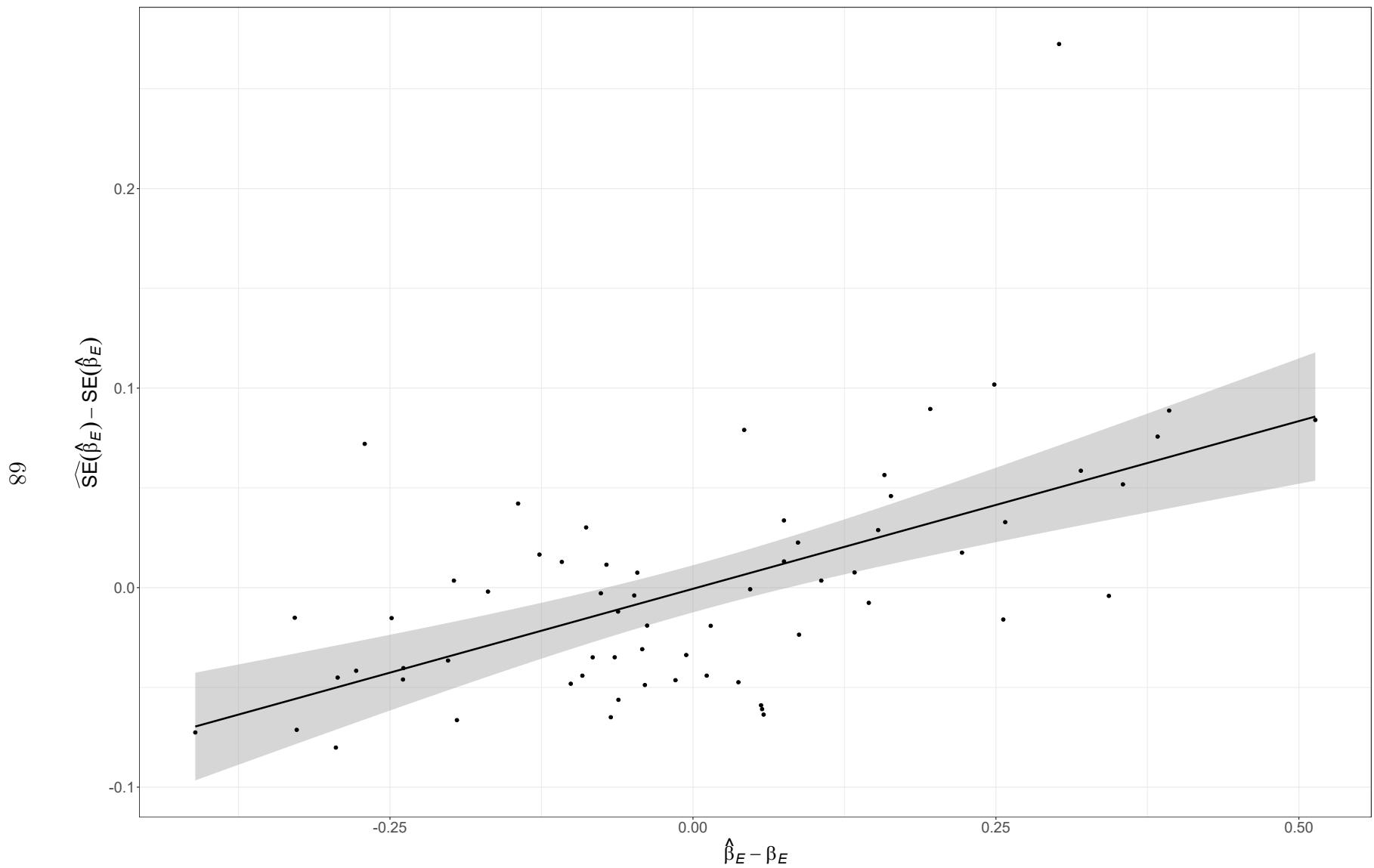


Fig. S24. Positive correlation of covariate effect estimates and Hessian-based standard error estimates for EM that also estimates the median natural mortality parameter and has correct R+M process error assumption fitted to simulated data from the OM with R+M process errors, temporal contrast in fishing pressure, low observation uncertainty for both population (*LowOE*) and covariate observations ( $\sigma_e = 0.1$ ), high and uncorrelated temporal variability in the true covariate ( $\sigma_E = 0.5$  and  $\rho_E = 0$ ), and the strongest covariate effect on natural mortality ( $\beta_E = 0.5$ ).

<sub>602</sub> Median Natural mortality parameter bias

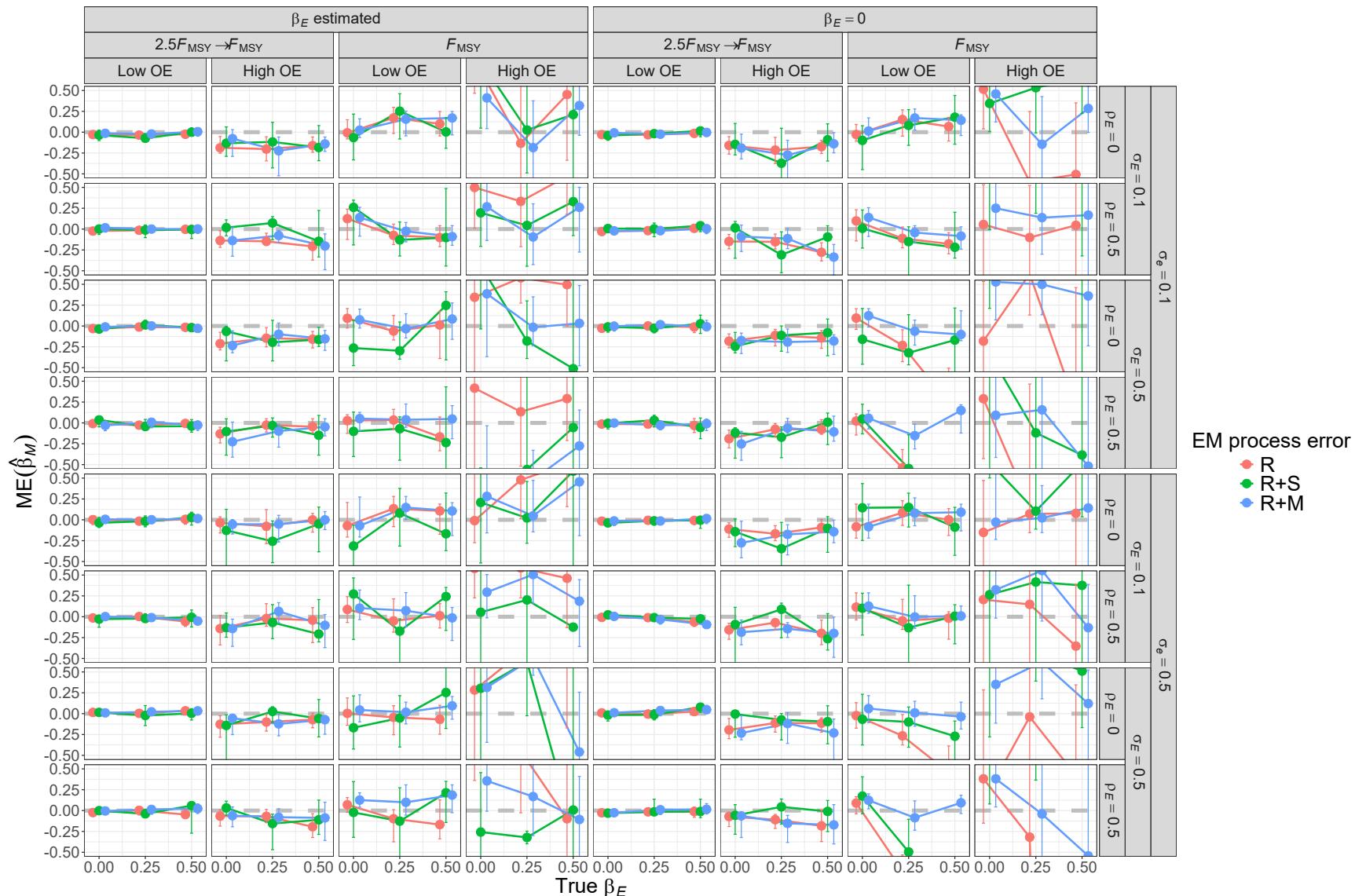


Fig. S25. For R OMs, median error (ME) of estimates of  $\beta_M$  from fitting EMs with alternative process error assumptions and treatment of covariate effect ( $\beta_E = 0$  or estimated). Vertical lines represent 95% confidence intervals.

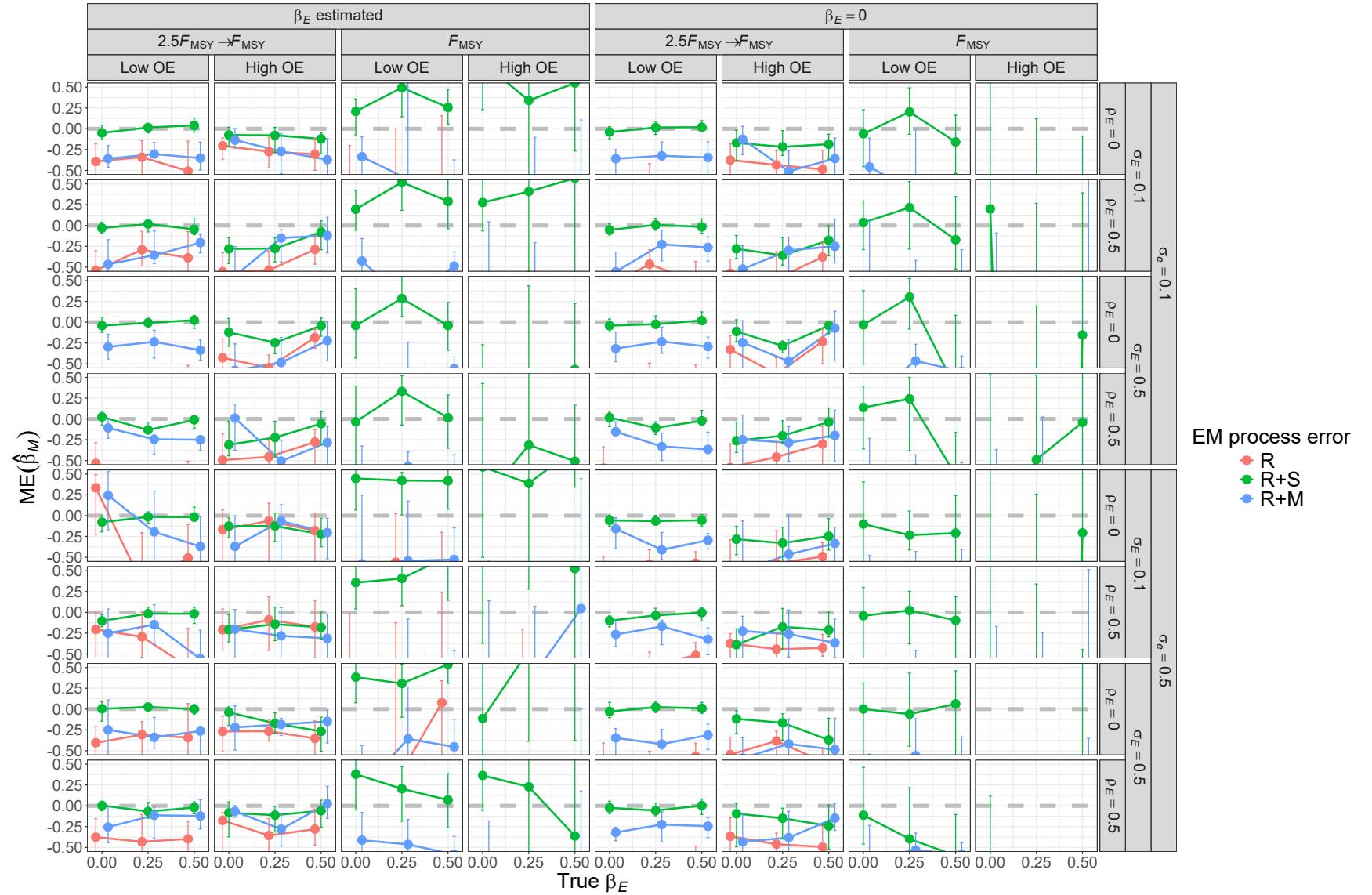


Fig. S26. For R+S OMs, median error (ME) of estimates of  $\beta_M$  from fitting EMs with alternative process error assumptions and treatment of covariate effect ( $\beta_E = 0$  or estimated). Vertical lines represent 95% confidence intervals.

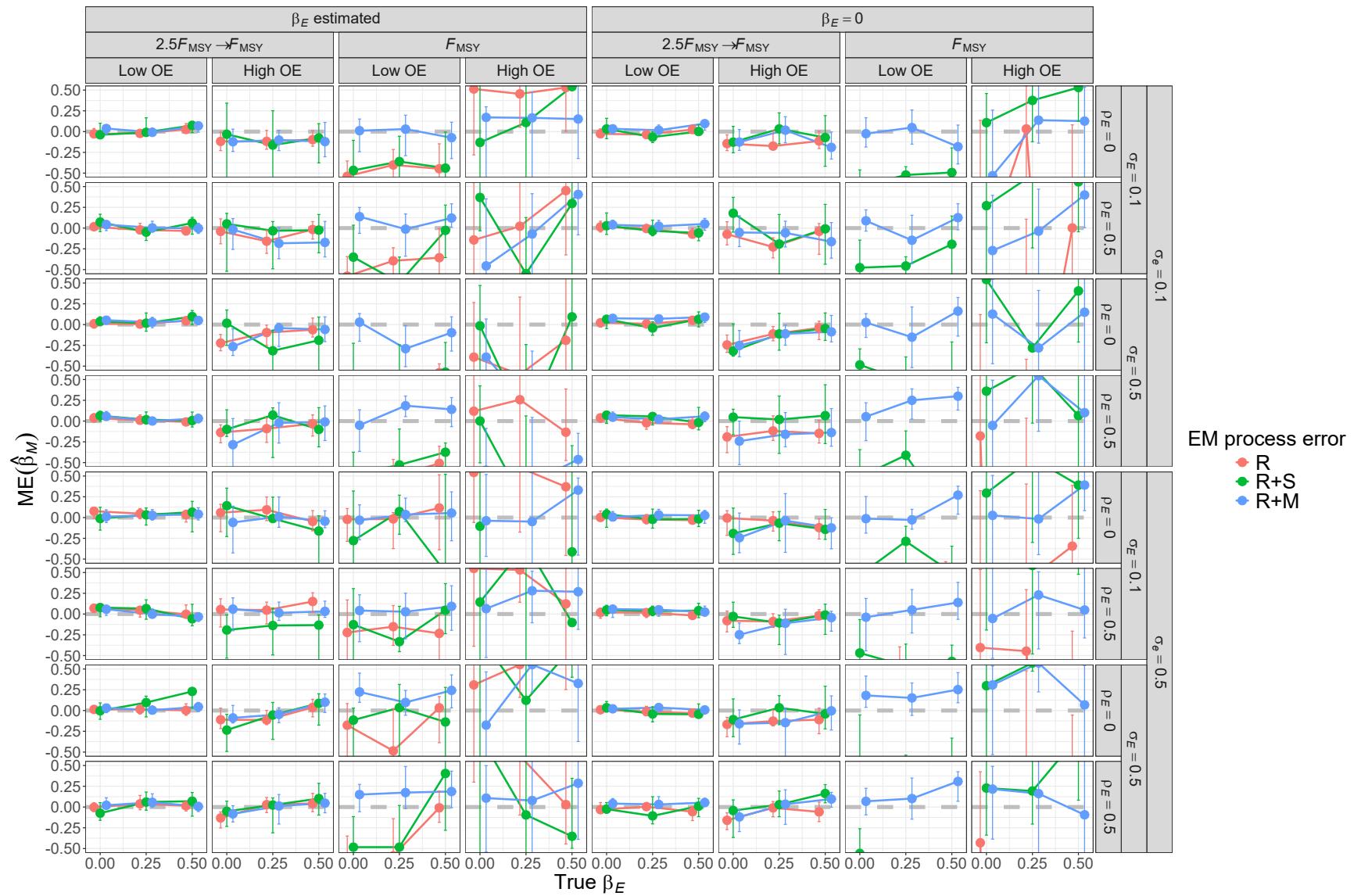


Fig. S27. For R+M OMs, median error (ME) of estimates of  $\beta_M$  from fitting EMs with alternative process error assumptions and treatment of covariate effect ( $\beta_E = 0$  or estimated). Vertical lines represent 95% confidence intervals.

<sup>603</sup> Median natural mortality parameter standard error estimation bias

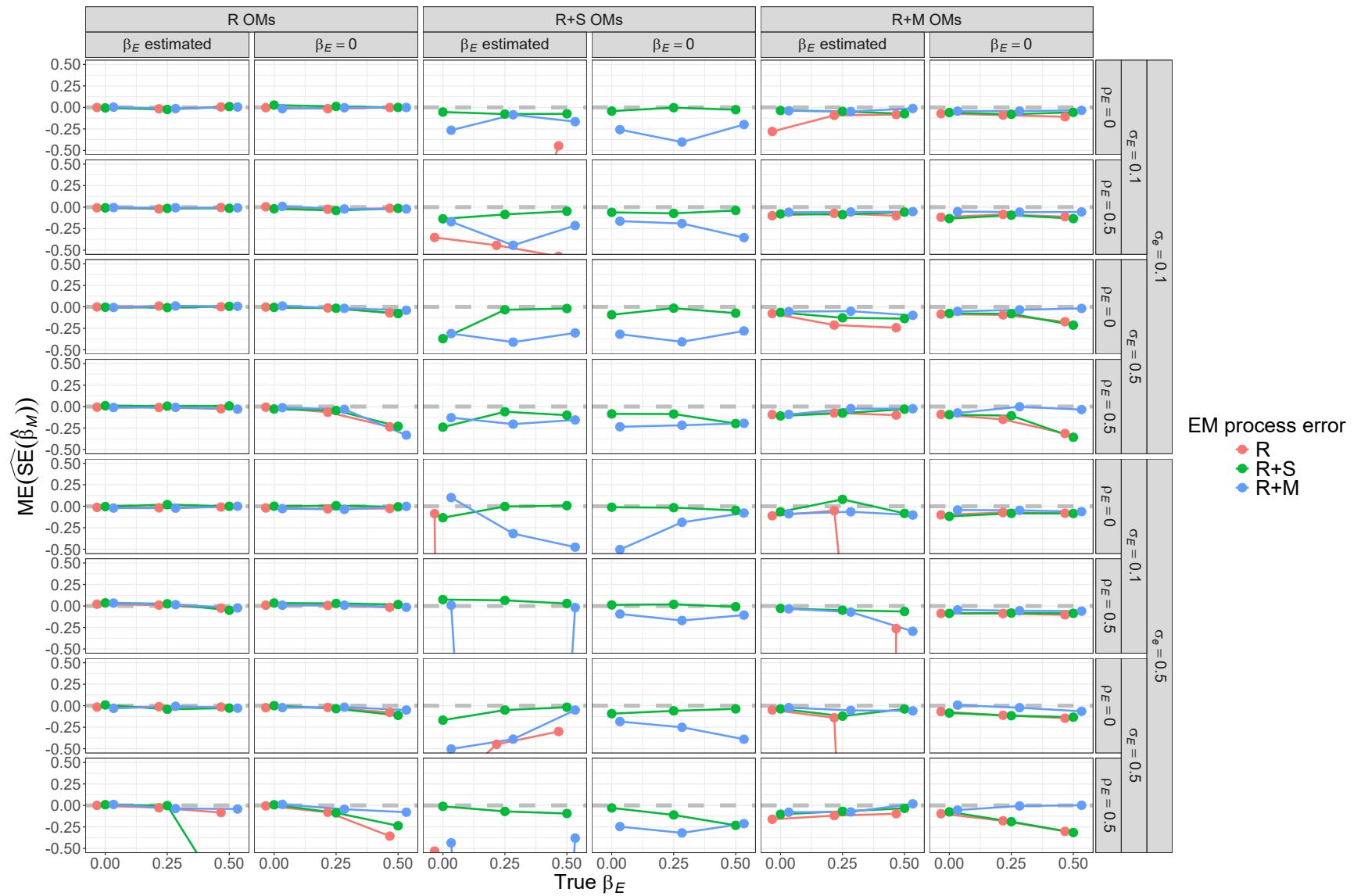


Fig. S28. Median error (ME) of Hessian-based estimates of standard error for median natural mortality parameter  $\beta_M$  from fitting EMs with alternative process error assumptions and treatment of the covariate effect ( $\beta_E = 0$  or estimated). All OM scenarios had low observation error and contrast in fishing mortality. True standard error is defined as the mean of the standard error estimates across converged fits to simulated data sets for a given OM scenario.

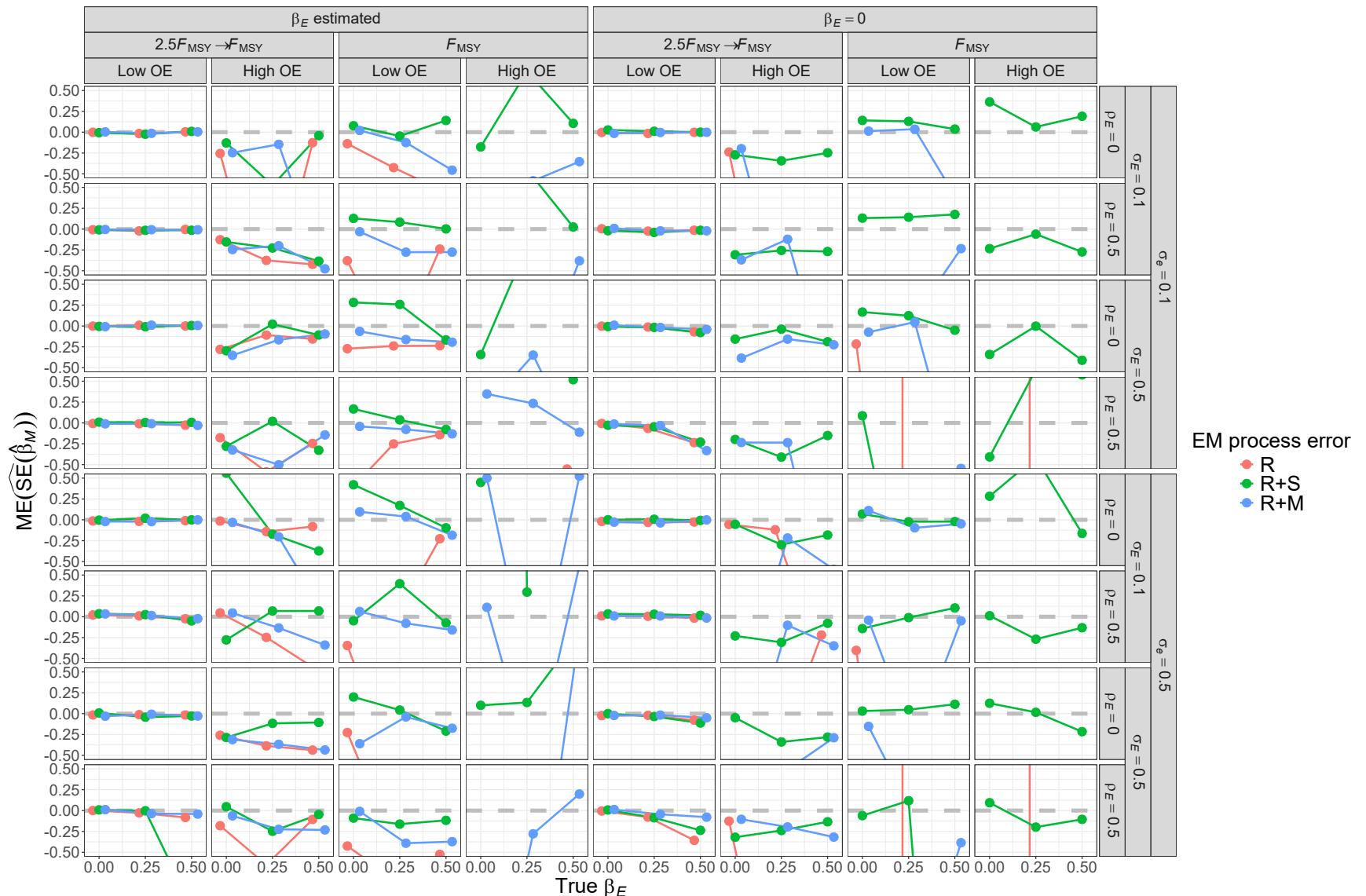


Fig. S29. For R OMs, median error (ME) of Hessian-based estimates of standard error for median natural mortality parameter  $\beta_M$  from fitting EMs with alternative process error assumptions and treatment of the covariate effect ( $\beta_E = 0$  or estimated). True standard error is defined as the mean of the standard error estimates across converged fits to simulated data sets for a given OM scenario.

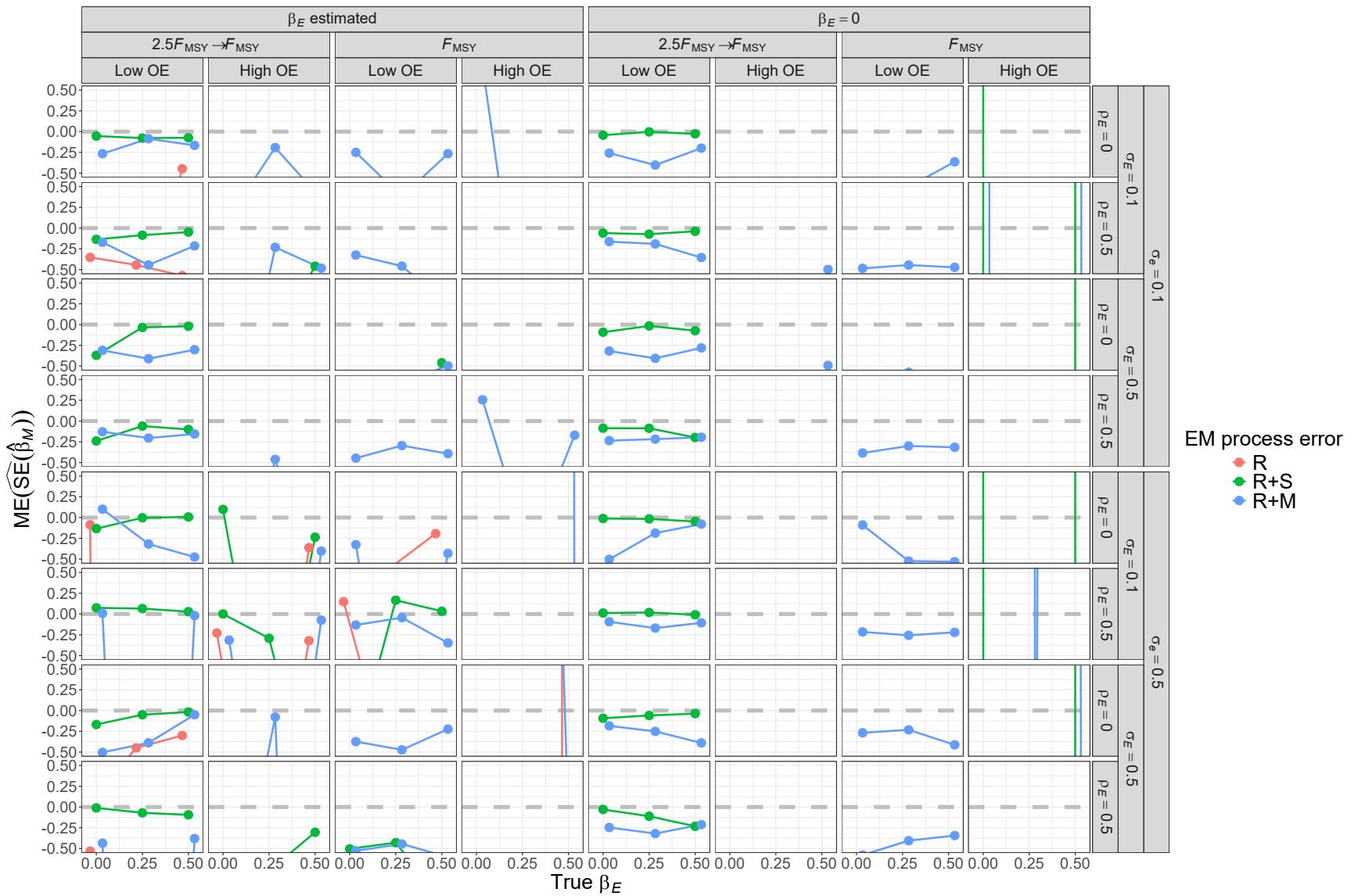


Fig. S30. For R+S OMs, median error (ME) of Hessian-based estimates of standard error for median natural mortality parameter  $\beta_M$  from fitting EMs with alternative process error assumptions and treatment of the covariate effect ( $\beta_E = 0$  or estimated). True standard error is defined as the mean of the standard error estimates across converged fits to simulated data sets for a given OM scenario.

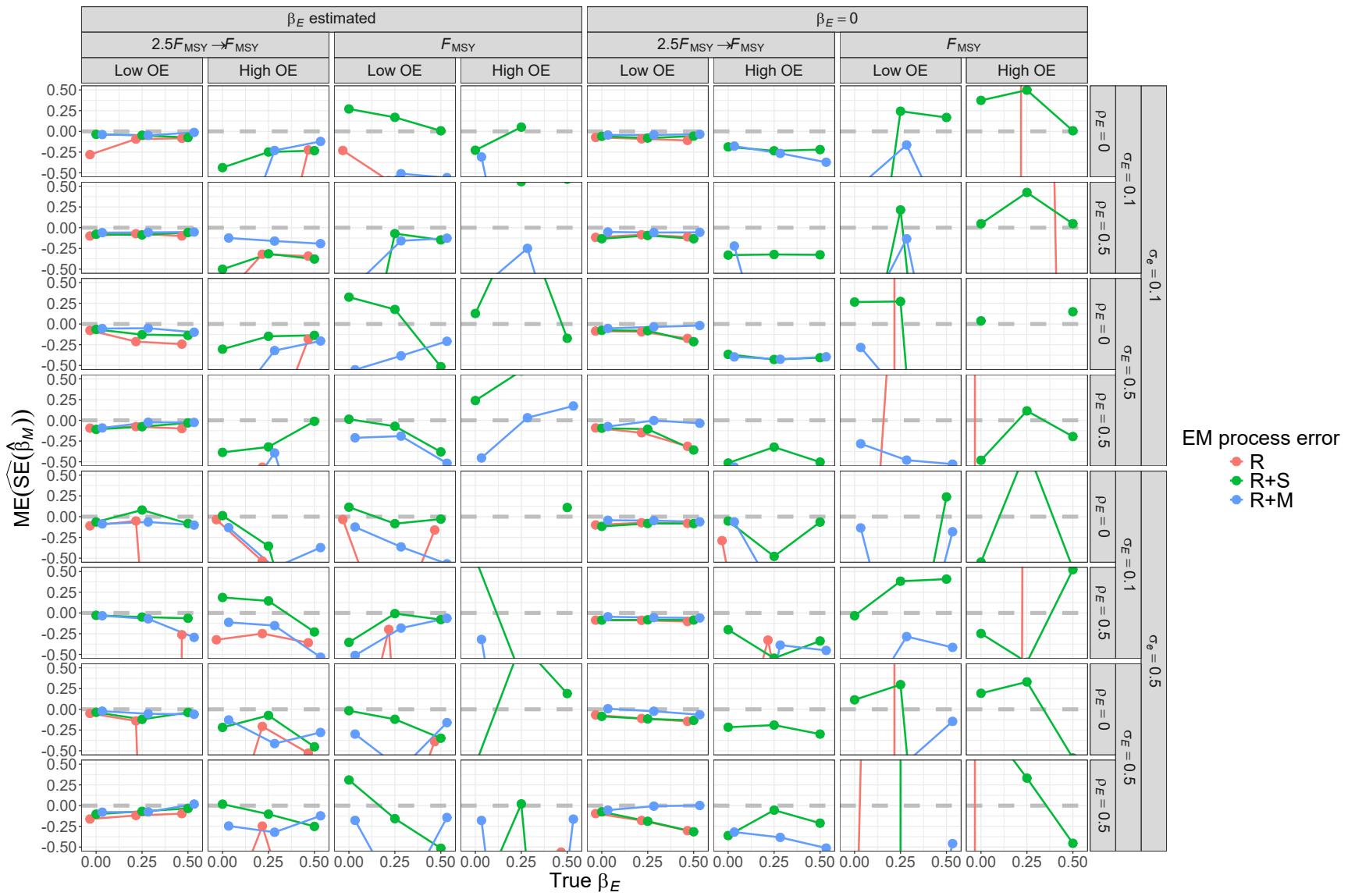


Fig. S31. For R+M OMs, median error (ME) of Hessian-based estimates of standard error for median natural mortality parameter  $\beta_M$  from fitting EMs with alternative process error assumptions and treatment of the covariate effect ( $\beta_E = 0$  or estimated). True standard error is defined as the mean of the standard error estimates across converged fits to simulated data sets for a given OM scenario.

<sup>604</sup> Median Natural mortality parameter confidence interval coverage

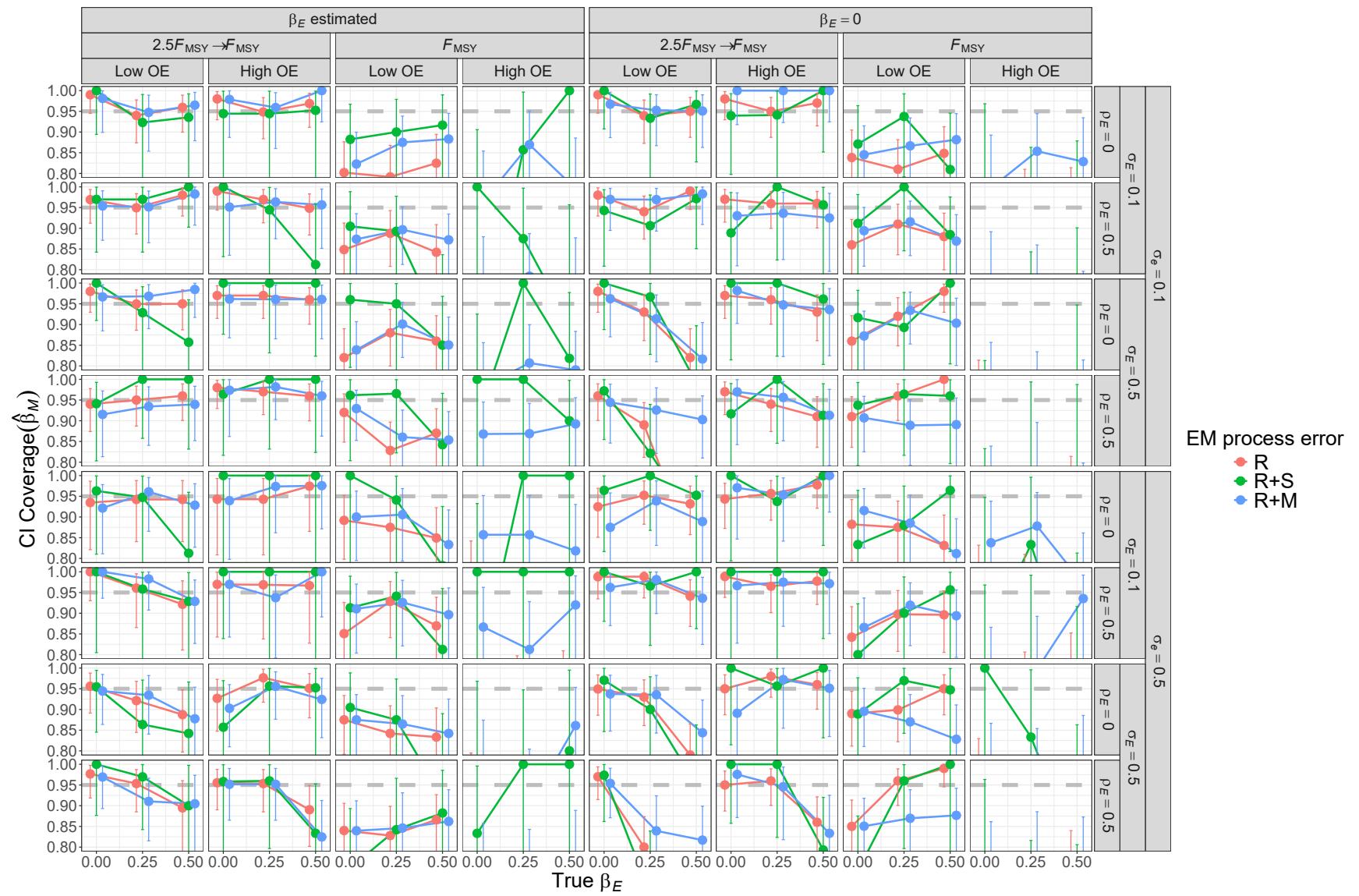


Fig. S32. For R OMs, probability of 95% confidence interval for  $\beta_M$  containing the true value for EMs with alternative process error assumptions and treatment of covariate effect ( $\beta_E = 0$  or estimated). Vertical lines represent 95% confidence intervals.

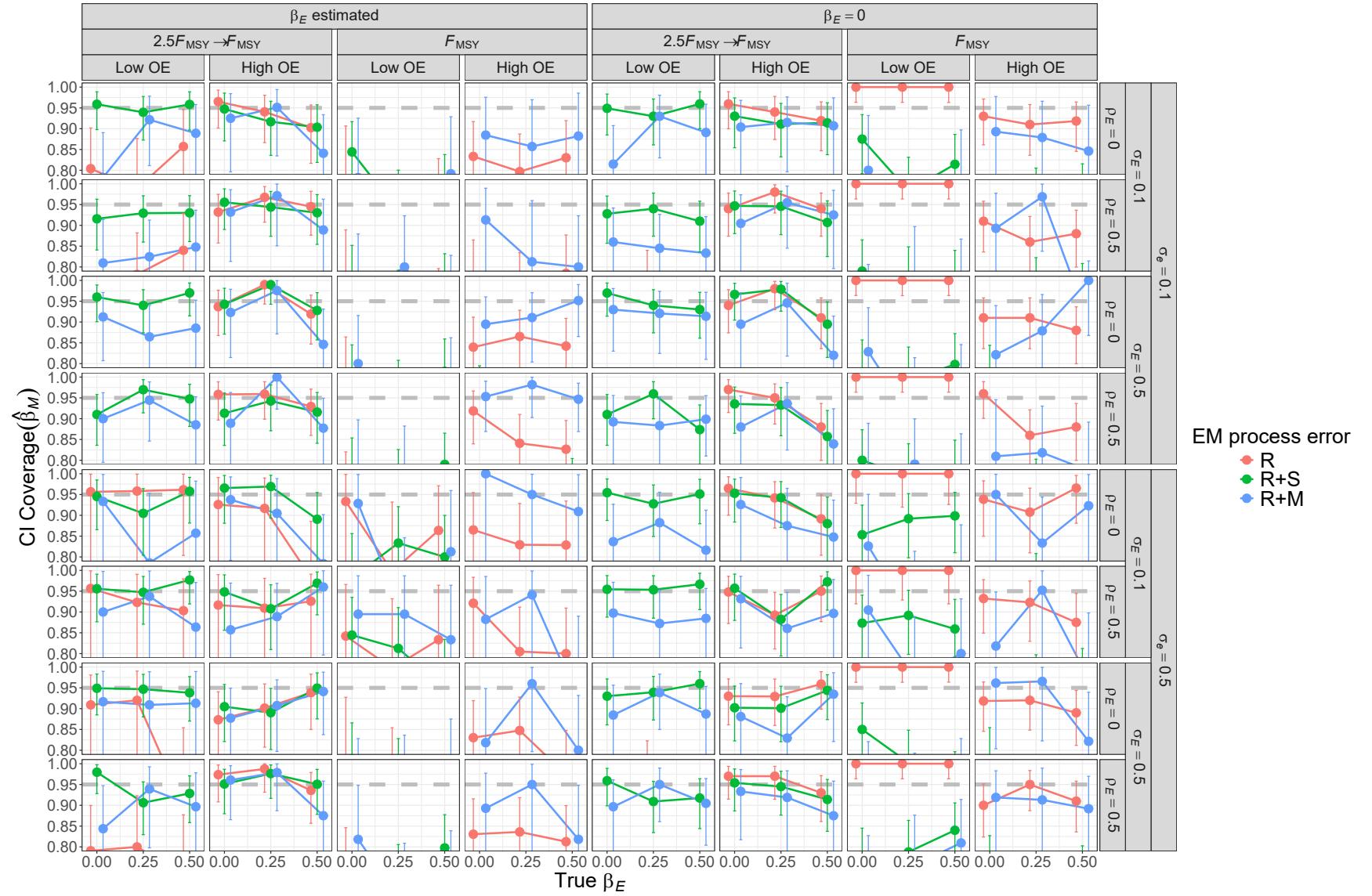


Fig. S33. For R+S OMs, probability of 95% confidence interval for  $\beta_M$  containing the true value for EMs with alternative process error assumptions and treatment of covariate effect ( $\beta_E = 0$  or estimated). Vertical lines represent 95% confidence intervals.

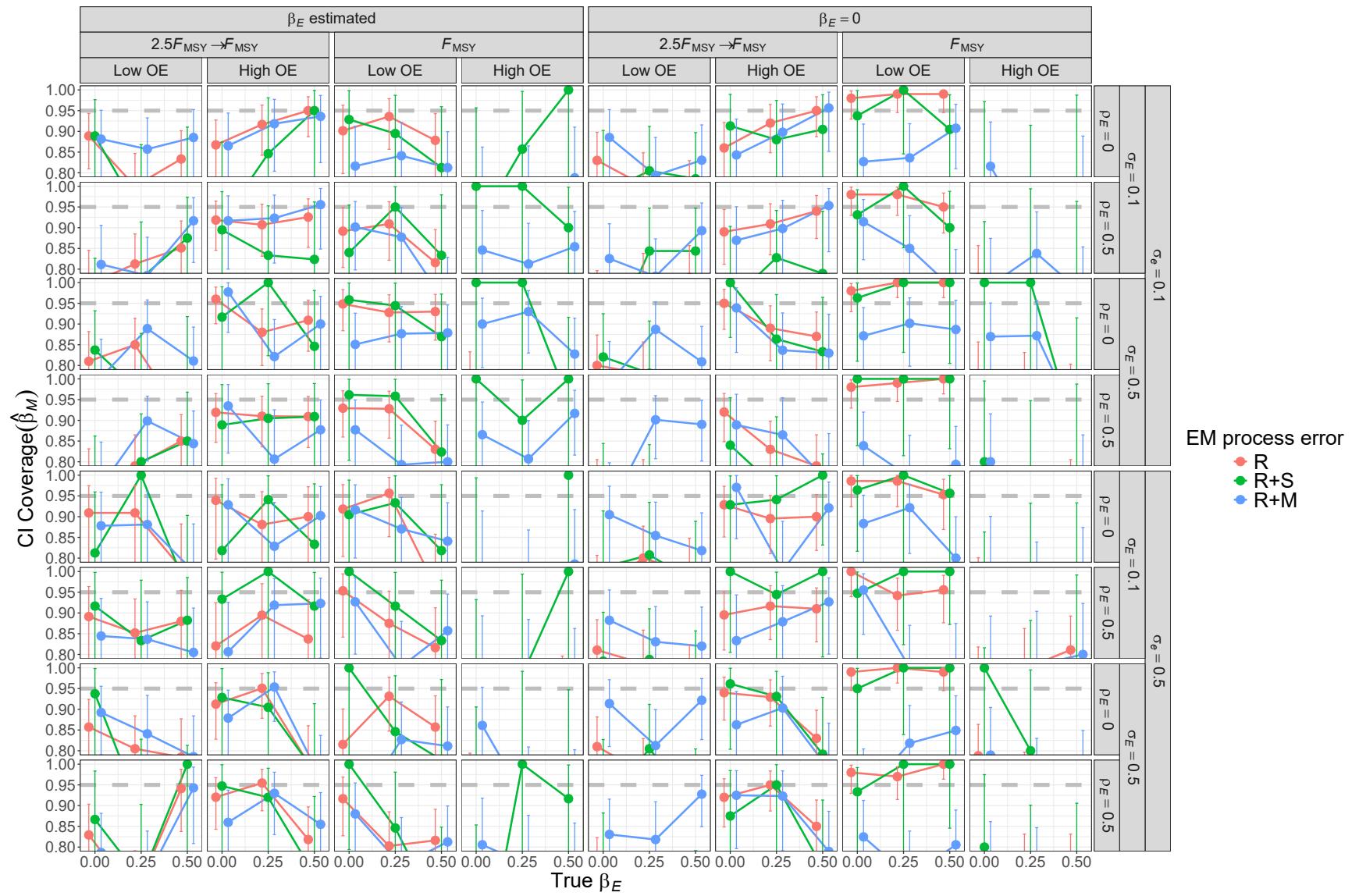


Fig. S34. For R+M OMs, probability of 95% confidence interval for  $\beta_M$  containing the true value for EMs with alternative process error assumptions and treatment of covariate effect ( $\beta_E = 0$  or estimated). Vertical lines represent 95% confidence intervals.

605 Median natural mortality parameter estimate and standard error

606 example

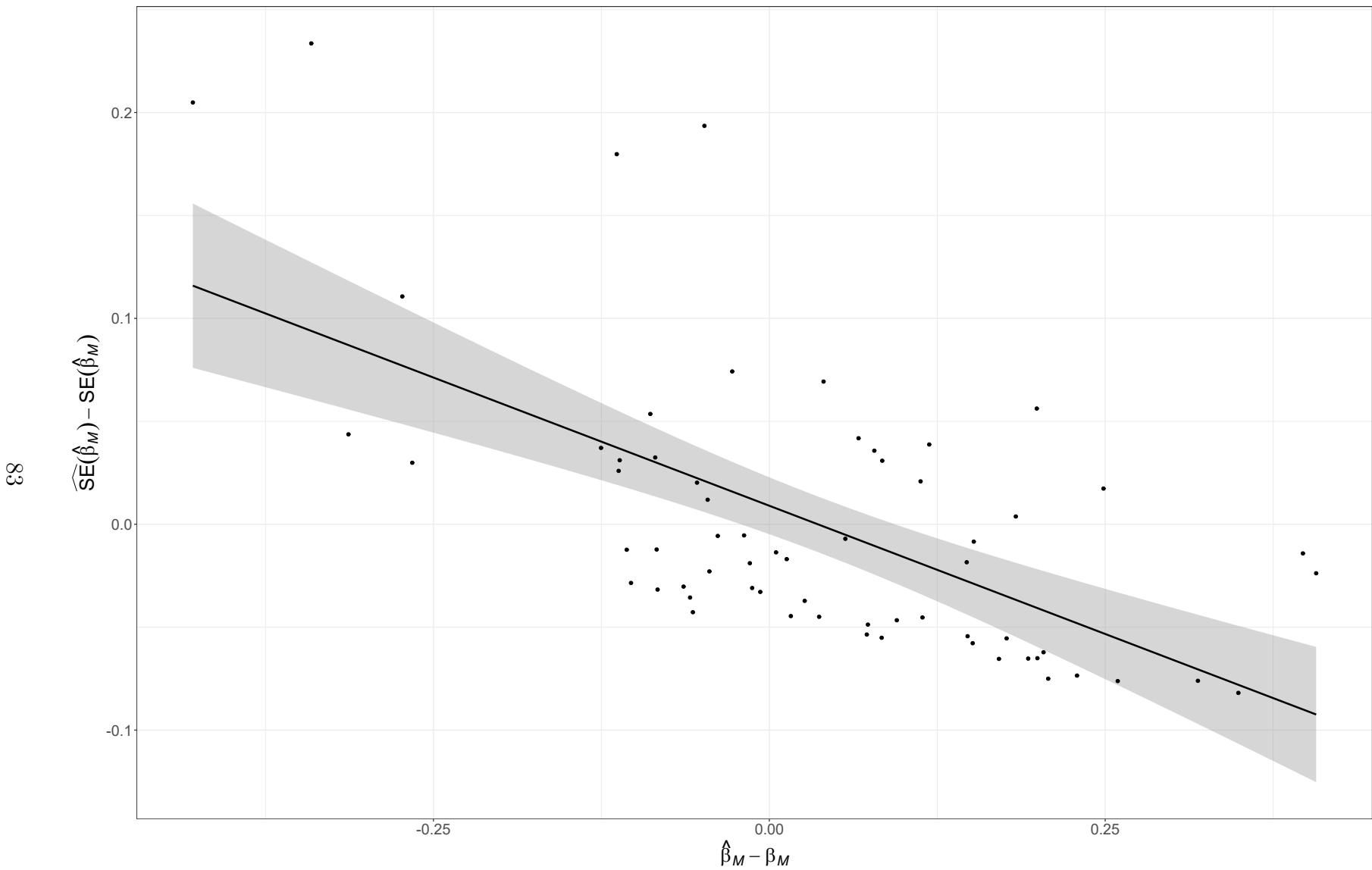


Fig. S35. Negative correlation of  $\beta_M$  estimates and Hessian-based standard error estimates for EM that also estimates the covariate effect and has correct R+M process error assumption fitted to simulated data from the OM with R+M process errors, temporal contrast in fishing pressure, low observation uncertainty for both population (*LowOE*) and covariate observations ( $\sigma_e = 0.1$ ), high and uncorrelated temporal variability in the true covariate ( $\sigma_E = 0.5$  and  $\rho_E = 0$ ), and the strongest covariate effect on natural mortality ( $\beta_E = 0.5$ ).

<sub>607</sub> Median Natural mortality parameter RMSE

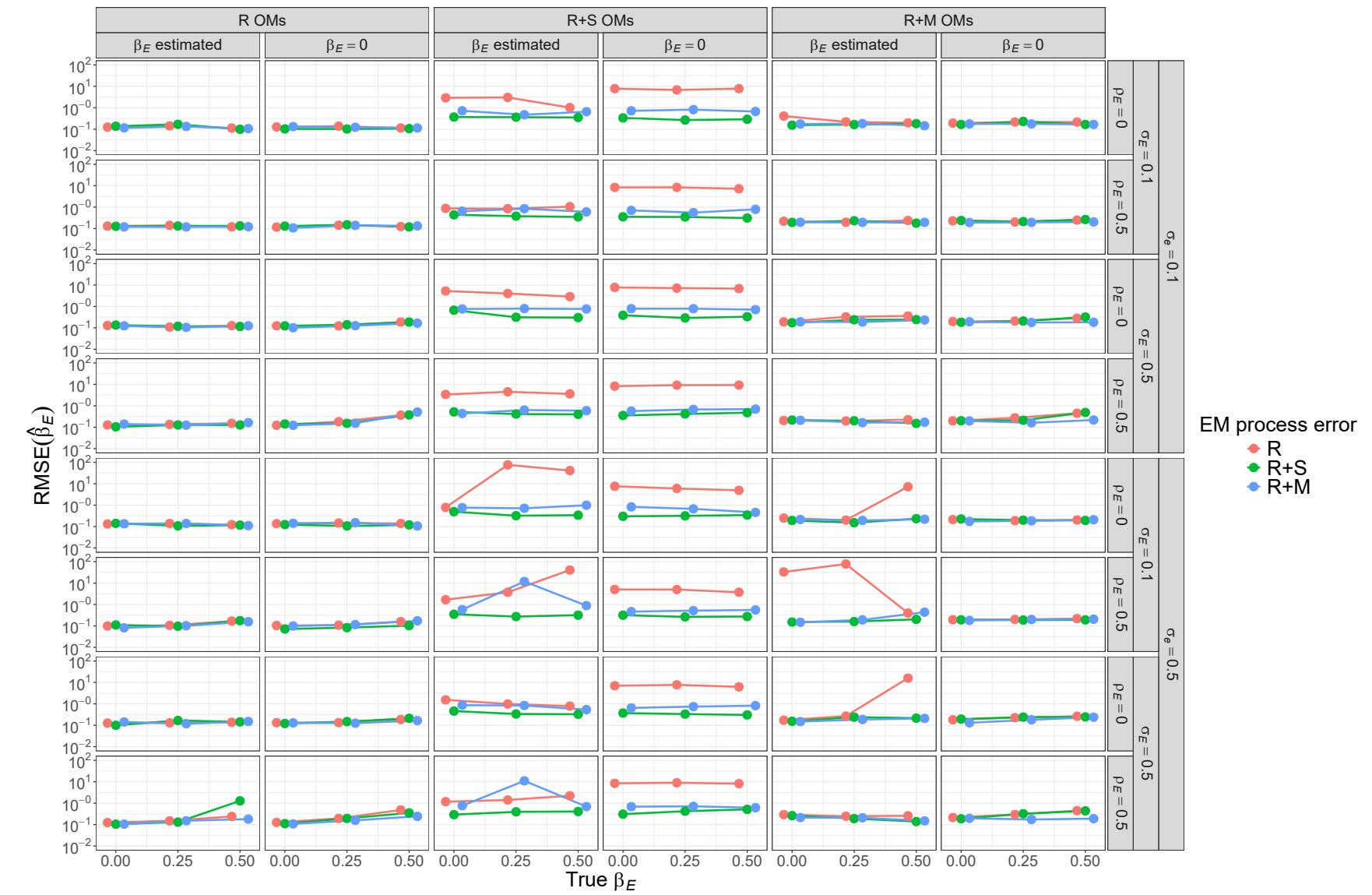


Fig. S36. Root mean square error (RMSE) of estimates of  $\beta_M$  from fitting EMs with alternative process error assumptions and treatment of covariate effect ( $\beta_E = 0$  or estimated). All OMs had low observation error for population observations and contrast in fishing mortality.

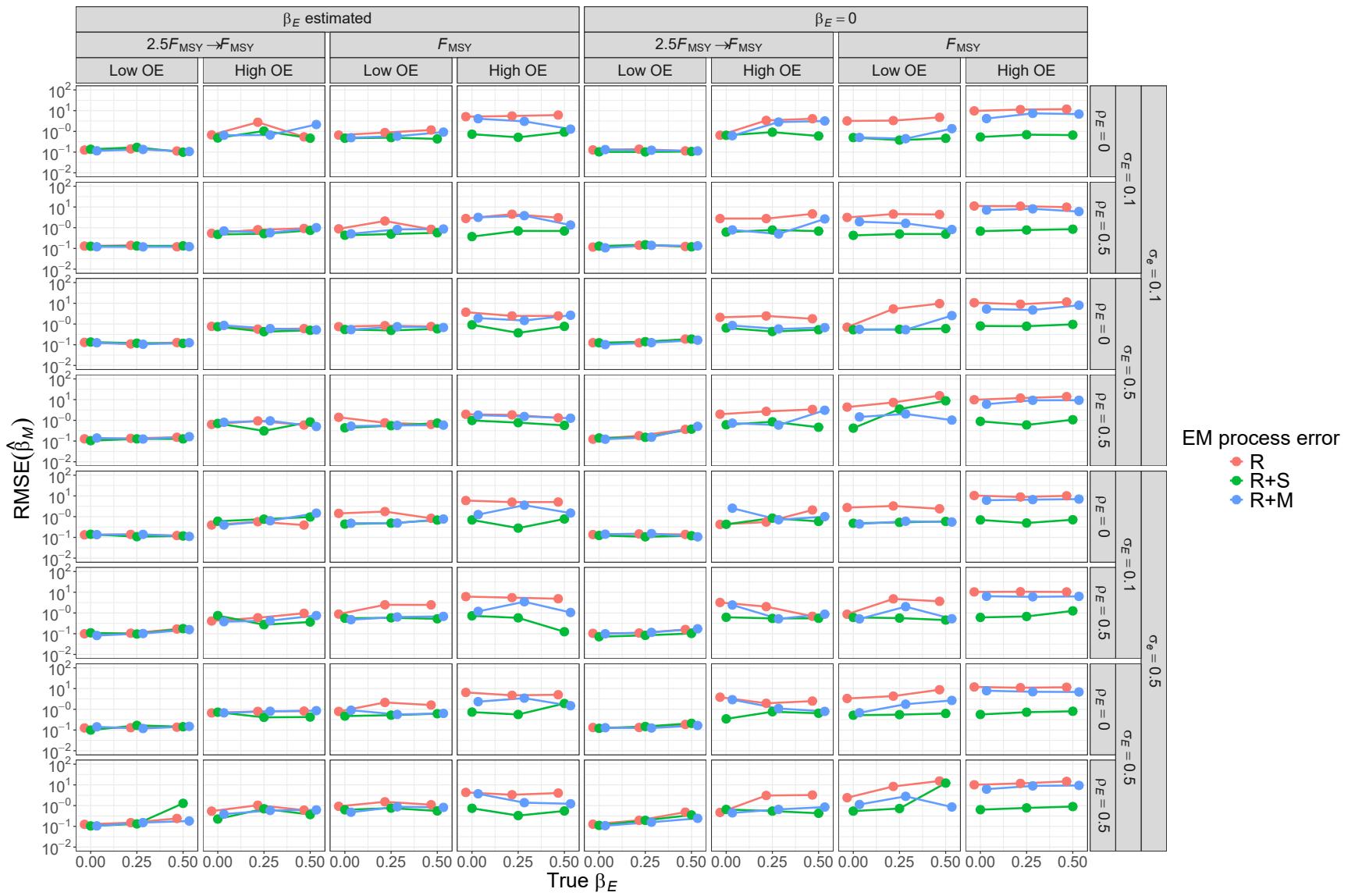


Fig. S37. For R OMs, root mean square error (RMSE) of estimates of  $\beta_M$  from fitting EMs with alternative process error assumptions and treatment of covariate effect ( $\beta_E = 0$  or estimated).

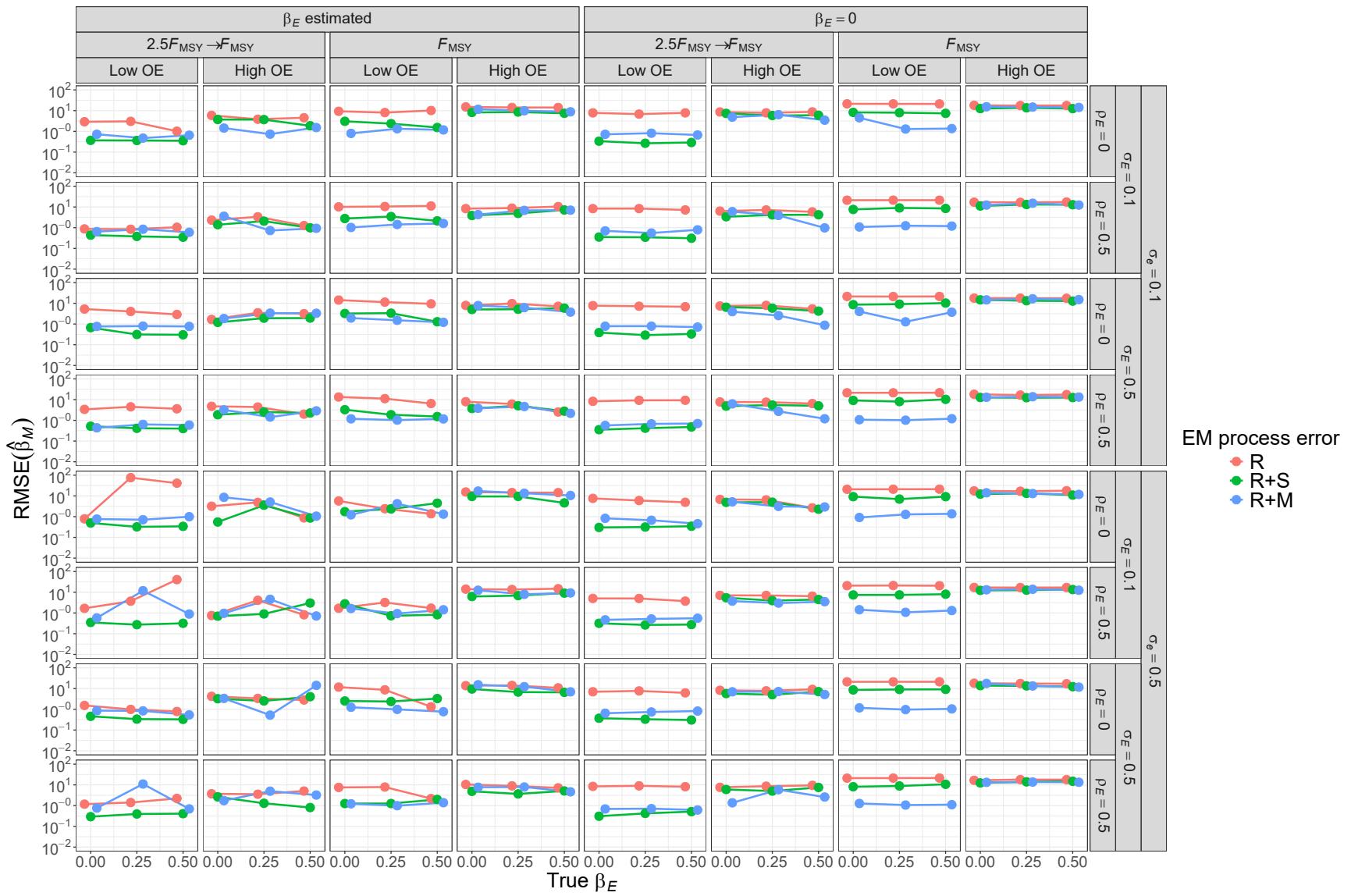


Fig. S38. For R+S OMs, root mean square error (RMSE) of estimates of  $\beta_M$  from fitting EMs with alternative process error assumptions and treatment of covariate effect ( $\beta_E = 0$  or estimated).

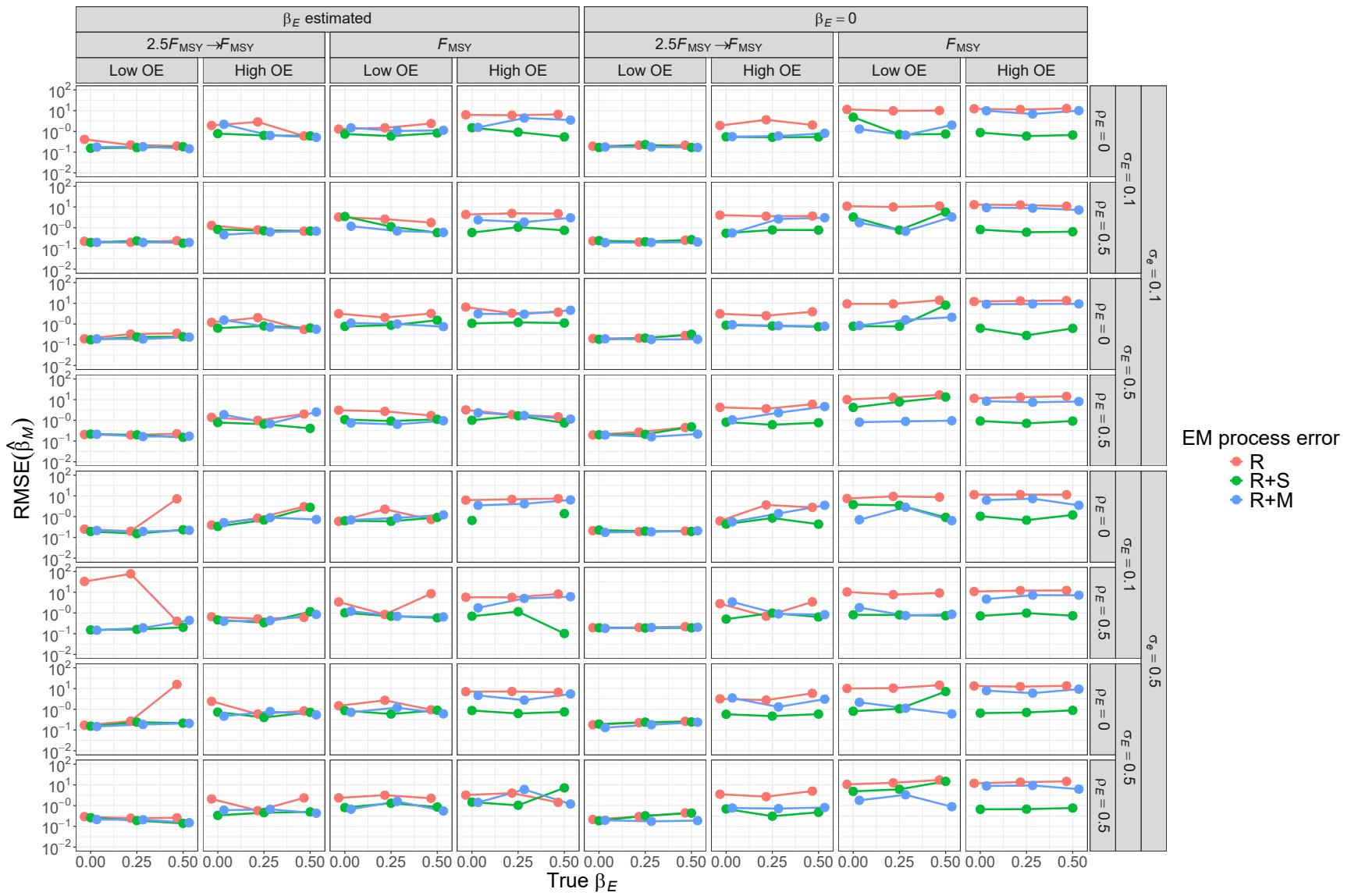


Fig. S39. For R+M OMs, root mean square error (RMSE) of estimates of  $\beta_M$  from fitting EMs with alternative process error assumptions and treatment of covariate effect ( $\beta_E = 0$  or estimated).

608 Terminal year natural mortality bias

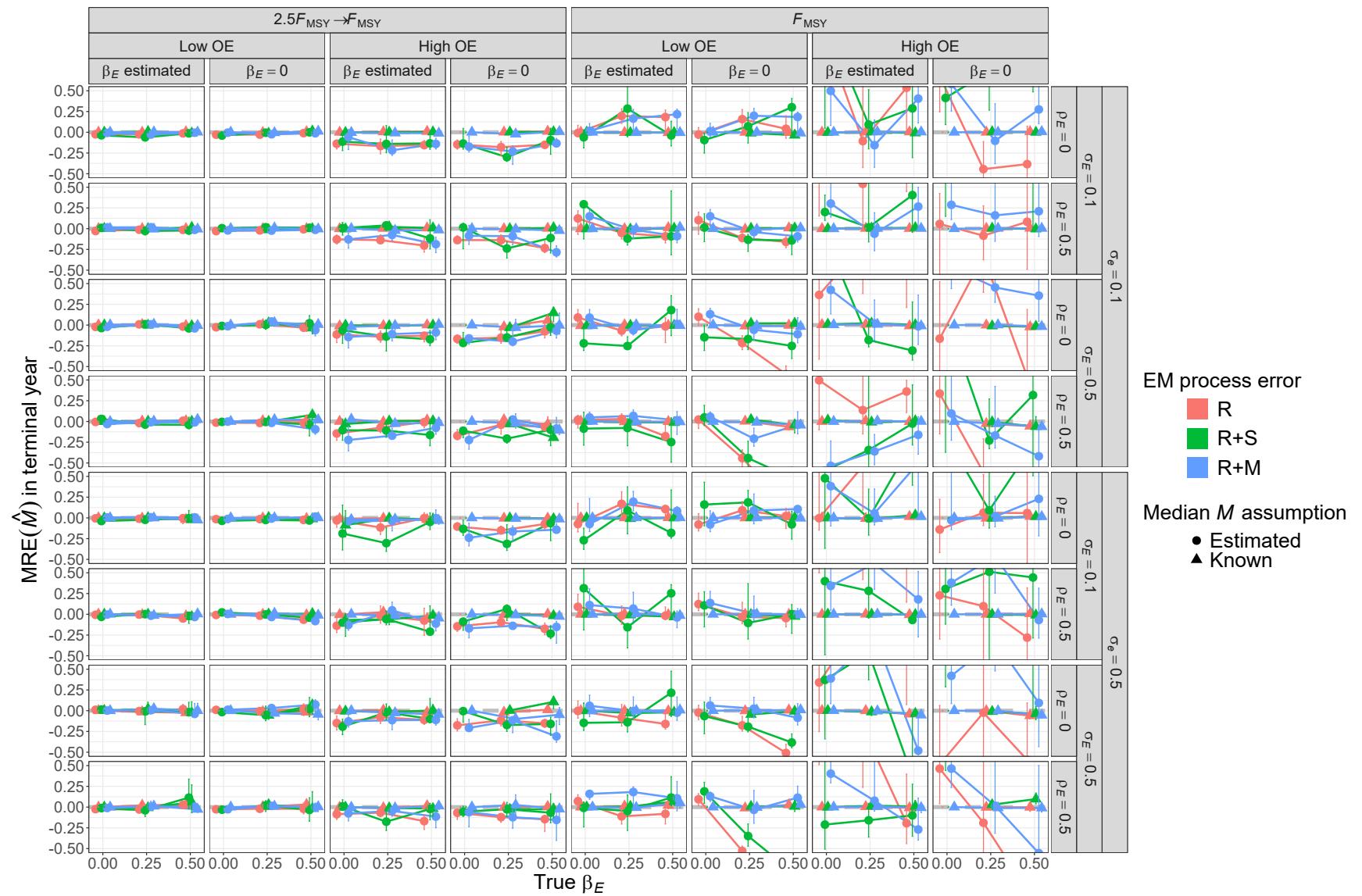


Fig. S40. For R OMs, median relative error (MRE) of estimates of natural mortality rate in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known).

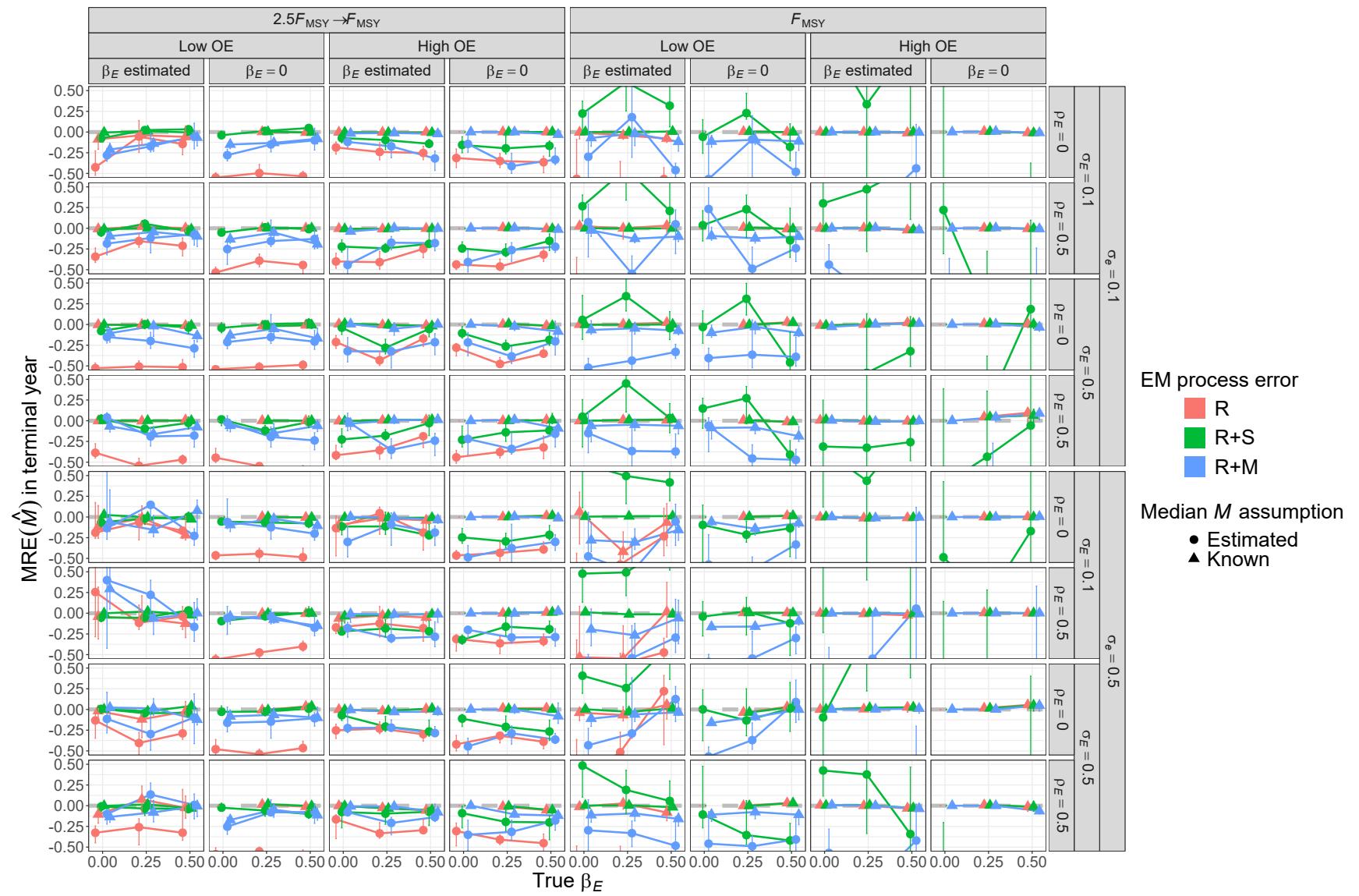


Fig. S41. For R+S OMs, median relative error (MRE) of estimates of natural mortality rate in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known).

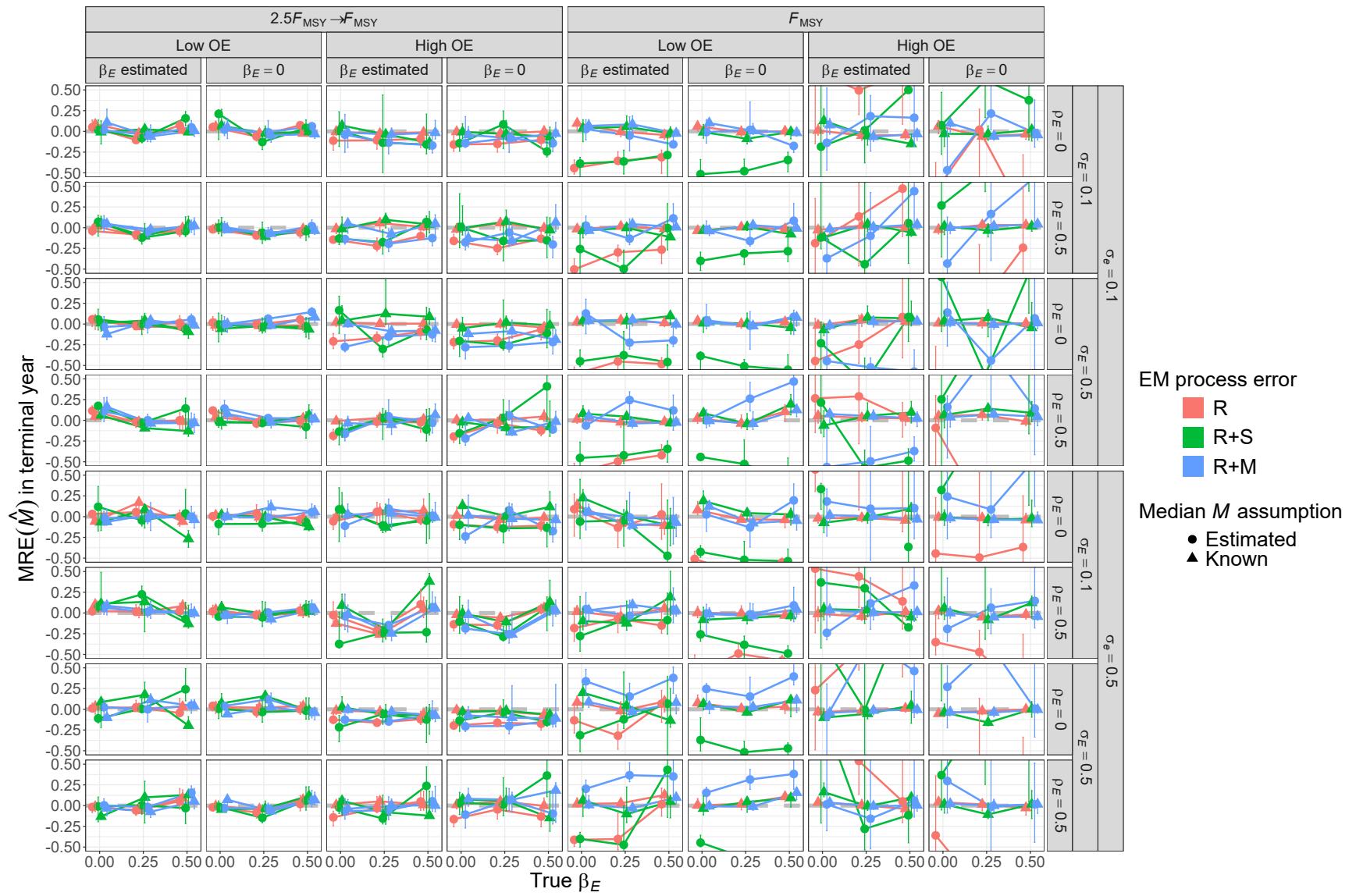


Fig. S42. For R+M OMs, median relative error (MRE) of estimates of natural mortality rate in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known).

<sup>609</sup> Terminal year natural mortality RMSE

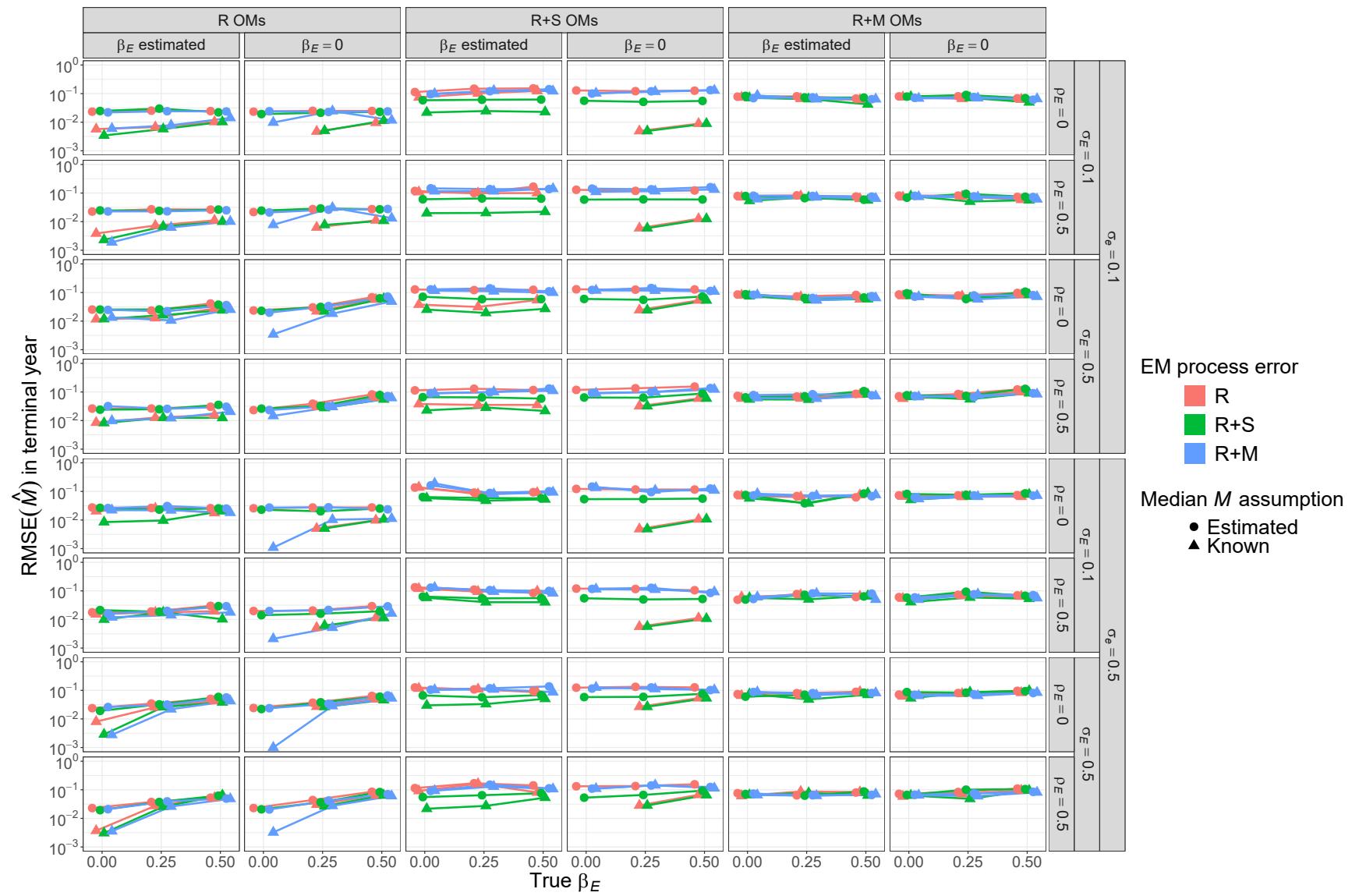


Fig. S43. Root mean square error (RMSE) of estimates of natural mortality rate in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known). All OMs had low population observation error and contrast in fishing mortality.

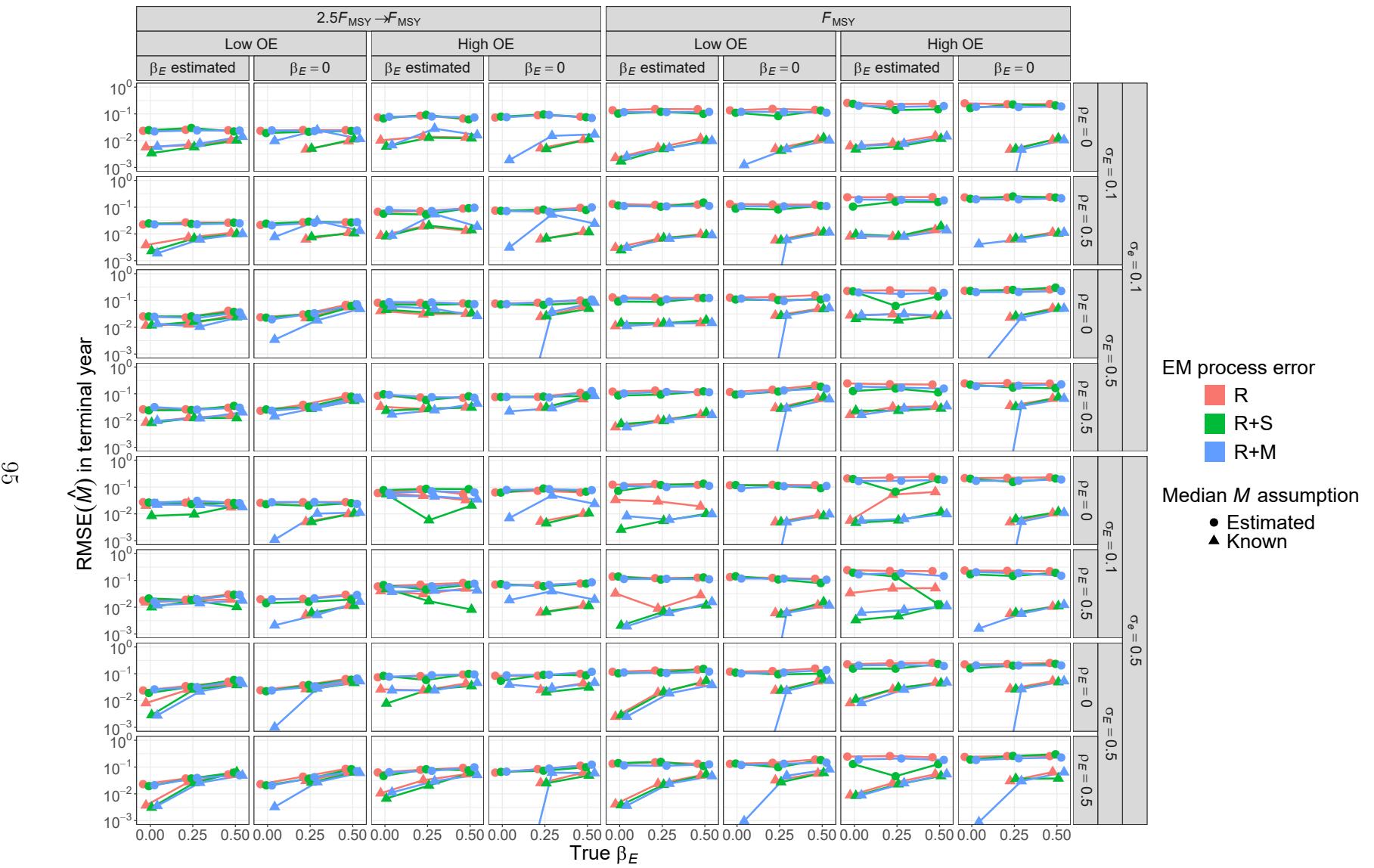


Fig. S44. For R OMs, root mean square error (RMSE) of estimates of natural mortality rate in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known).

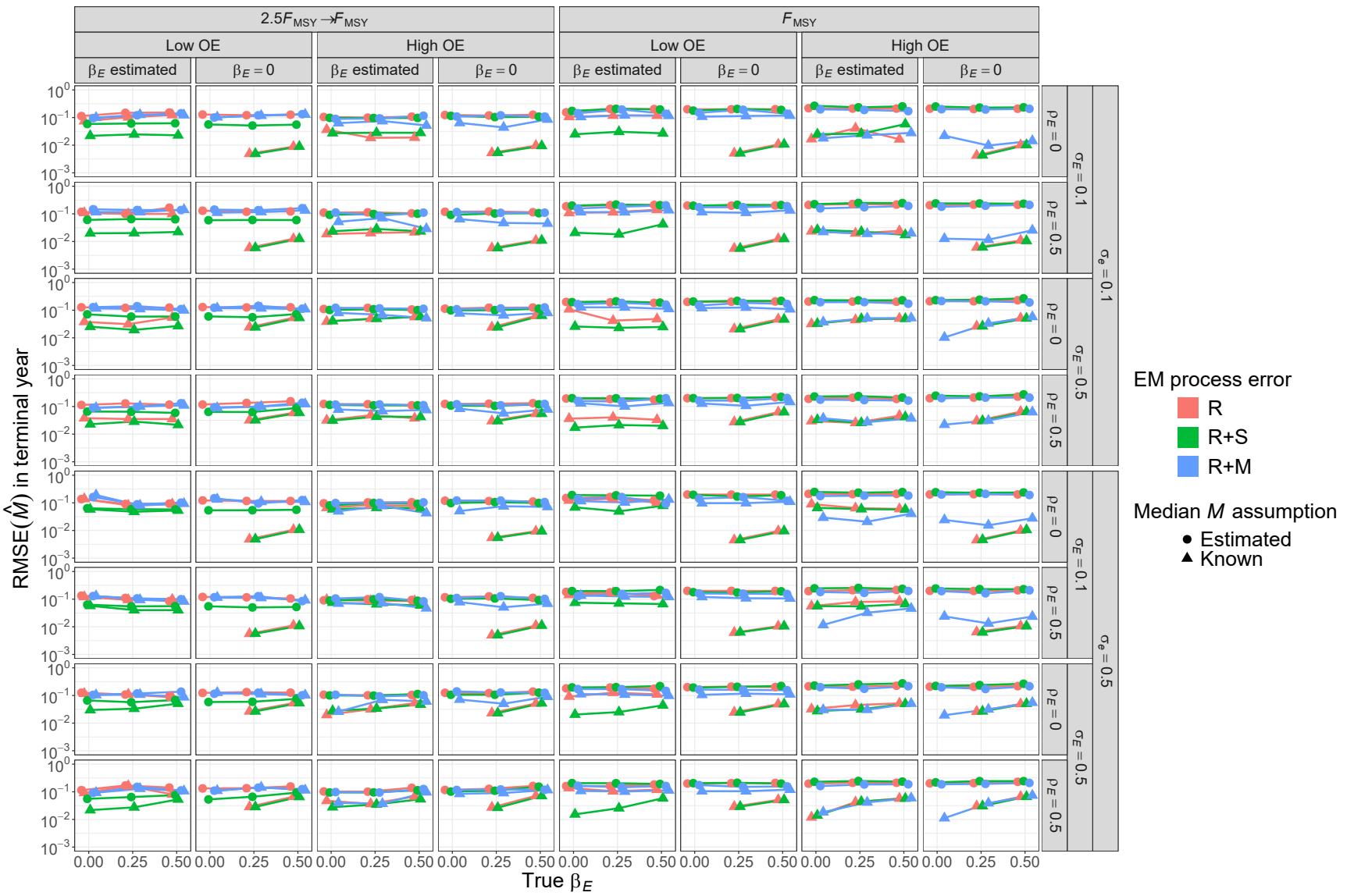


Fig. S45. For R+S OMs, root mean square error (RMSE) of estimates of natural mortality rate in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known).

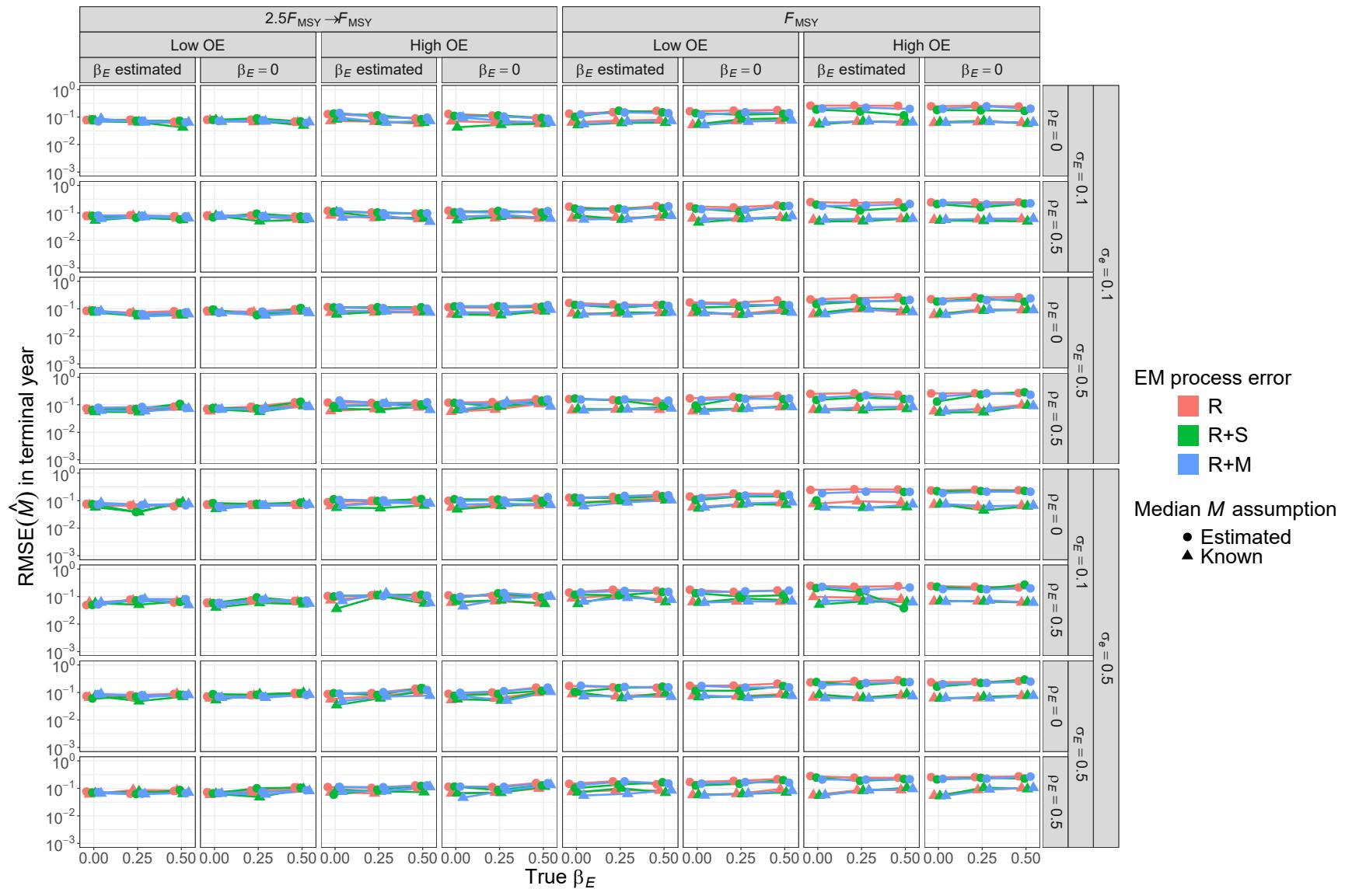


Fig. S46. For R+M OMs, root mean square error (RMSE) of estimates of natural mortality rate in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known).

<sup>610</sup> Terminal year spawning stock biomass bias

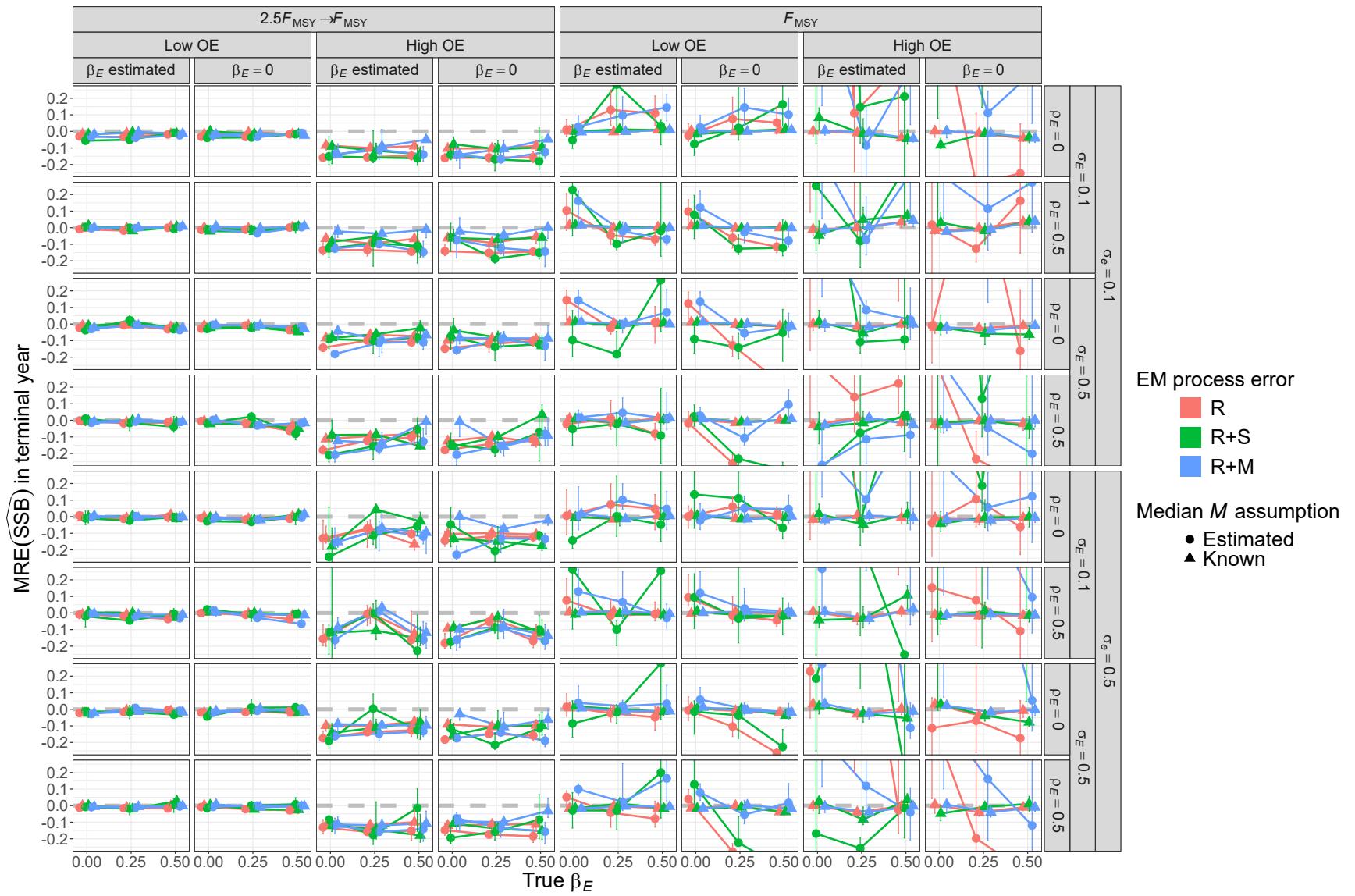


Fig. S47. For R OMs, median relative error (MRE) of estimates of spawning stock biomass (SSB) in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known).

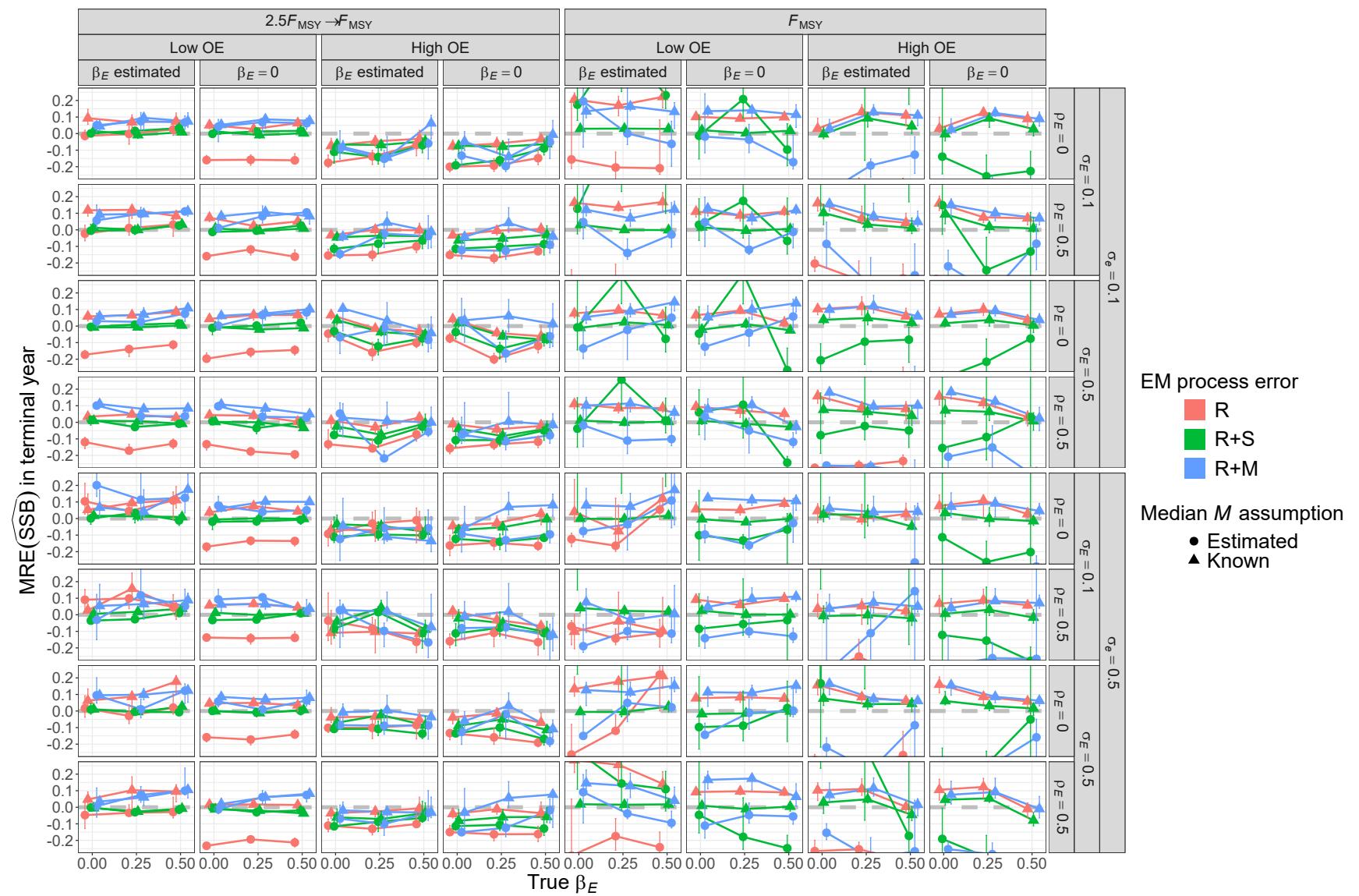


Fig. S48. For R+S OMs, median relative error (MRE) of estimates of spawning stock biomass (SSB) in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known).

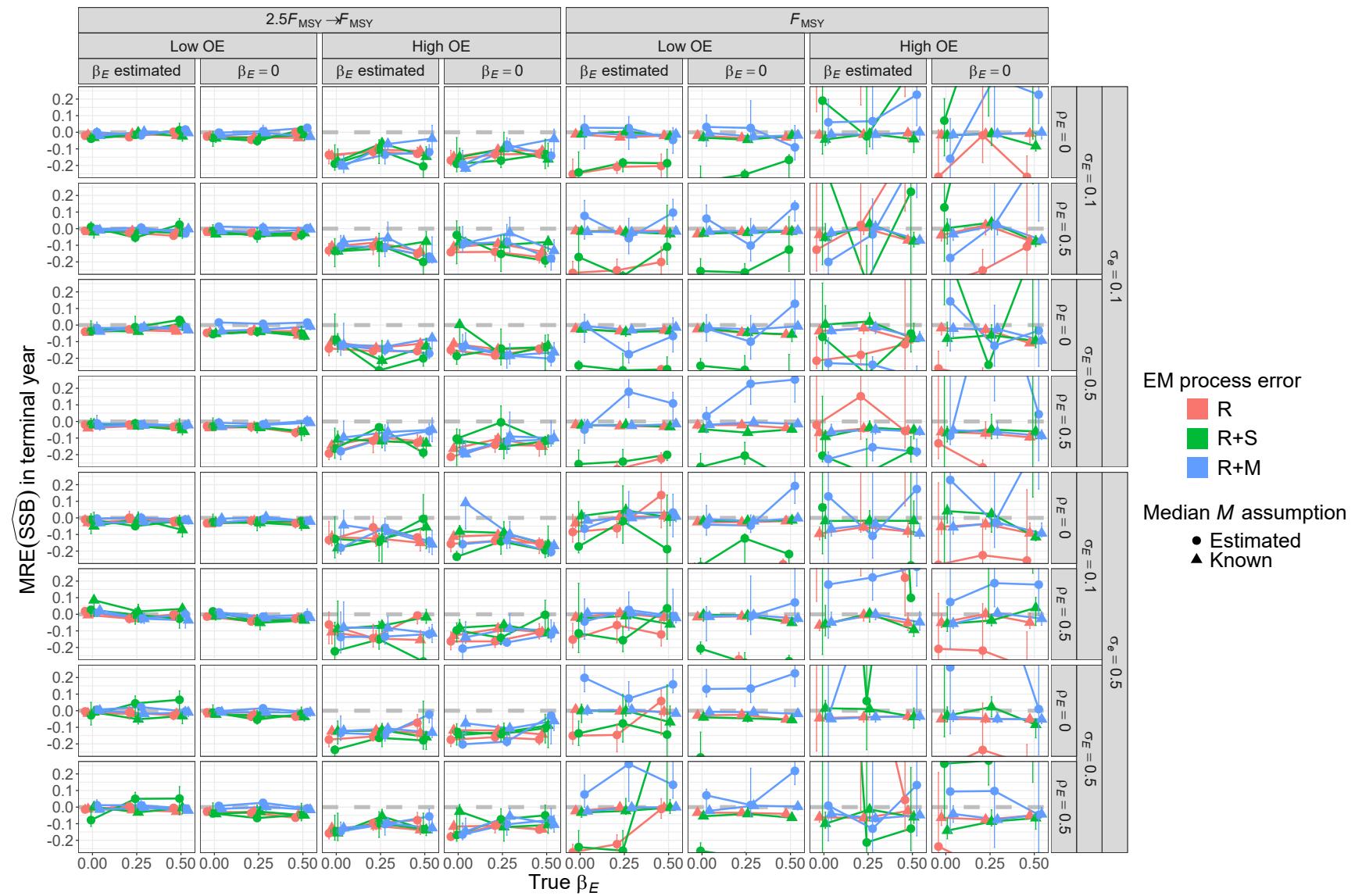


Fig. S49. For R+M OMs, median relative error (MRE) of estimates of spawning stock biomass (SSB) in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known).

<sup>611</sup> Terminal year spawning stock biomass RMSE

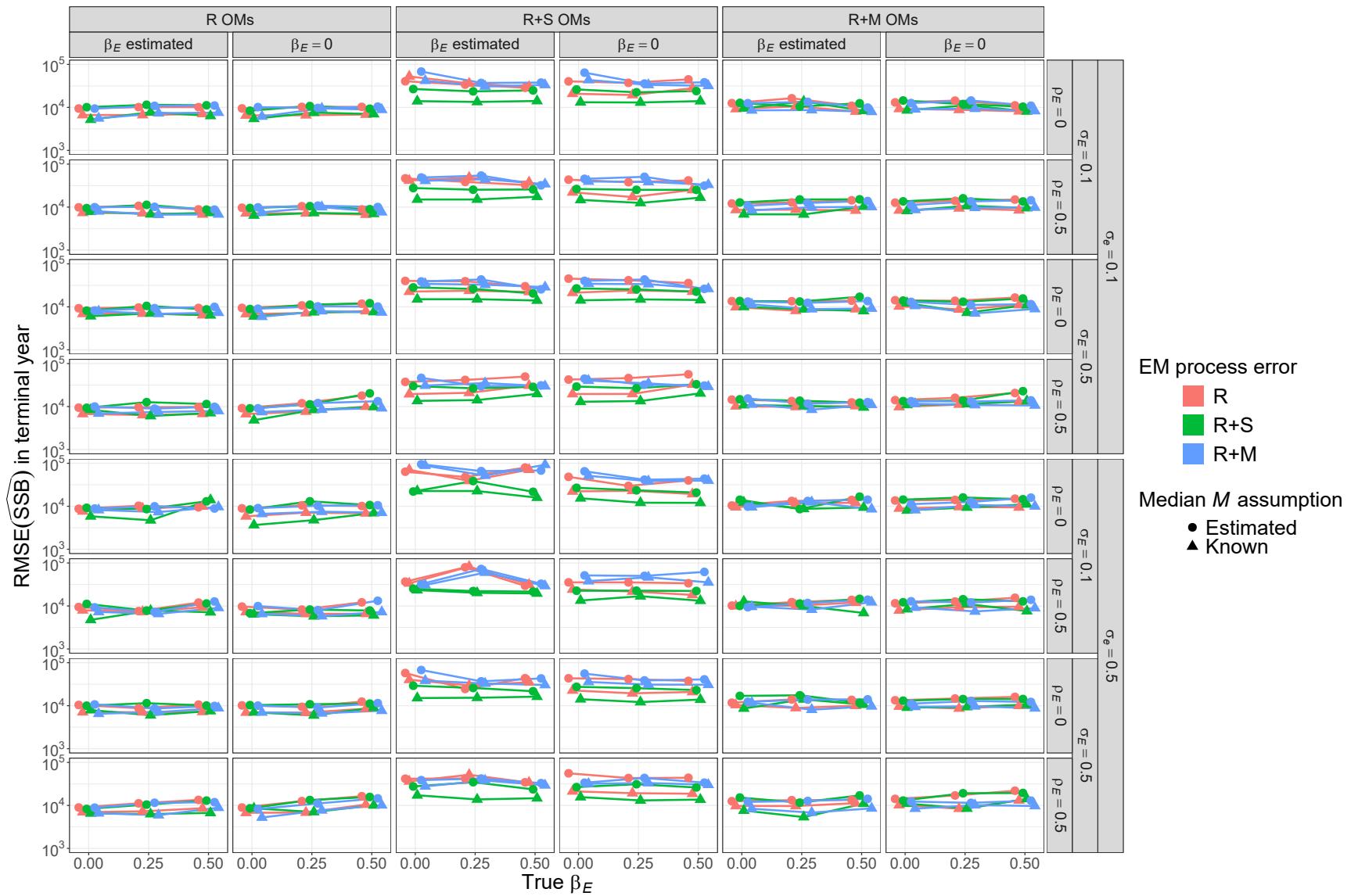


Fig. S50. Root mean square error (RMSE) of estimates of spawning stock biomass in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known). All OMs had low population observation error and contrast in fishing mortality.

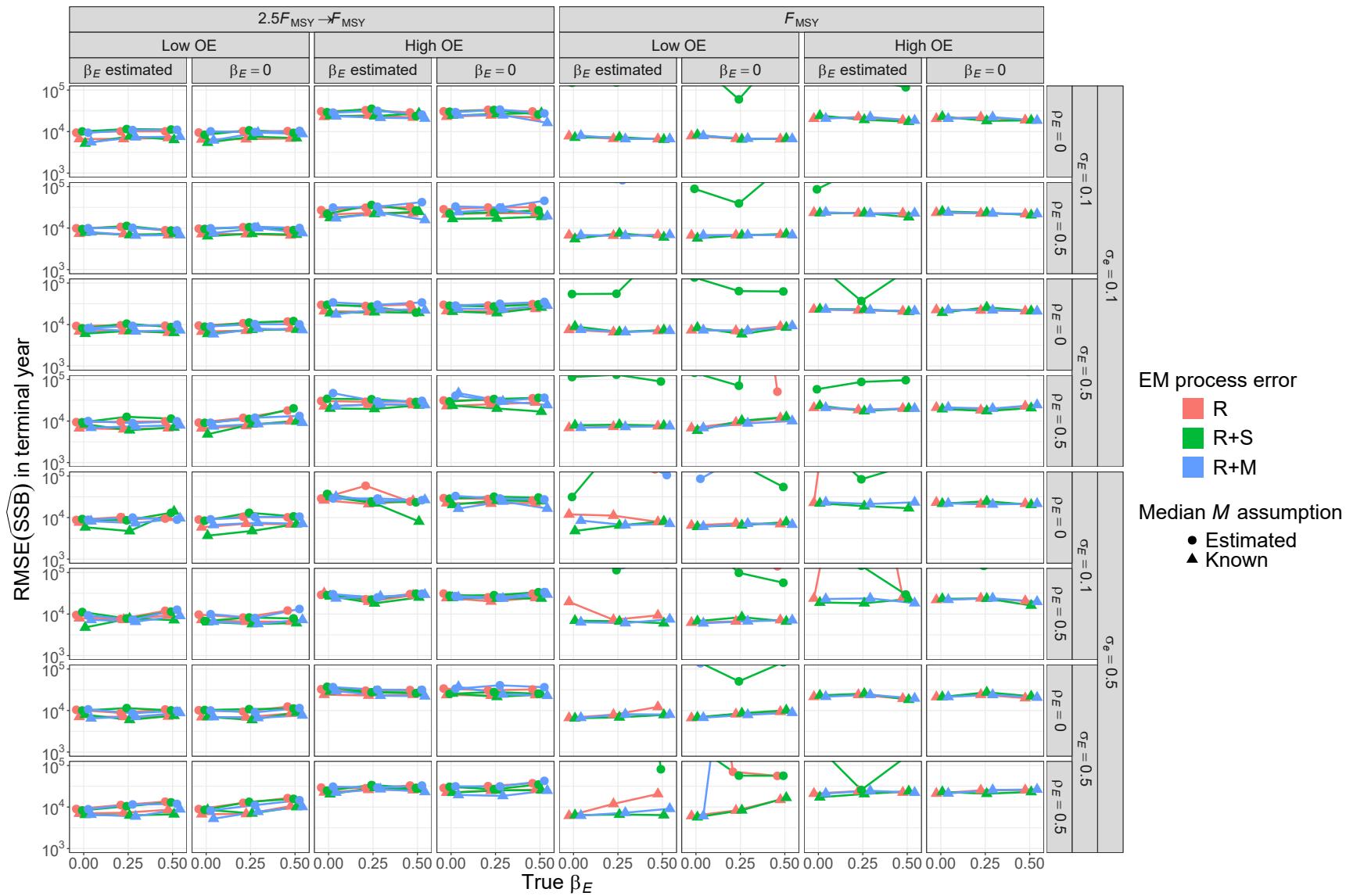


Fig. S51. For R OMs, root mean square error (RMSE) of estimates of spawning stock biomass in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known).

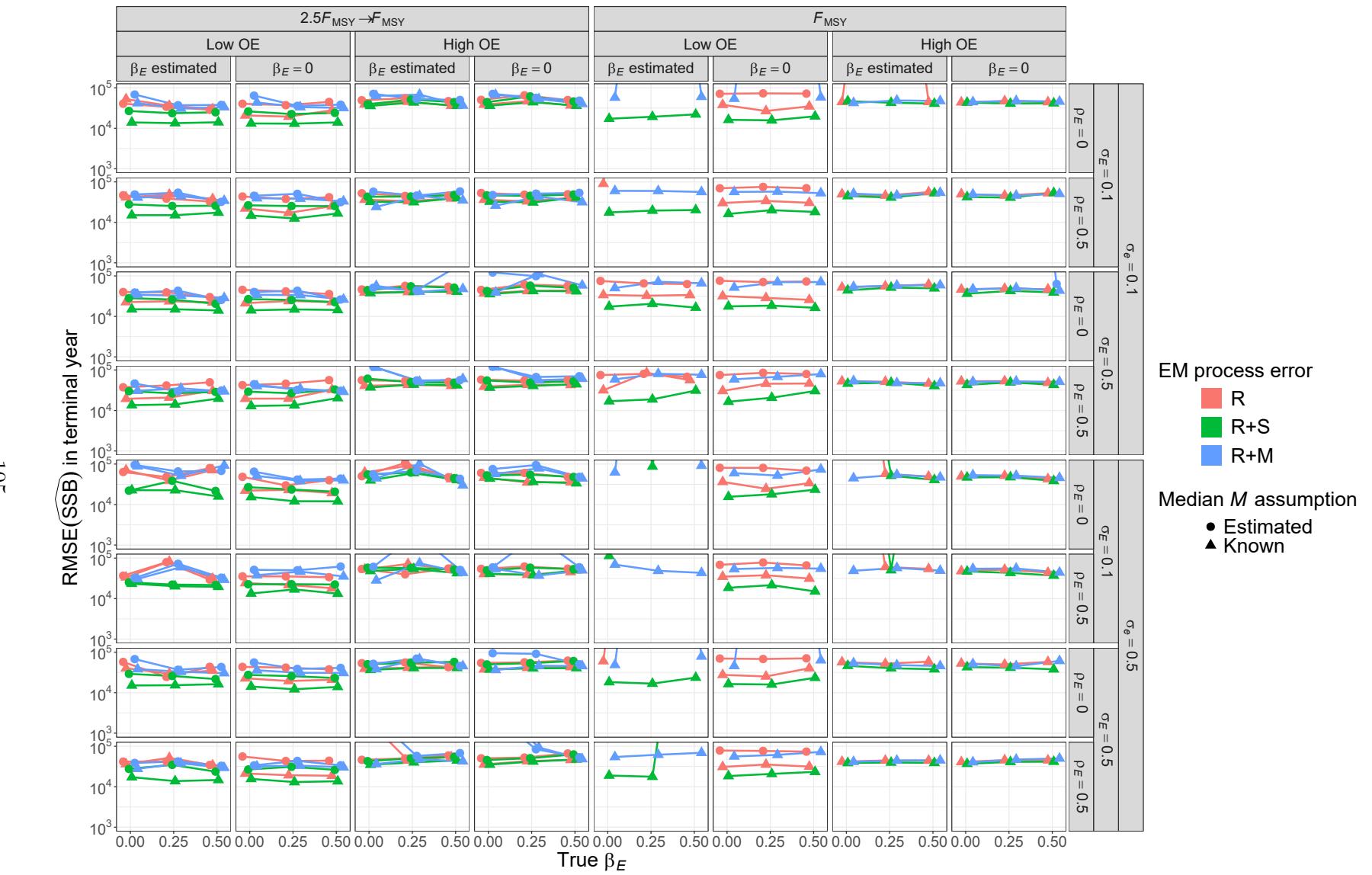


Fig. S52. For R+S OMs, root mean square error (RMSE) of estimates of spawning stock biomass in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known).

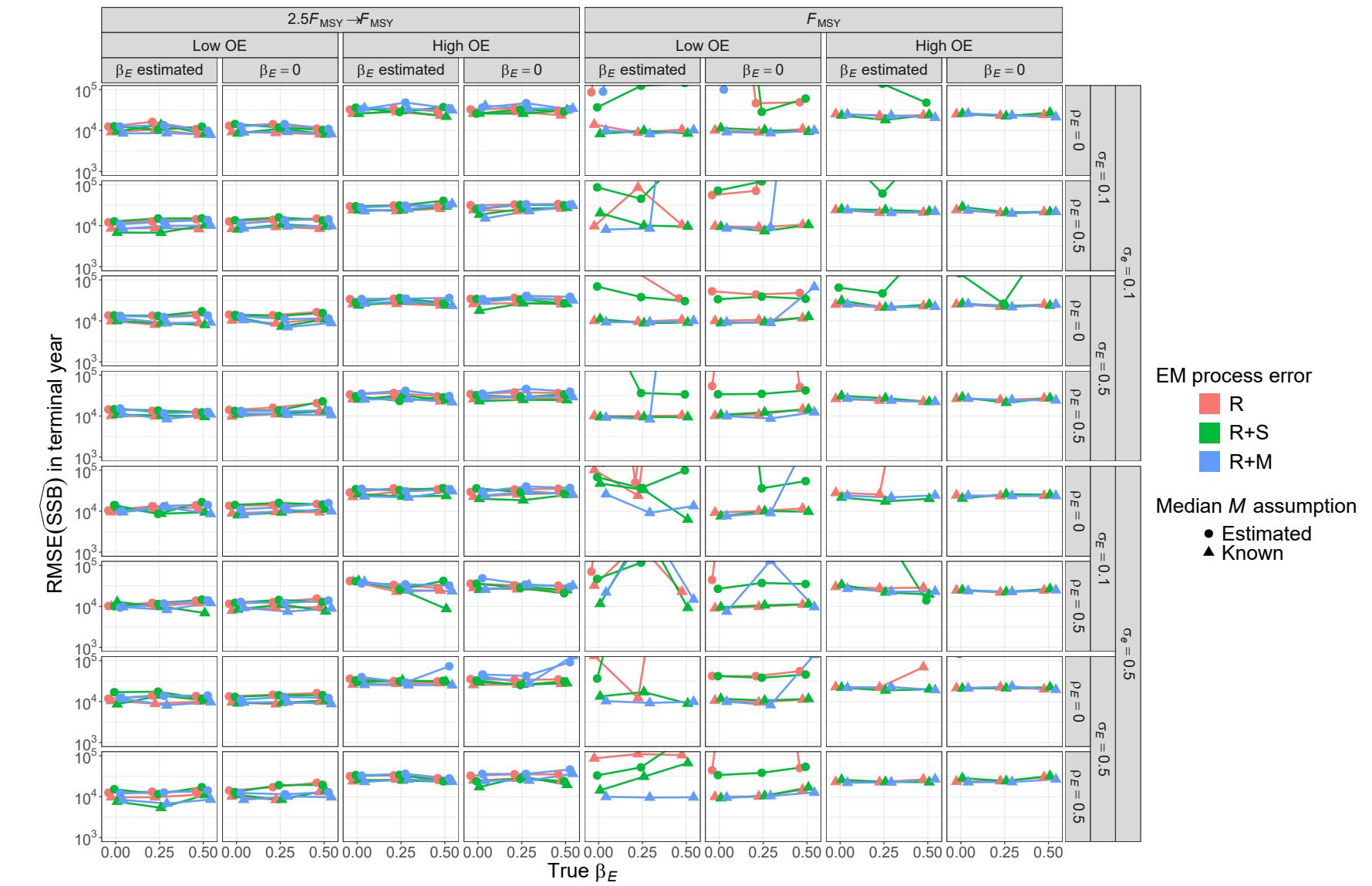


Fig. S53. For R+M OMs, root mean square error (RMSE) of estimates of spawning stock biomass in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known).

<sup>612</sup> Terminal year fishing mortality bias

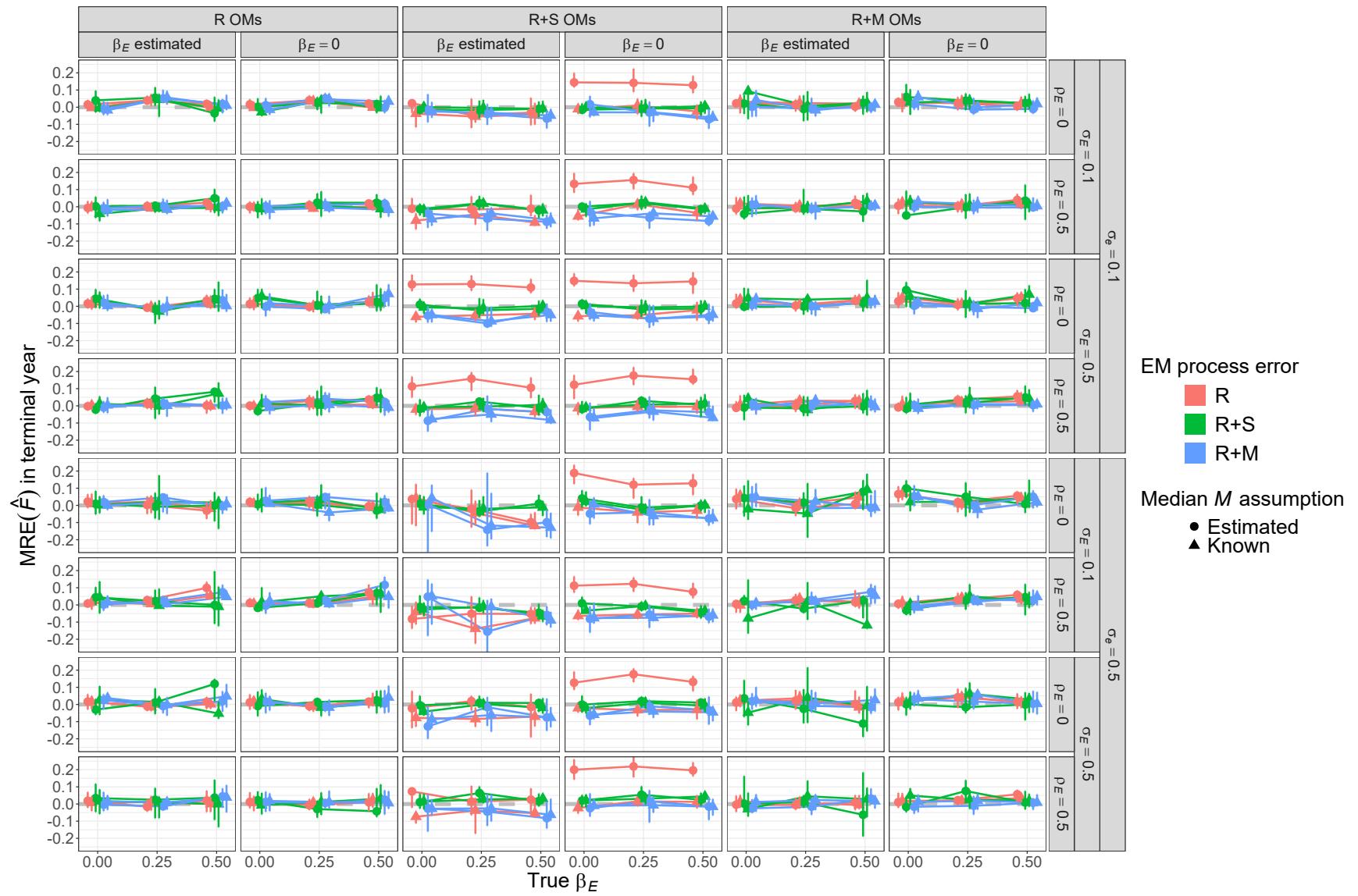


Fig. S54. Median relative error (MRE) of estimates of fully-selected fishing mortality ( $F$ ) in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known). All OMs had low population observation error and contrast in fishing mortality.

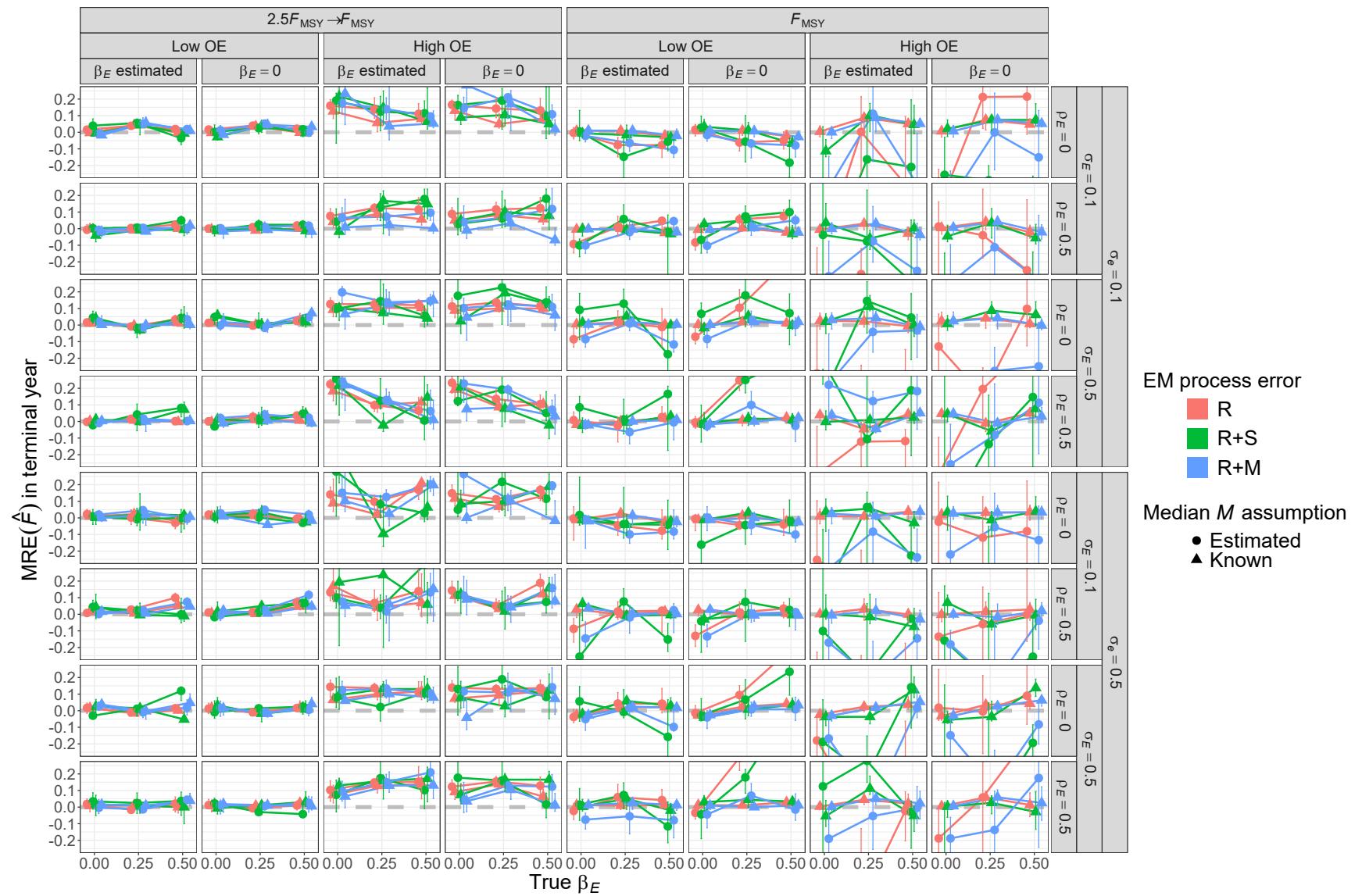


Fig. S55. For R OMs, median relative error (MRE) of estimates of fully-selected fishing mortality ( $F$ ) in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known). All OMs had low observation error and contrast in fishing mortality.

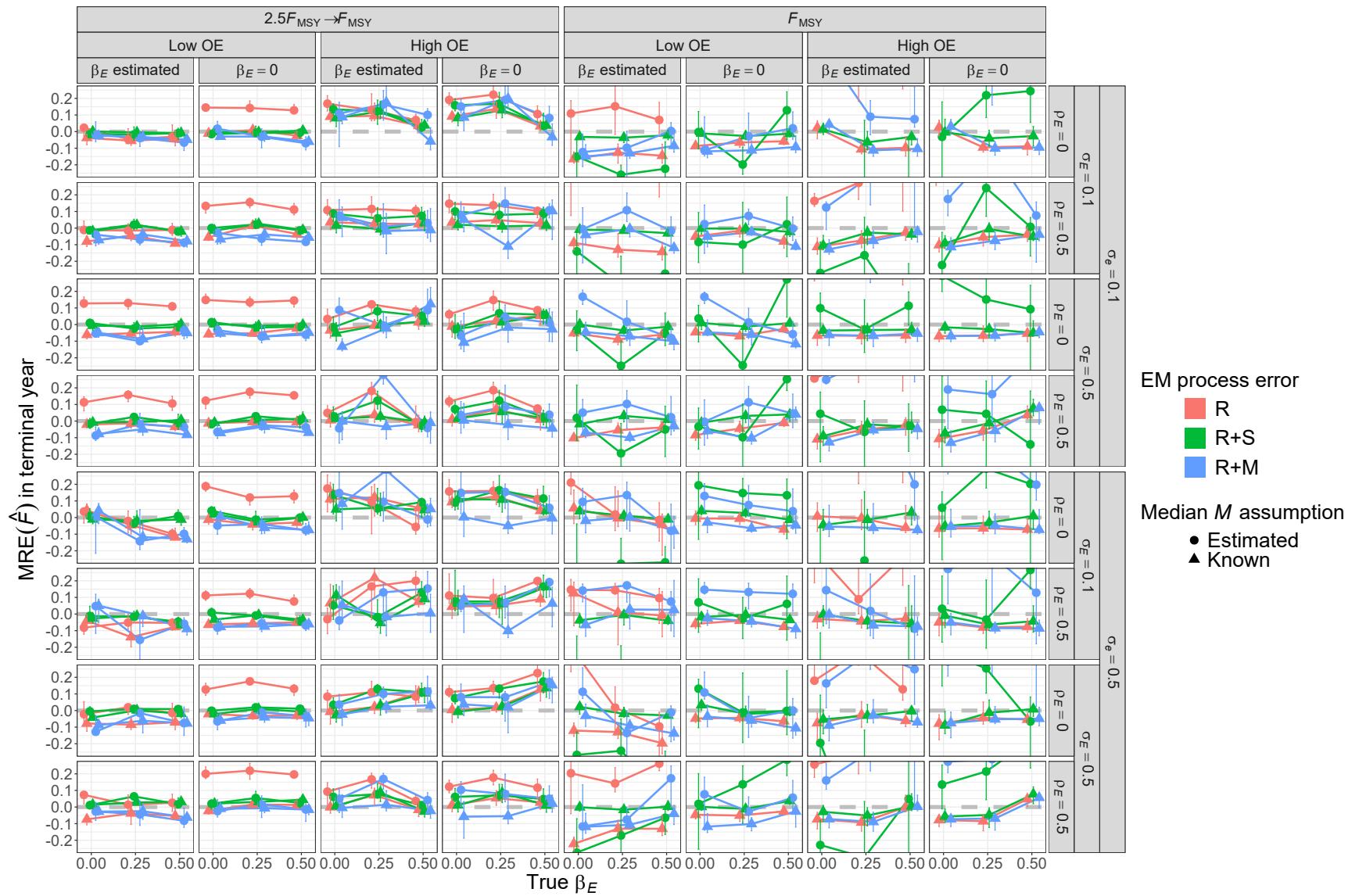


Fig. S56. For R+S OMs, median relative error (MRE) of estimates of fully-selected fishing mortality ( $F$ ) in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known). All OMs had low observation error and contrast in fishing mortality.

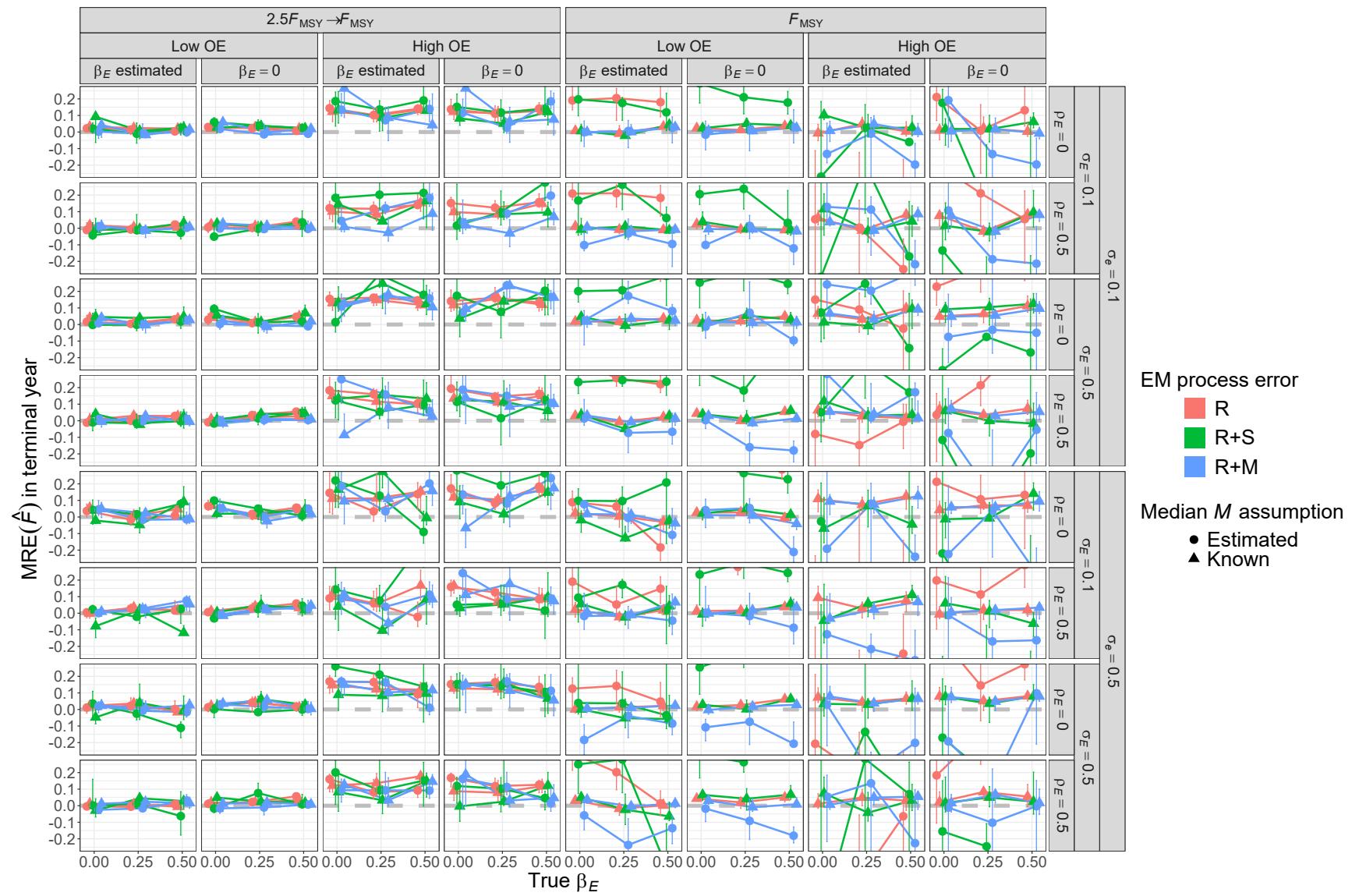


Fig. S57. For R+M OMs, median relative error (MRE) of estimates of fully-selected fishing mortality ( $\hat{F}$ ) in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known). All OMs had low observation error and contrast in fishing mortality.

<sub>613</sub> Terminal year fishing mortality RMSE

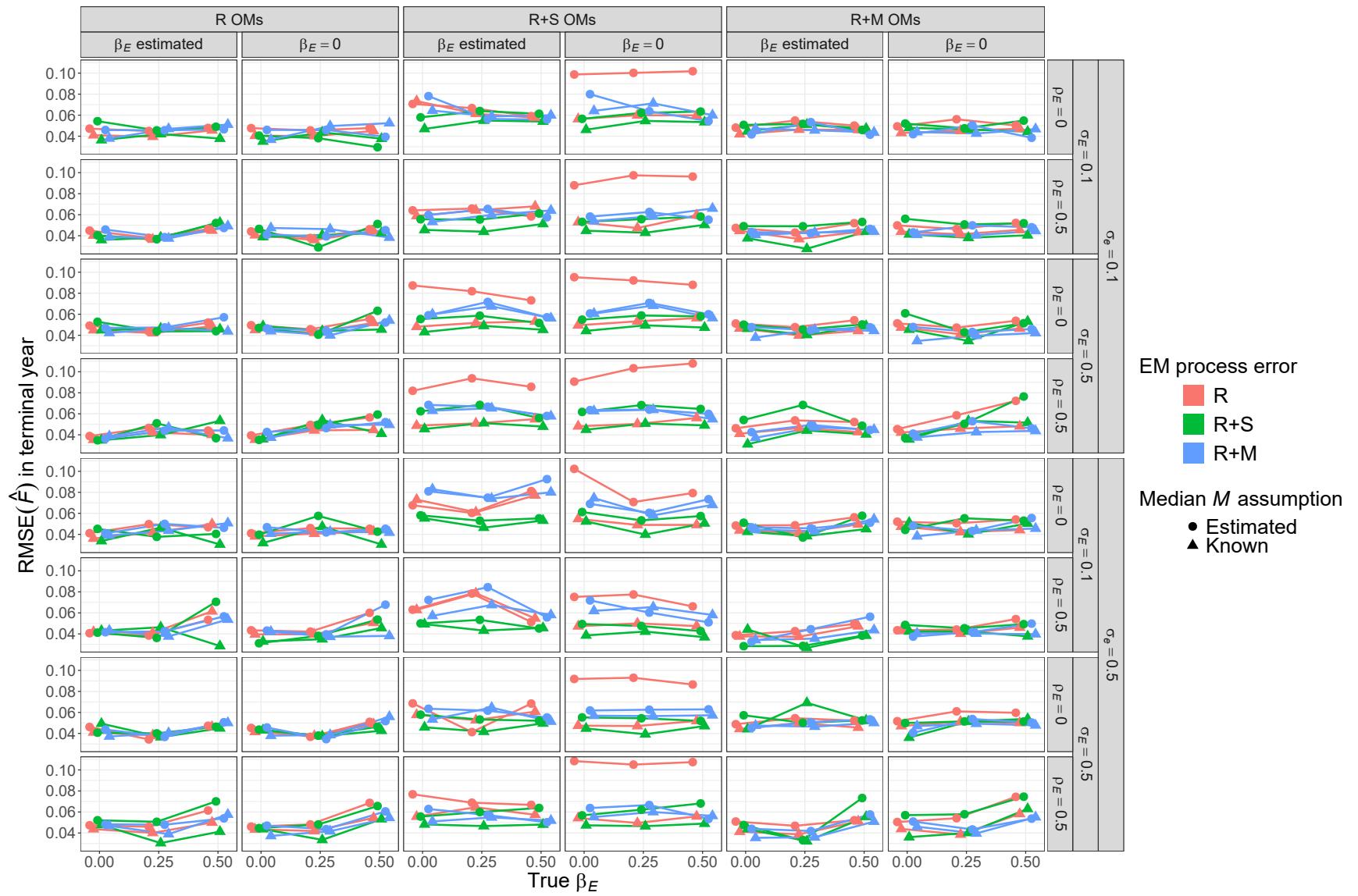


Fig. S58. Root mean square error (RMSE) of estimates of fully-selected fishing mortality ( $F$ ) in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known). All OMs had low population observation error and contrast in fishing mortality.

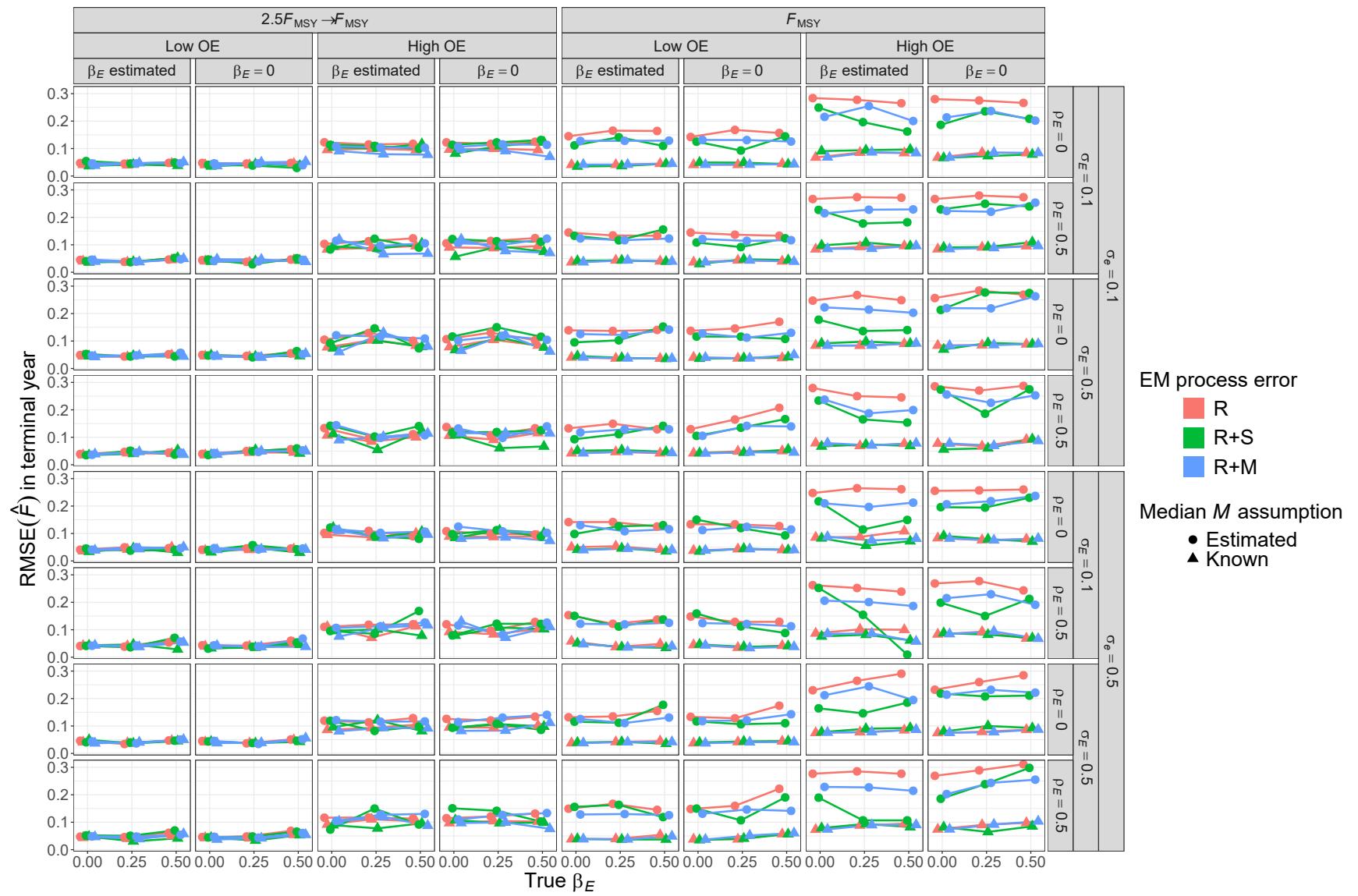


Fig. S59. For R OMs, root mean square error (RMSE) of estimates of fully-selected fishing mortality ( $F$ ) in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known).

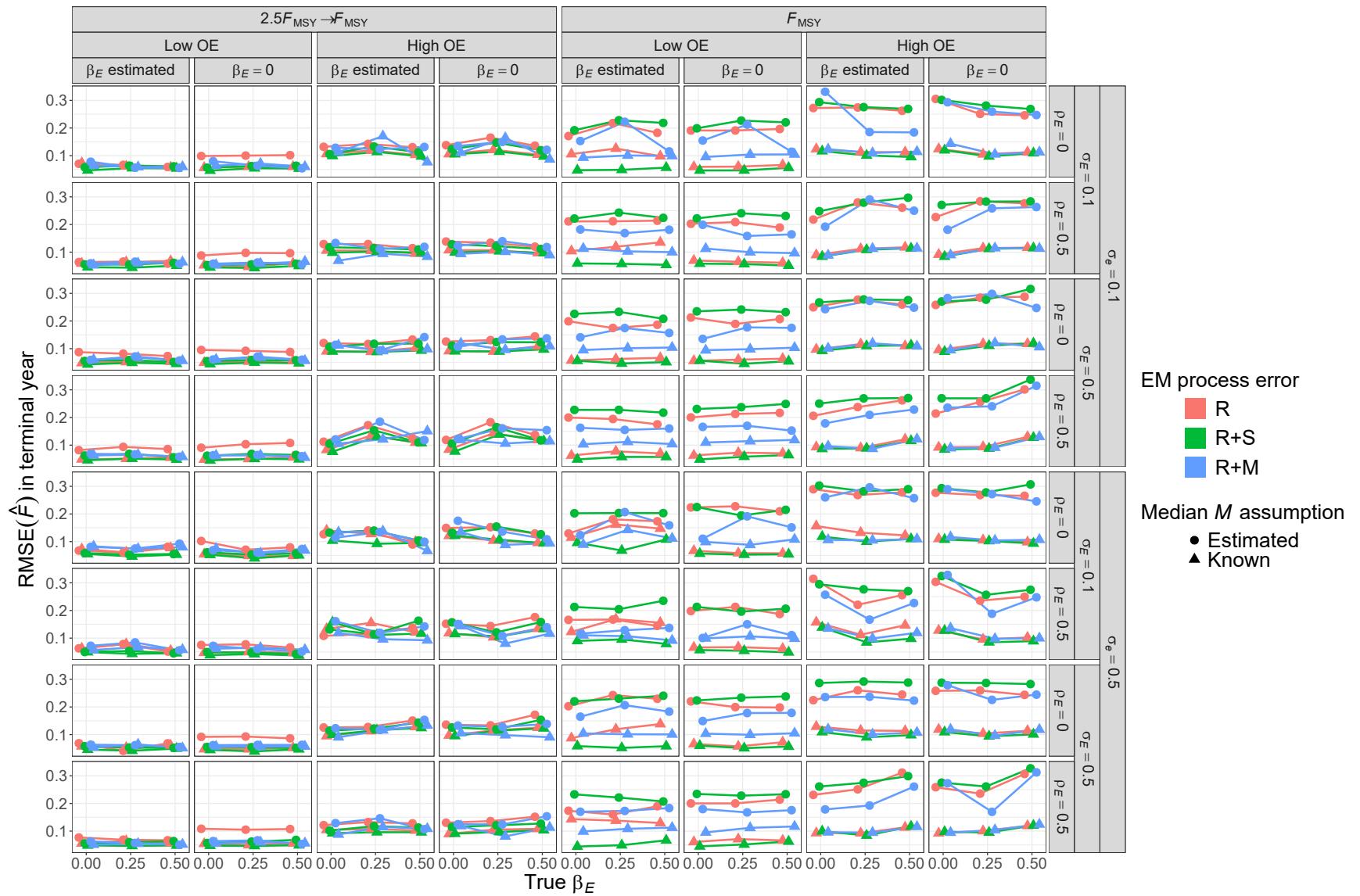


Fig. S60. For R+S OMs, root mean square error (RMSE) of estimates of fully-selected fishing mortality ( $F$ ) in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known).

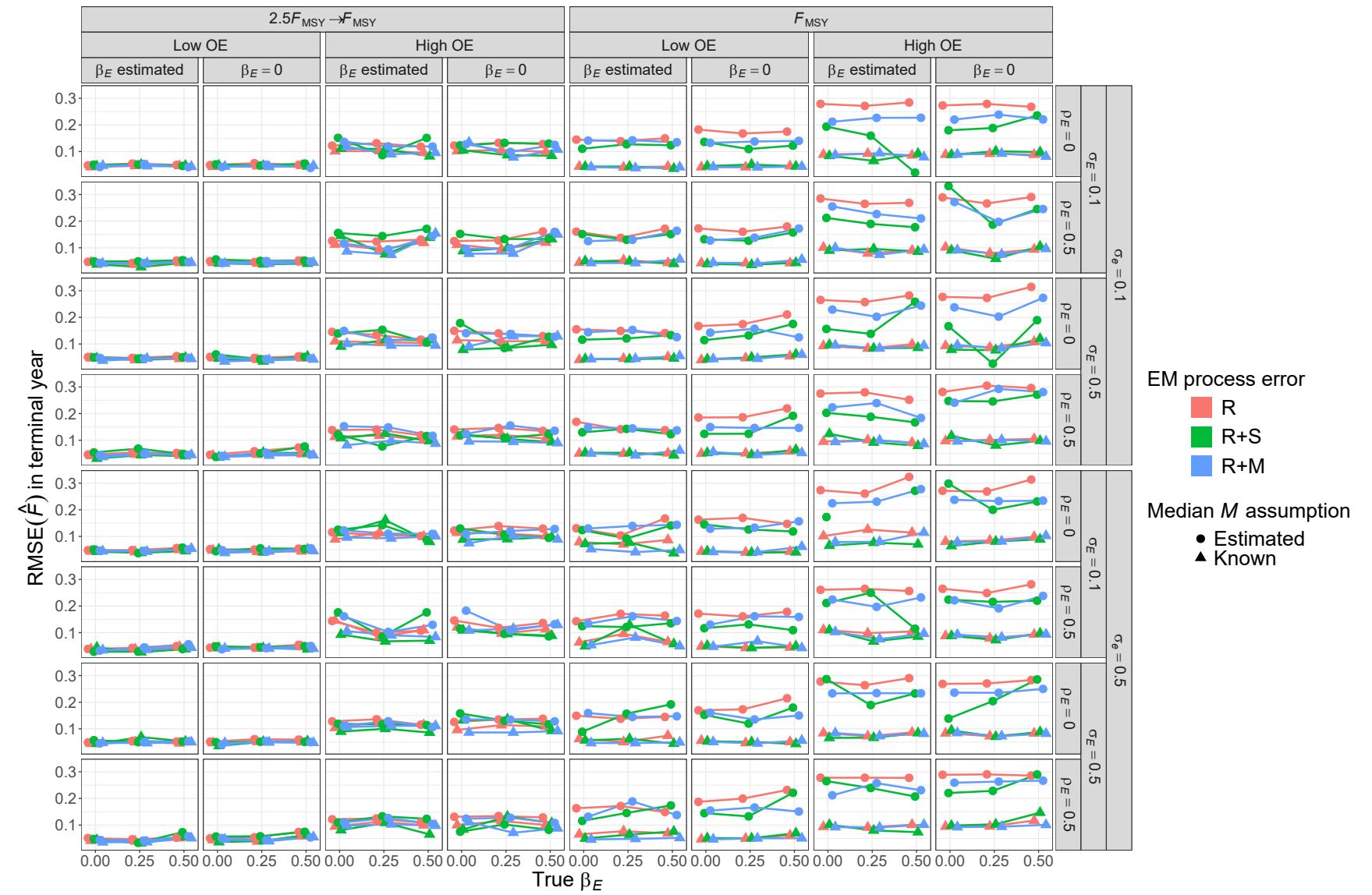


Fig. S61. For R+M OMs, root mean square error (RMSE) of estimates of fully-selected fishing mortality ( $F$ ) in the terminal year for EMs with alternative process error assumptions, treatment of covariate effect ( $\beta_E = 0$  or estimated), and treatment of median natural mortality parameter ( $\beta_M$  estimated or known).