**Supplemental Information**

# The Woods Hole Assessment Model (WHAM): a general state-space assessment framework that incorporates time- and age-varying processes via random effects and links to environmental covariates

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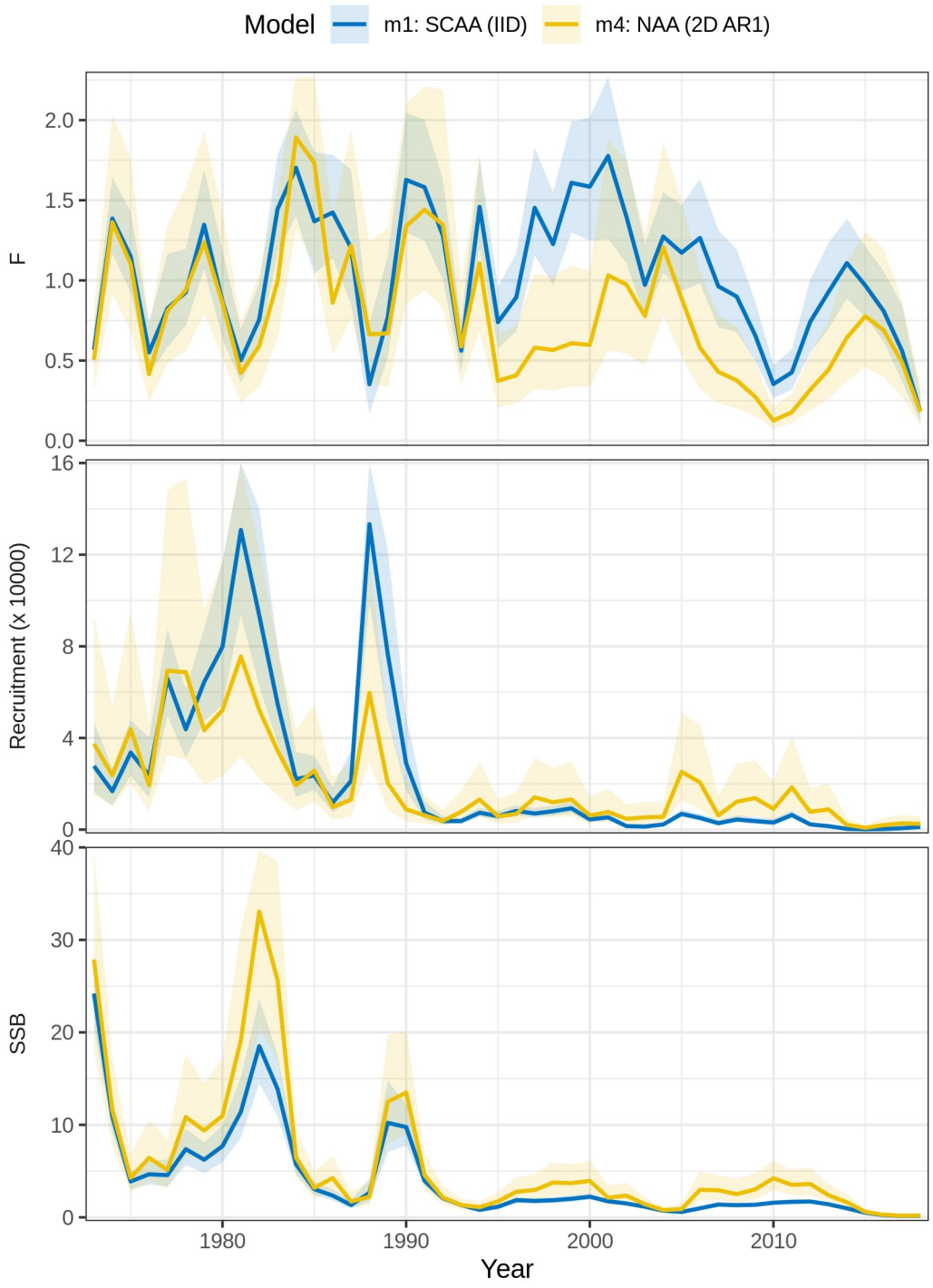


**Figure S1**. Trends in *F*, recruitment, and SSB estimated for SNEMA yellowtail flounder using two models of numbers-at-age (NAA) random effects. m1 = only recruitment deviations are random effects (most similar to traditional statistical catch-at-age, SCAA), and deviations are independent and identically distributed (IID). m4 = all NAA deviations are random effects correlated by age and year (2D AR1). Model m4 had the lowest AIC (Fig. 1 in main text).

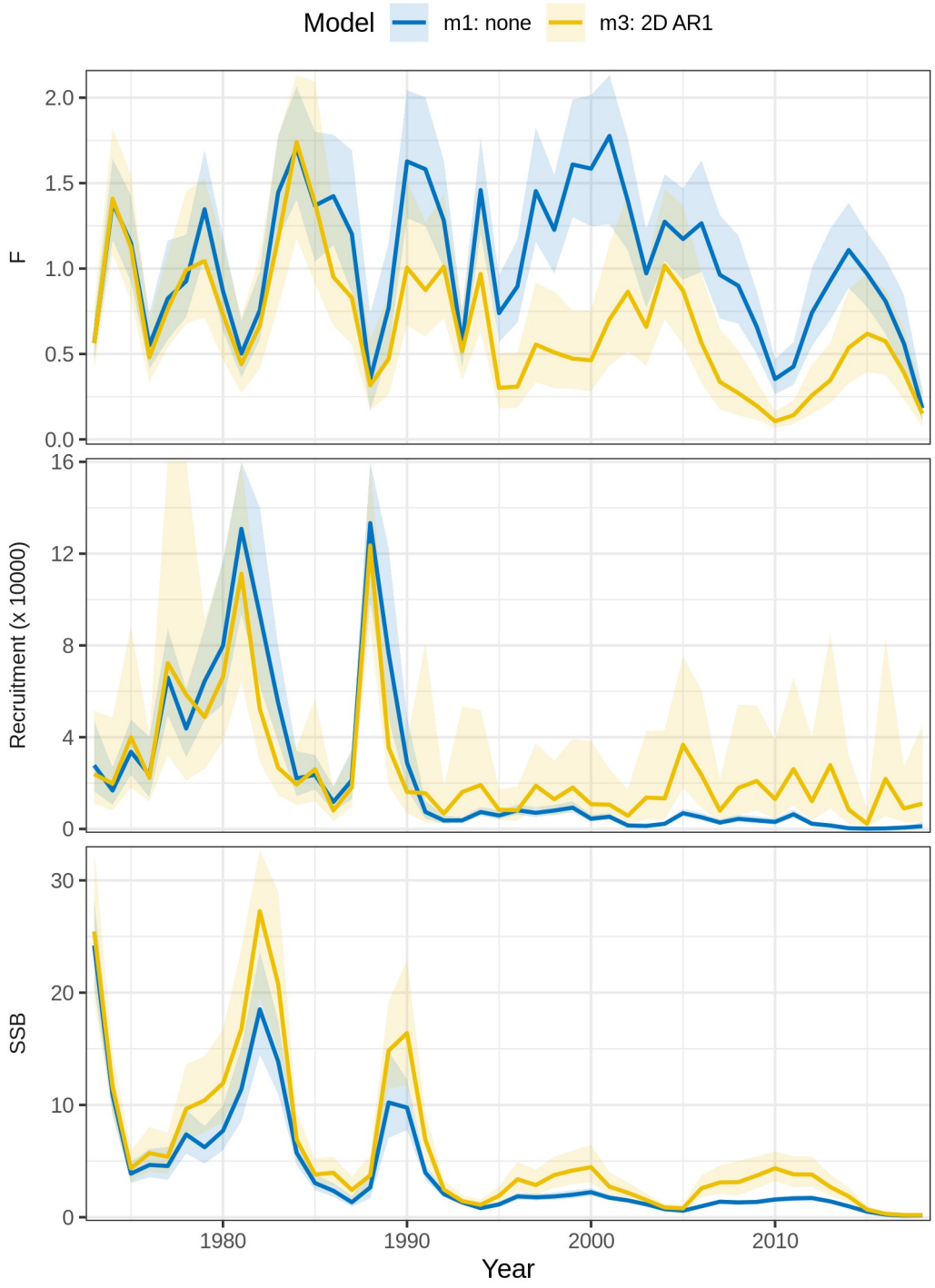


**Figure S2**. Trends in *F*, recruitment, and SSB estimated for SNEMA yellowtail flounder using two models of natural mortality (*M*) random effects. m1 = no random effects on *M*. m3 = *M* deviations are random effects correlated by age and year (2D AR1). Model m3 had the lowest AIC (Fig. 1 in main text).

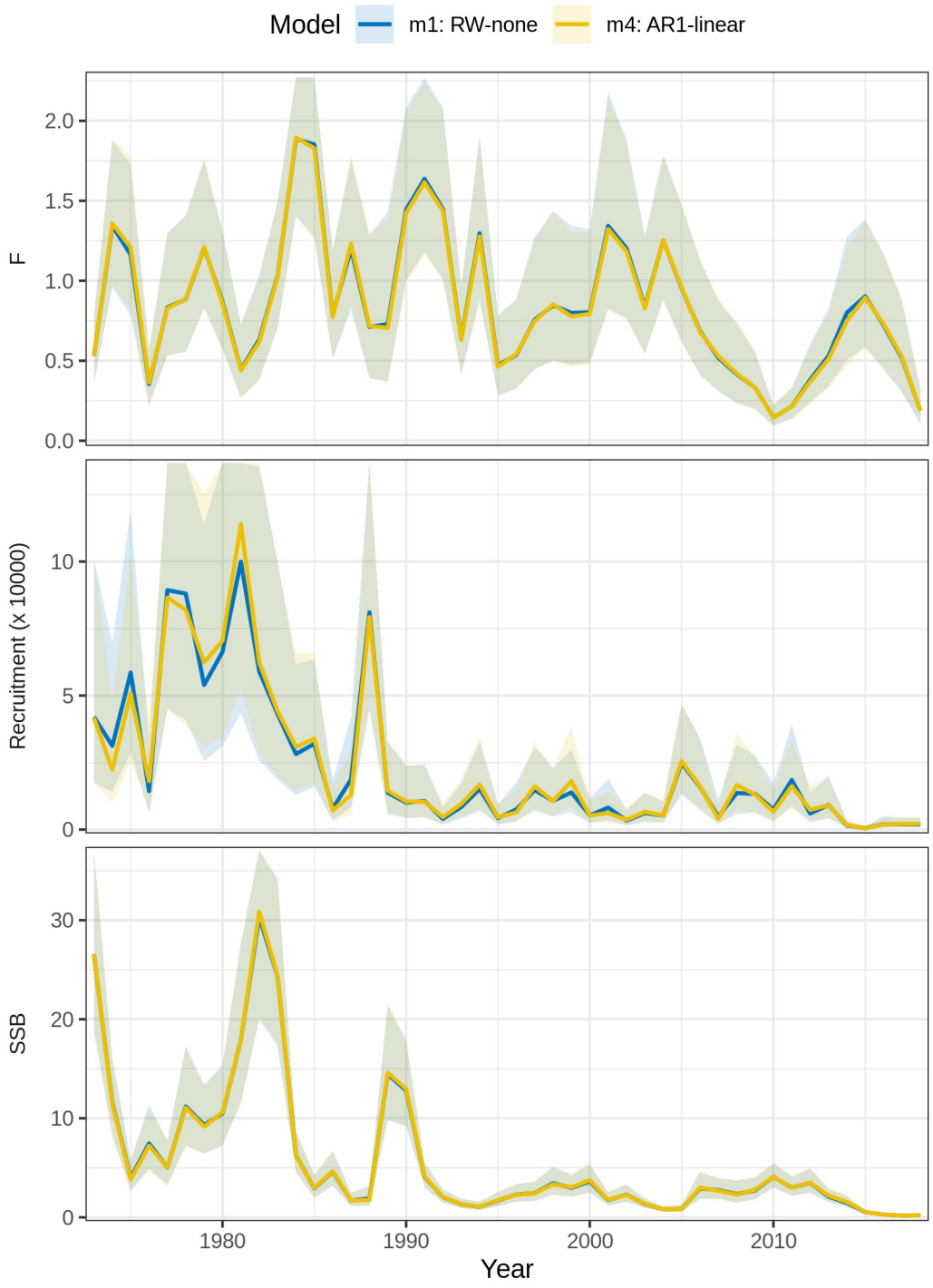


**Figure S3**. Trends in *F*, recruitment, and SSB estimated for SNEMA yellowtail flounder using models with and without effects of the Cold Pool Index (CPI) on recruitment. m1 = no CPI effect. m4 = CPI modeled as AR1 process with linear effect on the Beverton-Holt $\beta$ parameter. Model m4 had the lowest AIC (Fig. 1 in main text).

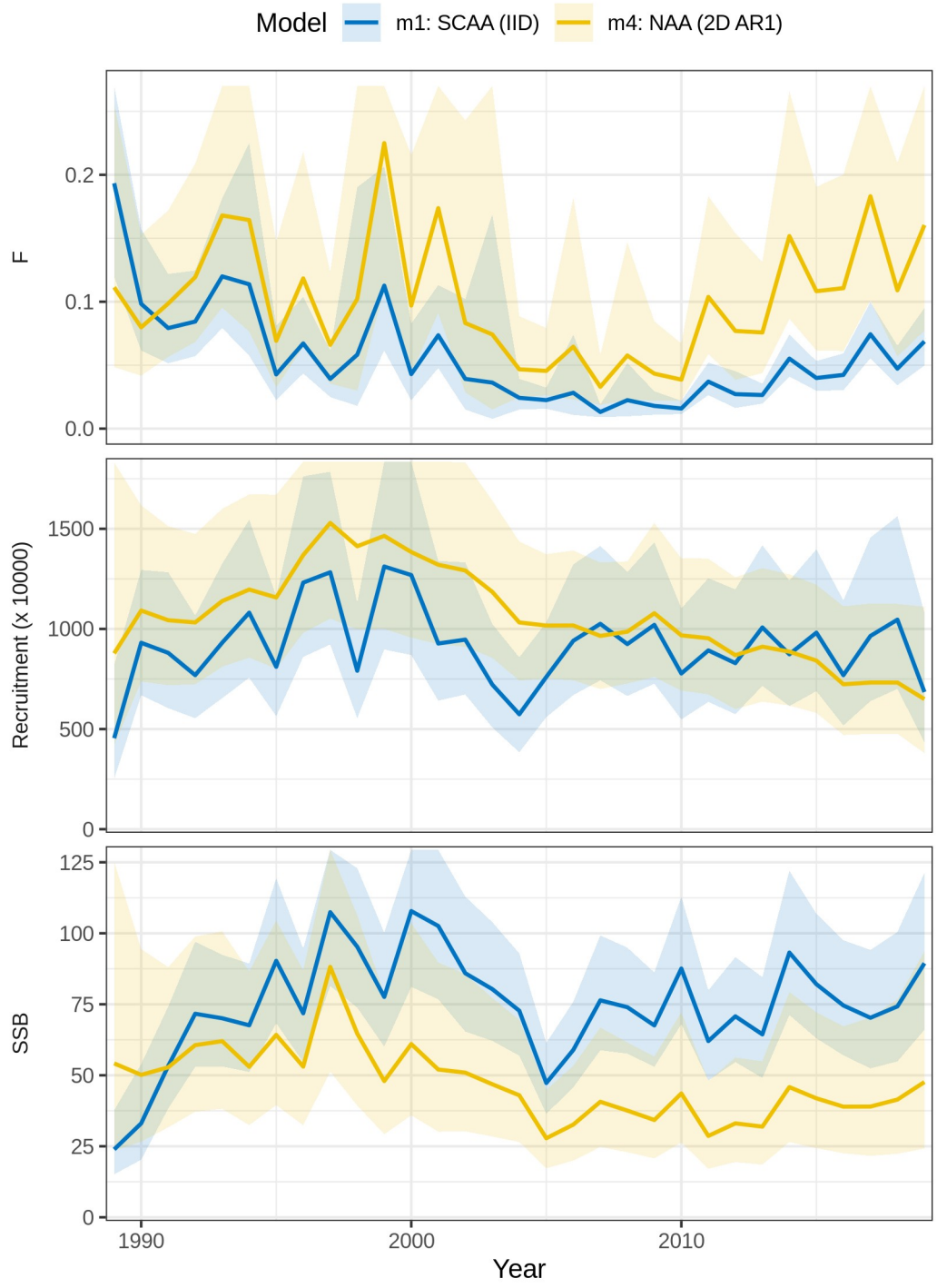


**Figure S4**. Trends in *F*, recruitment, and SSB estimated for butterfish using two models of numbers-at-age (NAA) random effects. m1 = only recruitment deviations are random effects (most similar to traditional statistical catch-at-age, SCAA), and deviations are independent and identically distributed (IID). m4 = all NAA deviations are random effects correlated by age and year (2D AR1). Model m4 had the lowest AIC (Fig. 1 in main text).

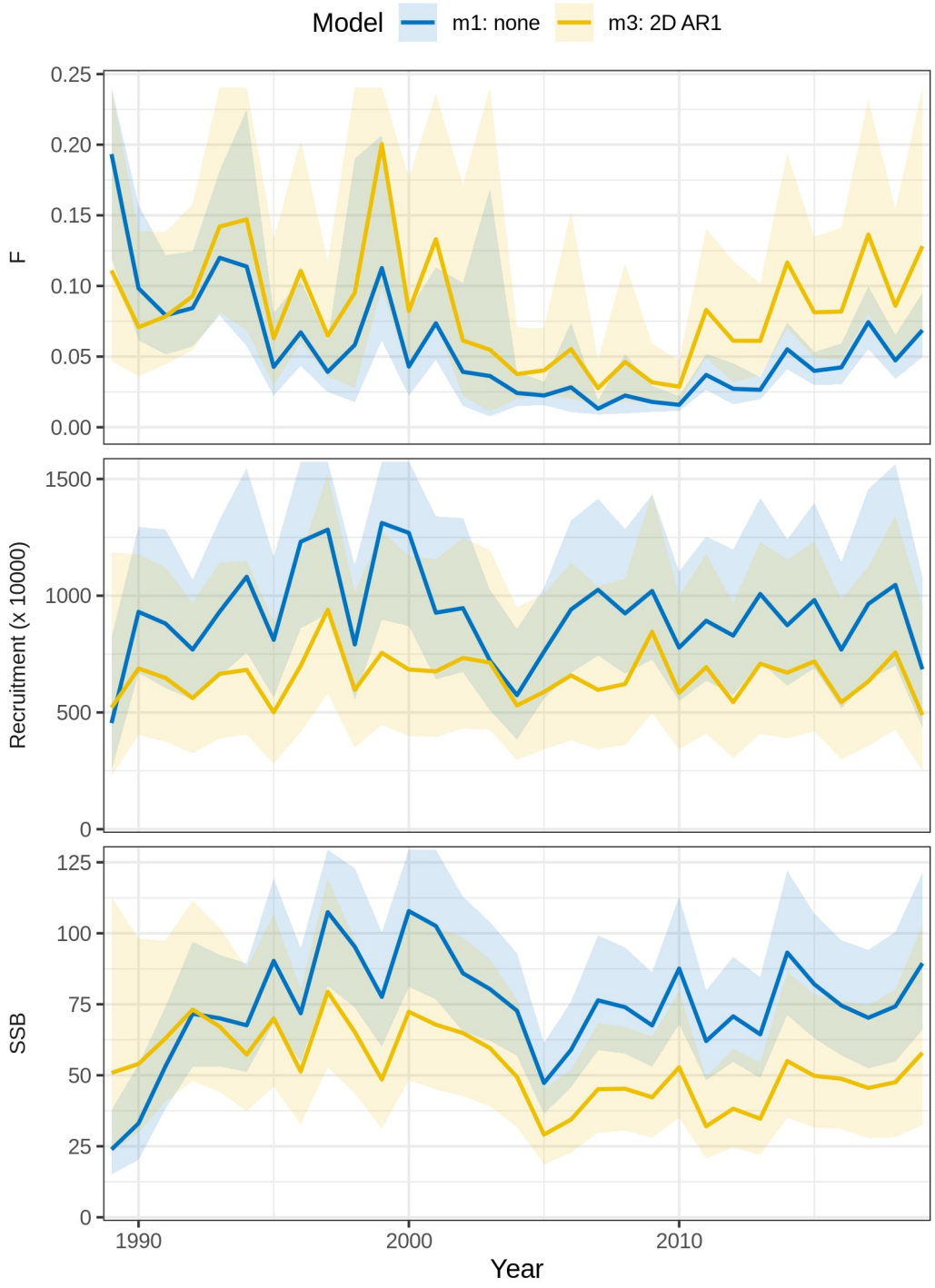


**Figure S5**. Trends in *F*, recruitment, and SSB estimated for butterfish using two models of natural mortality (*M*) random effects. m1 = no random effects on *M*. m3 = *M* deviations are random effects correlated by age and year (2D AR1). Model m3 had the lowest AIC (Fig. 1 in main text).

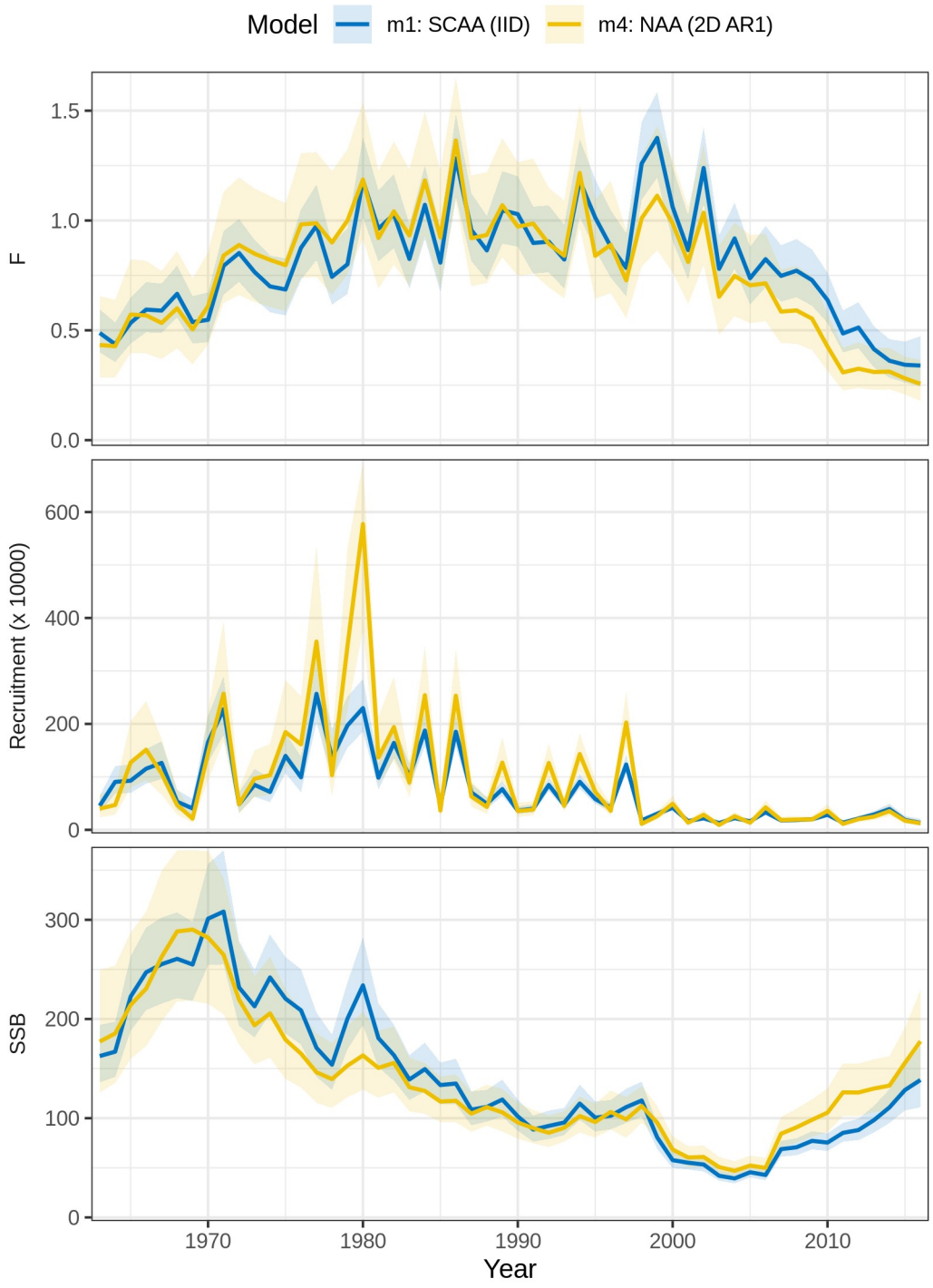


**Figure S6**. Trends in *F*, recruitment, and SSB estimated for North Sea cod using two models of numbers-at-age (NAA) random effects. m1 = only recruitment deviations are random effects (most similar to traditional statistical catch-at-age, SCAA), and deviations are independent and identically distributed (IID). m4 = all NAA deviations are random effects correlated by age and year (2D AR1). Model m4 had the lowest AIC (Fig. 1 in main text).

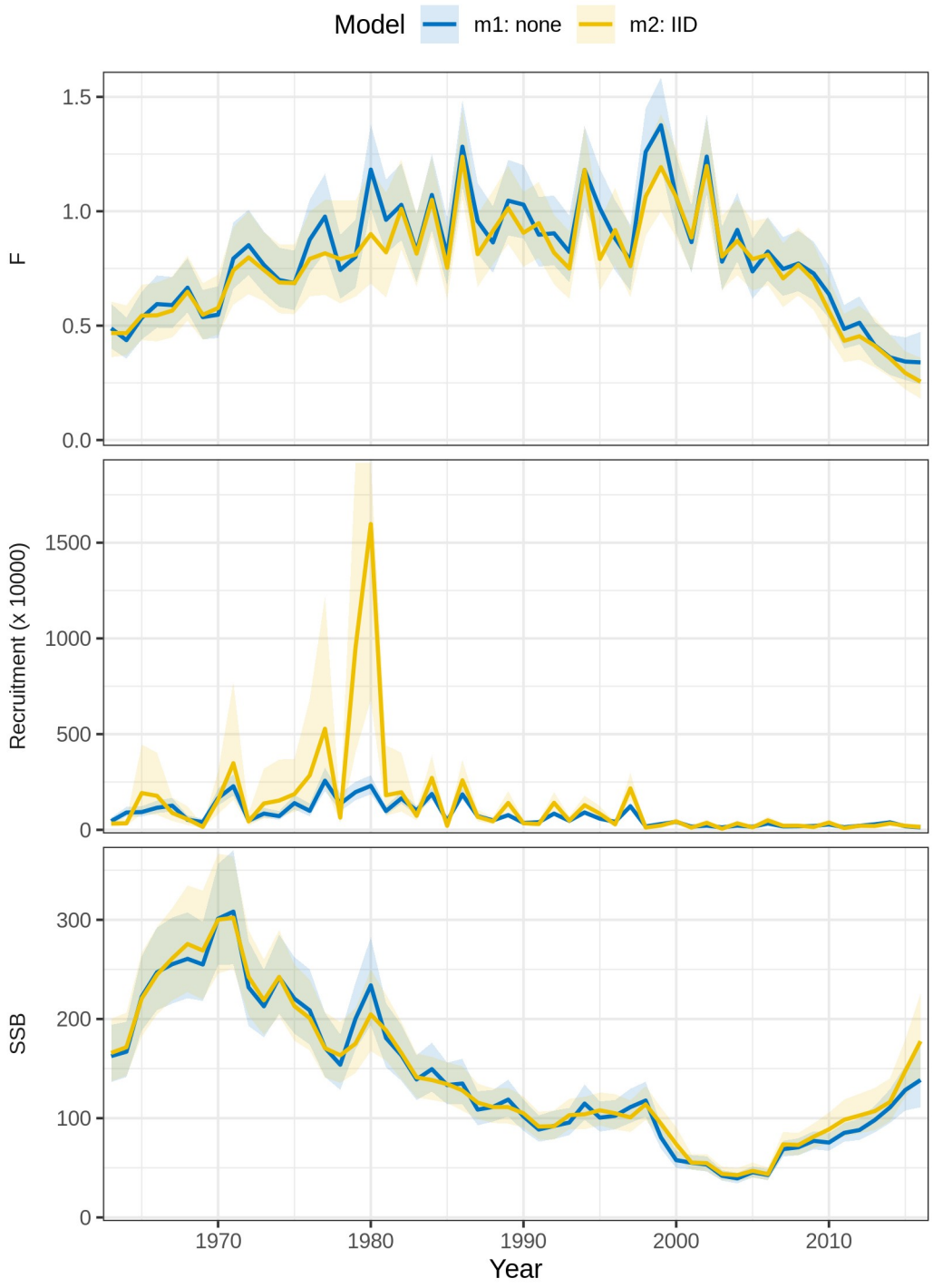


**Figure S7**. Trends in *F*, recruitment, and SSB estimated for North Sea cod using two models of natural mortality (*M*) random effects. m1 = no random effects on *M*. m2 = *M* deviations are independent random effects (IID). Model m2 had the lowest AIC (Fig. 1 in main text).

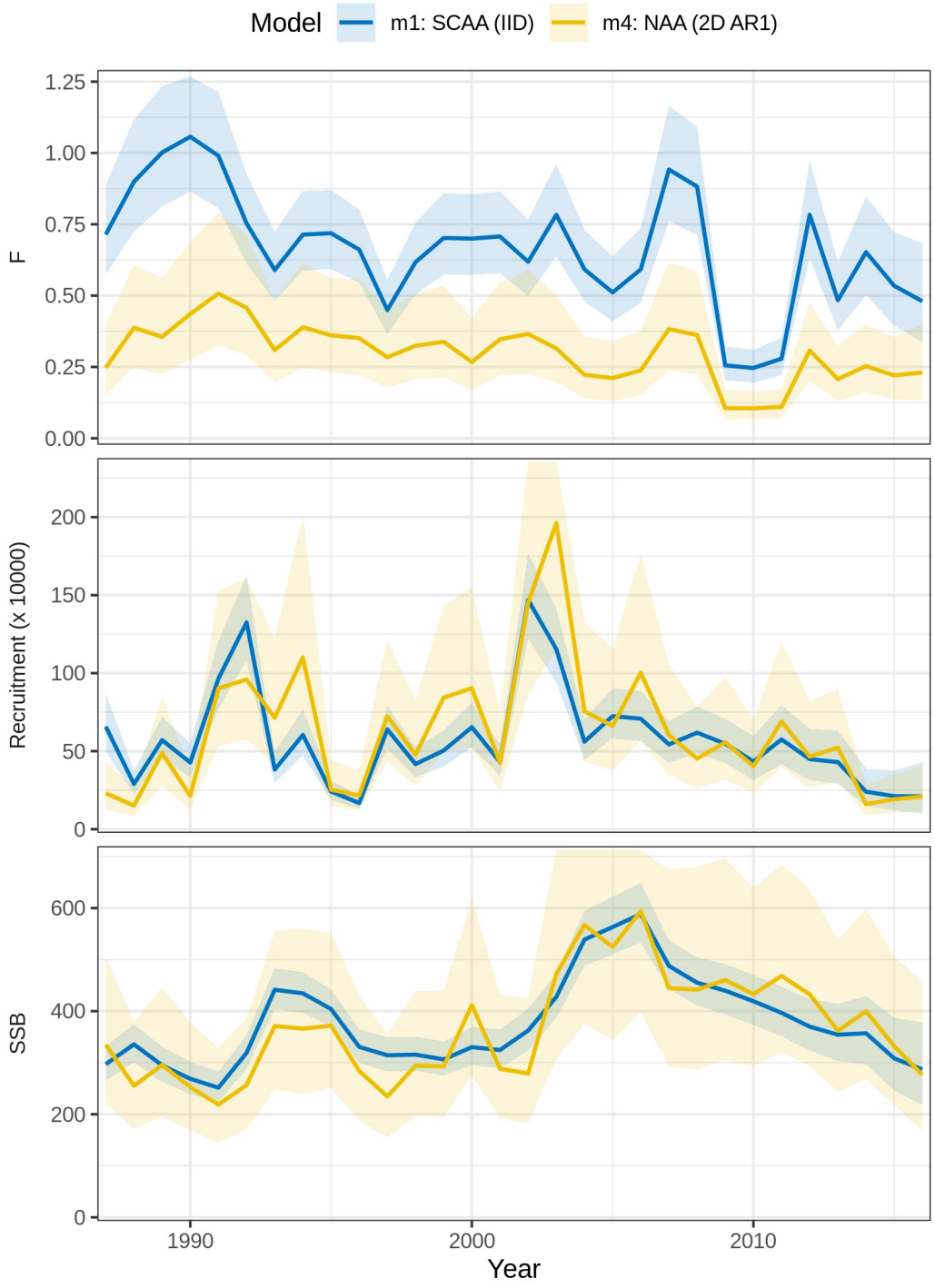


**Figure S8**. Trends in *F*, recruitment, and SSB estimated for Icelandic herring using two models of numbers-at-age (NAA) random effects. m1 = only recruitment deviations are random effects (most similar to traditional statistical catch-at-age, SCAA), and deviations are independent and identically distributed (IID). m4 = all NAA deviations are random effects correlated by age and year (2D AR1). Model m4 had the lowest AIC (Fig. 1 in main text).


**Figure S9**. Trends in *F*, recruitment, and SSB estimated for Georges Bank haddock using two models of numbers-at-age (NAA) random effects. m1 = only recruitment deviations are random effects (most similar to traditional statistical catch-at-age, SCAA), and deviations are independent and identically distributed (IID). m4 = all NAA deviations are random effects correlated by age and year (2D AR1). Model m4 had the lowest AIC (Fig. 1 in main text).

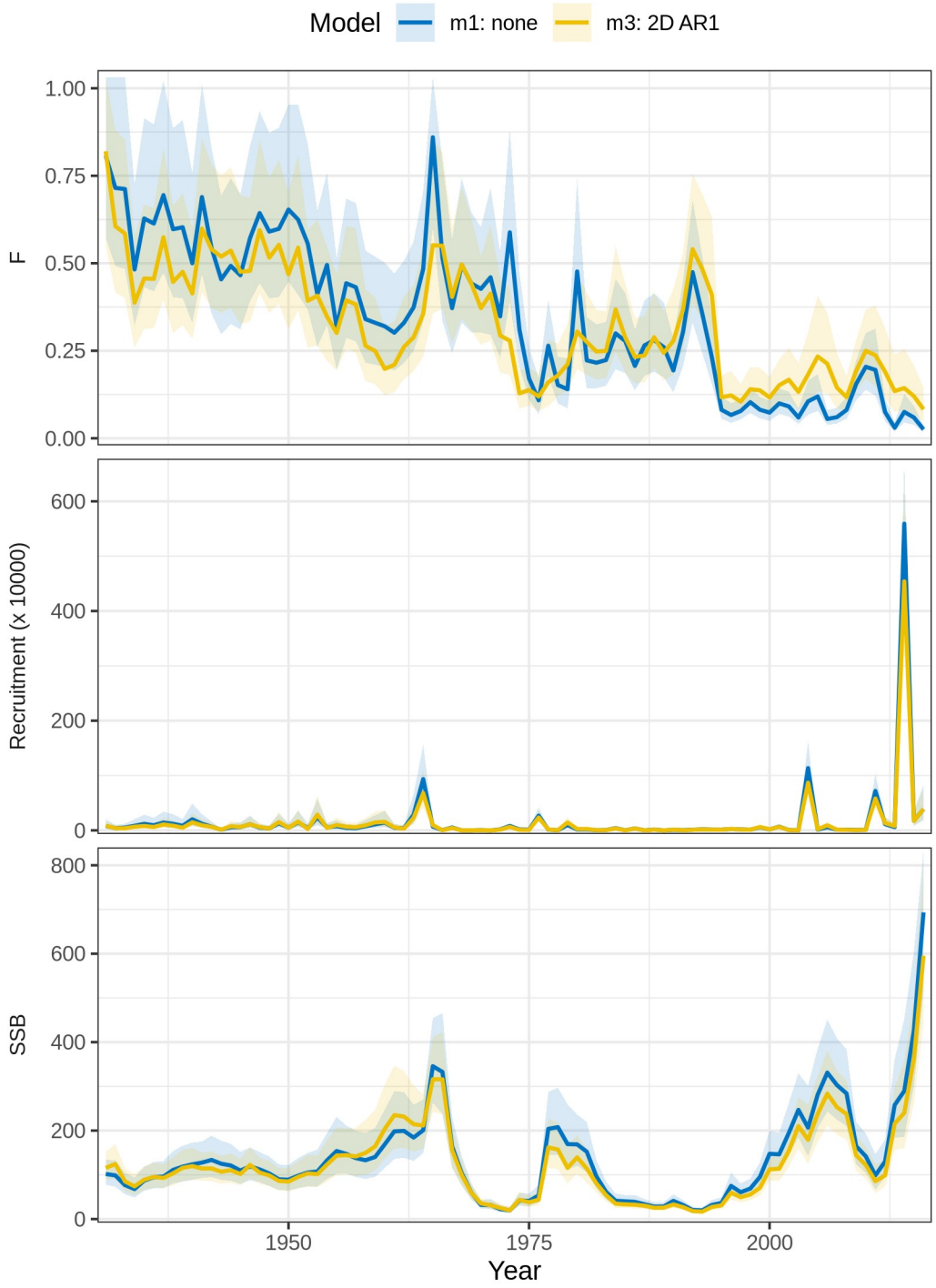


**Figure S10**. Trends in *F*, recruitment, and SSB estimated for Georges Bank haddock using two models of selectivity random effects. m1 = time-constant selectivity. m3 = random effect deviations in the logistic selectivity parameters correlated by parameter and year (2D AR1). Model m3 had the lowest AIC (Fig. 1 in main text).

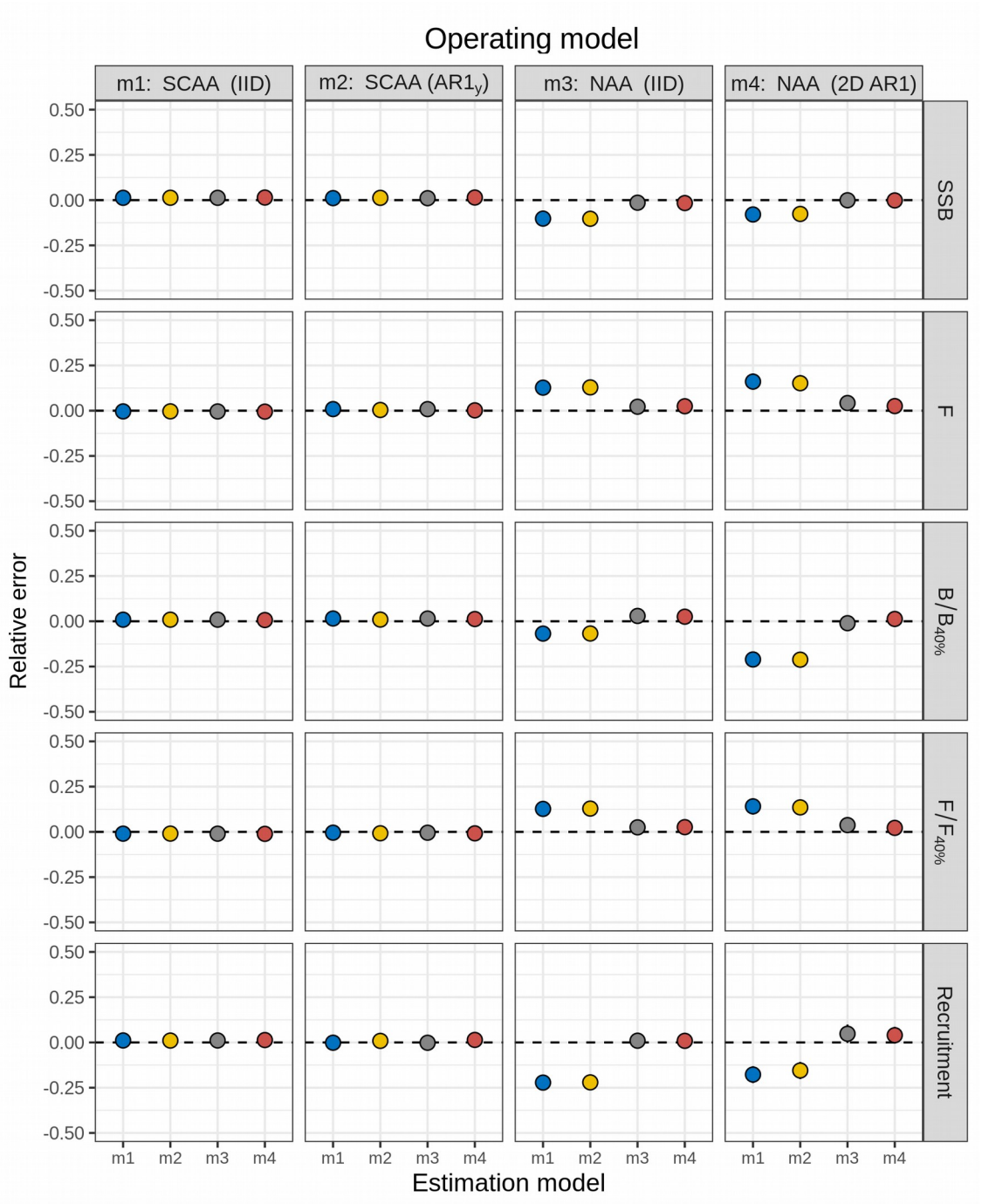


**Figure S11**. Relative error of key quantities estimated for SNEMA yellowtail flounder using four models of numbers-at-age (NAA) random effects. m1 = only recruitment deviations are random effects (most similar to traditional statistical catch-at-age, SCAA), and deviations are independent and identically distributed (IID). m2 = as m1, but with autocorrelated recruitment deviations (AR1). m3 = all NAA deviations are IID random effects. m4 = as m3, but deviations are correlated by age and year (2D AR1).

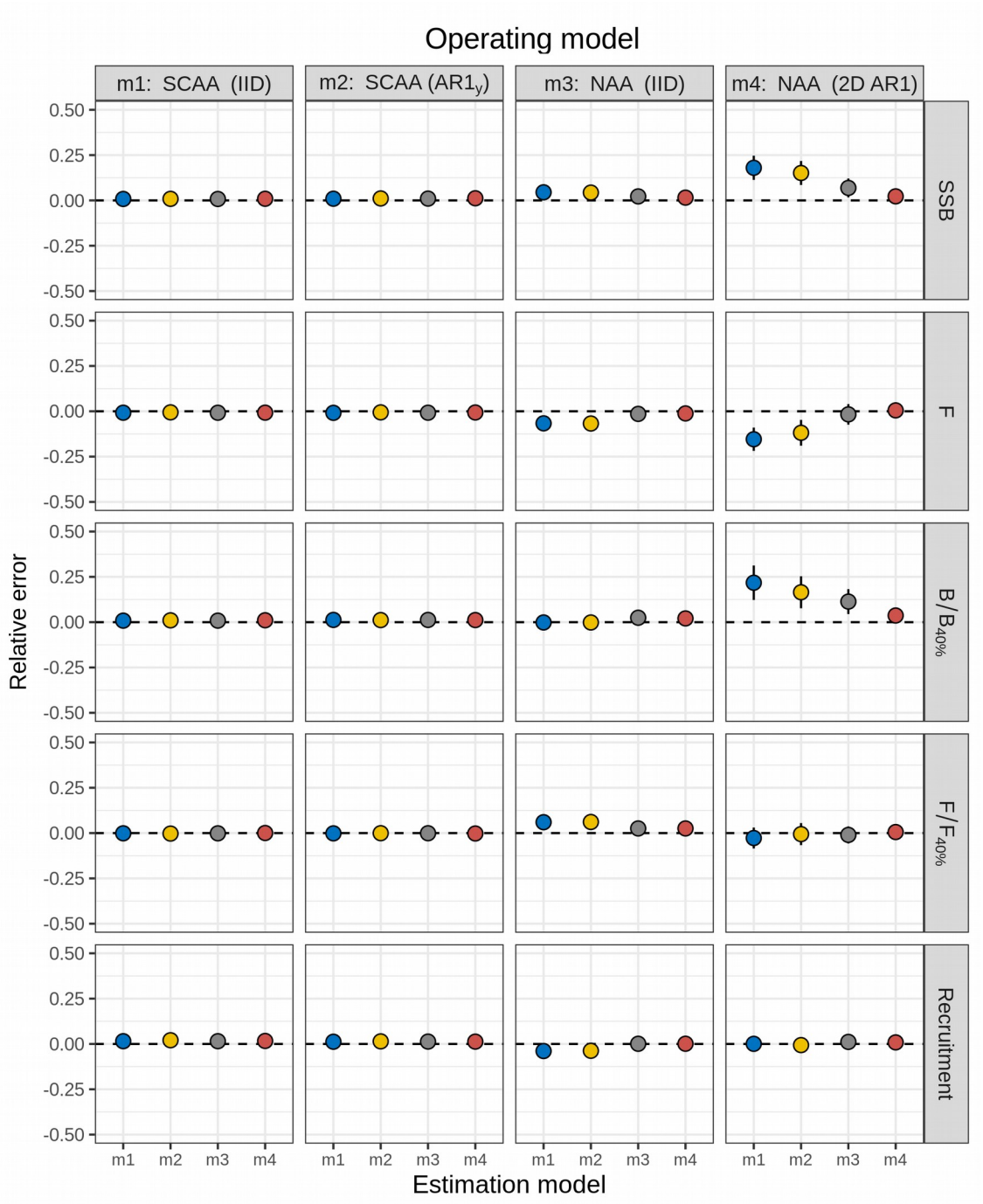


**Figure S12**. Relative error of key quantities estimated for butterfish using four models of numbers-at-age (NAA) random effects. m1 = only recruitment deviations are random effects (most similar to traditional statistical catch-at-age, SCAA), and deviations are independent and identically distributed (IID). m2 = as m1, but with autocorrelated recruitment deviations (AR1). m3 = all NAA deviations are IID random effects. m4 = as m3, but deviations are correlated by age and year (2D AR1).

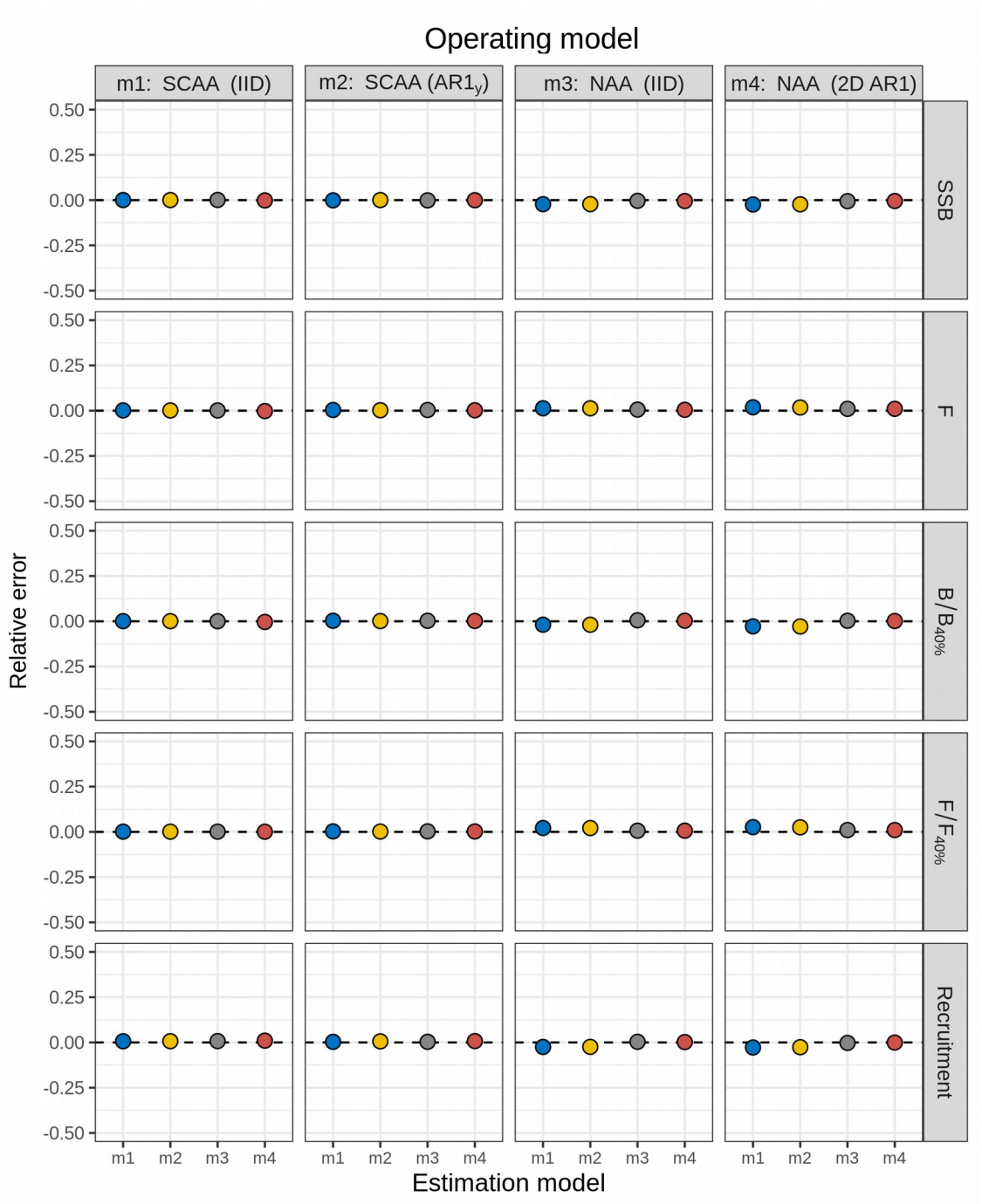


**Figure S13**. Relative error of key quantities estimated for North Sea cod using four models of numbers-at-age (NAA) random effects. m1 = only recruitment deviations are random effects (most similar to traditional statistical catch-at-age, SCAA), and deviations are independent and identically distributed (IID). m2 = as m1, but with autocorrelated recruitment deviations (AR1). m3 = all NAA deviations are IID random effects. m4 = as m3, but deviations are correlated by age and year (2D AR1).

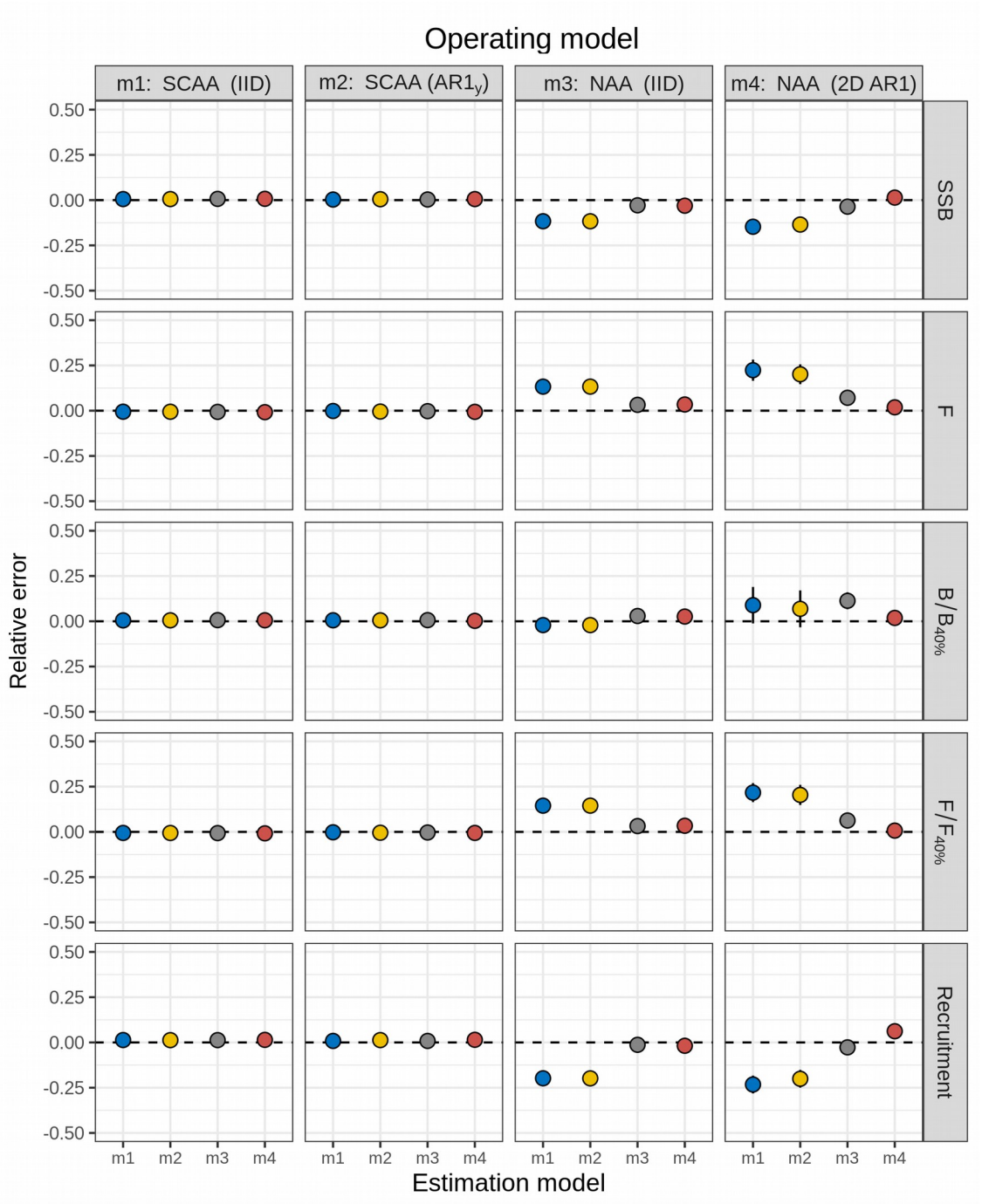


**Figure S14**. Relative error of key quantities estimated for Georges Bank haddock using four models of numbers-at-age (NAA) random effects. m1 = only recruitment deviations are random effects (most similar to traditional statistical catch-at-age, SCAA), and deviations are independent and identically distributed (IID). m2 = as m1, but with autocorrelated recruitment deviations (AR1). m3 = all NAA deviations are IID random effects. m4 = as m3, but deviations are correlated by age and year (2D AR1).

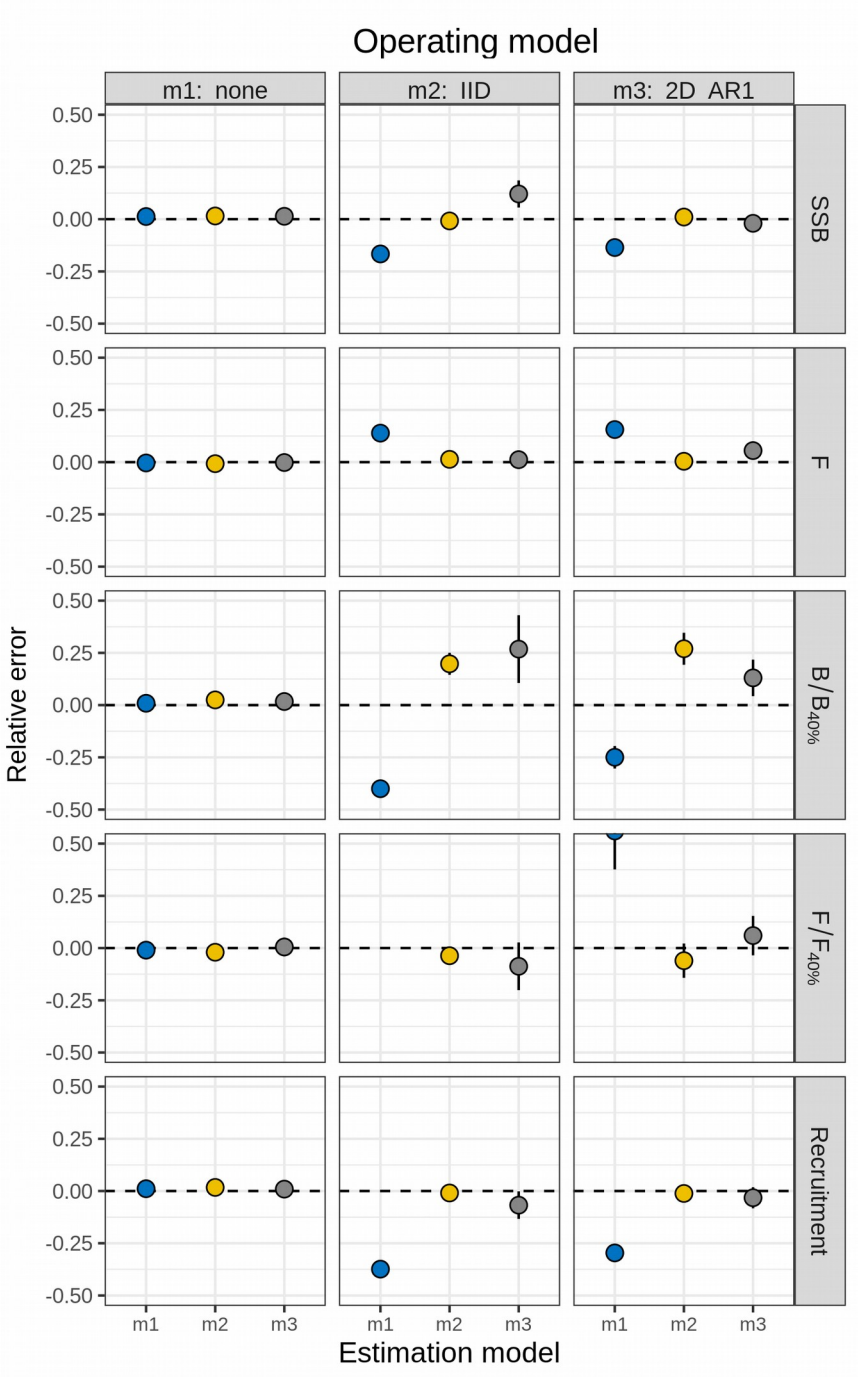


**Figure S15**. Relative error of key quantities estimated for SNEMA yellowtail flounder using three models of natural mortality (*M*) random effects. m1 = no random effects on *M*. m2 = M deviations are independent and identically distributed (IID). m3 = *M* deviations are correlated by age and year (2D AR1).

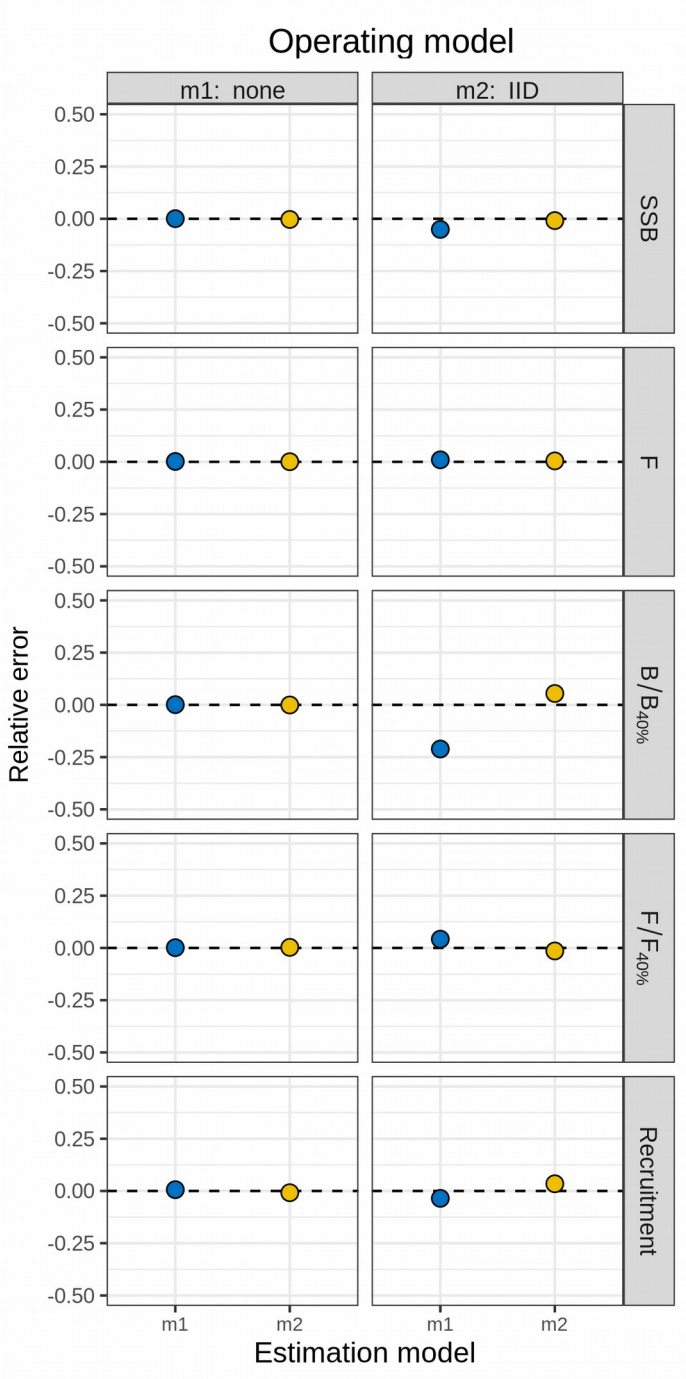


**Figure S16**. Relative error of key quantities estimated for North Sea cod using two models of natural mortality (*M*) random effects. m1 = no random effects on *M*. m2 = M deviations are independent and identically distributed (IID). The model with *M* deviations correlated by age and year (m3, 2D AR1) did not converge.

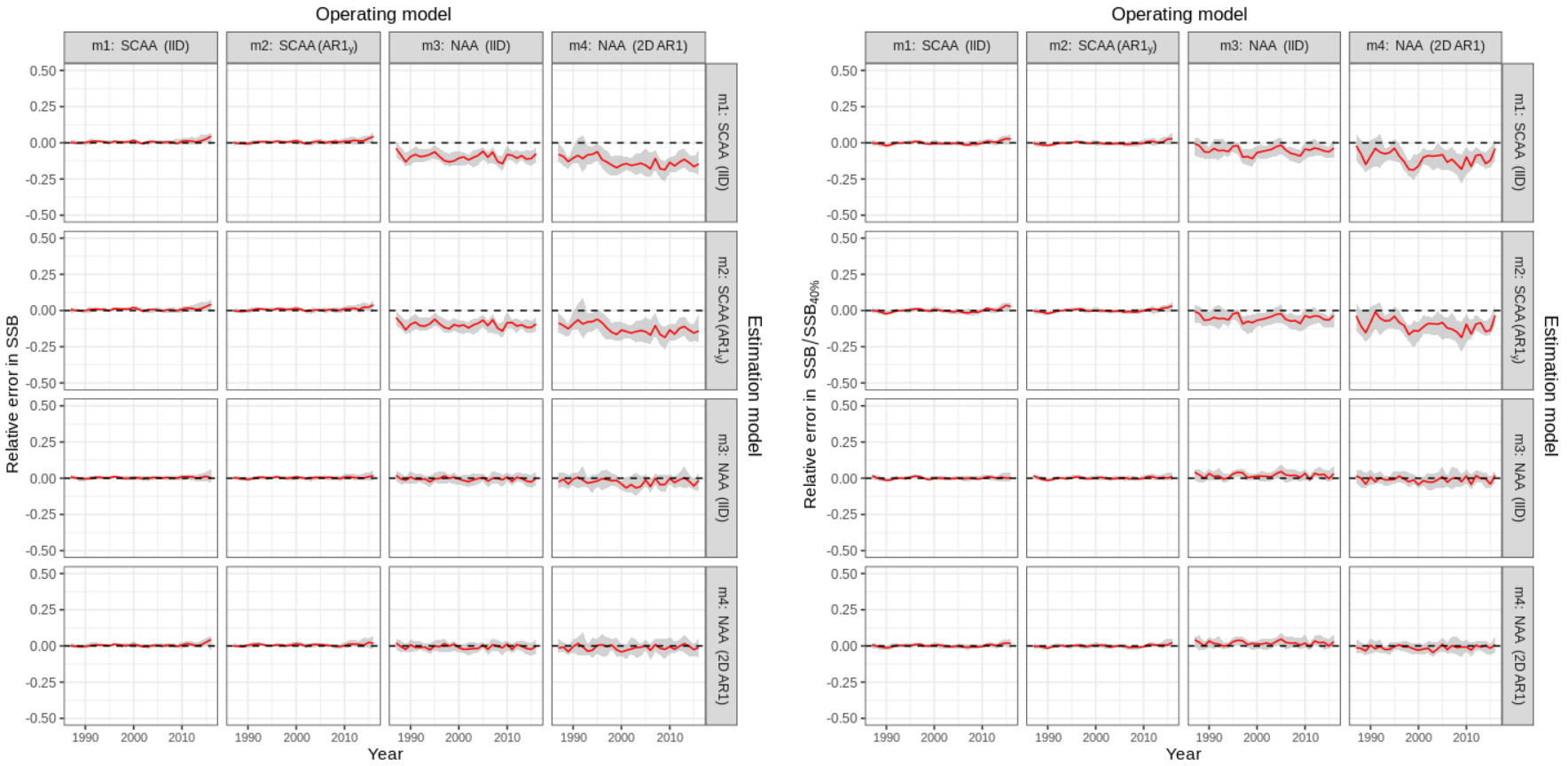


**Figure S17**. Relative error in SSB (left) and SSB / $SSB_{40\%}$ (right) in simulation cross-tests for the four numbers at age (NAA) models fit to Icelandic herring data. Relative error was calculated as $\frac{\hat{\theta}_i}{\theta_i} - 1$, where $\hat{\theta}_i$ was the estimate in simulation *i* for parameter $\theta$, and $\theta_i$ was the true value (estimate from original dataset). Solid red lines and grey shading show the median relative error with 95% CI, calculated within year and across simulations.

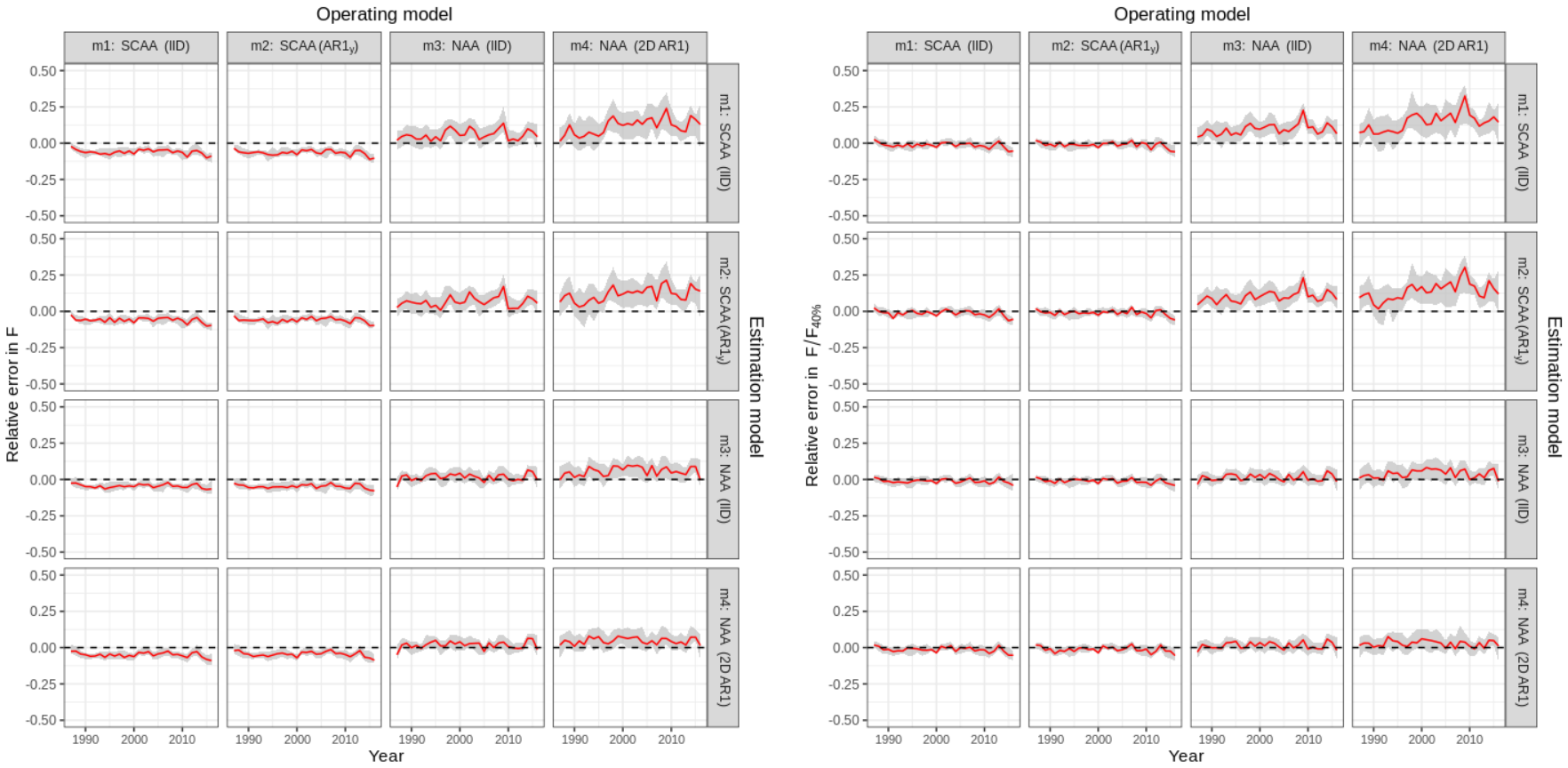


**Figure S18**. Relative error in $F$ (left) and $F$ / $F_{40\%}$ (right) in simulation cross-tests for the four numbers at age (NAA) models fit to Icelandic herring data. Relative error was calculated as $\frac{\hat{\theta}_i}{\theta_i} - 1$, where $\hat{\theta}_i$ was the estimate in simulation $i$ for parameter $\theta$, and $\theta_i$ was the true value (estimate from original dataset). Solid red lines and grey shading show the median relative error with 95% CI, calculated within year and across simulations.

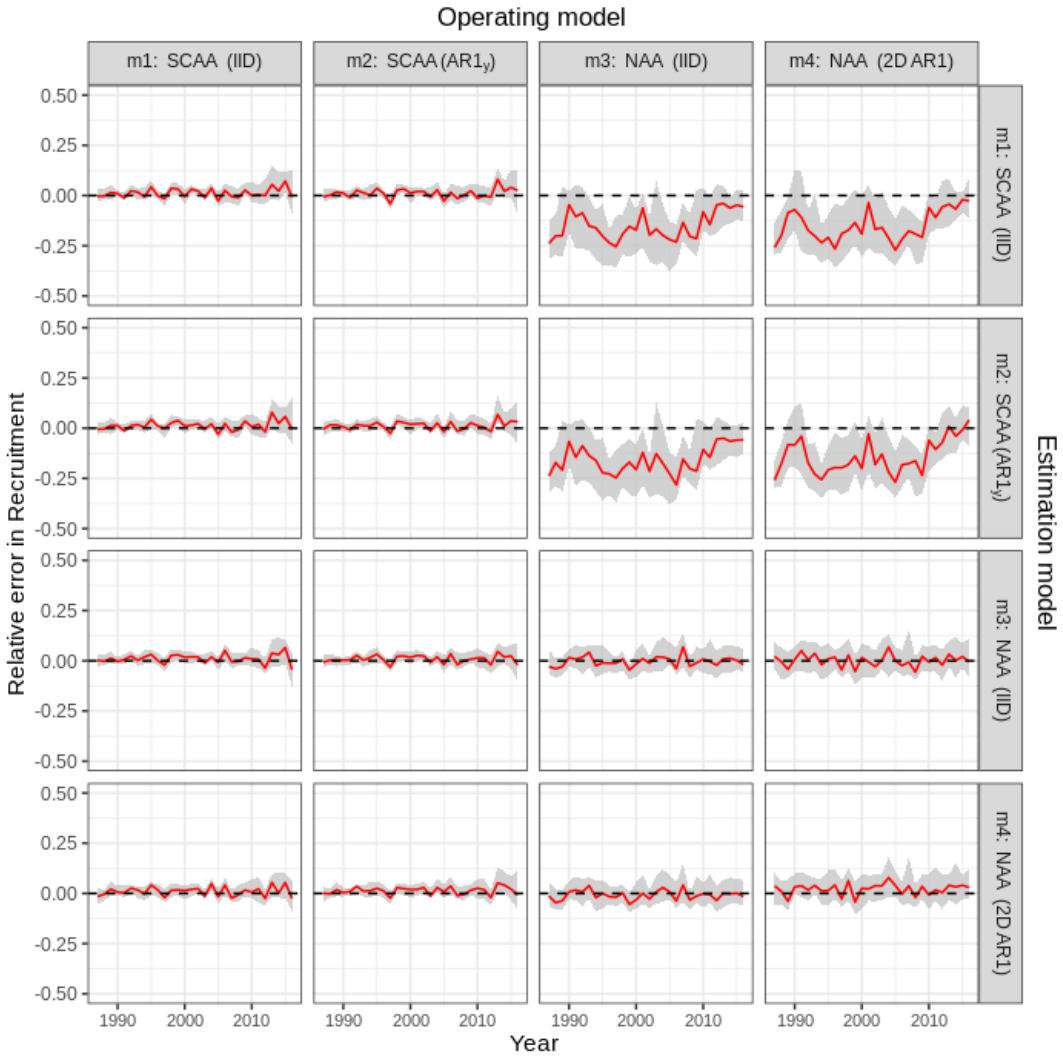


**Figure S19**. Relative error in recruitment in simulation cross-tests for the four numbers at age (NAA) models fit to Icelandic herring data. Relative error was calculated as $\frac{\hat{\theta}_i}{\theta_i} - 1$, where $\hat{\theta}_i$ was the estimate in simulation *i* for parameter $\theta$, and $\theta_i$ was the true value (estimate from original dataset). Solid red lines and grey shading show the median relative error with 95% CI, calculated within year and across simulations.

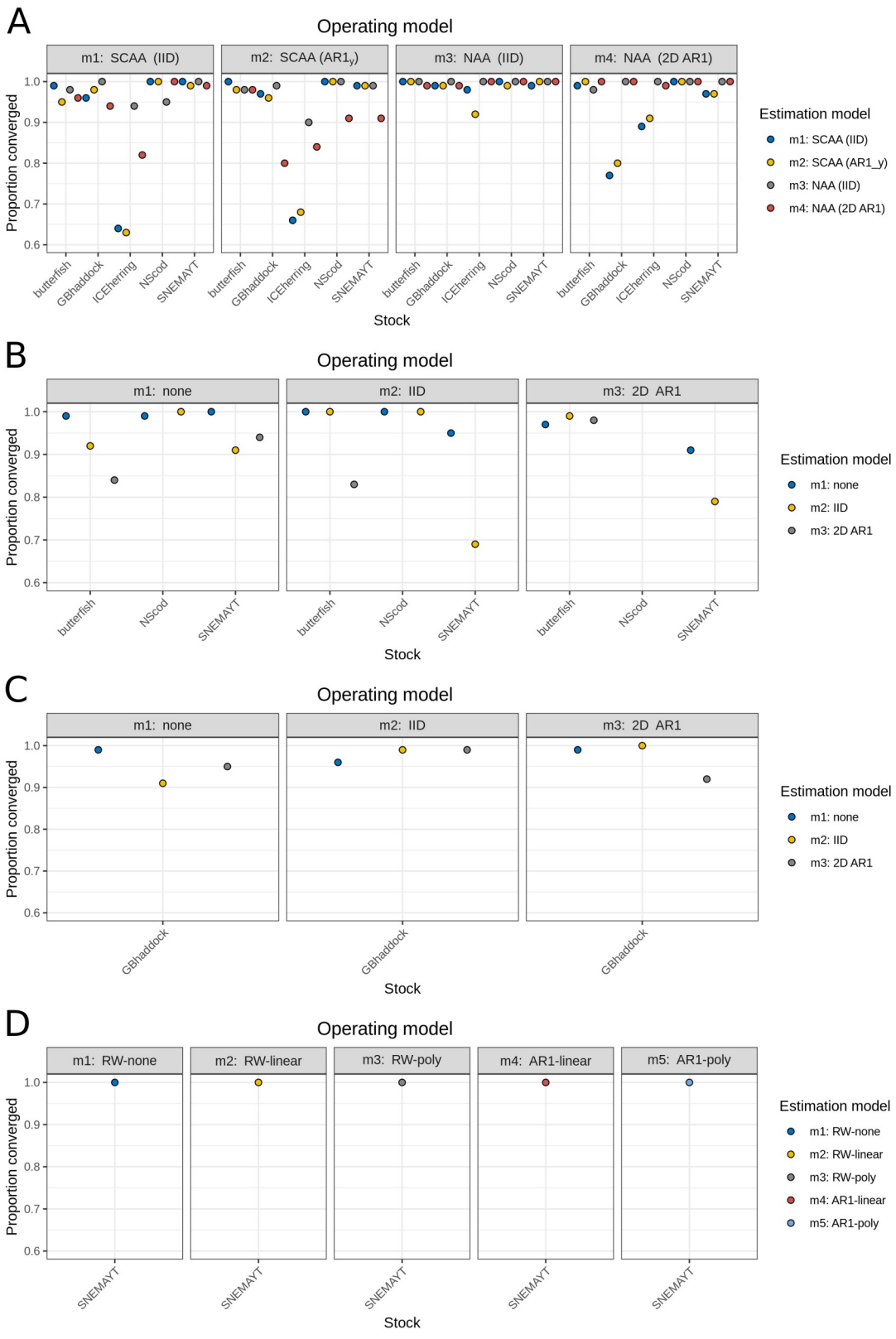


**Figure S20**. Proportion of cross-test simulations in which models converged for a) NAA, b) *M*, c) selectivity, and d) *Ecov* random effects. Models with positive definite Hessian matrix were considered to be converged.