A multi-stock, multi-region state-space age-structured assessment model with an application to black sea bass

Timothy J. Miller[1] Kierstin Curti Alex Hansell

16 September, 2024

[1]timothy.j.miller@noaa.gov, Northeast Fisheries Science Center, National Marine Fisheries Service, 166 Water Street, Woods Hole, MA 02543, USA

Main Message

A multi-region, multi-stock generalization of WHAM with an application evaluating evidence of alternative hypotheses about temperature effects on black sea bass

Abstract

The Woods Hole Assessment Model (WHAM) is a state-space age-structured assessment model that is used to assess and manage many stocks in the Northeast US. We first describe a multi-stock, multi-region extension of WHAM that treats the population and fleet dynamics seasonally and allows movement by season and region to be functions of time- and age-varying autocorrelated random effects and environmental covariates. We then illustrate the model with northern and southern components of the NEUS black sea bass stock and evaluate alternative hypotheses of bottom temperature and random effects on recruitment and natural mortality. We show strong evidence for temperature effects on recruitment and no random effects or temperature effects on natural mortality, primarily for the northern stock component.

Introduction

Why is this model needed. Models ignoring movement can provide biased catch advice. Using random effects for time-varying processes is considered best practice.

The Woods Hole Assessment Model (WHAM) is an R package developed at the Northeast Fisheries Science Center (https://timjmiller.github.io/wham, Miller and Stock 2020; Stock and Miller 2021). Primary inputs and observation types are similar to ASAP and there is functionality built into WHAM to move information from ASAP input files to R. WHAM can be configured to fit SCAA models often identically to ASAP, so that existing assessments in the U.S. Northeast can be replicated and tested against state-space models with process errors and environmental effects in a single framework. WHAM models are built using the Template Model Builder package (TMB, Kristensen et al. 2016) which provides efficient fitting of models that include random effects similar to the State-space Assessment Model (SAM, Nielsen and Berg 2014), which is used to manage many stocks within ICES. WHAM is currently used to manage 9 stocks in the NEUS.

The Woods Hole Assessment Model (WHAM) software package is maintained and developed at the Northeast Fisheries Science Center to enable state-space stock assessments, i.e. where processes such as the annual transitions in numbers-at-age (survival), natural mortality, selectivity, and catchability are treated as time- and(or) age-varying random effects. WHAM can also be configured without random effects as a traditional statistical catch-at-age model in order to bridge from current assessments which use Age-Structured Assessment Program (ASAP). Here, we present an extension of WHAM (Multi-WHAM) to model multiple stocks and(or) regions and allow movement of stocks among regions.

It has been reviewed in the literature and simulation tested (Stock et al. 2021; Stock and Miller 2021). It has also been used as the operating model in the index-based assessment methods research track assessment (Legault et al. 2023) and is also the main tool for simulation studies in the research track on state-space assessment models.

A review of the essential features for next-generation stock assessments concluded that only WHAM and a similar State-Space Assessment Method (Nielsen and Berg 2014) model random effects correctly (Punt et al. 2020). WHAM is now being used as the management model for 9 stocks in the northeast U.S., but the standard version of WHAM can only be applied to a single stock and area (Miller and Stock 2020).

Here, we describe a multi-stock and multi-region extension of the WHAM package (version 2.0 https://github.com/timjmiller/wham/tree/lab). Numerous new configuration options are also included in this extension that can be useful even for single-stock or single region models. Any number of stocks or regions can be configured in this version of WHAM. The multi-stock extension was developed to accommodate movement of stocks or stock components in anticipation of black sea bass and Atlantic cod stocks where new modeling frameworks where recently examined through Northeast research track assessment process (cite RT docs). We demonstrate capabilities of the new version of WHAM with black sea bass and evaluate evidence for alternative hypotheses of temporal variation in natural mortality and effects of bottom temperature on recruitment and natural mortality of age 1 individuals.

Methods

Model description

Many of the options and equations of version 2.0 are are the same as those described by Stock and Miller (2021), so we will only describe extensions and differences that have occurred since the first description of WHAM. The new version of WHAM can model multiple stocks, but survival, movement, harvest and natural mortality is are tracked for each stock. Therefore, much of the description below assumes a specific stock s, but this subscript is only explicitly used when necessary.

The probability transition matrix

Because individuals may be alive in one of several regions or harvested in one of several fleets, it is helpful to consider these as distinct categories or states and treat the number of individuals occurring in each category over time as a multi-state model (reference). Multi-state models use a probability transition matrix (PTM) that describes the probability of individuals living and moving among regions or dying due to fishing or natural mortality over a time interval i with duration δ_i in year y for individuals at age a on January 1. Each row and column of the PTM correspond to one the states: alive in region r, dead in fleet f, or dead from natural causes. The probabilities in each row sum to unity and assume an individual is in the corresponding state at the beginning of the interval. Given n_R regions and n_F fleets, the square PTM ($n_R + n_F + 1$ rows and columns) as a function of sub-matrices is

$$\mathbf{P}_{y,a,i} = \begin{bmatrix} \mathbf{O}_{y,a,i} & \mathbf{H}_{y,a,i} & \mathbf{D}_{y,a,i} \\ 0 & \mathbf{I}_H & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where

$$\mathbf{O}_{y,a,i} = \begin{bmatrix} O_{y,a,i}(1,1) & \cdots & O_{y,a,i}(1,n_R) \\ \vdots & \ddots & \vdots \\ O_{y,a,i}(n_R,1) & \cdots & O_{y,a,i}(n_R,n_R) \end{bmatrix}$$

is the $n_R \times n_R$ matrix defining survival and occurring in each region at the end of the interval,

$$\mathbf{H}_{y,a,i} = \begin{bmatrix} H_{y,a,i}(1,1) & \cdots & H_{y,a,i}(1,n_F) \\ \vdots & \ddots & \vdots \\ H_{y,a,i}(n_R,1) & \cdots & H_{y,a,i}(n_R,n_F) \end{bmatrix}$$

is the $n_R \times n_F$ matrix defining probabilities of being captured in each fleet during the interval, and $\mathbf{D}_{y,a,i}$ is the $n_R x 1$ matrix of probabilities of dying due to natural mortality during the interval. We have the identity matrix $\mathbf{I}_H$ for the states for capture by each fleet and a 1 for the state for natural mortality because the probabilities of being in one of the mortality states given starting the interval in that state is unity.

WHAM uses these PTMs to model abundance proportions in each state rather than true probabilities where numbers in each state would be multinomial distributed. The PTMs determine the expected numbers 1) in each state on January 1 of year $t+1$ at age $a+1$ given the abundances at age a on January 1 of year t, 2) captured over the year in each fleet, 3) available to each index, and 4) alive at the time and in the region where spawning occurs.

Single region PTMs

When there is only one region,

$$\mathbf{P}_{y,a,i} = \begin{bmatrix} S_{y,a,i} & \mathbf{H}_{y,a,i} & D_{y,a,i} \\ 0 & \mathbf{I}_H & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

where $S_{y,a,i} = e^{-Z_{y,a,i}\delta_i}$, $\mathbf{H}_{y,a,i}$ is a 1 x n_F matrix with elements for each fleet f: $\frac{F_{y,a,i,f}}{Z_{y,a,i}}\left(1 - e^{-Z_{y,a,i}\delta_i}\right)$, $D_{y,a,i} = \frac{M_{y,a}}{Z_{y,a,i}}\left(1 - e^{-Z_{y,a,i}\delta_i}\right)$, and $Z_{y,a,i} = M_{y,a} + \sum_{f=1}^{n_F} F_{y,a,i,f}$ is the total mortality rate.

Multi-region PTMs

When there is more than 1 region, WHAM can model survival and movement as processes occurring sequentially or simultaneously. The sequential assumption is used widely in spatially explicit model (e.g., SS3). Under the sequential assumption, survival and death occur over the interval and movement among regions occurs instantly at either the beginning or the end of the interval. WHAM is configured to have movement occur after survival and mortality:

$$\mathbf{O}_{y,a,i} = \mathbf{S}_{y,a,i}\boldsymbol{\mu}_{y,a,i}$$

where $\mathbf{S}_{y,a,i}$ is a $n_R \times n_R$ diagonal matrix of proportions surviving in each region (given they start in that region)

$$\mathbf{S}_{y,a,i} = \begin{bmatrix} e^{-Z_{y,a,i,1}\delta_i} & 0 & \cdots & 0 \\ 0 & e^{-Z_{y,a,i,2}\delta_i} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & e^{-Z_{y,a,i,n_R}\delta_i} \end{bmatrix}$$

and $\boldsymbol{\mu}_{y,a,i}$ is a $n_R \times n_R$ matrix of probabilities of moving from one region to another or staying in the region they occurred at the beginning of the interval:

$$\boldsymbol{\mu}_{y,a,i} = \begin{bmatrix} 1 - \sum_{r' \neq 1} \mu_{1 \to r',y,a,i} & \mu_{1 \to 2,y,a,i} & \cdots & \mu_{1 \to R,y,a,i} \\ \mu_{2 \to 1,y,a,i} & 1 - \sum_{r' \neq 2} \mu_{2 \to r',y,a,i} & \cdots & \mu_{2 \to R,y,a,i} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{R \to 1,y,a,i} & \cdots & \mu_{R \to R-1,y,a,i} & 1 - \sum_{r' \neq R} \mu_{R \to r',y,a,i} \end{bmatrix}$$

WHAM assumes each fleet f can harvest in only 1 region (r_f) during specified seasons. So, for each fleet f, row r_f and column f of $\mathbf{H}_{y,a,i}$ will be $F_{y,a,i,f}\left(1 - e^{-Z_{y,a,i,r}\delta_i}\right)/Z_{y,a,i,r}$ when fleet f is harvesting during the interval δ_i and all other elements will be zero. Row r of the single-column matrix $\mathbf{D}_{y,a,i}$ is $M_{y,a,r}\left(1 - e^{-Z_{y,a,i,r}\delta_i}\right)/Z_{y,a,i,r}$

When survival and movement are assumed to occur simultaneously, all movement and mortality parameters are instantaneous rates. We obtain the probability transition matrix over an interval δ_i by exponentiating the instantaneous rate matrix (Miller and Andersen 2008)

$$\mathbf{P}_{y,a,i} = e^{\mathbf{A}_{y,a,i}\delta_i}$$

The instantaneous rate matrix takes rates of movement between regions and the mortality rates for each fleet and region. Along the diagonal is the negative of the sum of the other rates (the hazard) so each row sums to zero. For two regions and one fleet operating in each region:

$$\mathbf{A}_{y,a,i} = \begin{bmatrix} a_{y,a,i,1} & \mu_{1\to 2,y,a,i} & F_{y,a,i,1} & 0 & M_{y,a,1} \\ \mu_{2\to 1,y,a,i} & a_{y,a,i,2} & 0 & F_{y,a,i,2} & M_{y,a,2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

where $a_{y,a,i,r} = -(\mu_{r\to r',y,a,i} + F_{y,a,i,r_f} + M_{y,a,r})$. When there is one region, n_f fleets, and $\delta_i = 1$, exponentiating the instantaneous rate matrix results in the PTM defined in Eq. 2.

Seasonality

Seasonality can be configured to accommodate characteristics of spawning, movement, and fleet-specific behavior. The annual time step can be divided into K (any number) seasons and the interval size δ_i for each season i need not be equal to any other seasonal interval. Under the Markov assumption, the PTM of surviving and moving and dying over K intervals $\delta_1, \dots, \delta_K$ (i.e., the entire year) is just the product of the PTMs for each interval:

$$\mathbf{P}_{y,a}(\delta_1, \dots, \delta_K) = \prod_{i=1}^{K} \mathbf{P}_{y,a,i}(\delta_i).$$

For a stock spawning at some fraction of the year $0 < t_s < 1$ in interval δ_j, the fraction of time in season j is

$$\delta_{s,j} = t_s - \sum_{i=0}^{j-1} \delta_i$$

and the PTM defining the proportions in each state at time t_s for age a is

$$\mathbf{P}_{y,a}\left(\delta_1, \dots, \delta_{j-1}, \delta_{s,j}\right) = \mathbf{P}_{y,a}\left(t_s\right) = \left[\prod_{i=1}^{j-1} \mathbf{P}_{y,a,i}(\delta_i)\right] \mathbf{P}_{y,a,j}(\delta_{s,j}). \quad (3)$$

Similarly, for an index m occurring at fraction of the year t_m in interval δ_j the proportions in each state at the time of the observation is

$$\mathbf{P}_{y,a}\left(\delta_1, \dots, \delta_{j-1}, \delta_{m,j}\right) = \mathbf{P}_{y,a}\left(t_m\right) = \left[\prod_{i=1}^{j-1} \mathbf{P}_{y,a,i}(\delta_i)\right] \mathbf{P}_{y,a,j}(\delta_{m,j}). \quad (4)$$

Numbers at age

When there are n_R regions and n_F fleets, The vector of abundance in each state at age $a > 1$ on January 1 is $\mathbf{N}_{y,a} = (\mathbf{N}'_{O,y,a}, \mathbf{0}')'$ where $\mathbf{N}_{O,y,a} = (N_{y,a,1}, \ldots, N_{y,a,n_R})'$ is the number in the states corresponding to being alive in each region and $\mathbf{0}$ is a vector $(n_F + 1)$ for the numbers captured in each fleet and dead from natural mortality because no age a fish have died yet on January 1.

Each stock s is assumed to spawn and recruit in one region r_s. So for age $a = 1$, $\mathbf{N}_{O,y,1}$ is 0 except for row $r = r_s$. Options for configuring recruitment $(N_{y,1,r_s})$ for each stock are the same as previous versions of WHAM. If recruitment is assumed to be a function of spawning stock biomass (SSB), it is only the spawning population in region r_s at the time of spawning constitutes the SSB in the stock-recruit function. However, models can configure spawning individuals to occur in other regions at the time of spawning. Aside from treating recruitment as a random walk, the general model for annual recruitment as random effects is

$$\log\left(N_{y,1,r_s}\right)|\text{SSB}_{y-1,r_s} = f\left(\text{SSB}_{y-1,r_s}\right) + \varepsilon_{y,1,r_s}$$

where

$$\text{SSB}_{s,y} = \sum_{a=1}^{A} w_{s,y,a} \text{mat}_{s,y,a} \mathbf{O}_{s,y,a,r_s}(t_s)' \mathbf{N}_{O,s,y,a}$$

where $w_{s,y,a}$ is the mean weight at age of spawning individuals, $\text{mat}_{s,y,a}$ is the maturity at age, and $\mathbf{O}_{s,y,a,r_s}(t_s)$, the r_s column of the upper-left submatrix of Eq. 3, are the probabilities of surviving and occurring in region r_s at time t_s given being alive in each region at the start of the year.

As in previous versions of WHAM, the transitions in numbers at age from one year to another after recruitment can be treated deterministically or as functions of random effects. The predicted numbers at age in year y at age a for a given stock are vector analogs $(\mathbf{N}_{O,y,a})$ of the equations for numbers at age in the standard WHAM model (Stock and Miller 2021). For ages $a = 2, \ldots, A-1$, where A is the plus group, the expected number alive in each region at the beginning of the following year and next age class age can be obtained from the first n_R elements of the vector

$$\mathbf{P}'_{y-1,a-1}\mathbf{N}_{y-1,a-1}.$$

The numbers alive in each region can also be modeled more simply using the sub-matrix $\mathbf{O}_{y,a}$. The general model for the transitions in abundance at age is

$$\log\left(\mathbf{N}_{O,y,a}\right)|\mathbf{N}_{O,y-1,a-1} = \log\left(\mathbf{O}'_{y-1,a-1}\mathbf{N}_{O,y-1,a-1}\right) + \boldsymbol{\varepsilon}_{y,a}$$

for ages $a = 2, \ldots, A-1$, and for the plus group

$$\log\left(\mathbf{N}_{O,y,A}\right)|\mathbf{N}_{O,y-1,a-1}, \mathbf{N}_{O,y-1,A} = \log\left(\mathbf{O}'_{y-1,A-1}\mathbf{N}_{O,y-1,A-1} + \mathbf{O}'_{y-1,A}\mathbf{N}_{O,y-1,A}\right) + \boldsymbol{\varepsilon}_{y,A}.$$

When the transitions in abundance at age are treated deterministically, $\boldsymbol{\varepsilon}_{y,a} = 0$. The stock- and region-specific errors $\boldsymbol{\varepsilon}_{y,a}$ are independent, but the same correlation structures as previous versions are possible across ages and years for a given stock and region. When there is autocorrelation with age, WHAM now assumes this applies only to ages $a > 1$ by default so that recruitment random effects are independent of those for for the annual transitions of older age classes. So the general covariance structure for a given stock at ages $a > 1$ in region r is

$$Cov\left(\epsilon_{y,a,r}, \epsilon_{y',a',r}\right) = \frac{\rho_{N,\text{age},r}^{|a-a'|}\rho_{N,\text{year},r}^{|y-y'|}\sigma_{N,r}^2}{\left(1-\rho_{N,\text{age},r}^2\right)\left(1-\rho_{N,\text{year},r}^2\right)}.$$

Initial numbers at age

Initial numbers at age for each stock and region can be treated as age-specific fixed effects or with an equilibrium assumption as in previous versions of WHAM. For the equilibrium option there are two parameters for each stock: the stock-specific fully-selected fishing mortality rate $\log \widetilde{F}$ and the recruitment in year 1 $\log N_{1,1,r_s}$. A stock-specific equilibrium fishing mortality at age by fleet $\tilde{F}_{a,f}$ is the product of $\widetilde{F}$ and the selectivity across fleets in the first year

$$\text{sel}_{1,a,f} = \frac{F_{1,a,f}}{\max_a \sum_{f=1}^{n_F} F_{1,a,f}}.$$

We use $\widetilde{F}_{a,f}$ to define an equilibrium abundance per recruit by region at age a conditional on recruiting to each region

$$\widetilde{\mathbf{O}}_a = \begin{cases} \prod_{j=0}^{a-1}\mathbf{O}_j & 1 \le a < A \\ \left[\prod_{j=0}^{a-1}\mathbf{O}_j\right]\left(\mathbf{I}-\mathbf{O}_A\right)^{-1} & a = A \end{cases} \tag{5}$$

where $\mathbf{O}_j$ is the equilibrium probability of surviving age a and occurring in each region and $\mathbf{O}_0 = \mathbf{I}$. Natural mortality and movement rates in the first year of the model are also used in Eq. 5. For the plus group $a = A$,

$(\mathbf{I} - \mathbf{O}_A)^{-1}$ is a “fundamental matrix’ ’ derived using the matrix version of the geometric series (Kemeny and Snell 1960). Recall that recruitment for stock s only occur in region r_s so, the equilibrium initial numbers at age a by region are

$$\mathbf{N}_{O,1,a} = \widetilde{\mathbf{O}}'_a \mathbf{N}_{O,1,1}.$$

The initial abundances at age can also be treated as independent or autocorrelated (with age) random effects. Defining the vector of initial abundance at age in region r as $\mathbf{N}_{O,1,r}$, the general model is

$$\log \mathbf{N}_{O,1,r} = \theta_{N_1,r} + \boldsymbol{\varepsilon}_{N_1,r}$$

where

$$Cov\left(\varepsilon_{N_1,a,r}, \varepsilon_{N_1,a',r}\right) = \frac{\rho_{N_1,r}^{|a-a'|}\sigma_{N_1,r}^2}{\left(1-\rho_{N_1,r}^2\right)}.$$

Parametizing movement

For each season, there are at most $n_R - 1$ parameters determining movement among regions given starting an the season in region r in either the sequential or simultaneous configurations. Movement parameters are estimated on a transformed scale via a link function $g(\cdot)$. If survival and movement are occur simultaneously, the parameters are estimated a log link function is used and if they are separable, an additive logit link function (like a multinomial regression) is used. On the transformed scale, the general model for the movement parameter from region r to r' in season i and year y for individuals of age a is a linear function of both random and environmental effects:

$$g(\mu_{r\to r',y,a,i}) = \theta_{r\to r',i} + \epsilon_{r\to r',y,a,i} + \sum_{k=1}^{n_E} \beta_{r\to r',a,i,k} E_{k,y}.$$

The random effects $\epsilon_{r\to r',y,a,i}$ are season-, and(or) region-to-region-specific and modeled most generally as a two-dimensional first-order autoregressive random effects with age and(or) year where the covariance is

$$Cov\left(\epsilon_{r\to r',y,a,i}, \epsilon_{r\to r',y',a',i}\right) = \frac{\rho_{r\to r',\text{age},i}^{|a-a'|}\rho_{r\to r',\text{year},i}^{|y-y'|}\sigma_{r\to r',i}^2}{\left(1-\rho_{r\to r',\text{age},i}^2\right)\left(1-\rho_{r\to r',\text{year},i}^2\right)}$$

similar to how WHAM models variation in survival, natural mortality, and selectivity. Effects of covariate E_k can be age-, season-, and(or) region-to-region-specific $\beta_{r\to r',a,i,k}$ and the same orthogonal polynomial options in the previous versions of WHAM for effects on recruitment and natural mortality are available.

There is currently no likelihood component for tagging data. Therefore, movement parameters would generally either need to be fixed or assumed to have some prior distribution, possibly based on external parameter estimates. We include prior distributions for the season and region-to-region specific (mean) movement parameters which are treated as random effects with the mean defined by the initial value of the fixed effect counterpart and standard deviation

$$\gamma_{r \to r',i} \sim \mathrm{N}\left(\theta_{r \to r',i}, \sigma^2_{r \to r',i}\right).$$

When priors are used, the movement is defined instead as

$$g(\mu_{r \to r',y,a,i}) = \gamma_{r \to r',i} + \epsilon_{r \to r',y,a,i} + \sum_{k=1}^{n_E} \beta_{r \to r',a,i,k} E_{k,y}$$

Natural mortality

Natural mortality options have been expanded in WHAM. When not estimated, (mean) mortality rates may be stock-, region-, and age-specific. When random effects are used, the same autocorrelation structures with age and year as described by Stock and Miller (2021) can be configured for a given stock and region. Any environmental covariate effects can be stock-, region-, and age-specific. So the general model for natural mortality is

$$\log M_{y,a,r} = \theta_{M,r} + \epsilon_{M,r,y,a} + \sum_{k=1}^{n_E} \beta_{M,r,a,k} E_{k,y}.$$

The general covariance structure for random effects are modeled most generally as a two-dimensional first-order autoregressive random effects with age and(or) year where the covariance is

$$Cov\left(\epsilon_{M,y,a,r}, \epsilon_{M,y',a',r}\right) = \frac{\rho^{|a-a'|}_{M,\text{age},r} \rho^{|y-y'|}_{M,\text{year},r} \sigma^2_{M,r}}{\left(1 - \rho^2_{M,\text{age},r}\right)\left(1 - \rho^2_{M,\text{year},r}\right)}.$$

Catch observations

The log-normal distributional assumption for aggregate catch observations is the same as Stock and Miller (2021), but the predicted catch is now a function of catch from each stock starting the year in each region. For a given stock and age, the numbers captured in each fleet over the year are

$$\widehat{\mathbf{N}}_{H,s,y,a} = \mathbf{H}'_{s,y,a} \mathbf{N}_{O,s,y,a}$$

The predicted numbers caught be each fleet across stocks is

$$\widehat{\mathbf{N}}_{H,y,a} = \sum_{s=1}^{n_S} \widehat{\mathbf{N}}_{H,s,y,a}$$

and the predicted aggregate catch at age a is

$$\widehat{\mathbf{C}}_{y,a} = \mathrm{diag}\left(\mathbf{c}_{y,a}\right) \widehat{\mathbf{N}}_{H,y,a}$$

where $\mathbf{c}_{y,a}$ is the vector of mean individual weight at age a for each fleet and the aggregate catch by fleet is

$$\widehat{\mathbf{C}}_{y} = \sum_{a=1}^{A} \widehat{\mathbf{C}}_{y,a}$$

The log-aggregate catch observations for fleet f are normally distributed

$$\log C_{y,f} \sim \mathrm{N}\left(\log \widehat{C}_{y,f}, \sigma^2_{y,f}\right).$$

The predicted numbers caught for each fleet f (row f of $\widehat{\mathbf{N}}_{H,y,a}$) are used to make predicted age composition observations as described by Stock and Miller (2021). Since then, three additional likelihood options for age composition observations have been added: a logistic-normal with AR(1) correlation structure (Francis 2014), the alternative Dirichlet-multinomial parameterization described by Thorson et al. (2017), and the multivariate Tweedie (Thorson et al. 2023).

Index observations

For index m occurring in region r_m at fraction of the year t_m, the predicted abundance at t_m in region r_m is

$$\widehat{N}_{s,y,a,m} = \mathbf{O}_{s,y,a,r_m}(t_m)'\mathbf{N}_{O,s,y,a}$$

where $\mathbf{O}_{s,y,a,r_m}(t_m)$, the r_m column of the upper-left submatrix of Eq. 4, are the probabilities of surviving and occurring in region r_m at time t_m given being alive in each region at the start of the year. The predicted index at age is

$$\widehat{I}_{m,y,a} = q_{m,y}\mathrm{sel}_{m,y,a} w_{m,y,a} \sum_{s=1}^{n_S} \widehat{N}_{s,y,a,m}$$

where $q_{m,y}$ is the catchability of the index in year y, $\mathrm{sel}_{m,y,a}$ is the selectivity and $w_{m,y,a}$ is the average weight of individuals at age a if the index is quantified in biomass and $w_{m,y,a} = 1$ if the index is quantified in numbers. Predicted age composition observations are functions of $\widehat{I}_{m,y,a}$ as described by Stock and Miller (2021) and the likelihood options are the same as those for catch explained above.

Catchability of the index can also be treated as functions of normal random effects and(or) environmental covariate effects

$$log\frac{q_{m,y}-l_m}{u_m-q_{m,y}} = \theta_{q,m} + \varepsilon_{q,m,y} + \sum_{k=1}^{n_E} \beta_{q,m,k} E_{k,y}$$

where u_m and l_m are the upper and lower bounds of catchability for index m (defaults are 0 and 1000) and the general covariance structure for the annual random effects is first-order autoregressive

$$Cov\left(\epsilon_{q,m,y}, \epsilon_{q,m,y'}\right) = \frac{\rho_{q,m}^{|y-y'|}\sigma_{q,m}^2}{1-\rho_{q,m}^2}.$$

Weight and Maturity at age

Weight and maturity at age are treated similarly to Stock and Miller (2021). Annual weight at age matrices for each fleet are used to calculate total catch and the weight at age is applied to catch numbers at age for any stocks caught by the fleet. Similarly, when indices and(or) associated age composition observations are measured in biomass, the weight at age matrices for the index are applied to predicted numbers at age of all stocks observed by the survey. Unique weight at age and maturity at age matrices are allowed for each stock to calculate spawning stock biomass.

Reference points

Currently a single F reference point $\widetilde{F}$ is estimated across stocks and regions and F by fleet and age is $\widetilde{F}_{f,a} = \widetilde{F}\mathrm{sel}_{f,a}$. Selectivity is determined as before when there are multiple fleets where $\mathrm{sel}_{f,a}$ is determined by averaging F at age over a user-defined set of years

$$\mathrm{sel}_{a,f} = \frac{\overline{F}_{a,f}}{\max_a \sum_{f=1}^{n_F} \overline{F}_{a,f}}$$

The equilibrium spawning stock biomass per recruit for stock s in region r_s is defined as

$$\phi_s(\widetilde{F}) = \sum_{a=1}^{A} \widetilde{\mathbf{O}}_{s,a,r_s,\cdot} \mathbf{O}_{s,a,\cdot,r_s}(t_s) w_{s,a} m_{s,a} \tag{6}$$

where $w_{s,a}$ and $m_{s,a}$ are the mean individual weight and probability of maturity at age a, $\widetilde{\mathbf{O}}_{s,a}$ are as described in Eq. 5, and $\mathbf{O}_{s,a}(t_s)$ is the $n_R \times n_R$ upper-left sub-matrix of eq. 3 with the probabilities of surviving and occurring in each region r' at age $a + t_s$ given starting in region r at age a. The further subscripts $r_s, \cdot$ and $\cdot, r_s$ indictate row or column r_s, respectively. Using these rows and columns is required because of the assumption that spawning and recruitment only occur in region r_s.

The equilibrium spawning biomass per recruit (eq. 6) is conditional on the region of recruitment r_s. The equilibrium recruitment in each region $\widetilde{\mathbf{N}}_{s,1}$, depends on the stock dynamics. This version of WHAM currently only allows complete spawning region fidelity so that a stock only spawns and recruits in a single region. In this case, $\widetilde{\mathbf{N}}_{s,1}$ will be positive in the spawning region (r_s) and zero elsewhere. Similarly, the row r_s of $\mathbf{O}_s$ will be zero off of the diagonal. The matrices of probabilities of surviving and occurring in each region, $\widetilde{\mathbf{O}}_{s,a}$ and $\mathbf{O}_{s,a}(\delta_s)$, are functions of the fishing mortality rates for fleets in each region $\widetilde{F}_{f,a}$.

The matrix equilibrium yield per recruit as a function of $\widetilde{F}$ is calculated as

$$\widetilde{Y}_s(\widetilde{F}) = \sum_{a=1}^{A} \widetilde{\mathbf{O}}_{s,a,r_s,\cdot} \mathbf{H}_{s,a} \mathbf{c}_{s,a} \tag{7}$$

where $\mathbf{c}_a$ is the vector of mean individual weight at age for each fleet, and $\mathbf{H}_{s,a}$ is the submatrix of the probabilities of being captured in each fleet over the interval from a to $a + 1$, defined in eq. 1.

As in previous versions of WHAM package, "static'' reference points, typically meant to be defined for prevailing conditions, average all of the inputs to the spawning biomass and yield per recruit calculations over the user-specified years (e.g., last 5 years of the model). This same averaging is also applied to possibly time-varying movement parameters.

For $X\%$ SPR-based reference points, we use a Newton method and iterate

$$\log \widetilde{F}^{(i)} = \log \widetilde{F}^{(i-1)} - \frac{g\left(\log \widetilde{F}^{(i-1)}\right)}{g'\left(\log \widetilde{F}^{(i-1)}\right)} \tag{8}$$

where $g(\log F)$ is the difference between the weighted sums of spawning biomass per recruit at F and $X\%$ of unfished spawning biomass per recruit across stocks:

$$g(\log F) = \sum_{s=1}^{n_s} \lambda_s \left[\phi_s\left(F = e^{\log F}\right) - \frac{X}{100} \phi_s\left(F = 0\right)\right].$$

where $\phi_s\left(F = 0\right)$ is the equilibrium unfished spawning biomass per recruit. $g'(\log F)$ is the derivative of g with respect to $\log F$, and the weights to use for each stock λ_s can be specified by the user or relative to the average of recruitment for each stock over the same years the user defines to calculate "static'' equilibrium spawning biomass and yield.

When a Beverton-Holt or Ricker stock recruit relationship is assumed, an analogous Newton method is used to find $\log F$ that maximizes yield for MSY-based reference points. which are also a functions of the equilibrium yield per recruit (7) and equilibrium recruitment. The function $g(\log F)$ in Eq. 8 is the first derivative of the yield curve with respect to $\log F$.

Projections

The projection options are generally the same as those for previous versions of WHAM. When there is movement of any stocks, the user has the option to project and use any random effects for time-varying movement or use the average over user specified years, analogous to how natural mortality can be treated in the projection period. The projections of any environmental covariates has been revised to better include error in the estimated latent covariate in any effects on the population in projection years.

Application to black sea bass

Prior to the most recent peer-reviewed assessment of black sea bass in the NEUS, the stock was assessed using the Age-Structured Assessment Program (ASAP) model (Legault and Restrepo 1999), a single-stock and -region statistical catch-at-age model that estimates all model parameters as fixed effects. Northern and southern components of the NEUS black sea bass stock divided by the Hudson Canyon were separately modeled in ASAP (reference map figure). Results from the separate ASAP models have been aggregated for a unit-stock assessment. Furthermore, the ASAP-based assessments have exhibited strong retrospective patterns (Mohn 1999), and exploring alternative modeling approaches for the northern and southern stock components has been a high priority for management.

In 2023, a working group composed of scientists from federal, state, and academic institutions determined an optimal data and model configuration for the black sea bass stock using the multi-stock and multi-region extension of WHAM described above (NEFSC 2023). This assessment included the spatial features and hypothesized environmental drivers that were prioritized research recommendations from previous black sea bass assessments.

Below we describe the assumptions and configuration of the assessment model as determined by the working group as well as the alternative assumptions for recruitment and natural mortality in the models we fit to evaluate alternative hypotheses of environmental effects.

Basic structure

The first year being modeled for the population is 1989 and the fishery and index data used in the model span from 1989 to 2021. There are two stock components (north and south) modeled as separate populations that spawn and recruit in respective regions separated by the Hudson Canyon. We have observations for each of four total fishing fleets, where two fishing fleets (Recreational and Commercial) operate in each region.

There are 11 seasonal intervals within each calendar year: five monthly time intervals from Jan 1 to May 31, a spawning season from June 1 to July 31, and five monthly intervals from August 1 to December 31. The southern stock component is assumed to never move to the northern region (Figure 1). For the northern component, a proportion μ_1 can move to the southern region each month during the last five months of the year, but no movement is allows from the south to the north during this period. During the first four intervals of the year a proportion μ_2 the northern component individuals in the south can move back to the north, but no movement from the north to south is allowed during this period. In the fifth interval (May), all northern component individuals remaining in the south are assumed to move back to the north for the subsequent spawning period. Survival and movement occur sequentially in each interval and each of the two movement proportions are assumed constant across intervals, ages, and years.

The monthly movement matrices are

$$\boldsymbol{\mu}_1 = \begin{bmatrix} 1-\mu_{1\to 2} & \mu_{1\to 2} \\ 0 & 1 \end{bmatrix}$$

for the portion of the year after spawning and

$$\boldsymbol{\mu}_2 = \begin{bmatrix} 1 & 0 \\ \mu_{2\to 1} & 1-\mu_{2\to 1} \end{bmatrix}$$

for the portion of the year before spawning.

The constant or mean log-natural mortality rate is assumed to be log(0.4)

As noted in the description of the general WHAM model, tagging data are not yet allowed. However, the working group also fit a Stock Synthesis model (Methot and Wetzel 2013) which provided estimated movement rate parameters and standard errors that were used to configure the priors for WHAM (see Supplementary Materials).

Initial abundance at age

With the movement configuration, the northern origin fish (ages 2+) can occur in the southern region on January 1. Estimating initial numbers at age as separate parameters can be challenging even in single-stock models. To avoid challenges with estimating initial numbers at age in each region for the northern component in the two stock model, we used the equilibrium assumption described above. Using this assumption two parameters are estimated for each stock component: an initial recruitment and an equilibrium fully-selected F.

Recruitment and Survival

For the northern population abundance at age 1 on January 1 (recruitment) is only allowed in the northern region, but given the monthly movement described above, older individuals that previously recruited in the northern region may occur in the southern region on January 1. Therefore, a model with survival random effects will model the transitions (survival/movement) of abundances at age of northern origin fish in each region. All of the initial runs assumed variance parameters for these random effects to be the same for northern origin fish occurring in both regions on January 1. The base model assumes very small variance for the transitions of northern fish in the southern region, which is essentially the same as the deterministic transition assumptions of a statistical catch at age model. We also allow 2DAR1 correlation for recruitment and survival for both the northern and southern components. Unique variance and correlation parameters for the recruitment and survival random effects are estimated for the northern and southern components.

Uncertainty recreational CPA index observations

The CVs provided by the analyses for the recreational catch per angler (RecCPA) ranged between 0.02 and 0.06 which the working group felt did not capture the true uncertainty in the index with regard to its relationship to stock abundance. In many of the initial runs as well as the base model we allowed a scalar multiple of the standard deviation of the log aggregate index to be estimated for these indices in the northern and southern regions. Models that successfully estimated these scalars indicated standard deviations for these surveys to be approximately 5 times the input value and this value was fixed in many preliminary runs to avoid dealing with convergence problems. However, the base model successfully estimated these scalars. The model estimates are negligibly affected by estimating these scalars, but we felt estimating these parameters allowed uncertainty in model output to be more properly conveyed.

Index and Catch age composition observations

Table 1

Bottom Temperature effects

All model fits also include bottom temperature observations for the northern and southern regions from 1963 to 2021 and estimated standard errors ranging between 0.03 and 0.09 degrees Celsius (NEFSC 2023). We retained the assumption from the peer-reviewed assessment that treated the latent bottom temperature covariates in each region as first-order autoregressive processes.

We fit 14 models with alternative assumptions about the effects of bottom temperature covariates, ranging from no effects to effects on both regions for either recruitment or natural mortality at age 1 (Table 2). These analyses derive from the hypothesis that bottom temperature affects overwinter survival of fish where the fish turn from age 0 to age 1 on January 1 (Miller et al. 2016a). This temperature may be a proxy for temperature prior to January 1 and affect survival during the end of the pre-recruit phase or natural mortality in the early part of of the year after becoming age 1. Furthermore, we have no direct observations of age 1 individuals from surveys until the spring season each year. Therefore, we fit models with effects of temperature on recruitment or natural mortality at age 1.

It is standard practice to treat annual recruitment as time-varying deviations from a mean model and all models here treat recruitment deviations as first-order autoregressive random effects. However, Miller et al. (2018) showed how inferences of temperature effects on growth or maturity parameters can be very different whether the null model without the effect and models with effects also include random effects representing residual temporal variation in parameters. Therefore, we also explored whether including temporal random effects on age 1 natural mortality M affected inferences on corresponding temperature effects. Initially, we included random effects on age 1 natural mortality for both the northern and southern stock components, but estimates of these random effects and corresponding variance for the southern component converged to 0 so these were not included in models presented here.

We assume the covariate in year y affects recruitment in the same years because the covariate observations are from months January to March. The fish are technically already 1 year old, but there are no observations of these individuals until later in the year except possibly in fishery catches which are accumulated over the whole year. Expected log-recruitment is a linear function of log-recruitment at the previous time step and bottom temperature

$$\log N_{s,r_s,y,1} = \mu_{R,s} + \beta_{R,s} x_y + \epsilon_{N,s,y,1} \tag{9}$$

$$\log M_{s,y,1} = \mu_{M,s,1} + \beta_{M,s} x_y + \epsilon_{M,s,y} \tag{10}$$

Model fitting, diagnostics, and comparisons

As in Miller et al. (2016b), we also assessed the stability of the AIC-based model selection over retrospective peels to guard against previously noted changes in perception of covariate effects on recruitment with increased length of the time series of observations (Myers 1998). This retrospective examination was also recommended by Brooks (2024).

We examined retrospective patterns by fitting models where the terminal year is reduced sequentially by one year (peel) for seven years. Therefore, there are 7 fits of the proposed base model with the time series reduced by one to seven years. We calculated Mohn's ρ for recruitment, SSB, and fully-selected F. Absolute values of Mohn's ρ near 0 imply no pattern in estimation of these quantities as the time-series is sequentially extended.

Specifically, we use version XX and commit XXX of the wham package (https://github.com/timjmiller/wham/tree/lab) for all results.

We performed a simulation self-test where new observations were simulated conditional on all random effects estimated in the proposed base model and the same model configuration was fit to each of the simulated data sets.

One-step-ahead residuals can now be calculated for all index, catch, environmental covariate observations using methods described by Thygesen et al. (2017) and Trijoulet et al. (2023).

One-step-ahead residuals

Retrospective patterns

Results

Although there was strong retrospective patterns in the most recent management track assessment for the northern component of the stock, there was no evidence of patterns for northern SSB (Mohn's ρ = -0.058) and F (Mohn's $\rho = 0.06$) for the proposed base model (Figures **??** and **??**). Similarly, no patterns were exhibited for the southern component of SSB (Mohn's ρ = -0.019) and regional F (Mohn's ρ = -0.05) (Figures **??** and **??**).

For 7 of the the simulated data sets the model failed to optimize. The maximum absolute gradient was $< 10^{-6}$ for only 9 and $< 10^{-4}$ for 52 of the 93 successfully fitted models. The poor convergence appeared to be attributable to the estimation of the scalar for the standard errors of the log-transformed northern Recreational CPA index for which estimates tended to 0 for nearly all of the fits (< 0.01 for 83 fits). However, even across all fits including those with poor convergence, the SSB estimates appeared to be reliable (Figure **??**).

Bottom temperature effect on recruitment

We found the best model of bottom temperature covariate effects included the effect only on the recruitment of the northern stock, which is the configuration in the proposed base model although the difference in AIC for the model including effects on both regional components was small suggesting some evidence for this hypothesis as well (Table **??**) . We also found AIC to strongly favor models that included at least an effect on the northern component across all retrospective peels (Table **??**). The posterior estimate of the bottom temperature covariate match the observations well because of the high precision of the observations (Figure **??**). The residual variation in the standard deviation of recruitment random effects is reduced because the expected recruitment (Eq. 9) is a function of the covariate (Figure **??**). This effect is included in the proposed base model (Figure **??**). Because the covariate is technically for temperature after the beginning of the calendar year when the fish are considered age 1 in the model, a comparison of the proposed base model to one with the bottom temperature effect on natural mortality at age 1 instead might be of interest in the future. An investigation of higher order orthogonal polynomial effects might also be of interest particularly for the southern component which might be experiencing more frequently higher temperatures less favorable to overwinter survival.

Discussion

The new features of Multi-WHAM include

- seasonal intervals within years
- variation in movement rates by stock, region-to-region, season, age, and year

- effects of environmental covariates on movement rates by stock, region-to-region, season, and age
- effects of environmental covariates on mortality rates by stock, region, and age
- effects of environmental covariates on recruitment by stock
- stock-specific stock-recruitment models
- priors for movement rates
- seasonal operation of fleets
- mortality and movement modeled sequentially or simultaneously
- options for weighting of stock-specific SSB/R for global SPR-based reference points

References

Brooks, E.N. 2024. Pragmatic approaches to modeling recruitment in fisheries stock assessment: A perspective. Fisheries Research **270**: 106896. doi:10.1016/j.fishres.2023.106896.

Francis, R.I.C.C. 2014. Replacing the multinomial in stock assessment models: A first step. Fisheries Research **151**: 70–84. doi:10.1016/j.fishres.2013.12.015.

Kemeny, J.G., and Snell, J.L. 1960. Finite markov chains. D. Van Nostrand Company, Princeton, New Jersey.

Kristensen, K., Nielsen, A., Berg, C., Skaug, H., and Bell, B.M. 2016. TMB: Automatic differentiation and Laplace approximation. Journal of Statistical Software **70**: 1–21. doi:10.18637/jss.v070.i05.

Legault, C.M., and Restrepo, V.R. 1999. A flexible forward age-structured assessment program. Col. Vol. Sci. Pap. ICCAT **49**(2): 246–253.

Legault, C.M., Wiedenmann, J., Deroba, J.J., Fay, G., Miller, T.J., Brooks, E.N., Bell, R.J., Langan, J., Cournane, J., Jones, A.W., and Muffley, B. 2023. Data rich but model resistant: An evaluation of data-limited methods to manage fisheries with failed age-based stock assessments. Canadian Journal of Fisheries and Aquatic Sciences **80**(1): 27–42. doi:10.1139/cjfas-2022-0045.

Methot, R.D., and Wetzel, C.R. 2013. Stock Synthesis: A biological and statistical framework for fish stock assessment and fishery management. Fisheries Research **142**(1): 86–99.

Miller, A.S., Shepherd, G.R., and Fratantoni, P.S. 2016a. PLOS ONE **11**(1): e0147627. doi:10.1371/journal.pone.0147627.

Miller, T.J., and Andersen, P.K. 2008. A finite-state continuous-time approach for inferring regional migration and mortality rates from archival tagging and conventional tag-recovery experiments. Biometrics **64**(4): 1196–1206. doi:10.1111/j.1541-0420.2008.00996.x.

Miller, T.J., Hare, J.A., and Alade, L.A. 2016b. A state-space approach to incorporating environmental effects on recruitment in an age-structured assessment model with an application to southern New England yellowtail flounder. Can. J. Fish. Aquat. Sci. **73**(8): 1261–1270. doi:10.1139/cjfas-2015-0339.

Miller, T.J., O'Brien, L., and Fratantoni, P.S. 2018. Temporal and environmental variation in growth and maturity and effects on management reference points of Georges Bank Atlantic cod. Can. J. Fish. Aquat. Sci. **75**(12): 2159–2171. doi:10.1139/cjfas-2017-0124.

Miller, T.J., and Stock, B.C. 2020. The Woods Hole Assessment Model (WHAM). https://timjmiller.github.io/wham.

Mohn, R. 1999. The retrospective problem in sequential population analysis: An investigation using cod fishery and simulated data. ICES Journal of Marine Science **56**(4): 473–488.

Myers, R.A. 1998. When do environment–recruitment correlations work? Reviews in Fish Biology and Fisheries **8**(3): 229–249. doi:10.1023/A:1008828730759.

NEFSC. 2023. Report of the black sea bass (*Centropristis striata*) research track stock assessment working group. Available at https://www.mafmc.org/s/a_2023_BSB_UNIT_RTWG_Report_V2_12_2_2023-1.pdf.

Nielsen, A., and Berg, C.W. 2014. Estimation of time-varying selectivity in stock assessments using state-space models. Fisheries Research **158**: 96–101. doi:10.1016/j.fishres.2014.01.014.

Stock, B.C., and Miller, T.J. 2021. The Woods Hole Assessment Model (WHAM): A general state-space assessment framework that incorporates time- and age-varying processes via random effects and links to environmental covariates. Fisheries Research **240**: 105967. doi:10.1016/j.fishres.2021.105967.

Stock, B.C., Xu, H., Miller, T.J., Thorson, J.T., and Nye, J.A. 2021. Implementing two-dimensional autocorrelation in either survival or natural mortality improves a state-space assessment model for Southern New England-Mid Atlantic yellowtail flounder. Fisheries Research **237**: 105873. doi:10.1016/j.fishres.2021.105873.

Thorson, J.T., Johnson, K.F., Methot, R.D., and Taylor, I.G. 2017. Model-based estimates of effective sample size in stock assessment models using the Dirichlet-multinomial distribution. Fisheries Research **192**: 84–93. doi:10.1016/j.fishres.2016.06.005.

Thorson, J.T., Miller, T.J., and Stock, B.C. 2023. The multivariate-tweedie: A self-weighting likelihood for age and length composition data arising from hierarchical sampling designs. ICES Journal of Marine Science **80**(10): 2630–2641. doi:10.1093/icesjms/fsac159.

Thygesen, U.H., Albertsen, C.M., Berg, C.W., Kristensen, K., and Nielsen, A. 2017. Validation of ecological state space models using the Laplace approximation. Environmental and Ecological Statistics **24**(2): 317–339. doi:10.1007/s10651-017-0372-4.

Trijoulet, V., Albertsen, C.M., Kristensen, K., Legault, C.M., Miller, T.J., and Nielsen, A. 2023. Model validation for compositional data in stock assessment models: Calculating residuals with correct properties. Fisheries Research **257**: 106487. doi:https://doi.org/10.1016/j.fishres.2022.106487.

Appendix A

Table A1: Definition of terms.

i	Seasonal time interval
δ_i	Length of seasonal time interval i
a	Age class
y	Year
A	Last age class ("plus group")
r	Region
f	Fishing fleet
n_F	Number of fishing fleets
s	Stock
n_R	Number of regions
$\mathbf{P}_{y,a,i}$	Probability transition matrix for year y, age a, and season i
$\mathbf{O}_{y,a,i}$	submatrix of $\mathbf{P}_{y,a,i}$ of probabilities of surviving and occuring in each region for year y, age a, and season i
$\mathbf{H}_{y,a,i}$	submatrix of $\mathbf{P}_{y,a,i}$ of probabilities of being captured in each fishing fleet for year y, age a, and season i
$\mathbf{I}_H$	n_f x n_f identity matrix
m	Index observation
$O_{y,a,i}(r, r')$	For year y, age a and season i, the probability of surviving and occurring in region r' given beginning the interval alive in region r
$H_{y,a,i}(r, f)$	For year y, age a and season i, the probability of being captured in fleet f given beginning the interval alive in region r
$S_{y,a,i}$	For year y, age a and season i, the probability of surviving the interval (1 region model)
$F_{y,a,i,f}$	Fishing mortality rate for fleet f in year y at age a in seasonal interval i
$M_{y,a}$	Natural mortality rate in year y at age a (single region)
$M_{y,a,r}$	Natural mortality rate in region r and year y at age a
$Z_{y,a,i}$	Total mortality rate in year y at age a in seasonal interval i (single region)
$Z_{y,a,i,r}$	Total mortality rate in region r and year y at age a in seasonal interval i
$\mathbf{S}_{y,a,i}$	matrix of of probabilities of surviving in each region over the interval for season i, year y, age a

Table A1: (Continued)

$\boldsymbol{\mu}_{y,a,i}$	matrix of of probabilities of moving or staying in each region at the end of season i in year y and age a
$\mu_{r \rightarrow r',y,a,i}$	For year y, age a and seasonal interval i, either the probability of moving at the end of the interval or instantanteous rate of movement from region r to region r'
r_f	region where fleet f operates
$\mathbf{N}_{y,a}$	Column vector of abundances by region at age a in year y
$\mathbf{A}_{y,a,i}$	instantaneous rate matrix for seasonal interval i, year y, and age a
$a_{y,a,i,r}$	For year y, age a and seasonal interval i, the hazard or negative sum of the instantaneous rates of mortality and movement from the state corresonding to being alive in region r
$\mathbf{P}_{y,a}(\delta_1, \ldots, \delta_K)$	Probability transition matrix for year y and age a over seasonal intervals $\delta_1, \ldots, \delta_K$
K	Number of seasons in the annual time step
t_s	fraction of the annual time step when spawning occurs for stock s
$\delta_{s,j}$	fraction of the annual time step between t_s and the end of season $j-1$
t_m	fraction of the annual time step when index m observers the population
$\delta_{m,j}$	fraction of the annual time step between t_m and the end of season $j-1$
$\mathbf{N}_{y,a}$	Abundance at age a in year y in each of the living and mortality states on January 1
$\mathbf{N}_{O,y,a}$	Abundance at age a in year y alive in each region on January 1
$N_{y,a,r}$	Abundance at age a in year y alive in region r on January 1
r_s	region where stock s spawns and recruits
$\mathrm{SSB}_{s,y}$	Spawning stock biomass for stock s in year y
$\varepsilon_{y,a,r}$	Random error for abundance at age a in year y in region r
$w_{s,y,a}$	mean individual weight for stock s at age a in year y
$\mathrm{mat}_{s,y,a}$	proportion mature at age a in year y for stock s
$\mathbf{O}_{s,y,a,r_s}(t_s)$	Probabilities of surviving and occurring in region r_s at time t_s given being alive in each region at the start of the year
$\boldsymbol{\varepsilon}_{y,a}$	vector of random errors for abundance alive in each region on January 1 of year y at age a
$\sigma_{N,r}$	standard deviation parameter for abundance at age random effects in region r
$\rho_{N,\mathrm{age},r}$	first order auto-regressive correlation parameter across age for abundance at age random effects in region r
$\rho_{N,\mathrm{year},r}$	first order auto-regressive correlation parameter across year for abundance at age random effects in region r

Table A1: (Continued)

$\text{sel}_{1,a,f}$	selectivity at age a for fleet f in the first year
$F_{1,a,f}$	fishing mortality rate at age a for fleet f in the first year
$\widetilde{F}_{a,f}$	equilibrium fishing mortality rate at age a for fleet f
$\mathbf{O}_a$	proportion surviving the year at age j and occuring in each region (columns) given alive on January 1 in each region (rows)
$\widetilde{\mathbf{O}}_a$	equilibrium proportions alive in each region at age a (columns) given recruitment in each region (rows)
$\mathbf{N}_{O,1,r}$	vector of abundance by age in region r in the first year
$\theta_{N_1,r}$	mean parameter for initial numbers at age random effects in the first year for region r
$\boldsymbol{\varepsilon}_{N_1,r}$	vector of random effects by age for initial numbers at age in the first year for region r
$\sigma_{N_1,r}$	standard deviation parameter for initial numbers at age random effects in the first year for region r
$\rho_{N_1,r}$	first order auto-regressive correlation parameter for initial numbers at age random effects in the first year for region r
$\mu_{r\to r',y,a,i}$	movement from region r to region r' in seasonal interval i and year y at age a
$g(\mu_{r\to r',y,a,i})$	link function for movement $\mu_{r\to r',y,a,i}$
$\theta_{r\to r',i}$	mean parameter across age and year for movement from region r to region r' in seasonal interval i
$\epsilon_{r\to r',y,a,i}$	random effect parameter for movement from region r to region r' in seasonal interval i and year y at age a
n_E	number of environmental covariates
$\beta_{r\to r',a,i,k}$	effect of environmental covariate k on movement from region r to r' at age a in seasonal interval i
$E_{k,y}$	latent environmetal covariate k affecting the population in year y
$\sigma_{r\to r',i}$	standard deviation parameter for movement random effects from region r to r' in seasonal interval i
$\rho_{r\to r',\text{age},i}$	first order auto-regressive correlation parameter across age for movement random effects from region r to r' in seasonal interval i
$\rho_{r\to r',\text{year},i}$	first order auto-regressive correlation parameter across year for movement random effects from region r to r' in seasonal interval i

Table A1: (Continued)

$\gamma_{r\rightarrow r',i}$	random effect for link-transformed mean movement from region r to r' in seasonal interval i when a prior distribution is assumed
$\sigma_{r\rightarrow r',i}$	standard deviation parameter for prior distribution of $\gamma_{r\rightarrow r',i}$
$M_{y,a,r}$	natural mortality rate for age a in year y in region r
$\theta_{M,r}$	mean parameter across age and year for nutural mortality in region r
$\epsilon_{M,r,y,a}$	random effect parameter for natural mortality in region r and year y at age a
$\beta_{M,r,a,k}$	effect of environmental covariate k on natural mortality in region r at age a
$\sigma_{M,r}$	standard deviation parameter for natural mortality random effects in region r
$\rho_{M,\text{age},r}$	first order auto-regressive correlation parameter across age for natural mortality random effects in region r
$\rho_{M,\text{year},r}$	first order auto-regressive correlation parameter across year for natural mortality random effects in region r
$\widehat{\mathbf{N}}_{H,s,y,a}$	vector of predicted numbers of stock s at age a in year y captured by each fleet
$\widehat{\mathbf{N}}_{H,y,a}$	vector of predicted numbers at age a in year y captured by each fleet across all stocks
$\widehat{\mathbf{C}}_{y,a}$	vector of predicted biomass captured at age a in year y by each fleet across all stocks
$\mathbf{c}_{y,a}$	vector of mean individual weight at age a in year y for each fleet
$\widehat{\mathbf{C}}_{y}$	vector of predicted aggregate catch for each fleet in year y
$C_{y,f}$	observed aggregate catch for fleet f in year y
$\widehat{C}_{y,f}$	predicted aggregate catch for fleet f in year y
$\sigma_{y,f}$	standard deviation of observed log-aggregate catch for fleet f in year y
$\widehat{N}_{s,y,a,m}$	predicted abundance at t_m in region r_m
$\mathbf{O}_{s,y,a,r_m}(t_m)$	the probabilities of surviving and occurring in region r_m at time t_m given being alive in each region at the start of the year which is the r_m column of the upper-left submatrix of Eq. 4
$\widehat{I}_{m,y,a}$	Predicted relative abundance index for survey d in year y at age a
$q_{m,y}$	catchability of index m in year y
$\text{sel}_{m,y,a}$	selectivity of index m at age a in year y
$w_{m,y,a}$	average weight of individuals at age a for index m if the index is quantified in biomass, otherwise it is unity
u_m	upper bound for index m catchability
l_m	lower bound for index m catchability
$\theta_{q,m}$	mean index m catchability parameter

Table A1: (Continued)

$\varepsilon_{q,m,y}$	index m catchability random effect in year y
$\beta_{q,m,k}$	effect of environmental covariate k on index m catchability
$\sigma_{q,r}$	standard deviation parameter for index m catchability random effects
$\rho_{q,m}$	first order auto-regressive correlation parameter across year for index m catchability random effects

<table>
<tr><th></th><th>Jan</th><th>Feb</th><th>Mar</th><th>Apr</th><th>May</th><th>Jun</th><th>Jul</th><th>Aug</th><th>Sep</th><th>Oct</th><th>Nov</th><th>Dec</th></tr>
<tr><td>Northern Component</td><td colspan="5">Monthly survival and movement south</td><td colspan="2">Survival and spawning</td><td colspan="5">Monthly survival and movement north</td></tr>
<tr><td>Southern Component</td><td colspan="5">Monthly survival</td><td colspan="2">Survival and spawning</td><td colspan="5">Monthly survival</td></tr>
</table>

Figure 1: Diagram of intervals within the year and configuation of the dynamics of each component of the BSB population.

Table 1: Configuration of Age composition likelihood, mean selectivity model, and selectivity random effects for each age composition data component.

Data component	Age Composition Likelihood	Mean Selectivity model	Random effects configuration
North Commercial	Dirichlet-Multinomial	age-specific (flat-topped at ages > 3)	2D-AR1 (age and year)
North Recreational	Logistic-normal (0s as missing)	age-specific (flat-topped at ages > 6)	2D-AR1 (age and year)
South Commercial	Logistic-normal (AR1, 0s as missing)	logistic	None
South Recreational	Logistic-normal (AR1, 0s as missing)	logistic	None
North Recreational CPA	Logistic-normal (0s as missing)	age-specific (flat-topped at ages > 1)	AR1 (year)
North VAST	Dirichlet-Multinomial	age-specific (flat-topped at ages > 4)	2D-AR1 (age and year)
South Recreational CPA	Logistic-normal (AR1, 0s as missing)	age-specific (flat-topped at ages > 2)	None
South VAST	Logistic-normal (AR1, 0s as missing)	age-specific (flat-topped at ages > 1)	None

Table 2: Assumptions for temperature effects and random effects for age 1 natural mortality for each model.

Model	Temperature Effect		
	North	South	M at age 1 random effects
M_0	–	–	none
M_1	Recruitment	–	none
M_2	–	Recruitment	none
M_3	Recruitment	Recruitment	none
M_4	M at age 1	–	none
M_5	–	M at age 1	none
M_6	M at age 1	M at age 1	none
M_7	–	–	time-varying
M_8	Recruitment	–	time-varying
M_9	–	Recruitment	time-varying
M_{10}	Recruitment	Recruitment	time-varying
M_{11}	M at age 1	–	time-varying
M_{12}	–	M at age 1	time-varying
M_{13}	M at age 1	M at age 1	time-varying

Table 3: Difference between AIC and the lowest AIC for each model by retrospective peel.

Model	Peel							
	0	1	2	3	4	5	6	7
M_0	14.53	16.09	13.78	13.69	13.79	17.02	8.47	8.22
M_1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
M_2	15.29	16.77	14.80	14.71	14.85	18.11	9.20	9.22
M_3	3.49	5.42	4.42	4.69	5.02	8.75	0.73	1.00
M_4	16.51	18.05	15.63	15.45	15.63	18.79	10.08	9.85
M_5	16.09	17.51	15.30	15.16	15.29	18.41	9.86	9.71
M_6	18.07	19.46	17.14	16.92	17.12	20.18	11.46	11.33
M_7	17.05	18.48	15.57	14.96	15.39	18.19	9.71	9.42
M_8	5.02	6.87	4.90	4.66	5.28	8.59	0.98	0.99
M_9	17.82	19.17	16.59	15.98	16.45	19.28	10.44	10.43
M_{10}	5.78	7.56	5.92	5.68	6.34	9.68	1.71	1.99
M_{11}	19.02	20.48	17.57	16.96	17.39	20.19	11.68	11.39
M_{12}	18.61	19.90	17.09	16.43	16.88	19.58	11.10	10.92
M_{13}	20.58	21.89	19.09	18.43	18.88	21.57	13.07	12.88

Table 4: Model AIC weights for each retrospective peel.

	Peel							
Model	0	1	2	3	4	5	6	7
M_0	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
M_1	0.76	0.89	0.80	0.80	0.83	0.97	0.36	0.38
M_2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
M_3	0.13	0.06	0.09	0.08	0.07	0.01	0.25	0.23
M_4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
M_5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
M_6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
M_7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
M_8	0.06	0.03	0.07	0.08	0.06	0.01	0.22	0.23
M_9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
M_{10}	0.04	0.02	0.04	0.05	0.04	0.01	0.15	0.14
M_{11}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
M_{12}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
M_{13}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 5: Mohn's ρ for SSB, R, average F at ages 6 and 7 in northern and southern regions.

	SSB		Average F		Recruitment	
Model	North	South	North	South	North	South
M_0	-0.031	-0.026	0.032	-0.043	0.211	-0.007
M_1	-0.040	-0.023	0.041	-0.048	0.181	-0.024
M_2	-0.031	-0.027	0.032	-0.043	0.211	-0.007
M_3	-0.031	-0.027	0.031	-0.043	0.193	-0.007
M_4	-0.032	-0.026	0.032	-0.044	0.268	-0.006
M_5	-0.031	-0.026	0.032	-0.045	0.210	-0.008
M_6	-0.032	-0.025	0.032	-0.045	0.267	-0.007
M_7	-0.033	-0.026	0.033	-0.043	0.225	-0.007
M_8	-0.035	-0.027	0.035	-0.043	0.186	-0.007
M_9	-0.033	-0.027	0.033	-0.043	0.224	-0.008
M_{10}	-0.035	-0.027	0.035	-0.043	0.186	-0.008
M_{11}	-0.032	-0.026	0.032	-0.044	0.253	-0.007
M_{12}	-0.033	-0.026	0.033	-0.045	0.224	-0.008
M_{13}	-0.032	-0.025	0.032	-0.045	0.253	-0.008

Supplemental Materials

Deriving the prior distribution for movement parameters

The working group fit a Stock Synthesis model (Methot and Wetzel 2013) that included tagging data with 2 seasons (6 months each) and 2 regions where a proportion μ_1^* of the northern component moves to the south in one season and some proportion $\mu_{2\to1}^*$ move back to the south in the second season (NEFSC 2023). The seasonal movement matrices for each season are

$$\boldsymbol{\mu}_1^* = \begin{bmatrix} 1-\mu_{1\to2}^* & \mu_{1\to2}^* \\ 0 & 1 \end{bmatrix}$$

and

$$\boldsymbol{\mu}_2 = \begin{bmatrix} 1 & 0 \\ \mu_{2\to1}^* & 1-\mu_{2\to1}^* \end{bmatrix}.$$

To obtain estimates of movement proportions for the monthly intervals in the WHAM model, the half-year movement matrices were converted to monthly movement matrices by taking the root z_k of $\boldsymbol{\mu}_k^*$ which are defined by the number of months of movement for each season (5 and 4, respectively). The roots of the matrices are calculated using an eigen decomposition of the matrices

$$\boldsymbol{\mu}_k = (\boldsymbol{\mu}_k^*)^{z_k} = \mathbf{V}_k \mathbf{D}_k^{z_k} \mathbf{V}_k^{-1}$$

where $z_1 = 1/5$ for and $z_2 = 1/4$, and $\mathbf{V}_k$ and $\mathbf{D}_k$ are the matrix of eigenvectors (columnwise) and the diagonal matrix of corresponding eigenvalues of $\boldsymbol{\mu}_k^*$. The working group used a parametric bootstrap approach to determine an appropriate standard deviation for the prior distribution for the movement parameters. Stock Synthesis also estimates parameters on a transformed scale, but different from WHAM:

$$\mu_{r\to r'}^* = \frac{1}{1+2e^{-x_{r\to r'}}}$$

The estimated parameters and standard errors from the Stock Synthesis model were $x_{1\to2} = -1.44$ and $x_{2\to1} = 1.94$ and $SE(x_{1\to2})) = 0.21$ and $SE(x_{2\to1})) = 0.37$. The resulting in the estimated proportions were $\mu_{1\to2}^* = 0.11$ and $\mu_{2\to1}^* = 0.78$.

In WHAM, an additive logit transformation is used which is simply a logit transformation when there are only two regions:

$$\mu_{r\to r'} = \frac{1}{1+e^{-y_{r\to r'}}}.$$

We simulated 1000 values from a normal distribution with mean and standard deviation defined by the parameter estimate and standard error $\tilde{x}_{r\to r',b} \sim N(x_{r\to r'}, SE(x_{r\to r'}))$ from the Stock Synthesis model. For each simulated value we constructed $\tilde{\boldsymbol{\mu}}^*_{r\to r',b}$, took the appropriate root and calculated inverse logit for $\tilde{y}_{r\to r',b}$. We calculated the mean and standard deviation of the values $y_{i,b}$. The mean values did not differ meaningfully from the transformation of the original estimates ($y_{1\to 2} = -3.79$ and $y_{2\to 1} = -0.79$) and the standard deviation was approximately 0.2 for both parameters.