

1 State-space age-structured assessment models provide  
2 reliable inferences(?)

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<sup>7</sup> **Abstract**

<sup>8</sup> **Keywords**

# 1 Introduction

Application of state-space models in fisheries stock assessment and management has expanded dramatically within ICES, Canada and the Northeast US (Nielsen and Berg, 2014; Cadigan, 2016; Stock and Miller, 2021).

Much is known about the reliability of state-space models that are linear or Gaussian (Aeberhard et al., 2018), but applications in fisheries management are nonlinear and typically include multiple types of observations with varying distributional assumptions. We know relatively little about the statistical reliability of such models. Also, there is a wide range of potential random effects structures in assessment models and we know little about the ability of information criteria to distinguish among such alternative structures.

But those studies focus primarily on Gaussian Reliability of hidden process models

## 2 Methods

Used the WHAM package (Stock and Miller, 2021, commit 77bbd94) (Miller and Stock 2020). This packages has also been used to configure operating and estimating models for closed loop simulations evaluating index-based assessment methods (Legault et al., In press).

We completed a simulation study with a number of operating models that can be categorized based on where random effects are assumed: abundance at age, natural mortality, fleet selectivity, or index catchability. For each operating model assumptions about variance of process errors and observations are required and the values we used were based on a review of the range of estimates from recent applications of WHAM in management of stocks of haddock, butterfish, and American plaice in the NE US.

We simulated 100 data sets for each operating model. For each simulated data from each operating model we fit a set of estimating models.

32 Y estimating models fit to each

## 33 **2.1 Operating models**

34 common to all:

35 ages = 1 to 10+, M maturity

36 marginal standard deviations for random effects are defined in tables of operating models.

### 37 **2.1.1 Population**

38 There are 10 age classes: ages 1 to 10+.

39 Spawning was assumed to occur 1/4 of the way through the year.

40 Natural mortality rate was assumed 0.2 when it was constant and the mean of the time series

41 process for operating models with M random effects. maturity a50 = 2.89, slope = 0.88

42 Weight at age was generated with a LVB growth

$$L_a = L_\infty \left(1 - e^{-k(a-t_0)}\right)$$

43 where  $t_0 = 0$ ,  $L_\infty = 85$ , and  $k = 0.3$ , and a L-W relationship such that

$$W_a = \theta_1 L_a^{\theta_2}$$

44 where  $\theta_1 = e^{-12.1}$  and  $\theta_2 = 3.2$ .

45 We assumed a Beverton-Holt stock recruit function with constant pre-recruit mortality pa-

46 rameters for all operating models. All post-recruit productivity components are constant in

47 the NAA and survey catchability process error operating models. Therefore steepness and

48 unfished recruitment are also constant over the time period for those operating models (Miller

and Brooks 2021). We specified unfished recruitment =  $R_0 = e^{10}$  and  $F_{\text{MSY}} = F_{40} = 0.348$  equated to a steepness of 0.69 and  $\alpha = 0.60$  and  $\beta = 2.4 \times 10^{-5}$  for the

$$N_{1,y} = \frac{\alpha \text{SSB}_{y-1}}{1 + \beta \text{SSB}_{y-1}}$$

Beverton-Holt parameterization.

The magnitude of the overfishing assumptions is based on average estimates of overfishing for NE groundfish stocks from Wiedenmann et al. (20XX). Legault et al. (2023) also used similar approaches to defining fishing mortality histories for operating models.

Initial population was configured at the equilibrium distribution fishing at either  $F = 2.5 \times F_{\text{MSY}}$  or  $F = F_{\text{MSY}}$  for the two alternative fishing histories. That is for a deterministic model, the age composition would not change over time when the fishing mortality was constant at the respective level.

For operating models with time-varying random effects for M, steepness is not constant, but we used the same alpha and beta parameters as other operating models this equates to a steepness and  $R_0$  at the mean of the time series process for M. For operating models with time-varying random effects for fishery selectivity,  $F_{\text{msy}}$  is also not constant however we use the same F history as other operating models which corresponds to  $F_{\text{msy}}$  at the mean selectivity parameters.

### 2.1.2 Fleets

We assumed a single fleet operating year round for catch observations with logistic selectivity for the fleet with  $a_{50} = 5$  and slope = 1. This selectivity is was used to define  $F_{\text{MSY}}$  for the Beverton-Holt stock recruitment parameters above. We assumed a logistic-normal distribution for the age-composition observations for the fleet.

### 2.1.3 Fishing histories

All operating models assumed one of two different fishing histories. One : Fishing mortality is equal to  $F_{msy}$  (0.348) for the whole 40 year period. Two : Fishing mortality is 2.5 times  $F_{msy}$  for the first 20 years then changes to  $F_{msy}$  for the last 20 years.

### 2.1.4 Indices

Two time series of surveys are assumed and observed in numbers rather than biomass for the entire 40 year period with one occurring in the spring (0.25 way through the year) and one in the fall (0.75 way through the year). Actually we have it currently configured that both occur 0.5 way through the year. Catchability of both surveys are assumed to be 0.1. We assumed logistic selectivity for both indices with  $a_{50} = 5$  and slope = 1. We assumed a logistic-normal distribution for the age-composition observations.

### 2.1.5 Observation Uncertainty

Standard deviation for log-aggregate catch was 0.1. There were two levels of observation error variance for indices and age composition for both indices and fleet catch. A low uncertainty specification assumed standard deviation of both series of log-aggregate index observations was 0.1 and the standard deviation of the logistic-normal for age composition observations was 0.3 In the high uncertainty specification the standard deviation for log-aggregate indices was 0.4 and that for the age composition observations was 1.5.

### 2.1.6 Operating models with random effects on numbers at age

24 operating models, 16 Sel re operating models and 16 q re operating models. Table of process error assumptions

91 **2.1.7 Operating models with random effects on natural mortality**

92 16 operating models Table of process error assumptions

93 NOTE: inv\_trans\_rho function in set\_M.R is mis-defined. Will affect correlation parame-  
94 ters assigned in operating models?

95 **2.1.8 Operating models with random effects on fleet selectivity**

96 16 operating models Table of process error assumptions

97 **2.1.9 Operating models with random effects on index catchability**

98 16 operating models Table of process error assumptions

99 **2.2 Estimating models**

100 32 estimating models Table of estimating models

101 1-20 fit to each NAA RE operating model 5-24 fit to each M RE operating model 5-20,25-28  
102 to each sel RE operating model 5-20, 29-32 to each q RE operating model

103 SR estimation or not

104 Make plot of S-R curve, Fmsy = F40 Initial values for BH parameters are the true values.  
105 Initial values for mean R model = true R0.

106 M estimation or not

107 NAA\_re Random effects on Recruitment only or random effects on recruitment and transi-  
108 tions among older numbers at age.

109 M\_re Random effects on Recruitment only, M constant across age .

110 sel\_re Random effects on Recruitment only, fleet logistic selectivity RE model?

111 q\_re Random effects on Recruitment only, one survey catchability RE model?  
112 Simulations were all carried out on the University of Massachusetts Green High-Performance  
113 Computing Cluster. Code for completing the simulations and summarization of results can be  
114 found at [github.com/timjmiller/SSRTWG/Project\\_0](https://github.com/timjmiller/SSRTWG/Project_0). We used the wham package version  
115 1.X.X (commit XXXXX).

## 116 **3 Results**

117 Do each of these by type of operating model (Naa, M, sel, q) Convergence performance  
118 AIC performance  
119 SR estimation? M estimation?  
120 Bias, Mean Square error  
121 Certain basic parameters (stock-recruit pars, M, variance parameters) SSB, F, R



122 **3.1 Numbers at age operating models**

123 **3.1.1** Estimating models include alternative random effects options: NAA, M,  
124 sel, q

Table 1. NAA operating models, estimating models all assume a B-H stock recruit relationship and M is fixed at the true value.

$\sigma_R$	$\sigma_N$	F-history	Obs Error	R only	NAA	M	Sel	q
0.5		H-MSY	L	96	0	0	0	4
1.5		H-MSY	L	96	0	0	0	4
0.5	0.25	H-MSY	L	0	100	0	0	0
1.5	0.25	H-MSY	L	0	100	0	0	0
0.5	0.50	H-MSY	L	0	96	4	0	0
1.5	0.50	H-MSY	L	0	100	0	0	0
0.5		MSY	L	97	0	0	0	3
1.5		MSY	L	96	0	0	0	4
0.5	0.25	MSY	L	0	99	1	0	0
1.5	0.25	MSY	L	0	99	1	0	0
0.5	0.50	MSY	L	0	100	0	0	0
1.5	0.50	MSY	L	0	99	1	0	0
0.5		H-MSY	H	94	0	0	0	6
1.5		H-MSY	H	94	0	0	0	6
0.5	0.25	H-MSY	H	46	50	0	0	4
1.5	0.25	H-MSY	H	65	30	0	0	5
0.5	0.50	H-MSY	H	1	99	0	0	0
1.5	0.50	H-MSY	H	0	98	0	0	2
0.5		MSY	H	94	0	0	0	6
1.5		MSY	H	95	0	0	0	5
0.5	0.25	MSY	H	45	52	0	0	3
1.5	0.25	MSY	H	63	28	0	0	9
0.5	0.50	MSY	H	0	100	0	0	0
1.5	0.50	MSY	H	0	98	1	0	1

Table 2. NAA operating models, estimating models all assume a B-H stock recruit relationship and M is estimated.

$\sigma_R$	$\sigma_N$	F-history	Obs Error	R only	NAA	M	Sel	q
0.5		H-MSY	L	96	0	0	0	4
1.5		H-MSY	L	96	0	0	0	4
0.5	0.25	H-MSY	L	0	98	1	1	0
1.5	0.25	H-MSY	L	0	100	0	0	0
0.5	0.50	H-MSY	L	0	97	3	0	0
1.5	0.50	H-MSY	L	0	96	2	2	0
0.5		MSY	L	95	1	0	0	4
1.5		MSY	L	93	3	0	0	4
0.5	0.25	MSY	L	0	94	1	5	0
1.5	0.25	MSY	L	0	85	5	3	0
0.5	0.50	MSY	L	0	91	7	1	1
1.5	0.50	MSY	L	0	77	20	0	1
0.5		H-MSY	H	94	0	0	0	6
1.5		H-MSY	H	96	0	0	0	4
0.5	0.25	H-MSY	H	50	47	0	0	3
1.5	0.25	H-MSY	H	68	28	0	0	4
0.5	0.50	H-MSY	H	1	99	0	0	0
1.5	0.50	H-MSY	H	0	97	1	0	2
0.5		MSY	H	78	15	0	1	4
1.5		MSY	H	69	21	0	2	6
0.5	0.25	MSY	H	45	41	0	0	6
1.5	0.25	MSY	H	37	44	1	0	8
0.5	0.50	MSY	H	3	79	0	0	11
1.5	0.50	MSY	H	4	69	7	1	13

Table 3. NAA operating models, estimating models all estimate a mean recruitment and M is fixed at the true value.

$\sigma_R$	$\sigma_N$	F-history	Obs Error	R only	NAA	M	Sel	q
0.5		H-MSY	L	96	0	0	0	4
1.5		H-MSY	L	96	0	0	0	4
0.5	0.25	H-MSY	L	0	99	0	1	0
1.5	0.25	H-MSY	L	0	100	0	0	0
0.5	0.50	H-MSY	L	0	99	1	0	0
1.5	0.50	H-MSY	L	0	97	3	0	0
0.5		MSY	L	97	0	0	0	3
1.5		MSY	L	96	0	0	0	4
0.5	0.25	MSY	L	0	100	0	0	0
1.5	0.25	MSY	L	0	100	0	0	0
0.5	0.50	MSY	L	0	100	0	0	0
1.5	0.50	MSY	L	0	100	0	0	0
0.5		H-MSY	H	94	0	0	0	6
1.5		H-MSY	H	94	0	0	0	6
0.5	0.25	H-MSY	H	48	48	0	0	4
1.5	0.25	H-MSY	H	65	30	0	0	5
0.5	0.50	H-MSY	H	0	99	1	0	0
1.5	0.50	H-MSY	H	0	99	0	0	1
0.5		MSY	H	94	0	0	0	6
1.5		MSY	H	95	0	0	0	5
0.5	0.25	MSY	H	46	51	0	0	3
1.5	0.25	MSY	H	63	28	0	0	9
0.5	0.50	MSY	H	0	100	0	0	0
1.5	0.50	MSY	H	0	98	1	0	1

Table 4. NAA operating models, estimating models all estimate a mean recruitment and M estimated.

$\sigma_R$	$\sigma_N$	F-history	Obs Error	R only	NAA	M	Sel	q
0.5		H-MSY	L	96	0	0	0	4
1.5		H-MSY	L	96	0	0	0	4
0.5	0.25	H-MSY	L	0	99	0	1	0
1.5	0.25	H-MSY	L	0	100	0	0	0
0.5	0.50	H-MSY	L	0	99	1	0	0
1.5	0.50	H-MSY	L	0	97	3	0	0
0.5		MSY	L	97	0	0	0	3
1.5		MSY	L	96	0	0	0	4
0.5	0.25	MSY	L	0	100	0	0	0
1.5	0.25	MSY	L	0	100	0	0	0
0.5	0.50	MSY	L	0	100	0	0	0
1.5	0.50	MSY	L	0	100	0	0	0
0.5		H-MSY	H	94	0	0	0	6
1.5		H-MSY	H	94	0	0	0	6
0.5	0.25	H-MSY	H	48	48	0	0	4
1.5	0.25	H-MSY	H	65	30	0	0	5
0.5	0.50	H-MSY	H	0	99	1	0	0
1.5	0.50	H-MSY	H	0	99	0	0	1
0.5		MSY	H	94	0	0	0	6
1.5		MSY	H	95	0	0	0	5
0.5	0.25	MSY	H	46	51	0	0	3
1.5	0.25	MSY	H	63	28	0	0	9
0.5	0.50	MSY	H	0	100	0	0	0
1.5	0.50	MSY	H	0	98	1	0	1

Fig. 1. Median relative bias of for SSB for estimating models that estimate mean recruitment and  $M$  is fixed at the true value.

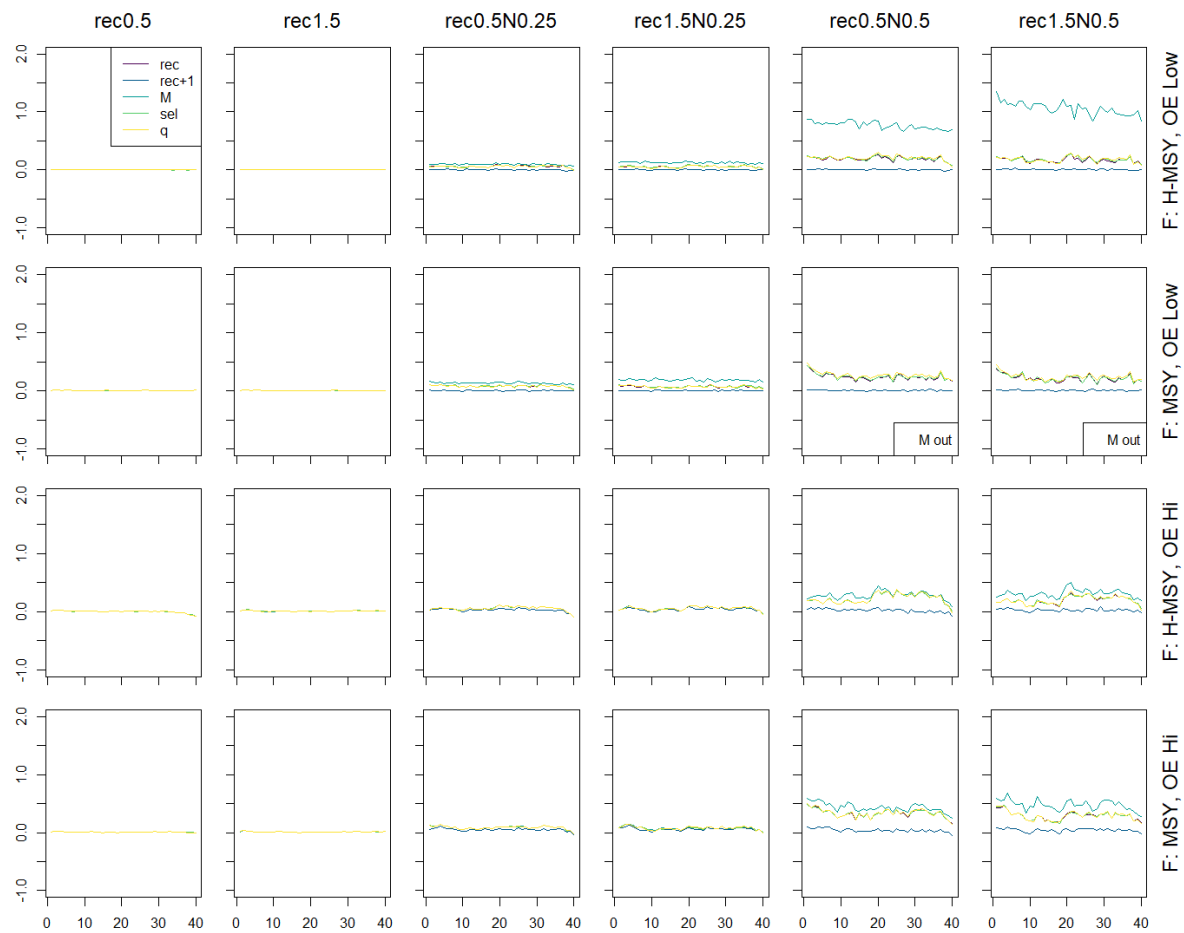


Fig. 2. Median relative bias of for SSB for estimating models that estimate mean recruitment and M is estimated.

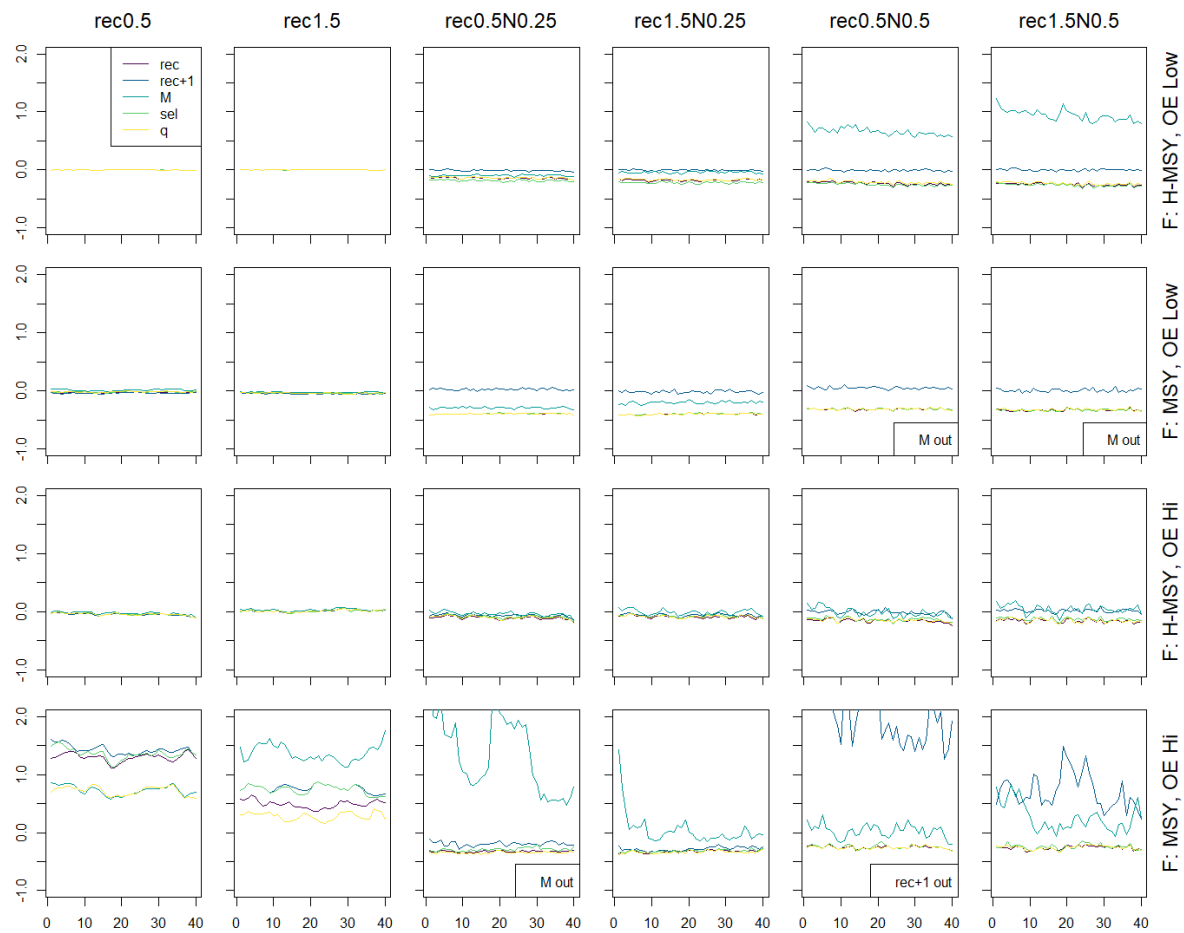


Fig. 3. Median relative bias of for SSB for estimating models that estimates a BH stock-recruitment function and M is fixed at the true value.

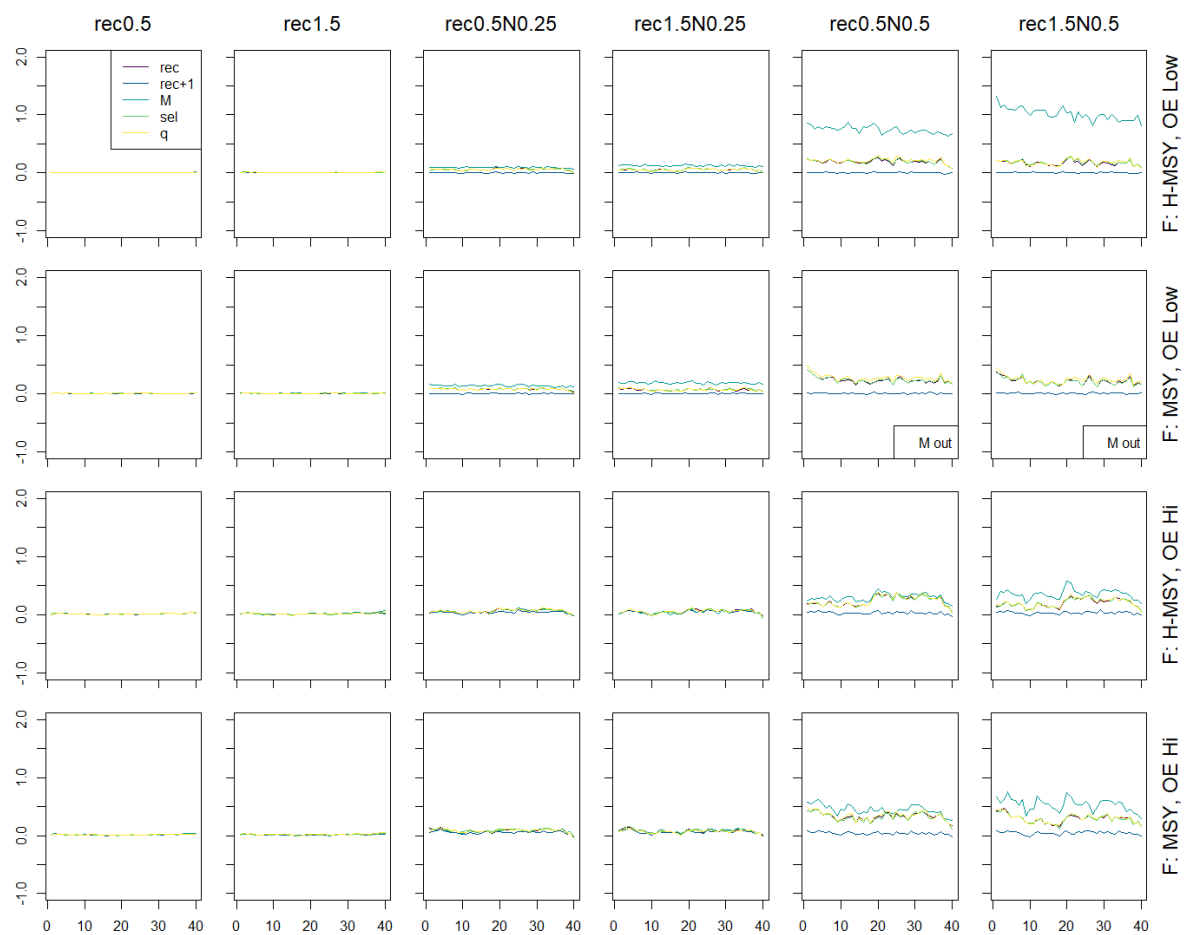
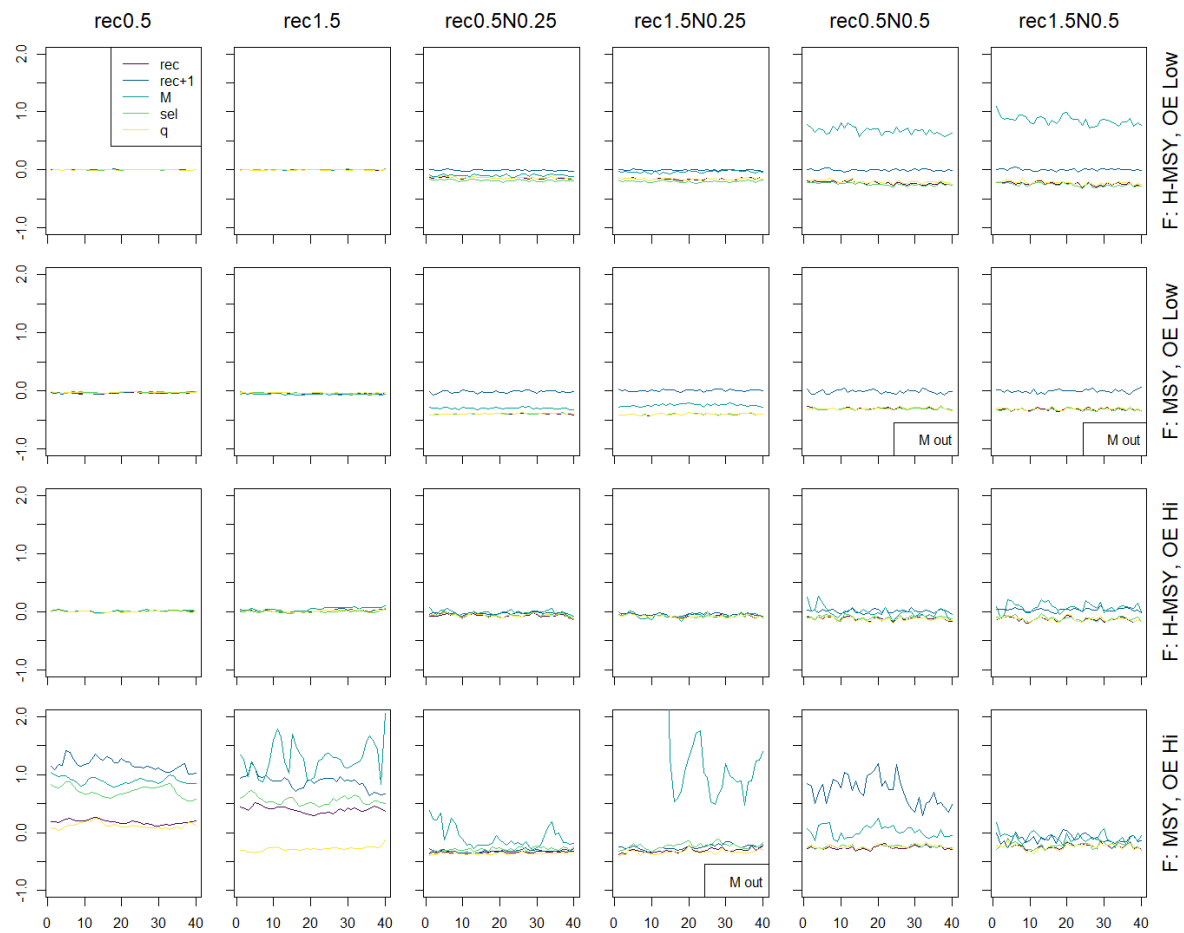




Fig. 4. Median relative bias of for SSB for estimating models that estimates a BH stock-recruitment function and M is estimated.



125 **3.1.2** Estimating models include NAA random effects and estimation assumes  
126 mean R or BH SR

Table 5. Operating models and estimation models all assume RE on recruitment only, estimating models assume mean recruitment or a B-H stock recruit relationship and M is fixed at the true value.

$\sigma_R$	$\sigma_N$	F-history	Obs Error	R only	BH
0.5		H-MSY	L	46	54
1.5		H-MSY	L	82	18
0.5		MSY	L	71	29
1.5		MSY	L	85	15
0.5		H-MSY	H	51	49
1.5		H-MSY	H	82	18
0.5		MSY	H	72	28
1.5		MSY	H	86	14

Table 6. Operating models and estimation models all assume RE on recruitment only, estimating models assume mean recruitment or a B-H stock recruit relationship and M is estimated.

$\sigma_R$	$\sigma_N$	F-history	Obs Error	R only	BH
0.5		H-MSY	L	45	55
1.5		H-MSY	L	82	18
0.5		MSY	L	70	30
1.5		MSY	L	87	13
0.5		H-MSY	H	56	44
1.5		H-MSY	H	82	18
0.5		MSY	H	75	25
1.5		MSY	H	84	16

Table 7. Operating models and estimation models all assume RE on all abundances at age, estimating models assume mean recruitment or a B-H stock recruit relationship and M is fixed at the true value.

$\sigma_R$	$\sigma_N$	F-history	Obs Error	R only	BH
0.5	0.25	H-MSY	L	43	57
1.5	0.25	H-MSY	L	84	16
0.5	0.50	H-MSY	L	33	67
1.5	0.50	H-MSY	L	77	23
0.5	0.25	MSY	L	69	31
1.5	0.25	MSY	L	88	12
0.5	0.50	MSY	L	55	45
1.5	0.50	MSY	L	87	13
0.5	0.25	H-MSY	H	57	43
1.5	0.25	H-MSY	H	84	16
0.5	0.50	H-MSY	H	66	34
1.5	0.50	H-MSY	H	79	21
0.5	0.25	MSY	H	78	22
1.5	0.25	MSY	H	88	12
0.5	0.50	MSY	H	73	27
1.5	0.50	MSY	H	83	17

Table 8. Operating models and estimation models all assume RE on all abundances at age, estimating models assume mean recruitment or a B-H stock recruit relationship and M is estimated.

$\sigma_R$	$\sigma_N$	F-history	Obs Error	R only	BH
0.5	0.25	H-MSY	L	44	56
1.5	0.25	H-MSY	L	84	16
0.5	0.50	H-MSY	L	31	69
1.5	0.50	H-MSY	L	80	20
0.5	0.25	MSY	L	68	32
1.5	0.25	MSY	L	88	12
0.5	0.50	MSY	L	55	45
1.5	0.50	MSY	L	86	14
0.5	0.25	H-MSY	H	59	41
1.5	0.25	H-MSY	H	81	19
0.5	0.50	H-MSY	H	67	33
1.5	0.50	H-MSY	H	80	20
0.5	0.25	MSY	H	66	34
1.5	0.25	MSY	H	74	26
0.5	0.50	MSY	H	74	26
1.5	0.50	MSY	H	87	13

## 4 Discussion

The estimating models assumed variances of aggregate catch and index observations was known. This approximation may be appropriate for indices where we have a reliable estimate of uncertainty based on the survey design (), but there may be better approaches for the aggregate catch such as an informed prior on the standard errors with realistic bounds.

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