State-space models and WHAM Overview

WHAM Workshop
NEFSC Woods Hole Lab
9-13 February 2026

Tim Miller and Emily Liljestrand
Northeast Fisheries Science Center

Outline

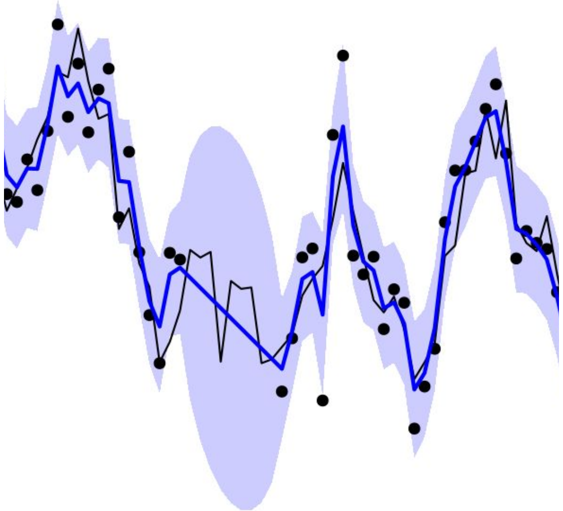
- State-space model overview
- WHAM model features
  - Random effects options
  - Environmental covariate effect options
  - Observation likelihood options
  - Biological Reference Point options
  - Projection options
  - Useful features: OSA residuals, auto-generated output, simulations

What is a “state-space” model?

x_t **True process**

$\widehat{x}_t$ **Process estimate**

y_t **Observations**



$$x_t = \mu\left(1-\rho\right) + \rho x_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim \mathrm{N}\left(0, \sigma^2\right)$$

$$y_t = x_t + \delta_t$$

$$\delta_t \sim \mathrm{N}\left(0, \tau^2\right)$$

$$l\left(\boldsymbol{\theta}|\boldsymbol{y}\right) = \int \prod_{t=0}^{T-1} f\left(x_{t+1}|x_t, \boldsymbol{\theta}\right) g\left(y_{t+1}|x_{t+1}, \boldsymbol{\theta}\right) d\boldsymbol{x}$$

$$\widehat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmax}}\, l\left(\boldsymbol{\theta}|\boldsymbol{y}\right)$$

$$\widehat{\boldsymbol{x}}|\widehat{\boldsymbol{\theta}} = \underset{\boldsymbol{x}}{\operatorname{argmax}} \prod_{t=0}^{T-1} f\left(x_{t+1}|x_t, \widehat{\boldsymbol{\theta}}\right) g\left(y_{t+1}|x_{t+1}, \widehat{\boldsymbol{\theta}}\right)$$

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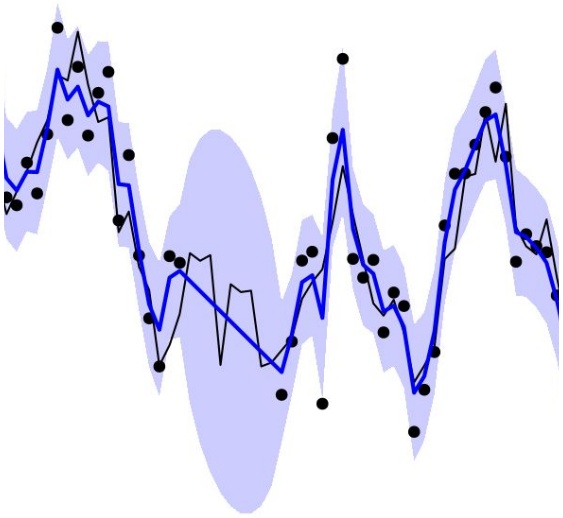

$$x_t = \mu(1-\rho) + \rho x_{t-1} + \boxed{\varepsilon_t}$$

$$\boxed{\varepsilon_t \sim \mathrm{N}\left(0, \sigma^2\right)}$$

Process errors

$$y_t = x_t + \delta_t$$

$$\delta_t \sim \mathrm{N}\left(0, \tau^2\right)$$

$$l(\boldsymbol{\theta}|\boldsymbol{y}) = \int \prod_{t=0}^{T-1} f\left(x_{t+1}|x_t, \boldsymbol{\theta}\right) g\left(y_{t+1}|x_{t+1}, \boldsymbol{\theta}\right) d\boldsymbol{x}$$

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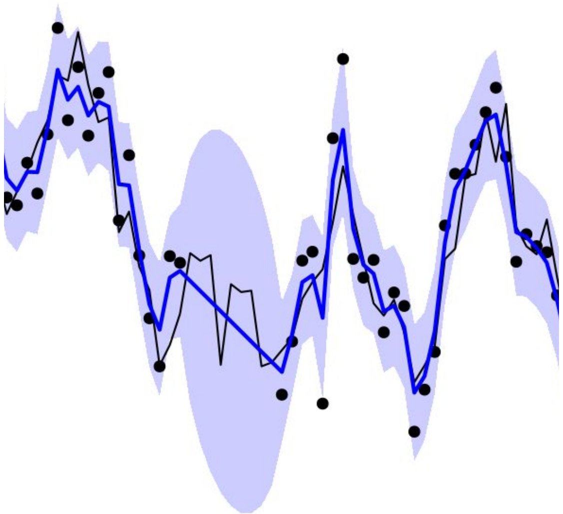

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Observation errors

$$l(\boldsymbol{\theta}|\boldsymbol{y}) = \int \prod_{t=0}^{T-1} f(x_{t+1}|x_t, \boldsymbol{\theta})\, g(y_{t+1}|x_{t+1}, \boldsymbol{\theta})\, d\boldsymbol{x}$$

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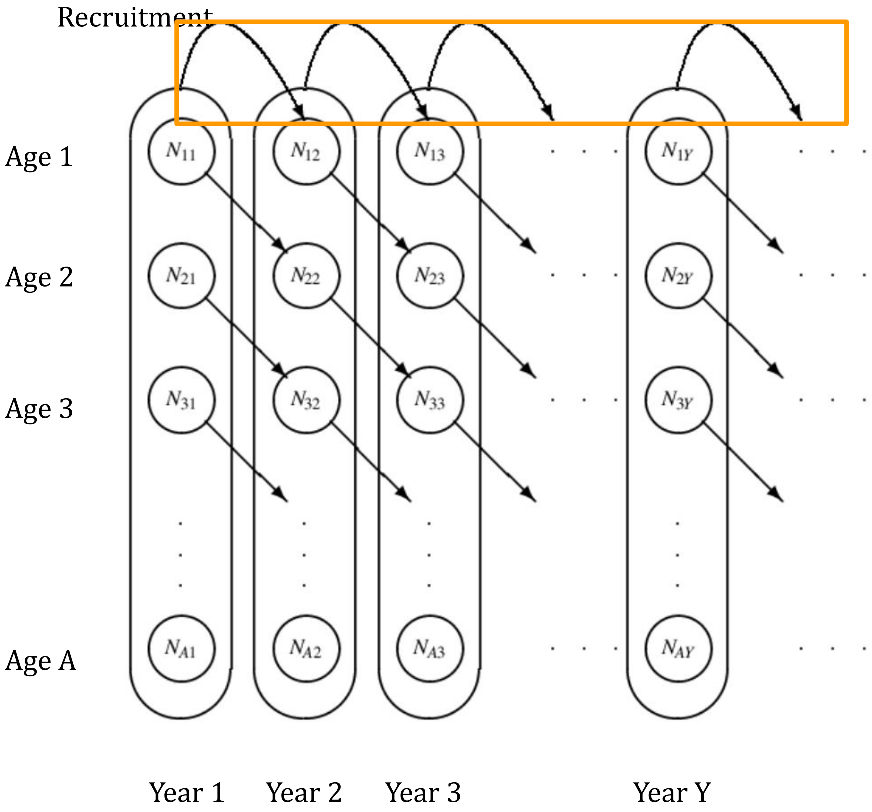
Traditional age-structured model



Age-structured population model

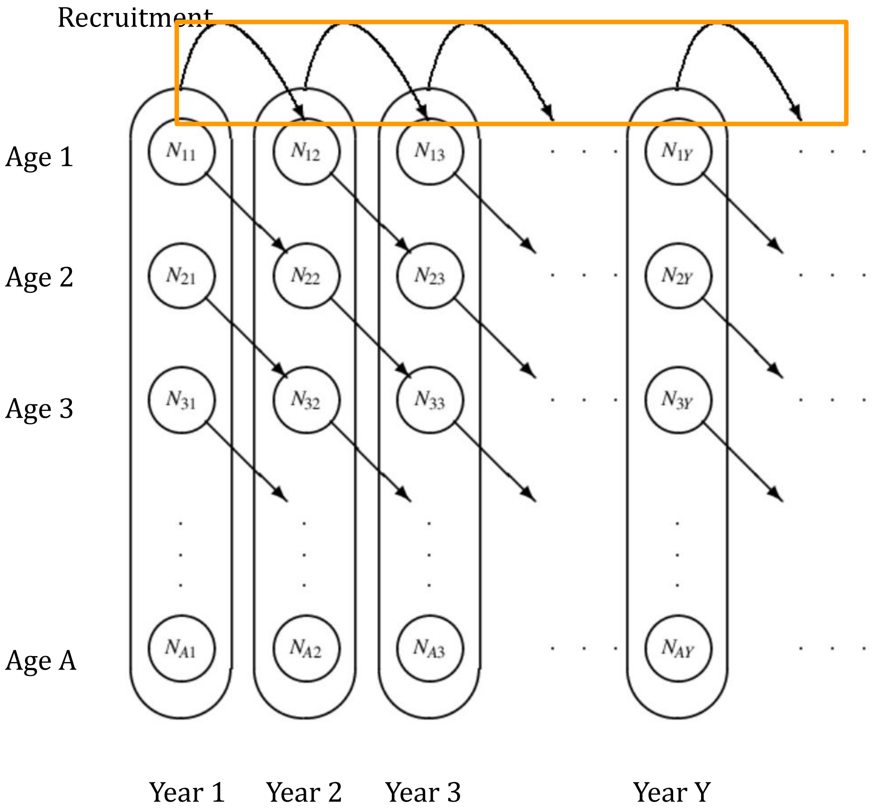
Keep track of the numbers of each age in each year, $N_{a,y}$

Recruitment may or may not be explicitly related to spawning population

$$\boxed{\log N_{1,y} = \log\left(f\left(\mathrm{SSB}_{y-1}\right)\right) + \boxed{\varepsilon_{1,y}}}$$

Annual recruitment or deviations historically estimated as fixed effects, possibly with a penalty.

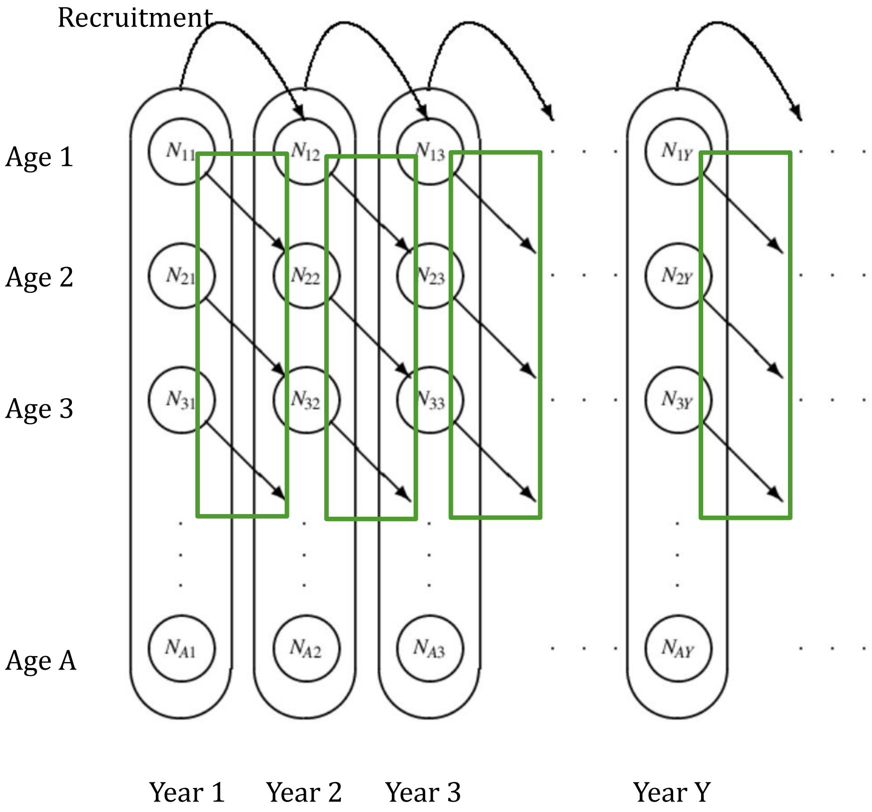
Simplest state-space model



Treat recruitment or deviations as random effects with some distribution that are integrated out

$$\boxed{\log N_{1,y} = \log\left(f(\mathrm{SSB}_{y-1})\right) + \boxed{\varepsilon_{1,y}}}$$

Traditional age-structured model



Transitions between ages/years is a function of

- Natural mortality ($M_{a,y}$)
- Fishing mortality ($F_{a,y}$)
- Total mortality $Z_{a,y} = F_{a,y} + M_{a,y}$

$$\boxed{\log N_{a,y} = \log(N_{a-1,y-1}) - Z_{a-1,y-1}}$$

Full state-space

Z includes natural mortality and fishing mortality

$\varepsilon_{a,y}$ terms may account for

- various time-varying processes: selectivity, natural mortality, migration
- mis-specification of how mortality affects population

Other aspects of the assessment model can be modeled in similar ways

Statistical catch-at-age

$$\log N_{a,y} = f(\log N_{a-1,y-1})$$

Statistical catch-at-age, random recruitment

$$\log N_{1,y} = \log\left(f(\mathrm{SSB}_{y-1})\right) + \varepsilon_{1,y}$$

“Full state-space”

$$\boxed{\log N_{a,y} = \log\left(N_{a-1,y-1}\right) - Z_{a-1,y-1} + \boxed{\varepsilon_{a,y}}}$$

Pros and cons

Pros

- Reduced retrospective patterns
- More realistic perception of uncertainty in assessment output
- Ability to better model temporal changes in productivity and fisheries
- Allows more statistical rigor in comparing alternative assessment models
- Short-term projections can be done consistently within the assessment model

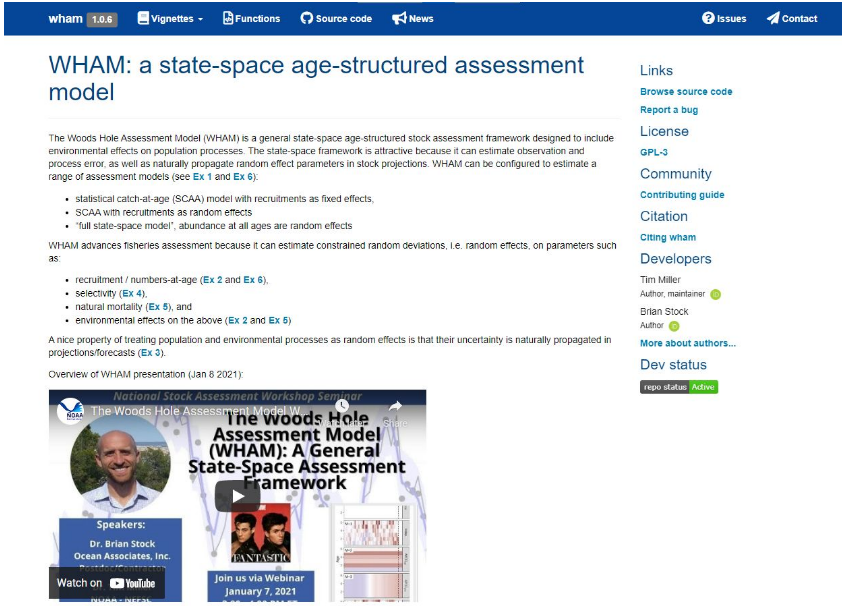
Cons

- Greater complexity in fitting and interpretation of results
- Research needed to evaluate
  - improvements over current assessment models
  - new diagnostics
  - reliability of particular configurations

An open-source state-space assessment framework

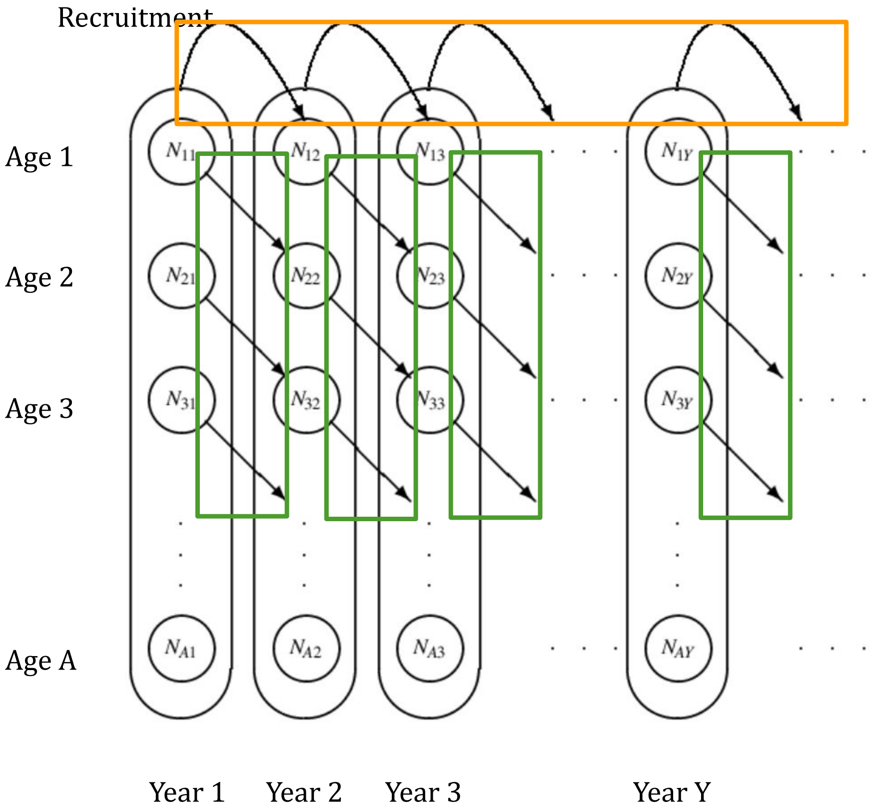
- An R package available from Github
- Models can be completely configured using R package functionality
- Several tutorial vignettes
- Automatically produce a variety of output useful for both evaluating models and providing management advice.
- Tests to check package development.
- Several collaborators: **Brian Stock** (IMR) and others

The Woods Hole Assessment Model
WHAM:



\+ TMB

WHAM is an age-structured model



Configuration options for abundance at age:

1) Statistical catch-at-age (no random effects)

$$\boxed{\log N_{a,y} = f(\log N_{a-1,y-1})}$$

2) Statistical catch-at-age, random recruitment

$$\boxed{\log N_{1,y} = \log\left(f\left(\mathrm{SSB}_{y-1}\right)\right) + \varepsilon_{1,y}}$$

3) "Full state-space" (survival random effects)

$$\boxed{\log N_{a,y} = \log\left(N_{a-1,y-1}\right) - Z_{a-1,y-1} + \boxed{\varepsilon_{a,y}}}$$

Random effects

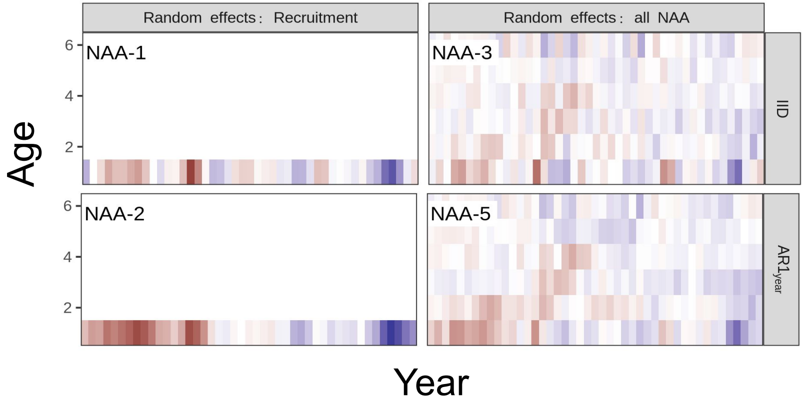
Options for alternative covariance structures (AR1, iid, etc)

- Recruitment (year)
- Interannual transitions in abundance at age (“survival”) (year, age)
- Natural mortality (year, age)
- Selectivity (fishery or index) (year, age)
- Catchability (year)
- Hidden (imperfectly observed) environmental/climate variables (year)
- Movement (year,age)(development branch)
- *Growth (development branch)*

Time- and age-varying processes

Biological processes are often **correlated by year and age**

- Recruitment
- Inter-annual transitions (“Survival”)
- Natural mortality
- Selectivity
- Catchability
- Movement (development branch)




```
NAA_re = list(sigma="rec+1", cor="iid"))
```

Code	Description	Parameters
"none"	time-constant (no deviation)	
"iid"	independent, identically-distributed	σ^2
"ar1"	autoregressive-1 (correlated across ages/parameters)	σ^2, ρ_a
"ar1_y"	autoregressive-1 (correlated across years)	σ^2, ρ_y
"2dar1"	2D AR1 (correlated across both years and ages/parameters)	σ^2, ρ_a, ρ_y

$$\mathrm{Cov}\left(\varepsilon_{a,y}, \varepsilon_{\tilde{a},\tilde{y}}\right) = \frac{\sigma_a \sigma_{\tilde{a}} \rho_a^{|a-\tilde{a}|} \rho_y^{|y-\tilde{y}|}}{\left(1-\rho_a^2\right)\left(1-\rho_y^2\right)}$$

Time- and age-varying processes

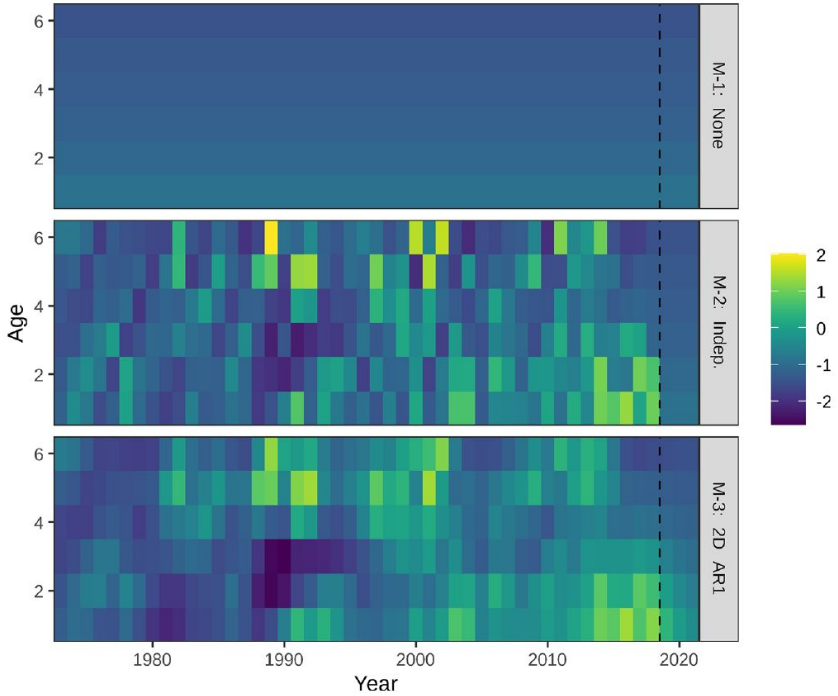
Biological processes are often **correlated by year and age**

- Recruitment
- Inter-annual transitions ("Survival")
- Natural mortality
- Selectivity
- Catchability
- Movement (development branch)

log(M) Gaussian random effects (iid, 2DAR1)

Estimate or fix mean M parameters:

- constant across ages
- age-specific
- function of weight-at-age

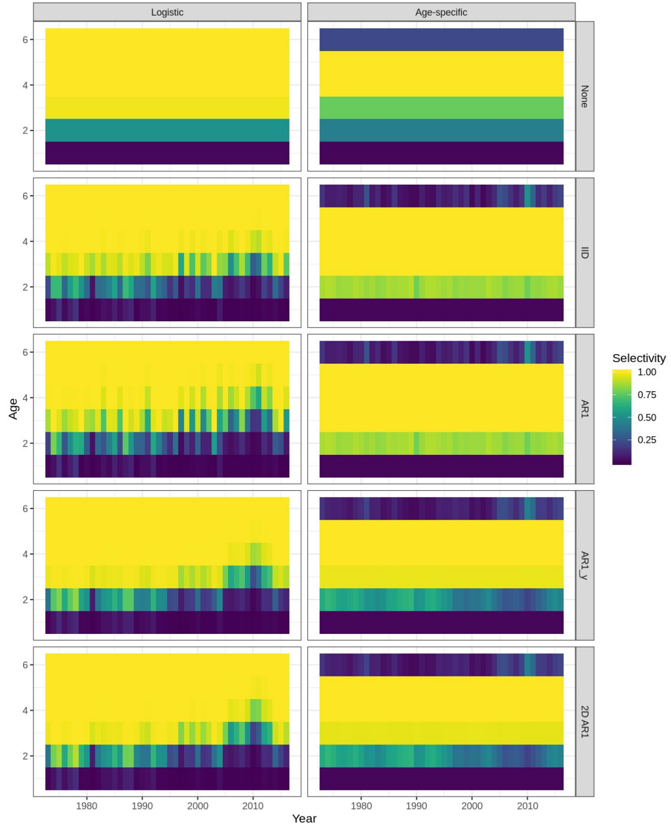


Time- and age-varying processes

- Recruitment
- Inter-annual transitions (“Survival”)
- Natural mortality
- Selectivity
- Catchability
- Movement (development branch)

“blocks” indexed to particular years of indices and fleets

- logistic (increasing or decreasing), double logistic, or age-specific
- constant, iid, or 1D or 2D AR1 processes for annual parameter values
- Gaussian on logit scale

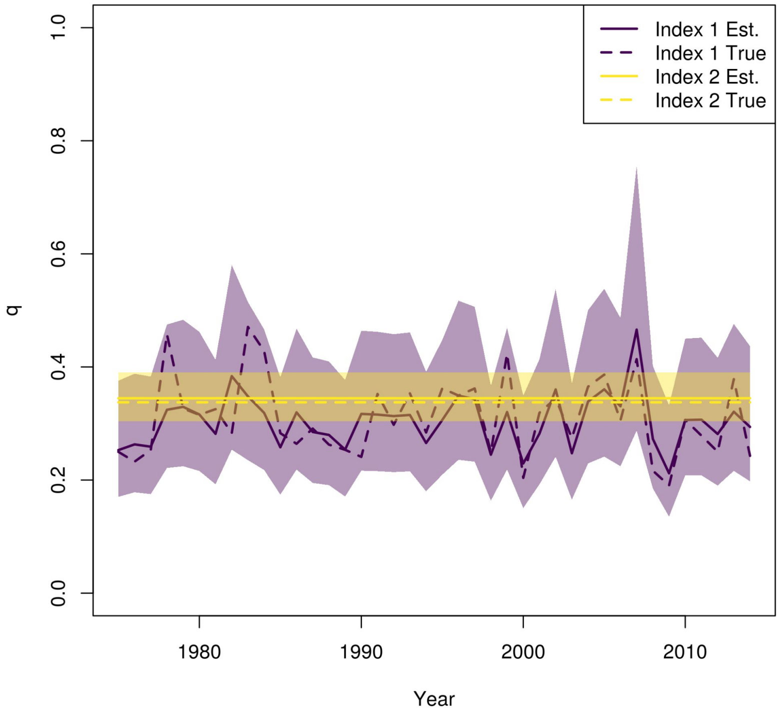


Time- and age-varying processes

- Recruitment
- Inter-annual transitions ("Survival")
- Natural mortality
- Selectivity
- Catchability
- Movement (development branch)

Gaussian iid, or AR1 processes on logit scale

$$\log\left(\frac{q_y - b_l}{b_u - q_y}\right) = \mu_q + \epsilon_{q,y}$$



Time- and age-varying processes

- Recruitment
- Inter-annual transitions ("Survival")
- Natural mortality
- Selectivity
- Catchability
- Movement (development branch)
  - Fixed effects (mean, variance, correlation parameters) are stock, region->region, season specific
  - random effects by year and/or age
  - movement can be modeled as
    - probabilities sequential to mortality
    - instantaneous rate simultaneous to mortality rates

$$f(\mu_{s,r,r',t,y,a}) = \theta_{s,r,r',t} + \epsilon_{s,r,r',t,y,a} \quad r \neq r'$$

$$Cov\left(\epsilon_{s,r,r',t,y,a}, \epsilon_{s,r,r',t,y',a'}\right) = \frac{\rho_{s,r,r',t,A}^{|a-a'|}\rho_{s,r,r',t,Y}^{|y-y'|}\sigma_{s,r,r',t}^2}{\left(1-\rho_{s,r,r',t,A}^2\right)\left(1-\rho_{s,r,r',t,Y}^2\right)}$$

additive logit transformation for probabilities sequential to survival:

$$f(\mu_{s,r,r',t,y,a}) = \log\left(\frac{\mu_{s,r,r',t,y,a}}{1-\sum_{r'}\mu_{s,r,r',t,y,a}}\right)$$

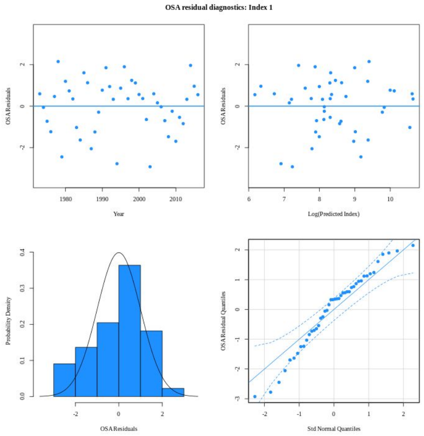
log transformation for instantaneous rates:

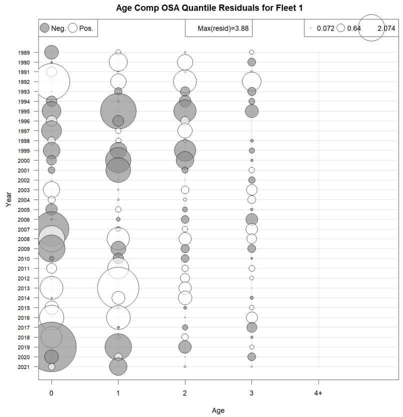
$$f(\mu_{s,r,r',t,y,a}) = \log(\mu_{s,r,r',t,y,a})$$

Data components

All observations have error

- Aggregate catch (fleet-specific)
  - log-normal
- Catch age composition (fleet-specific)
  - Several likelihood options
- Aggregate indices (biomass or numbers)
  - log-normal
- Index age composition (biomass or numbers)
  - Several likelihood options
- **Optional**: Environmental/Climate observations
  - normal
- *Tagging data not yet included*





State-space models for the covariate

- Imperfectly observed environmental variables can affect
  - Recruitment
  - Natural mortality (by age)
  - Index catchability
  - Movement (development)
- User-defined lag between covariate and population effect
- Effects options are "linear" or orthogonal polynomial
- Each covariate can have multiple effects
- Multiple covariates can be included

Covariate state-space models:

1. Random walk

$$\boldsymbol{\theta} = (x_1, \sigma_x^2, \sigma_y^2)$$
$$x_t = x_{t-1} + \mathcal{N}(0, \sigma_x^2)$$
$$y_t = x_t + \mathcal{N}(0, \sigma_y^2)$$

2. AR1

$-1 < \phi < 1$

$$\boldsymbol{\theta} = (\mu, \sigma_x^2, \sigma_y^2, \phi)$$
$$x_t = \mu + \phi x_{t-1} + \mathcal{N}(0, \sigma_x^2)$$
$$y_t = x_t + \mathcal{N}(0, \sigma_y^2)$$

Environmental effects on...

Recruitment models:

$\hat{R}_t =$

1. Random walk (No effects) $e^{\log R_{y-1}+\epsilon_y}$
2. Mean (no SRR) $e^{\mu_R + \boxed{\beta E_y} + \epsilon_y}$
3. Beverton-Holt

   Controlling $\frac{aS_{y-1}e^{\boxed{\beta E_y}+\epsilon_y}}{1+bS_{y-1}}$ Limiting $\frac{aS_{y-1}e^{\epsilon_y}}{1+bS_{y-1}e^{\boxed{\beta E_y}}}$ Masking $\frac{aS_{y-1}e^{\epsilon_y}}{e^{\boxed{\beta E_y}}+bS_{y-1}}$
4. Ricker

   $aS_{y-1}e^{-bS_{y-1}+\boxed{\beta E_y}+\epsilon_y}$ $aS_{y-1}e^{-bS_{y-1}(1+\boxed{\beta E_y})+\epsilon_y}$

   Controlling Masking

Iles & Beverton (1998)

Catchability models:

linear in logit space

$$\log\left(\frac{q_y - b_l}{b_u - q_y}\right) = \mu_q + \boxed{\beta E_y} + \epsilon_{q,y}$$

M models:

1. log-linear $\log M_{y,a} = \mu_{M,a} + \boxed{\beta_a E_y} + \epsilon_{y,a}$
2. allometric $\log M_{y,a} = \log(a) + b\log(W_{y,a}) + \boxed{\beta_a E_y} + \epsilon_{y,a}$

Movement models:

linear in (additive) logit space or log-space

$$f(\mu_{s,r,r',t,y,a}) = \theta_{s,r,r',t} + \boxed{\beta E_y} + \epsilon_{s,r,r',t,y,a}$$

Environmental and random effects on…

Recruitment models:

$$\hat{R}_t =$$

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2. Mean (no SRR) $e^{\mu_R + \beta E_y + \boxed{\epsilon_y}}$
3. Beverton-Holt
   - Controlling: $\dfrac{a S_{y-1} e^{\beta E_y + \boxed{\epsilon_y}}}{1 + b S_{y-1}}$
   - Limiting: $\dfrac{a S_{y-1} e^{\boxed{\epsilon_y}}}{1 + b S_{y-1} e^{\beta E_y}}$
   - Masking: $\dfrac{a S_{y-1} e^{\boxed{\epsilon_y}}}{e^{\beta E_y + b S_{y-1}}}$
4. Ricker
   - Controlling: $a S_{y-1} e^{-b S_{y-1} + \beta E_y + \boxed{\epsilon_y}}$
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Iles & Beverton (1998)

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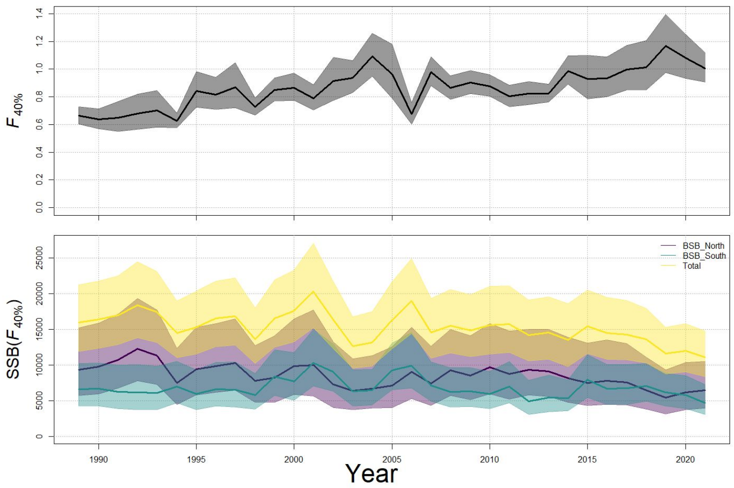
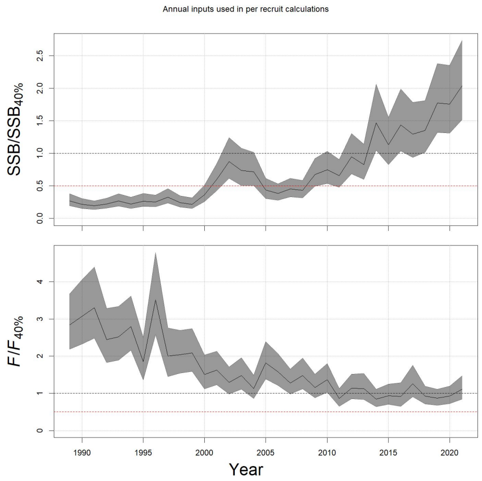
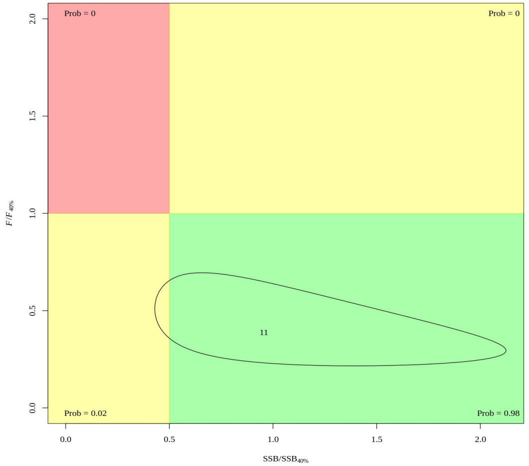
Annual and prevailing BRPs and status

- Internally calculated reference points and status
- Allows uncertainty in parameters to be propagated



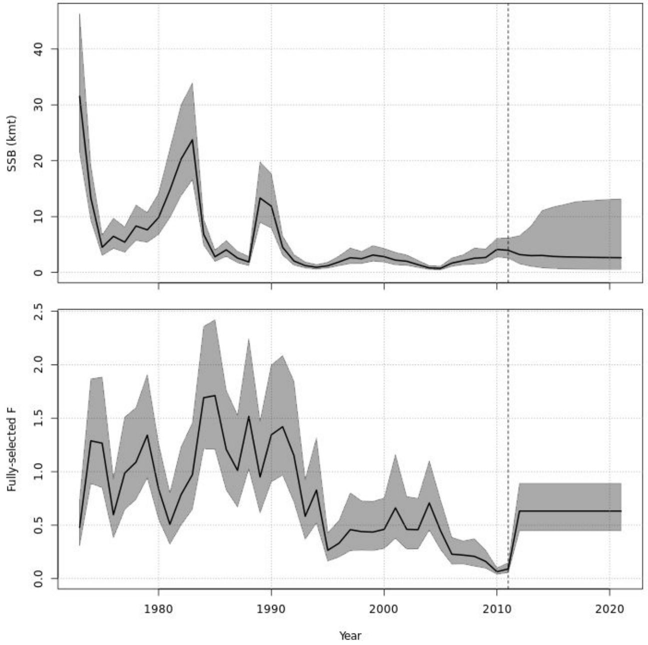


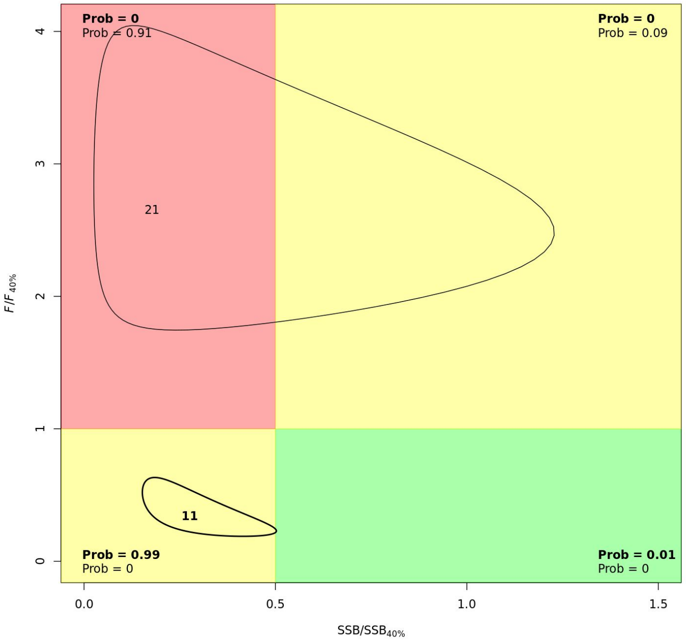


Projections

Random effects (and uncertainty) can be projected

Can specify catch, status quo F, average F, F(X%SPR), FMSY

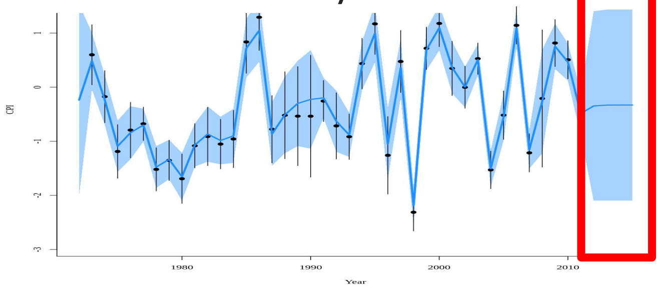




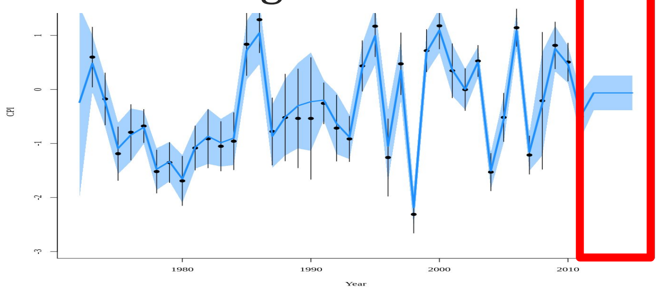
Projections

Several options for treating environmental covariates

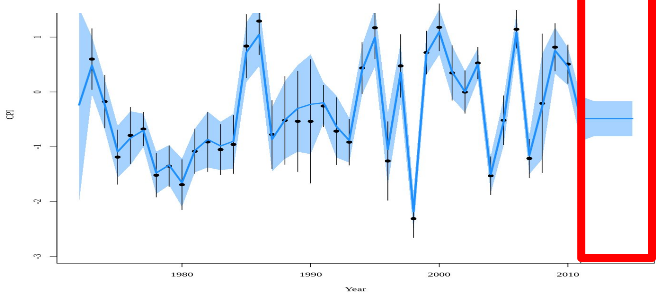
1. Continue RW/AR1



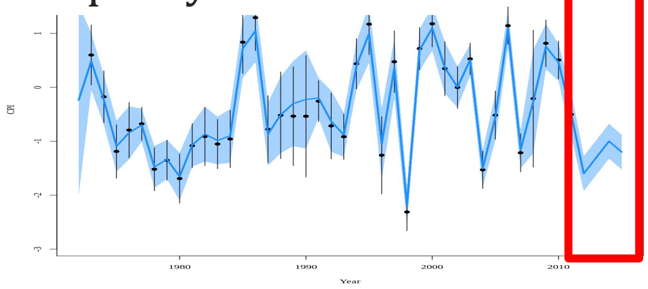
3. Use average value



2. Use last value



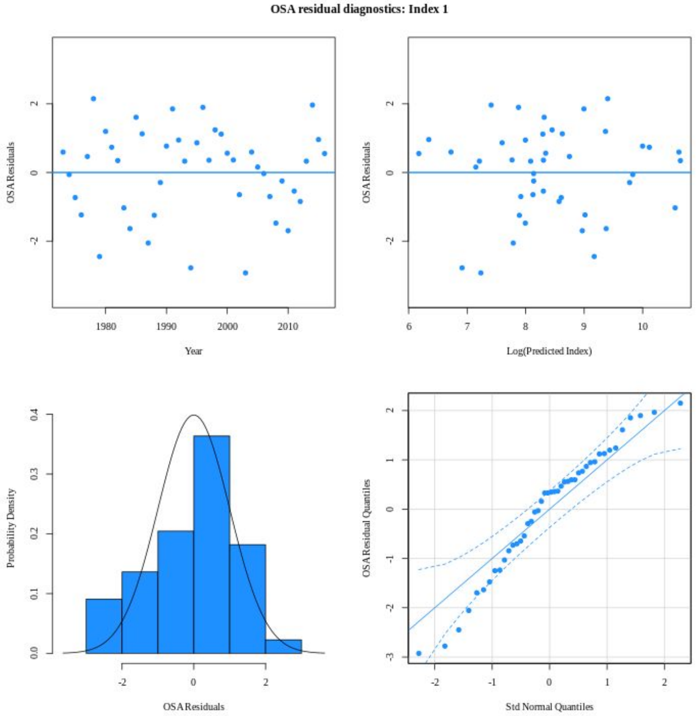
4. Specify values



OSA residual diagnostics

One step ahead (OSA) residuals

- provides independent residuals for correlated observations
- available for all observation types: aggregate catch and indices, age composition, environmental covariates



Environ Ecol Stat (2017) 24:317–339
DOI 10.1007/s10651-017-0372-4

Validation of ecological state space models using the Laplace approximation

Uffe Høgsbro Thygesen[1] · Christoffer Moesgaard Albertsen[1] · Casper Willestofte Berg[1] · Kasper Kristensen[1] · Anders Nielsen[1]

Fisheries Research 257 (2023) 106487

Contents lists available at ScienceDirect

Fisheries Research

urnal homepage: www.elsevier.com/locate/fishres

Model validation for compositional data in stock assessment models: Calculating residuals with correct properties

Vanessa Trijoulet[a,*], Christoffer Moesgaard Albertsen[a], Kasper Kristensen[a], Christopher M. Legault[b], Timothy J. Miller[b], Anders Nielsen[a]

[a] *National Institute of Aquatic Resources, Technical University of Denmark, Kemitorvet 201, DK-2800 Kgs. Lyngby, Denmark*
[b] *Northeast Fisheries Science Center, National Marine Fisheries Service, National Oceanic and Atmospheric Administration, 166 Water Street, Woods Hole, MA 02543, USA*

Automatically generated outputs

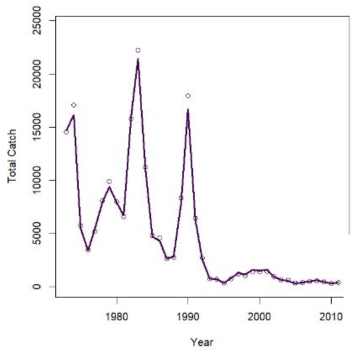
```r
plot_wham_output(mod=m4, out.type='html')
```

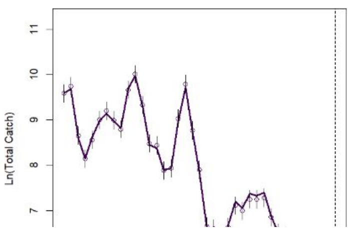
WHAM figures and tables

Figures | Tables

Input | Diagnostics | Results | Retro | Reference points | Miscelaneous

fleet_1 in region_1





WHAM figures and tables

Figures | Tables

Parameter estimates | Abundance at age | Fishing mortality at age by region | Fishing mortality at age by fleet

Parameter estimates, standard errors, and confidence intervals. Rounded to 3 decimal places.

	Estimate	Std. Error	95% CI lower	95% CI upper
index 1 fully selected q	2.441	0.272	1.962	3.037
index 2 fully selected q	0.490	0.038	0.421	0.571
index 3 fully selected q	0.297	0.042	0.225	0.393
index 4 fully selected q	0.006	0.001	0.005	0.009
index 5 fully selected q	0.051	0.008	0.037	0.070
Block 1: fleet 1 a_{50}	2.473	0.051	2.374	2.574
Block 1: fleet 1 1/slope (increasing)	0.417	0.017	0.385	0.452
Block 2: index 1 a_{50}	1.942	0.092	1.766	2.127

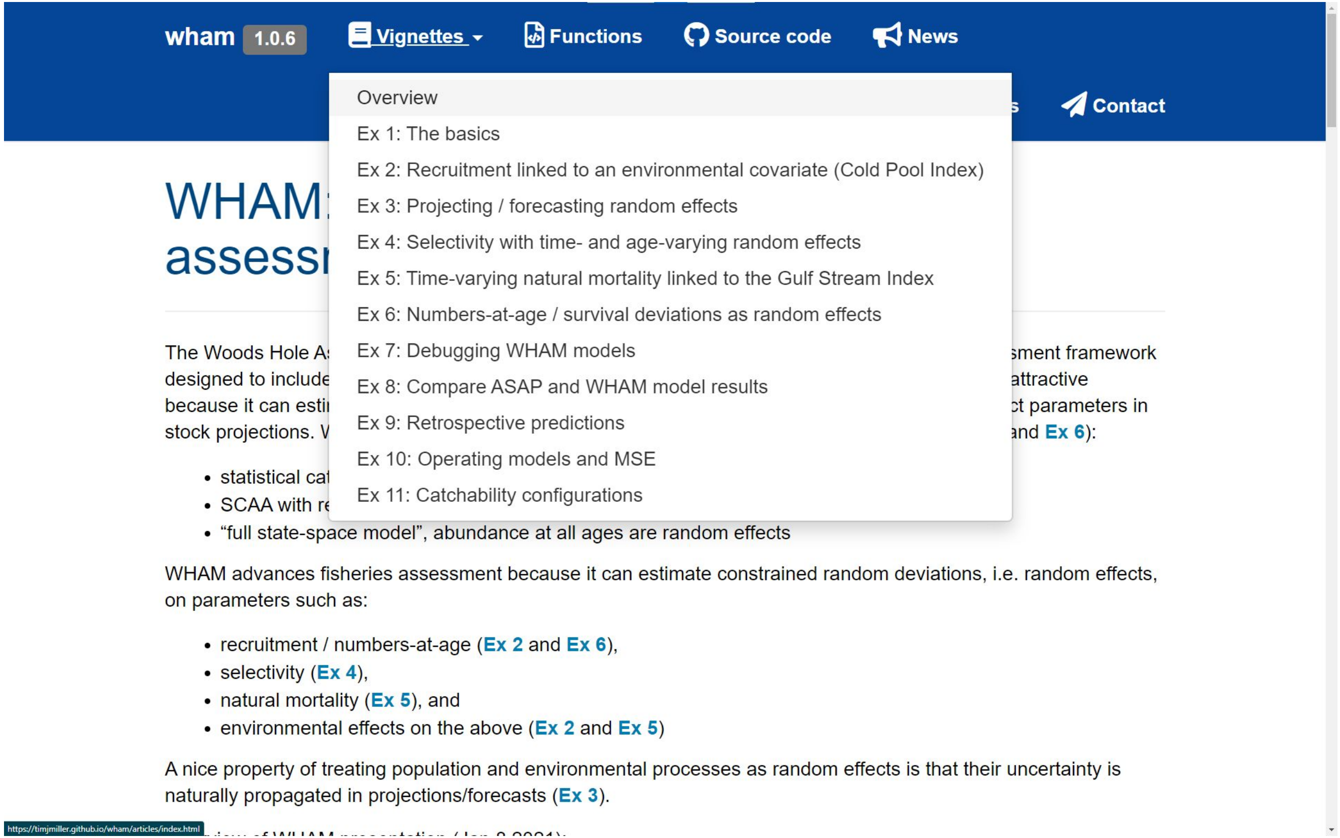
```r
check_convergence(m1)
```

```
#> stats:nlminb thinks the model has converged: mod$opt$conve
#> Maximum gradient component: 1.01e-07
#> Max gradient parameter: log_F1
#> TMB:sdreport() was performed successfully for this model
```

```r
res <- compare_wham_models(mods, fname=
```

```
#>          AIC   rho_R rho_SSB rho_Fbar
#> m4 -1466.9 0.3610  0.0091  -0.0106
#> m2 -1172.7 3.1589 -0.0735  -0.0167
#> m3  4107.1 0.1287  0.0304  -0.0162
#> m1  4846.5 0.8207  0.1905  -0.2322
```

Online Tutorials



Simulations, including MSE

Operating model/MSE usage

- Can be used for simulating populations and data as well as estimation
- Used this way in Index-based Methods Research Track and state-space Research Track
- Used for testing reliability of models in stock-specific research tracks.